

Analysis of the strong coupling constant $G_{D_s^* D_s \phi}$ and the decay width of $D_s^* \rightarrow D_s \gamma$ with QCD sum rules

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Abstract In this article, we calculate the form factors and the coupling constant of the vertex $D_s^* D_s \phi$ using the three-point QCD sum rules. We consider the contributions of the vacuum condensates up to dimension 7 in the operator product expansion. And all possible off-shell cases are considered, ϕ , D_s and D_s^* , resulting in three different form factors. Then we fit the form factors into analytical functions and extrapolate them into time-like regions, which giving the coupling constant for the process. Our analysis indicates that the coupling constant for this vertex is $G_{D_s^* D_s \phi} = 4.12 \pm 0.70 \text{ GeV}^{-1}$. The results of this work are very useful in the other phenomenological analysis. As an application, we calculate the coupling constant for the decay channel $D_s^* \rightarrow D_s \gamma$ and analyze the width of this decay with the assumption of the vector meson dominance of the intermediate $\phi(1020)$. Our final result about the decay width of this decay channel is $\Gamma = 0.59 \pm 0.15 \text{ keV}$.

1 Introduction

In relativistic heavy ion collisions J/ψ suppression has been recognized as an important tool to identify the possible phase transition to quark–gluon plasma [1]. The dissociation of J/ψ in the quark–gluon plasma due to color screening can lead to a reduction of its production. People usually explained this phenomenon as a process of the J/ψ absorption by π , ρ or ϕ mesons in a meson-exchange model [2–6]. And we can calculate the absorption cross sections based on the interactions among the quarkonia and mesons, where the hadronic coupling constants are basic input parameters. For example, $G_{D_s^* D_s \phi}$ will be used in the absorption process $J/\psi + \phi \rightarrow D_s^{*+} + D_s^-$. A detailed knowledge of this coupling constant is of great importance in understanding the

effects of heavy quarkonium absorptions in hadronic matter. Besides, the hadronic coupling constants about the heavy-light mesons can also help us understanding the final-state interactions in the heavy quarkonium decays [7–10]. Furthermore, some exotic mesons have been detected in recent years [11–13], which are beyond the usual quark-model description as $q\bar{q}$ pairs. And people interpret them as quark–gluon hybrids ($q\bar{q}g$), tetraquark states ($q\bar{q}q\bar{q}$), molecular states of two ordinary mesons, glueballs, states with exotic quantum numbers and many others [14–22]. The form factors and coupling constants play an important role in understanding the nature of these exotic mesons. For example, the coupling constants $G_{D_s^* D_s \phi}$ will be used in the analysis about the process $Y(4140) \rightarrow D_s^{*-} D_s^+ \rightarrow J/\psi \phi$.

However, the strong coupling constant used in the above questions can not be explained by perturbative theories, because the associate interactions lie in the low energy region. It is fortunate that the QCDSR approach can help us to solve the difficulty. The QCDSR is one of the most powerful non-perturbative methods, which is also independent of model parameters. In recent years, numerous research articles have been reported about the precise determination of the strong form factors and coupling constants via QCDSR, light-cone QCDSR or lattice calculation [23–37]. And many strong coupling constants have been determined by different groups, for example, $D^* D_s K$, $D_s^* D K$, $B_c^* B_c \Upsilon$, $B_c^* B_c \psi$, $B_s^* B K$, $J/\psi D_s^* D_s$, $J/\psi D_s D_s$, $J/\psi D_s^* D_s^*$, $D_s^* D_s \eta'$ [23–29, 38–43]. In this work, we use the QCDSR formalism to obtain the coupling constant of the meson vertex $D_s^* D_s \phi$, where the contributions of the vacuum condensates up to dimension 7 in the OPE are considered. The results of this work are very useful in these phenomenological analysis mentioned above.

It is indicated by the BaBar collaboration that $\Gamma(D_s^*) < 1.9 \text{ MeV}$ and $\frac{\Gamma(D_s^* \rightarrow D_s \gamma)}{\Gamma_{\text{total}}} \approx 0.94$ [44]. However, the exact value of the decay width have yet not been determined. A

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more exact result can help us understanding the nature of the meson and testing the validity of the theoretical model. As an application, we also give an analysis about the decay $D_s^* \rightarrow D_s \gamma$ in the end of this paper, where the electromagnetic coupling constant $G_{D_s^* D_s \gamma}$ will be used. This coupling constant can be easily obtained, when we set $Q^2 = 0$ in the analytical function of coupling constant $G_{D_s^* D_s \phi}(Q^2)$ in Sect. 3.

The outline of this paper is as follows. In Sect. 2, we study the $D_s^* D_s \phi$ vertices using the three-point QCDSR. In order to reduce the uncertainties of the result, we calculate the three-point correlation functions: one with the vector meson ϕ off-shell, another with the pseudoscalar meson D_s off-shell, and a third one with the vector meson D_s^* off-shell. Besides of the perturbative contribution, we also consider the contribution of $\langle q\bar{q} \rangle$, $\langle \bar{q} g \sigma \cdot G q \rangle$, $\langle g^2 G^2 \rangle$, $\langle f^3 G^3 \rangle$, $\langle q\bar{q} \rangle^2$ and $\langle q\bar{q} \rangle \langle GG \rangle$ at OPE side. In Sect. 3, we present the numerical results and discussions, and Sect. 4 is reserved for our conclusions.

2 QCD sum rules for the $D_s^* D_s \phi$

In this work, the $D_s^* D_s \phi$ is a vector-pseudoscalar-vector (VPV) vertex. With each meson off-shell, we write down the three-point correlation functions:

$$\Pi_{\mu\nu}^\phi(p, p') = i^2 \int d^4x d^4y e^{ip'x+i(p-p')y} \times \langle 0|T\{J_5(x)j_\mu(y)J_\nu^\dagger(0)\}|0\rangle \quad (1)$$

$$\Pi_{\mu\nu}^{D_s}(p, p') = i^2 \int d^4x d^4y e^{ip'x+i(p-p')y} \times \langle 0|T\{J_\nu(x)J_5(y)j_\mu^\dagger(0)\}|0\rangle \quad (2)$$

$$\Pi_{\mu\nu}^{D_s^*}(p, p') = i^2 \int d^4x d^4y e^{ip'x+i(p-p')y} \times \langle 0|T\{j_\mu(x)J_\nu(y)J_5^\dagger(0)\}|0\rangle \quad (3)$$

where T is the time ordered product and $J_\nu^\dagger(x)$, $J_5(x)$ and $j_\mu(x)$ are the interpolating currents of the mesons D_s^* , D_s and ϕ respectively:

$$J_\nu^\dagger(x) = \bar{s}(x)\gamma_\nu c(x) \quad (4)$$

$$J_5(x) = \bar{c}(x)i\gamma_5 s(x) \quad (5)$$

$$j_\mu(x) = \bar{s}(x)\gamma_\mu s(x) \quad (6)$$

According to the QCDSR, these correlation functions can be calculated in two different ways: using hadron degrees of freedom, called the phenomenological side, or using quark degrees of freedom, called the OPE side. In the following we will obtain the sum rule according to above formulations.

2.1 The phenomenological side

We insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_\nu^\dagger(x)$, $J_5(x)$ and $j_\mu(x)$ into the correlation functions $\Pi_{\mu\nu}^\phi(p, p')$, $\Pi_{\mu\nu}^{D_s}(p, p')$ and $\Pi_{\mu\nu}^{D_s^*}(p, p')$ to obtain the phenomenological representations. After isolating the ground-state contributions, we get the following functions for the mesons ϕ , D_s and D_s^* off-shell cases.

$$\Pi_{\mu\nu}^{\text{phen}(\phi)} = \frac{-CG_{D_s^* D_s \phi}^{(\phi)}(q^2)p^\alpha p'^\beta \varepsilon_{\mu\nu\alpha\beta}}{(m_{D_s^*}^2 - p^2)(m_\phi^2 - q^2)(m_{D_s}^2 - p'^2)} + h.r. \quad (7)$$

$$\Pi_{\mu\nu}^{\text{phen}(D_s)} = \frac{-CG_{D_s^* D_s \phi}^{(D_s)}(q^2)p^\alpha p'^\beta \varepsilon_{\mu\nu\alpha\beta}}{(m_\phi^2 - p^2)(m_{D_s}^2 - q^2)(m_{D_s^*}^2 - p'^2)} + h.r. \quad (8)$$

$$\Pi_{\mu\nu}^{\text{phen}(D_s^*)} = \frac{-CG_{D_s^* D_s \phi}^{(D_s^*)}(q^2)p^\alpha p'^\beta \varepsilon_{\mu\nu\alpha\beta}}{(m_{D_s}^2 - p^2)(m_{D_s^*}^2 - q^2)(m_\phi^2 - p'^2)} + h.r. \quad (9)$$

where $C = \frac{f_{D_s} m_{D_s}^2 f_{D_s^*} m_{D_s^*} f_\phi m_\phi}{(m_s + m_c)}$ and $h.r.$ stand for the contributions of higher resonances and continuum states of each meson. And in the derivation, we have used the following effective Lagrangian \mathcal{L} and definitions for the decay constants $f_{D_s^*}$, f_{D_s} and f_ϕ :

$$\mathcal{L} = G_{D_s^* D_s \phi} \varepsilon_{\alpha\beta\lambda\tau} (\partial^\alpha D_s^{*+\beta} D_s^- \partial^\lambda + \partial^\alpha D_s^{*-\beta} D_s^+ \partial^\lambda) \phi^\tau \quad (10)$$

$$\langle 0|J_\nu(0)|D_s^*(p)\rangle = f_{D_s^*} m_{D_s^*} \zeta_\nu \quad (11)$$

$$\langle 0|J_5(0)|D_s(p')\rangle = f_{D_s} m_{D_s}^2 / (m_s + m_c) \quad (12)$$

$$\langle 0|j_\mu(0)|\phi(q)\rangle = f_\phi m_\phi \xi_\mu \quad (13)$$

where ζ_ν and ξ_μ are the polarization vectors:

$$\xi_\mu \xi^\tau = -g_{\mu\tau} + \frac{q_\mu q_\tau}{q^2} \quad (14)$$

$$\zeta_\nu \zeta^\beta = -g_{\nu\beta} + \frac{p_\nu p_\beta}{p^2} \quad (15)$$

From Eqs. (8)–(10), we can see that there is only one tensor structure to work within the formalism of the QCDSR.

2.2 The OPE side

Now, we briefly outline the operator product expansion for the correlation functions Eqs. (1)–(3). Firstly, we contract the quark fields in the correlation functions with Wick's theorem.

$$\Pi_{\mu\nu}^{(\phi)} = -i^2 \int d^4x d^4y e^{ip'x+i(p-p')y} \text{tr} \times \{i\gamma_5 s^{mn}(x-y)\gamma_\mu s^{nk}(y-0)\gamma_\nu c^{km}(0-x)\} \quad (16)$$

$$\Pi_{\mu\nu}^{(D_s)} = -i^2 \int d^4x d^4y e^{ip'x+i(p-p')y} \text{tr} \times \{ \gamma_\nu c^{mn}(x-y) i\gamma_5 s^{nk}(y-0) \gamma_\mu s^{km}(0-x) \} \quad (17)$$

$$\Pi_{\mu\nu}^{(D_s^*)} = -i^2 \int d^4x d^4y e^{ip'x+i(p-p')y} \text{tr} \times \{ \gamma_\mu s^{mn}(x-y) \gamma_\nu c^{nk}(y-0) i\gamma_5 s^{km}(0-x) \} \quad (18)$$

Then, we replace the c and s quark propagators $c^{ij}(x)$ and $s^{ij}(x)$ with the corresponding full propagators [45–47],

$$S_{ij}(x) = \frac{i\delta_{ij}\not{x}}{2\pi^2x^4} - \frac{\delta_{ij}m_s}{4\pi^2x^4} - \frac{\delta_{ij}\langle\bar{s}s\rangle}{12} + \frac{i\delta_{ij}\not{x}m_s\langle\bar{s}s\rangle}{48} - \frac{\delta_{ij}x^2\langle\bar{s}g_s\sigma Gs\rangle}{192} + \frac{i\delta_{ij}x^2\not{x}m_s\langle\bar{s}g_s\sigma Gs\rangle}{1152} - \frac{ig_sG_{\alpha\beta}^at_{ij}^a(\not{x}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\not{x})}{32\pi^2x^2} - \frac{i\delta_{ij}x^2\not{x}g_s^2\langle\bar{s}s\rangle^2}{7776} - \frac{\delta_{ij}x^4\langle\bar{s}s\rangle\langle g_s^2GG\rangle}{27648} - \frac{\langle\bar{s}_j\sigma^{\mu\nu}s_i\rangle\sigma_{\mu\nu}}{8} - \frac{\langle\bar{s}_j\gamma^\mu s_i\rangle\gamma_\mu}{4} + \dots, \quad (19)$$

$$C_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ikx} \times \left\{ \frac{\delta_{ij}}{k-m_c} - \frac{g_sG_{\alpha\beta}^nt_{ij}^n\sigma^{\alpha\beta}(\not{k}+m_c) + (\not{k}+m_c)\sigma^{\alpha\beta}}{4(k^2-m_c^2)} + \frac{g_sD_\alpha G_{\beta\lambda}^nt_{ij}^n(f^{\lambda\beta\alpha} + f^{\alpha\beta\lambda})}{3(k^2-m_c^2)^4} - \frac{g_s^2(t^at^b)_{ij}G_{\alpha\beta}^aG_{\mu\nu}^b(f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\nu\mu\beta})}{4(k^2-m_c^2)^5} + \dots \right\}, \quad (20)$$

$$f^{\lambda\alpha\beta} = (\not{k}+m_c)\gamma^\lambda(\not{k}+m_c)\gamma^\alpha(\not{k}+m_c)\gamma^\beta(\not{k}+m_c) \quad (21)$$

$$f^{\alpha\beta\mu\nu} = (\not{k}+m_c)\gamma^\alpha(\not{k}+m_c)\gamma^\beta(\not{k}+m_c)\gamma^\mu(\not{k}+m_c)\gamma^\nu(\not{k}+m_c) \quad (22)$$

where $\langle g_s^2GG \rangle = \langle g_s^2G_{\alpha\beta}^nG^{n\alpha\beta} \rangle$, $t^n = \frac{\lambda^n}{2}$, the λ^n is the Gell–Mann matrix, $D_\alpha = \partial_\alpha - ig_sG_\alpha^n t^n$, and the i, j are color indices. Then we compute the integrals both in the coordinate and momentum spaces, and obtain the correlation functions. Finally, the correlation functions can be divided into two parts:

$$\Pi_{\mu\nu}^{\text{OPE}(M)} = \Pi_{\mu\nu}^{\text{pert}(M)} + \Pi_{\mu\nu}^{\text{non-pert}(M)} \quad (23)$$

where M is the off-shell meson ($M = \phi, D_s, D_s^*$). Using dispersion relations, the perturbative term for a given meson M off-shell can be written in the following form:

$$\Pi_{\mu\nu}^{\text{pert}(M)}(p, p') = -\frac{1}{4\pi^2} \int_{s_1}^{s_0} \int_{u_1}^{u_0} \frac{\rho_{\mu\nu}^{\text{pert}(M)}(s, u, q^2)}{(s-p^2)(u-p'^2)} ds du \quad (24)$$

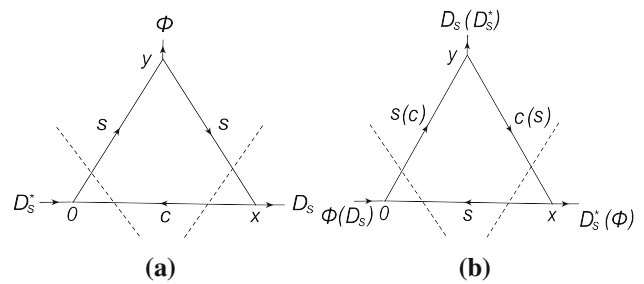


Fig. 1 The perturbative contributions for ϕ, D_s and D_s^* off-shell. The dashed lines denote the Cutkosky cuts

and the quantities $s = p^2, u = p'^2$ and $q = p - p'$. We put all quark lines on mass shell using Cutkosky’s rules (Fig. 1a, b) and obtain the spectral density $\rho_{\mu\nu}^{\text{pert}(M)}(s, u, q^2)$

$$\rho_{\mu\nu}^{\text{pert}(\phi)}(s, u, q^2) = -\frac{3}{\sqrt{\lambda}} \left[(m_c - m_s) \frac{q^2(s+u-q^2+2m_s^2-2m_c^2)}{\lambda(s, u, q^2)} - m_s \right] \times \varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta \quad (25)$$

$$\rho_{\mu\nu}^{\text{pert}(D_s)}(s, u, q^2) = -\frac{3}{\sqrt{\lambda}} \left[(m_c - m_s) \times \frac{(u - q^2)(s + u - q^2) - 2s(u + m_c^2 - m_s^2)}{\lambda(s, u, q^2)} - m_c \right] \times \varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta \quad (26)$$

$$\rho_{\mu\nu}^{\text{pert}(D_s^*)}(s, u, q^2) = -\frac{3}{\sqrt{\lambda}} \left[(m_c - m_s) \times \frac{u(s+u-q^2) - 2u(m_c^2 - m_s^2 + u - q^2)}{\lambda(s, u, q^2)} - m_s \right] \varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta \quad (27)$$

where $\lambda(s, u, q^2) = (s + u - q^2)^2 - 4su$. As to the non-perturbative contributions, we take into account the contribution of $\langle s\bar{s} \rangle, \langle \bar{s}g_s\sigma Gs \rangle, \langle g_s^2G^2 \rangle, \langle f^3G^3 \rangle, \langle s\bar{s} \rangle^2$ and $\langle s\bar{s} \rangle \langle GG \rangle$, which are showed explicitly in Figs. 2 and 3. It should be noticed that as the consequence of the use of the double Borel transform, the ϕ off-shell case has only the contributions of $\langle g_s^2G^2 \rangle$ and $\langle f^3G^3 \rangle$ (Fig. 2). Full expressions for these contributions of Figs. 2 and 3 for ϕ, D_s and D_s^* off-shell cases can be found in Appendix A, B and C, where the following representations will be used:

$$N_{m_1 m_2 m_3}^{abc} = \frac{(-1)^{a+b+c} \pi^2 i}{\Gamma(a)\Gamma(b)\Gamma(c)(M_1^2)^b(M_2^2)^c(M^2)^{a-2}} \times \int_0^\infty d\tau (\tau+1)^{a+b+c-4} \tau^{1-b-c} \exp \left\{ -\frac{1}{\tau} \frac{Q^2}{M_1^2 + M_2^2} - \frac{(\tau+1)m_1^2}{M^2} - \frac{(\tau+1)m_2^2}{\tau M_1^2} - \frac{(\tau+1)m_3^2}{\tau M_2^2} \right\} \quad (28)$$

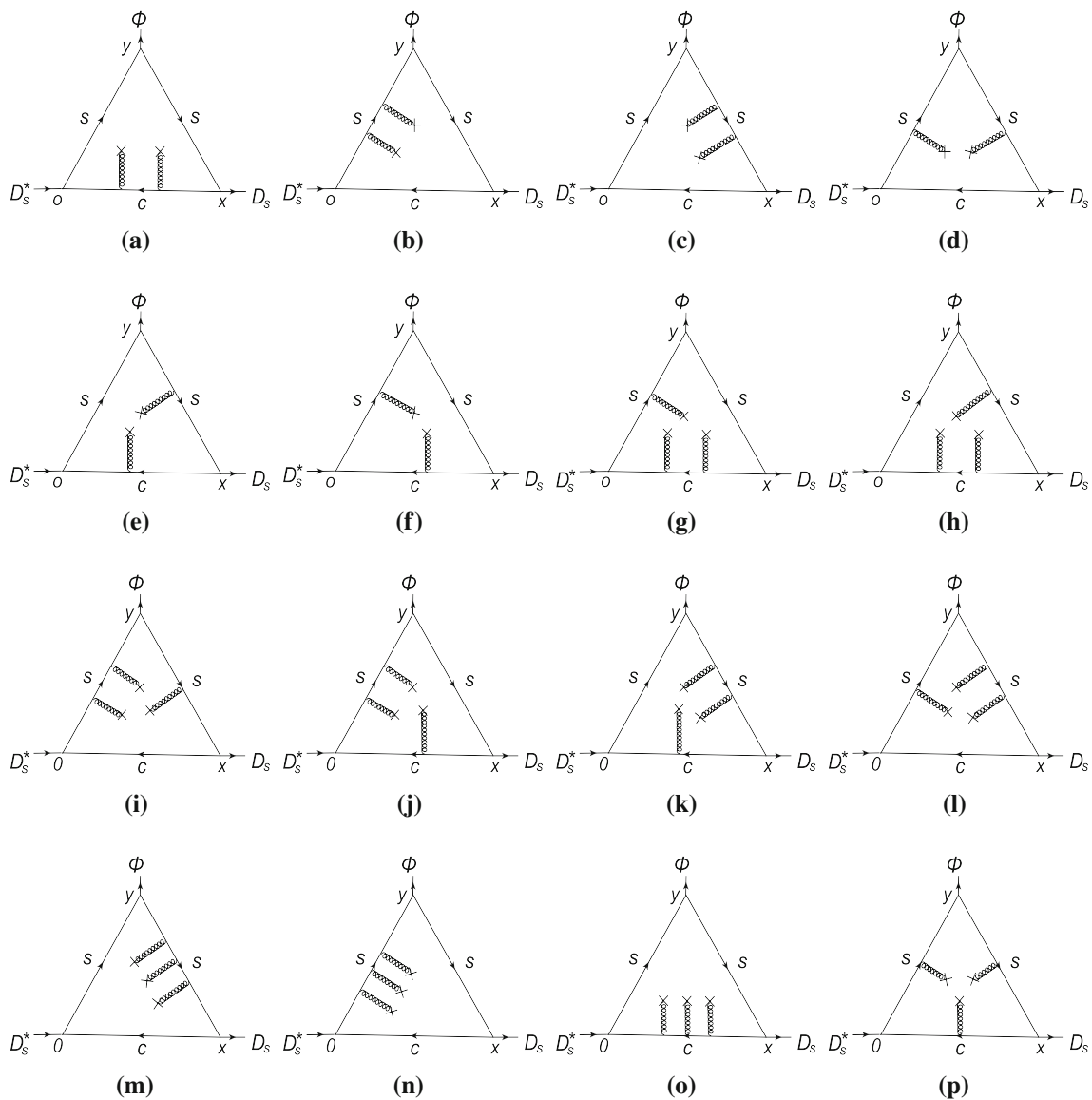


Fig. 2 Contributions of the condensate parts ($g^2 G^2$) and ($f^3 G^3$) for ϕ off-shell case

2.3 The coupling constant and the meson decay

$$I_{m_1 m_2 m_3}^{abc} = \frac{(-1)^{a+b+c} \pi^2 i}{\Gamma(a)\Gamma(b)\Gamma(c)(M_1^2)^b (M_2^2)^{c+1} (M^2)^{a-3}}$$

$$\times \int_0^\infty d\tau (\tau + 1)^{a+b+c-5} \tau^{1-b-c} \exp \left\{ -\frac{1}{\tau} \frac{Q^2}{M_1^2 + M_2^2} - \frac{(\tau + 1)m_1^2}{M^2} - \frac{(\tau + 1)m_2^2}{\tau M_1^2} - \frac{(\tau + 1)m_3^2}{\tau M_2^2} \right\} \quad (29)$$

$$\tilde{I}_{m_1 m_2 m_3}^{abc} = \frac{(-1)^{a+b+c} \pi^2 i}{\Gamma(a)\Gamma(b)\Gamma(c)(M_1^2)^{b+1} (M_2^2)^c (M^2)^{a-3}}$$

$$\times \int_0^\infty d\tau (\tau + 1)^{a+b+c-5} \tau^{1-b-c} \exp \left\{ -\frac{1}{\tau} \frac{Q^2}{M_1^2 + M_2^2} - \frac{(\tau + 1)m_1^2}{M^2} - \frac{(\tau + 1)m_2^2}{\tau M_1^2} - \frac{(\tau + 1)m_3^2}{\tau M_2^2} \right\} \quad (30)$$

We make the change of variables $p^2 \rightarrow -P^2, p'^2 \rightarrow -P'^2$ and $q^2 \rightarrow -Q^2$ and perform a double Borel transform [48] to the physical as well as the OPE sides, which involves the transformation: $P^2 \rightarrow M_1^2$ and $P'^2 \rightarrow M_2^2$, where M_1 and M_2 are the Borel parameters. Then, we equate the phenomenological and OPE sides, invoking the quark-hadron duality from which the sum rule is obtained.

In order to eliminate the *h.r.* terms from the phenomenological side in Eqs. (8)–(10), two continuum threshold parameters s_0 and u_0 in the OPE side are introduced. These parameters fulfill the following relations: $m_i^2 < s_0 < m_i'^2$ and $m_o^2 < u_0 < m_o'^2$, where m_i and m_o are the masses of the incoming and out-coming mesons respectively and m' is the mass of the first excited state of these mesons.

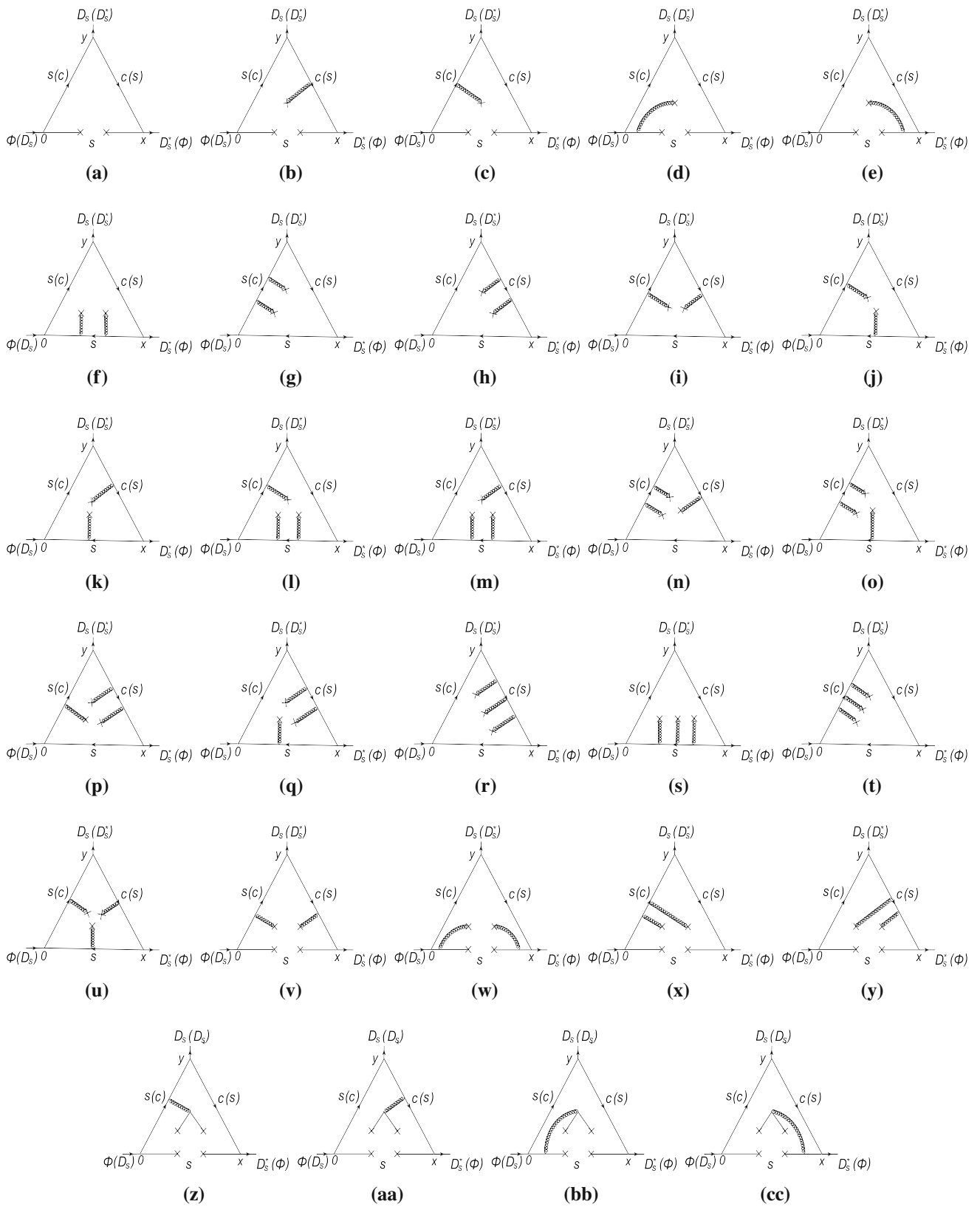


Fig. 3 Contributions of the non-perturbative parts for $D_s(D_s^*)$ off-shell case

After these performances, the form factors can be written as:

$$G_{D_s^* D_s \phi}(Q^2) = \frac{-\frac{1}{4\pi^2 M_1^2 M_2^2} \int_{s_1}^{s_0} \int_{u_1}^{u_0} \rho^{\text{pert}(\phi)}(s, u, Q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{u}{M_2^2}} ds du + \mathcal{BB}[\Pi^{\text{non-pert}(\phi)}]}{\frac{C}{(Q^2+m_\phi^2)M_1^2 M_2^2} e^{-m_{D_s^*}^2/M_1^2} e^{-m_{D_s}^2/M_2^2}}, \tag{31}$$

$$G_{D_s^* D_s \phi}^{D_s}(Q^2) = \frac{-\frac{1}{4\pi^2 M_1^2 M_2^2} \int_{s_1}^{s_0} \int_{u_1}^{u_0} \rho^{\text{pert}(D_s)}(s, u, Q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{u}{M_2^2}} ds du + \mathcal{BB}[\Pi^{\text{non-pert}(D_s)}]}{\frac{C}{(Q^2+m_{D_s}^2)M_1^2 M_2^2} e^{-m_\phi^2/M_1^2} e^{-m_{D_s^*}^2/M_2^2}}, \tag{32}$$

$$G_{D_s^* D_s \phi}^{D_s^*}(Q^2) = \frac{-\frac{1}{4\pi^2 M_1^2 M_2^2} \int_{s_1}^{s_0} \int_{u_1}^{u_0} \rho^{\text{pert}(D_s^*)}(s, u, Q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{u}{M_2^2}} ds du + \mathcal{BB}[\Pi^{\text{non-pert}(D_s^*)}]}{\frac{C}{(Q^2+m_{D_s^*}^2)M_1^2 M_2^2} e^{-m_{D_s}^2/M_1^2} e^{-m_\phi^2/M_2^2}}, \tag{33}$$

where $\mathcal{BB}[\]$ stands for the double Borel transform. Now, we can calculate the form factors in the space-like region according to these above equations. However, in order to obtain the coupling constants, it is necessary to extrapolate these results into physical regions ($Q^2 < 0$), which is realized by fit the form factors into suitable analytical functions. It is indicated that we should get the same values for the coupling constants $G_{D_s^* D_s \phi}^\phi$, $G_{D_s^* D_s \phi}^{D_s}$ and $G_{D_s^* D_s \phi}^{D_s^*}$ [49], when we take $Q^2 = -m_\phi^2$, $Q^2 = -m_{D_s}^2$ and $Q^2 = -m_{D_s^*}^2$ separately. This above procedure is used to minimize the uncertainties in the calculation of the coupling constant, which will be quite clear in the following section.

With the assumption of the vector meson dominance ($\phi(1020)$), the radiative decays $D_s^* \rightarrow D_s \gamma$ can be described by the following electromagnetic lagrangian \mathcal{L}' ,

$$\mathcal{L}' = -e Q_s \bar{s} \gamma_\mu s A^\mu \tag{34}$$

where the A_μ , Q_s are the electromagnetic field and the charge number. From the lagrangian \mathcal{L}' , we can obtain the decay amplitude [50,51],

$$\begin{aligned} &\langle D_s(p) \gamma(q, \varepsilon) | D_s^*(p', \xi) \rangle \\ &= \langle \gamma(q, \varepsilon) | \phi(q, \eta) \rangle \frac{i}{q^2 - m_\phi^2} \langle D_s(p) \phi(q, \eta) | D_s^*(p', \xi) \rangle \\ &= \langle D_s(p) \phi(q, \eta) | D_s^*(p', \xi) \rangle \frac{i}{q^2 - m_\phi^2} f_\phi m_\phi e Q_s (-i) \varepsilon_\mu^* \eta^\mu \\ &= G_{D_s^* D_s \gamma} \varepsilon^{\alpha\beta\lambda\tau} p'_\alpha q_\lambda \xi_\beta \eta_\tau^* \frac{i}{q^2 - m_\phi^2} f_\phi m_\phi e Q_s (-i) \varepsilon_\mu^* \eta^\mu \end{aligned} \tag{35}$$

The parameters $G_{D_s^* D_s \gamma}$ and f_ϕ are the coupling constant and the weak decay constant, respectively. p'_α and q_λ are the four momenta of the D_s and γ . η^μ , ε_μ^* and ξ_β are the polarization vectors of the ϕ , γ and D_s^* , respectively. The strong coupling constant $G_{D_s^* D_s \gamma}$ can be related to the effective coupling con-

stant in the heavy quark effective Lagrangian by Eq. (10) in this paper.

3 Results and discussions

Present section is devoted to the numerical analysis of the sum rules for the coupling constants. The decay constants and hadronic parameters used in this work are taken as $f_\phi = 0.229 \pm 0.003$ [52], $f_{D_s} = 0.257 \pm 0.006$ [52], $f_{D_s^*} = 0.301 \pm 0.013$ [52], $m_\phi = 1.019 \pm 0.020$ GeV [52], $m_{D_s} = 1.968 \pm 0.00032$ GeV [52], $m_{D_s^*} = 2.112 \pm 0.0005$ GeV [52]. The vacuum condensates are taken to be the standard values $\langle \bar{s}s \rangle = -(0.8 \pm 0.1) \times (0.24 \pm 0.01 \text{ GeV})^3$ [48,53], $\langle \bar{s}g_s \sigma Gs \rangle = m_0^2 < \bar{s}s >$ [48,53], $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle g_s^2 GG \rangle = (0.022 \pm 0.004) \text{ GeV}^4$ [54–56], $\langle f^3 G^3 \rangle = (8.8 \pm 5.5) \text{ GeV}^2 \langle g_s^2 GG \rangle$ [54–56]. And we also take the masses of quark $m_c = (1.275 \pm 0.025) \text{ GeV}$, $m_s = 0.095 \pm 0.005 \text{ GeV}$ from the Particle Data Group [52].

From Eqs. (31)–(33), we also know that the value of the form factor $G_{D_s^* D_s \phi}$ is the function of the input parameters, including the Borel parameters M_1^2 and M_2^2 , the continuum thresholds s_0 and u_0 , the momentum Q^2 . The working regions for the M_1^2 and M_2^2 are determined by requiring not only that the contributions of the higher states and continuum be effectively suppressed, but also that the contributions of the higher-dimensional operators are small. In other words, we should find a good plateau which will ensure OPE convergence and the stability of our results [48]. The plateau is often called ‘‘Borel window’’. In addition, the continuum parameters, $s_0 = (m_i + \Delta_i)^2$ and $u_0 = (m_o + \Delta_o)^2$ are employed to include the pole and to suppress the *h.r.* contributions for the cases of ϕ , D_s and D_s^* mesons off-shell. The values for Δ_ϕ , Δ_{D_s} and $\Delta_{D_s^*}$ can not be far from the experimental value of the distance between the pole and the first excited state [48]. In general, these two continuum thresholds s_0 and u_0 are determined by the relations $s_0 \sim (m_i + 0.5 \text{ GeV})^2$ and $u_0 \sim (m_o + 0.5 \text{ GeV})^2$.

The form factor $G_{D_s^* D_s \phi}^\phi$ on Borel parameter M_2^2 in the different values of s_0 and u_0 for the off-shell ϕ meson

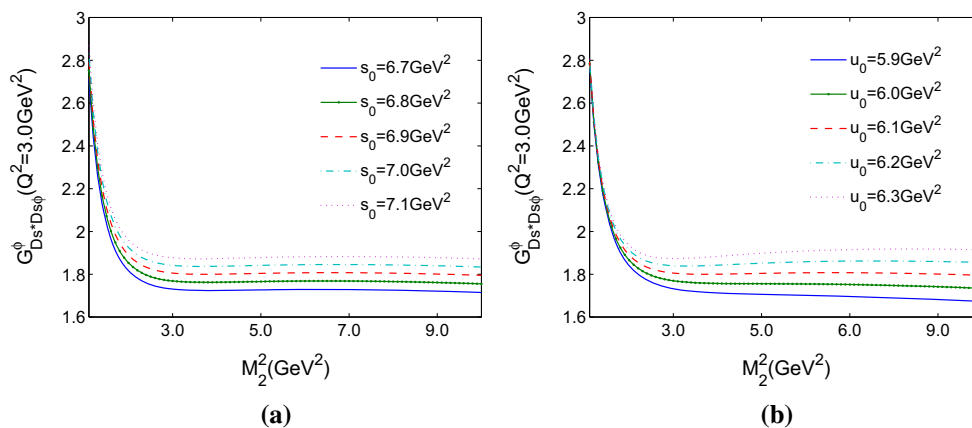


Fig. 4 The strong form factor, $G_{D_s^* D_s \phi}$ on Borel parameter M_2^2 , in the different values of s_0 and u_0 , for the off-shell ϕ meson

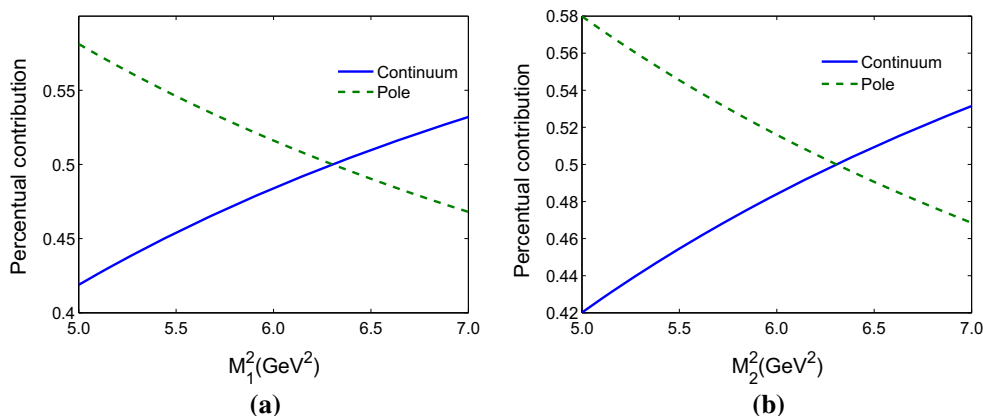


Fig. 5 Pole and continuum contributions for the ϕ off-shell, on Borel parameter M_1^2 (a) and M_2^2 (b)

are shown in Fig. 4. It can be seen that the results have more stability with $s_0 = 6.9 \text{ GeV}^2 (\Delta_{D_s^*} \approx 0.5 \text{ GeV})$ and $u_0 = 6.1 \text{ GeV}^2 (\Delta_{D_s} \approx 0.5 \text{ GeV})$. In Fig. 5 we show also the relative continuum versus pole contribution, from where we clearly see that the pole contribution is bigger than the continuum contribution for $M^2 < 6.3 \text{ GeV}^2$. Thus, we use the Borel region $5.0 \leq M_1^2 \leq 7.0 \text{ GeV}^2$ and $5.0 \leq M_2^2 \leq 7.0 \text{ GeV}^2$ ($Q^2 = 3.0 \text{ GeV}^2$) for ϕ off-shell. According to the same analysis with ϕ off-shell, we choose the Borel window $6.0 \leq M_1^2 \leq 8.0 \text{ GeV}^2$ and $6.0 \leq M_2^2 \leq 8.0 \text{ GeV}^2$ ($Q^2 = 1.0 \text{ GeV}^2$) for D_s and D_s^* off-shell with $\Delta_\phi = \Delta_{D_s} = \Delta_{D_s^*} = 0.5 \text{ GeV}$.

In Fig. 6, we show explicitly the contributions of different condensate terms in the OPE with variations of the Borel parameters M_1^2 and M_2^2 for ϕ , D_s and D_s^* off-shell. From the figure, we can see that the values are rather stable with variations of the Borel parameters, it is reliable to extract the form factors. Besides of the perturbative term, we can also see that $\langle s\bar{s} \rangle$ give a considerable contribution for D_s and D_s^* off-shell cases (Fig.6c–f). And

the contributions of the other condensate terms are small ($< 1\%$). To the case of ϕ off-shell, condensate parts $\langle g^2 G^2 \rangle$ and $\langle f^3 G^3 \rangle$ make up 1–2% of the total contributions. It should be noticed that although these condensates terms, all except for the perturbative term and $\langle s\bar{s} \rangle$, give small contributions to the form factors, they have a significant influence on the following analytical functions (Eqs. (36)–(38)), which are obtained by numerical fitting. Thus, these condensates contributions should not be neglected in the calculation.

The form factors $G_{D_s^* D_s \phi}^\phi$, $G_{D_s^* D_s \phi}^{D_s}$ and $G_{D_s^* D_s \phi}^{D_s^*}$ are shown explicitly in Fig. 7 and are fitted into the following analytical functions:

$$G_{D_s^* D_s \phi}^\phi = A \exp(-B Q^2), \tag{36}$$

$$G_{D_s^* D_s \phi}^{D_s} = \frac{C}{1 + D Q^2} \exp(-E Q^2), \tag{37}$$

$$G_{D_s^* D_s \phi}^{D_s^*} = \frac{C'}{1 + D' Q^2} \exp(-E' Q^2), \tag{38}$$

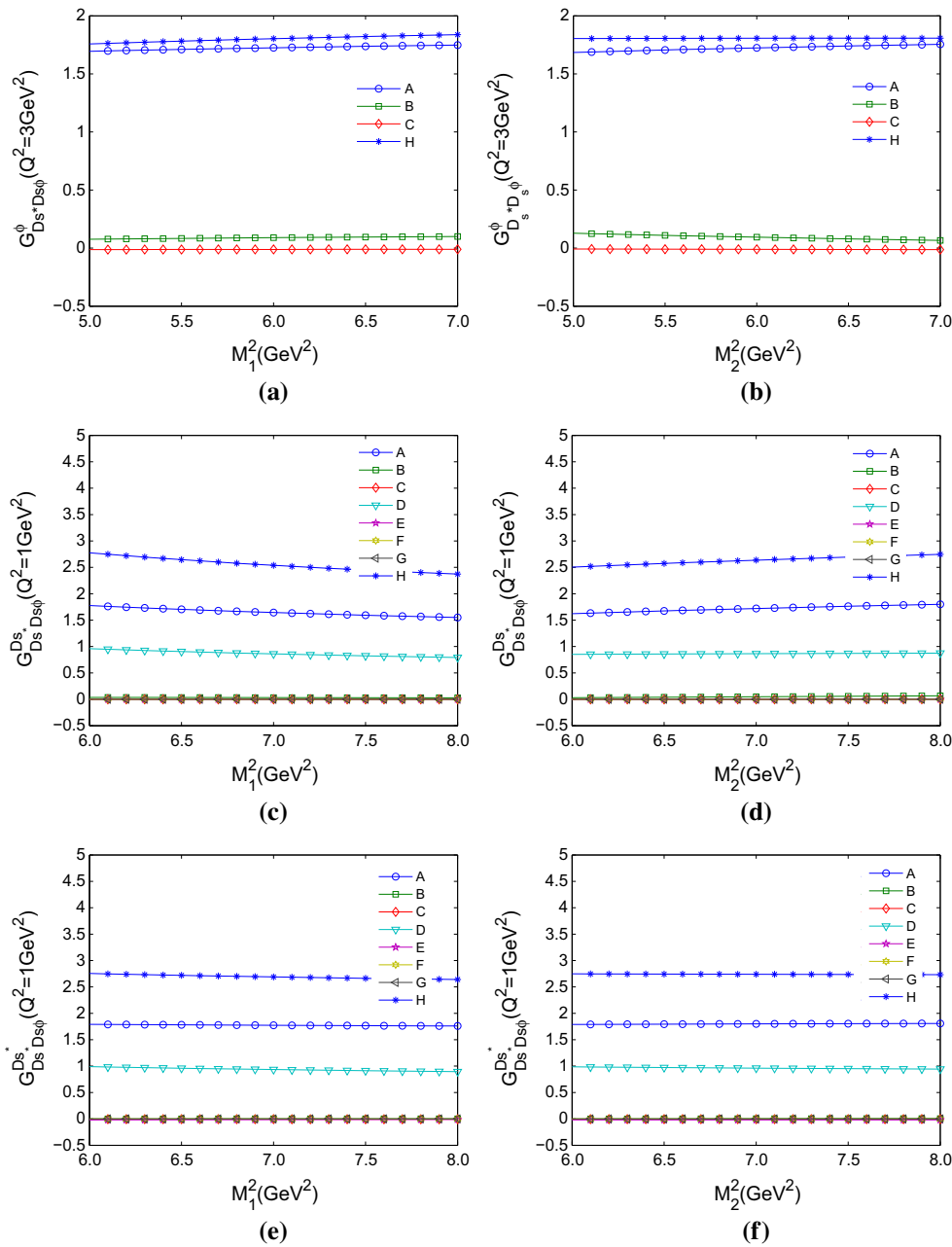


Fig. 6 The contributions of different condensate terms in the OPE with variations of the Borel parameters M_1^2 and M_2^2 for ϕ (a, b), D_s (c, d) and D_s^* (e, f) off-shell, where A–H denote the perturbative term, $\langle g^2 G^2 \rangle$, $\langle f^3 G^3 \rangle$, $\langle s\bar{s} \rangle$, $\langle \bar{s}g\sigma_s Gs \rangle$, $\langle \bar{s}\bar{s} \rangle^2$, $\langle \bar{s}\bar{s} \rangle \langle GG \rangle$ and total contributions

where $A = 2.964 \pm 0.089 \text{ GeV}^{-1}$, $B = 0.1621 \pm 0.0077 \text{ GeV}^{-2}$, $C = 2.755 \pm 0.008$, $D = -0.1944 \pm 0.0186$, $E = 0.256 \pm 0.0265$, $C' = 2.825 \pm 0.012 \text{ GeV}^{-1}$, $D' = -0.1855 \pm 0.0171 \text{ GeV}^{-2}$, $E' = 0.2593 \pm 0.0257$,

Considering uncertainties of all the input parameters, such as quark and mesons masses, decay constants and the values of different condensates terms, we plot the upper and lower bounds of the form factors in Fig. 7. We can see that although the uncertainties are large (about 20–30 % of the central values), the fitted functions can reproduce the central values of

the form factors well. Thus, it is reliable for us to extrapolate the Q^2 to the physical region $Q^2 < 0$ for ϕ , D_s and D_s^* off-shell cases to obtain the coupling constant for the vertex $D_s^* D_s \phi$. Substituting $Q^2 = -m_\phi^2$, $Q^2 = -m_{D_s}^2$ and $Q^2 = -m_{D_s^*}^2$ separately in Eqs. (36)–(38), we obtain the values for $G_{D_s^* D_s \phi}^\phi$, $G_{D_s^* D_s \phi}^{D_s}$ and $G_{D_s^* D_s \phi}^{D_s^*}$:

$$G_{D_s^* D_s \phi}^\phi = 3.51 \pm 0.11 \text{ GeV}^{-1} \tag{39}$$

$$G_{D_s^* D_s \phi}^{D_s} = 4.24 \pm 0.47 \text{ GeV}^{-1} \tag{40}$$

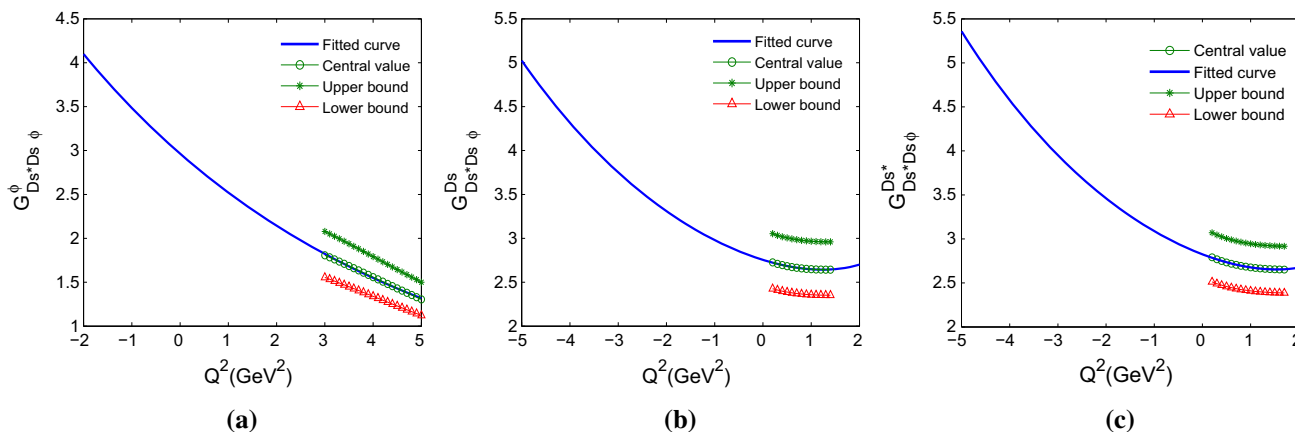


Fig. 7 Form factors of the $D_s^* D_s \phi$ vertex for ϕ off-shell (a), D_s off-shell (b) and D_s^* off-shell (c)

$$G_{D_s^* D_s \phi}^{D_s^*} = 4.61 \pm 0.51 \text{ GeV}^{-1} \tag{41}$$

Although the values for each off-shell case are different, they are roughly compatible when the uncertainties are taken into account, where the uncertainties in Eqs. (39)–(41) originate from the uncertainties of the fitted parameters $\delta A, \delta B, \delta C, \delta D, \delta E, \delta C', \delta D'$ and $\delta E'$. Taking the mean values between the numbers presented above, we obtain the strong coupling constant for $G_{D_s^* D_s \phi}$:

$$G_{D_s^* D_s \phi} = 4.12 \pm 0.70 \text{ GeV}^{-1} \tag{42}$$

In Ref. [57], Z.G.Wang studied the $D^* DV$ vertex with the light-cone QCD sum rules. Most of the results in this work is analyzed to be much smaller than others [57]. And the coupling constant $G_{D_s^* D_s \phi}$ is estimated to be about $0.82 \pm 0.16 \text{ GeV}^{-1}$ which is also much smaller than our result. This difference is most probably due to the different input parameters and the different methods employed. In Reference [42,43], it is indicated that the value of $G_{D_s^* D_s \phi}$ is $4.07 \pm 0.71 \text{ GeV}^{-1}$ in the framework of the three-point QCD sum rules. Besides of the perturbative part, the contributions of quark–quark, gluon–gluon, and quark–gluon condensate are considered in this work. It is clearly that our result is compatible well with that of Refs. [42,43], which indicates to some extent the reliability of our result.

As to the coupling constant $G_{D_s^* D_s \gamma}$ in the decay $D_s^* \rightarrow D_s \gamma$ in Eq. (35), we can easily obtain its value by setting $Q^2 = 0$ in the analytical function (Eq. 36):

$$G_{D_s^* D_s \gamma} = 2.96 \pm 0.09 \text{ GeV}^{-1} \tag{43}$$

Now, it is time for us to give an analysis of electromagnetic decay $D_s^* \rightarrow D_s \gamma$. As to its decay width, it can be written as the following representation:

$$\Gamma = \frac{1}{2J + 1} \sum \frac{p}{8\pi M_i^2} |T|^2$$

$$p = \frac{\sqrt{(M_i^2 - (M_f^2 + m^2)^2)(M_i^2 - (M_f^2 - m^2)^2)}}{2M_i} \tag{44}$$

where the i and f denote the initial and final state mesons, respectively, the J is the total angular momentum of the initial meson, the \sum denotes the summation of all the polarization vectors, and the T denotes the scattering amplitudes.

With the Eqs. (35) and (44), the decay width of $D_s^* \rightarrow D_s \gamma$ can be expressed as:

$$\Gamma = \alpha G_{D_s^* D_s \gamma}^2 \left[\frac{f_\phi}{m_\phi} Q_s \right]^2 \frac{|p|^3}{3M_{D_s^*}^2} \left[\frac{M_{D_s^*}^2 + M_{D_s}^2}{2M_{D_s^*}} \right]^2 \tag{45}$$

with $Q_s = \frac{1}{3}, \alpha = \frac{1}{137}$. Considering all the uncertainties of the input parameters, we finally get the decay width of the process $D_s^* \rightarrow D_s \gamma$:

$$\Gamma = 0.59 \pm 0.15 \text{ keV}$$

It is indicated by the Babar collaboration that the decay width of $\Gamma(D_s^*) < 1.9 \text{ MeV}$. And the ratio of the decay channel $D_s^* \rightarrow D_s \gamma$ is about 94.2% of the total width. This means that our result is compatible with the experimental data. Besides, Donald et al. predicted the value of this decay channel is $\Gamma(D_s^* \rightarrow D_s \gamma) = 0.066 \pm 0.026 \text{ keV}$ in Full Lattice QCD [58], which is much smaller than our result. Although these results are all compatible with the experimental data, it needs to be further testified by more experiments and theoretical calculations because of this difference.

4 Conclusions

In this article, we have calculated the form factors $G_{D_s^* D_s \phi}^\phi, G_{D_s^* D_s \phi}^{D_s}$ and $G_{D_s^* D_s \phi}^{D_s^*}$ in the space-like regions with ϕ, D_s and D_s^* off-shell cases by three different QCD sum rules. Then we fit the form factors into analytical functions, extrapolated

them into the time-like regions, and obtained the strong coupling constant $G_{D_s^* D_s \phi}$. This procedure help us to reduce the errors related to the method, leading to compatible coupling constants, as seen Eqs. (39)–(41). In addition, we also obtained the coupling constant $G_{D_s^* D_s \gamma}$ with the analytical function. With this coupling constant, we calculated the decay width of the electromagnetic decay $D_s^* \rightarrow D_s \gamma$ and compared our result with those of other groups.

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Appendix A: Full expressions of the $\langle g^2 G^2 \rangle$ and $\langle f^3 G^3 \rangle$ contributions for ϕ off-shell case

$$\begin{aligned} \Pi_{\mu\nu}^{(g^2 G^2)(\phi)} = & \frac{i \langle \frac{g^2 G^2}{4\pi^2} \rangle}{4\pi^2} \left\{ -m_c \left[\tilde{I}_{m_c m_s m_s}^{311} + m_c^2 \tilde{I}_{m_c m_s m_s}^{411} + I_{m_c m_s m_s}^{311} \right. \right. \\ & + m_c^2 I_{m_c m_s m_s}^{411} + N_{m_c m_s m_s}^{311} + m_c^2 N_{m_c m_s m_s}^{411} \left. \right] \\ & + m_c^2 m_s \left[\tilde{I}_{m_c m_s m_s}^{411} + m_c^2 I_{m_c m_s m_s}^{411} \right] + m_s I_{m_c m_s m_s}^{131} \\ & + m_s^2 (m_s - m_c) \left[I_{m_c m_s m_s}^{141} + \tilde{I}_{m_c m_s m_s}^{141} \right] \\ & - m_c m_s^2 N_{m_c m_s m_s}^{141} - m_s^2 (m_c - m_s) \\ & \times \left[I_{m_c m_s m_s}^{114} + \tilde{I}_{m_c m_s m_s}^{114} \right] \\ & + m_s \tilde{I}_{m_c m_s m_s}^{113} - m_c m_s^2 N_{m_c m_s m_s}^{114} \\ & + \frac{1}{6} \left[(m_c - m_s) I_{m_c m_s m_s}^{122} \right. \\ & + (m_c - m_s) \tilde{I}_{m_c m_s m_s}^{122} + m_c N_{m_c m_s m_s}^{122} \left. \right] \\ & + \frac{1}{6} \left[(m_c - m_s) I_{m_c m_s m_s}^{221} + (m_c - m_s) \tilde{I}_{m_c m_s m_s}^{221} \right. \\ & + m_c N_{m_c m_s m_s}^{221} \left. \right] + \frac{1}{6} \left[3(m_s - m_c) \tilde{I}_{m_c m_s m_s}^{212} + (m_s \right. \\ & \left. - 3m_c) I_{m_c m_s m_s}^{212} - 3m_c N_{m_c m_s m_s}^{212} \right] \left. \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \end{aligned} \tag{46}$$

$$\begin{aligned} \Pi_{\mu\nu}^{(f^3 G^3)(\phi)} = & \frac{i \langle f^3 G^3 \rangle}{32\pi^4} \left\{ \frac{m_c}{6} \left[N_{m_c m_s m_s}^{321} + 2m_c^2 N_{m_c m_s m_s}^{421} \right. \right. \\ & - 3\tilde{I}_{m_c m_s m_s}^{321} - 3I_{m_c m_s m_s}^{321} - 2m_c^2 \tilde{I}_{m_c m_s m_s}^{421} \\ & \left. \left. - 2m_c^2 I_{m_c m_s m_s}^{421} \right] \right. \\ & + \frac{m_c}{6} \left[11N_{m_c m_s m_s}^{312} + 11m_c^2 N_{m_c m_s m_s}^{412} \right. \end{aligned}$$

$$\begin{aligned} & \left. - 2I_{m_c m_s m_s}^{402} + I_{m_c m_s m_s}^{312} + m_c^2 I_{m_c m_s m_s}^{412} + 9\tilde{I}_{m_c m_s m_s}^{312} \right. \\ & \left. + 6m_c^2 \tilde{I}_{m_c m_s m_s}^{412} \right] \\ & - \frac{m_c}{3} \left[I_{m_c m_s m_s}^{132} + \tilde{I}_{m_c m_s m_s}^{132} + N_{m_c m_s m_s}^{132} \right] \\ & - \frac{m_c}{6} \left[I_{m_c m_s m_s}^{231} + \tilde{I}_{m_c m_s m_s}^{231} + N_{m_c m_s m_s}^{231} \right] \\ & - \frac{m_c}{6} \left[I_{m_c m_s m_s}^{123} + \tilde{I}_{m_c m_s m_s}^{123} + N_{m_c m_s m_s}^{123} \right] \\ & + m_c \left[I_{m_c m_s m_s}^{213} + \tilde{I}_{m_c m_s m_s}^{213} + N_{m_c m_s m_s}^{213} \right] \\ & + \frac{1}{2} \left[(6m_s - m_c) \left(\tilde{I}_{m_c m_s m_s}^{411} - I_{m_c m_s m_s}^{411} \right) \right. \\ & + 8m_s^2 (m_s - m_c) \left(\tilde{I}_{m_c m_s m_s}^{511} - I_{m_c m_s m_s}^{511} \right) \\ & + (m_s - m_c) \left(I_{m_c m_s m_s}^{411} + 8m_s^2 I_{m_c m_s m_s}^{511} \right) \\ & \left. - 6m_s N_{m_c m_s m_s}^{411} - 8m_s^3 N_{m_c m_s m_s}^{511} \right] \\ & + \frac{1}{2} \left[(m_s - 6m_c) I_{m_c m_s m_s}^{411} + 8m_c^2 (m_s - m_c) I_{m_c m_s m_s}^{511} \right. \\ & - (m_s - 6m_c) \tilde{I}_{m_c m_s m_s}^{411} - 8m_c^2 (m_s - m_c) \tilde{I}_{m_c m_s m_s}^{511} \\ & \left. - 6m_c N_{m_c m_s m_s}^{411} - 8m_c^3 N_{m_c m_s m_s}^{511} \right] \\ & + \frac{1}{2} \left[(m_s - m_c) \left(\tilde{I}_{m_c m_s m_s}^{141} - I_{m_c m_s m_s}^{141} + 9m_s^2 \tilde{I}_{m_c m_s m_s}^{151} \right. \right. \\ & \left. \left. - 9m_s^2 I_{m_c m_s m_s}^{151} \right) + (m_s - m_c) \right. \\ & \left. \times \left(I_{m_c m_s m_s}^{141} + 9m_s^2 I_{m_c m_s m_s}^{151} \right) \right. \\ & + 5m_s \left(I_{m_c m_s m_s}^{141} - m_s^2 I_{m_c m_s m_s}^{151} \right) \\ & \left. + m_s \left(N_{m_c m_s m_s}^{141} + 9m_s^2 N_{m_c m_s m_s}^{151} \right) \right] \\ & \left. - \frac{m_c}{6} \left[I_{m_c m_s m_s}^{222} + \tilde{I}_{m_c m_s m_s}^{222} + N_{m_c m_s m_s}^{222} \right] \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \end{aligned} \tag{47}$$

Appendix B: Full expressions about the condensate terms for D_s off-shell case

$$\begin{aligned} \Pi_{\mu\nu}^{(\bar{s}s)(D_s)} = & \frac{\langle \bar{s}s \rangle}{2} \left\{ -\frac{m_s^2}{(p^2 - m_c^2)(p^2 - m_s^2)^2} \right. \\ & \left. + \frac{m_c m_s}{(p^2 - m_c^2)^2(p^2 - m_s^2)} + \frac{2}{(p^2 - m_c^2)(p^2 - m_s^2)} \right\} \\ & \times p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{48} \\ \Pi_{\mu\nu}^{(\bar{s}g\sigma.Gs)(D_s)} = & \langle \bar{s}g\sigma.Gs \rangle \left\{ \frac{1}{12} \left[\frac{1}{(p^2 - m_c^2)^2(p^2 - m_s^2)} \right. \right. \\ & \left. \left. + \frac{1}{(p^2 - m_c^2)(p^2 - m_s^2)^2} \right] \right. \\ & - \frac{1}{96} \left[-2m_s^2 \left(\frac{12}{(p^2 - m_c^2)(p^2 - m_s^2)^3} \right. \right. \\ & \left. \left. + \frac{24m_s^2}{(p^2 - m_c^2)(p^2 - m_s^2)^4} + \frac{2}{(p^2 - m_c^2)^2(p^2 - m_s^2)^2} \right) \right. \\ & \left. \left. + \frac{8m_c^2}{(p^2 - m_c^2)^3(p^2 - m_s^2)^2} \right] + 2m_c m_s \left(\frac{12}{(p^2 - m_c^2)^3(p^2 - m_s^2)} \right) \right\} \end{aligned}$$

$$\begin{aligned}
 & + \frac{24m_c^2}{(p^2 - m_c^2)^4(p^2 - m_s^2)} + \frac{2}{(p^2 - m_c^2)^2(p^2 - m_s^2)^2} \\
 & + \frac{8m_s^2}{(p^2 - m_c^2)^2(p^2 - m_s^2)^3} \Big) + 6 \left(\frac{2}{(p^2 - m_c^2)(p^2 - m_s^2)^2} \right. \\
 & + \frac{8m_s^2}{(p^2 - m_c^2)(p^2 - m_s^2)^3} + \frac{2}{(p^2 - m_c^2)^2(p^2 - m_s^2)} \\
 & \left. + \frac{8m_c^2}{(p^2 - m_c^2)^3(p^2 - m_s^2)} \right) \Big] p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{49}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{\mu\nu}^{(\bar{s}s)^2(D_s)} = & \left\{ \frac{2 \langle \bar{s}s \rangle^2 m_c}{27(p^2 - m_s^2)(p^2 - m_c^2)^3} \right. \\
 & - \frac{\langle \bar{s}s \rangle^2}{162(p^2 - m_s^2)(p^2 - m_c^2)} \left[\frac{3m_s}{(p^2 - m_s^2)^2} + \frac{2m_s^3}{(p^2 - m_s^2)^3} \right. \\
 & - \frac{m_c}{(p^2 - m_c^2)(p^2 - m_s^2)} + \frac{2m_c m_s^2}{(p^2 - m_c^2)(p^2 - m_s^2)^2} \\
 & + \frac{m_s}{(p^2 - m_c^2)(p^2 - m_s^2)} + \frac{2m_s m_c^2}{(p^2 - m_c^2)^2(p^2 - m_s^2)} \\
 & \left. \left. - \frac{3m_c}{(p^2 - m_c^2)^2} - \frac{2m_c^3}{(p^2 - m_c^2)^3} \right] \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{50}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{\mu\nu}^{(\bar{s}s}GG)(D_s)} = & \frac{\langle g^2 \bar{s}s GG \rangle}{72} \left\{ - \frac{3m_s^2}{(p^2 - m_c^2)^2(p^2 - m_s^2)^3} \right. \\
 & + \frac{3m_c m_s}{(p^2 - m_c^2)^3(p^2 - m_s^2)^2} + \frac{1}{(p^2 - m_c^2)^2(p^2 - m_s^2)^2} \\
 & + \frac{2}{(p^2 - m_c^2)^2(p^2 - m_s^2)^2} + \frac{4m_s^2}{(p^2 - m_c^2)^2(p^2 - m_s^2)^3} \\
 & + \frac{4m_c^2}{(p^2 - m_c^2)^3(p^2 - m_s^2)^2} + \frac{4m_s^2 m_c^2}{(p^2 - m_c^2)^3(p^2 - m_s^2)^3} \\
 & + \frac{3}{(p^2 - m_c^2)(p^2 - m_s^2)^3} + \frac{12m_s^2}{(p^2 - m_c^2)(p^2 - m_s^2)^4} \\
 & + \frac{4m_s^4}{(p^2 - m_c^2)(p^2 - m_s^2)^5} + \frac{3}{(p^2 - m_c^2)^3(p^2 - m_s^2)} \\
 & + \frac{12m_c^2}{(p^2 - m_c^2)^4(p^2 - m_s^2)} + \frac{4m_c^4}{(p^2 - m_c^2)^5(p^2 - m_s^2)} \\
 & + \frac{m_s^2 m_c m_s}{2(p^2 - m_c^2)^2(p^2 - m_s^2)^4} - \frac{m_s^4}{2(p^2 - m_c^2)(p^2 - m_s^2)^5} \\
 & - \frac{m_s^2}{2(p^2 - m_c^2)(p^2 - m_s^2)^4} + \frac{m_c^2 m_c m_s}{2(p^2 - m_c^2)^5(p^2 - m_s^2)} \\
 & - \frac{m_c^2 m_s^2}{2(p^2 - m_c^2)^4(p^2 - m_s^2)^2} + \frac{3m_c m_s}{2(p^2 - m_c^2)^4(p^2 - m_s^2)} \\
 & \left. \left. + \frac{2m_c^2}{2(p^2 - m_c^2)^4(p^2 - m_s^2)} \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{51}
 \end{aligned}$$

$$\Pi_{\mu\nu}^{(g^2 G^2)(D_s)} = \frac{i \langle g^2 G^2 \rangle}{4\pi^2} \left\{ -m_s \left[(m_s^2 - m_s m_c) \tilde{I}_{m_s m_s m_c}^{411} \right. \right.$$

$$\begin{aligned}
 & + \tilde{I}_{m_s m_s m_c}^{311} + N_{m_s m_s m_c}^{311} + m_s^2 N_{m_s m_s m_c}^{411} + I_{m_s m_s m_c}^{311} \Big] \\
 & + m_s \left[m_s(m_c - m_s) \tilde{I}_{m_s m_s m_c}^{141} - m_s^2 N_{m_s m_s m_c}^{141} + I_{m_s m_s m_c}^{131} \right] \\
 & + m_c \left[\tilde{I}_{m_s m_s m_c}^{113} + (m_c^2 - m_s m_c) \tilde{I}_{m_s m_s m_c}^{114} - m_s m_c N_{m_s m_s m_c}^{114} \right] \\
 & + \frac{1}{6} \left[(3m_c - m_s) \tilde{I}_{m_s m_s m_c}^{122} + 2m_s I_{m_s m_s m_c}^{122} - m_s N_{m_s m_s m_c}^{122} \right] \\
 & + \frac{1}{6} \left[(m_s - m_c) \tilde{I}_{m_s m_s m_c}^{221} + m_s N_{m_s m_s m_c}^{221} \right] \\
 & \left. + \frac{1}{6} \left[(m_s - m_c) \tilde{I}_{m_s m_s m_c}^{212} + m_s N_{m_s m_s m_c}^{212} \right] \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{52}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{\mu\nu}^{(f^3 G^3)(D_s)} = & \frac{i \langle f^3 G^3 \rangle}{32\pi^4} \left\{ \frac{m_c}{6} \tilde{I}_{m_s m_s m_c}^{321} + \frac{m_c}{3} \tilde{I}_{m_s m_s m_c}^{312} \right. \\
 & - m_c \tilde{I}_{m_s m_s m_c}^{132} + \frac{m_c}{6} \tilde{I}_{m_s m_s m_c}^{231} + \frac{m_c}{6} \left[-5 \tilde{I}_{m_s m_s m_c}^{123} \right. \\
 & + 2I_{m_s m_s m_c}^{123} + 2I_{m_s m_s m_c}^{114} - 2m_c^2 \tilde{I}_{m_s m_s m_c}^{124} + 2(m_s^2 + m_c^2 \\
 & + q^2) I_{m_s m_s m_c}^{124} \Big] + \frac{m_c}{6} \left[4N_{m_s m_s m_c}^{213} + 4m_c^2 N_{m_s m_s m_c}^{214} \right. \\
 & + \tilde{I}_{m_s m_s m_c}^{213} + 2\tilde{I}_{m_s m_s m_c}^{114} \Big] + m_c \left[3\tilde{I}_{m_s m_s m_c}^{114} + 4m_c^2 \tilde{I}_{m_s m_s m_c}^{115} \right] \\
 & + \frac{m_c}{2} \tilde{I}_{m_s m_s m_c}^{411} + \frac{1}{2} \left[m_c \tilde{I}_{m_s m_s m_c}^{141} + 8m_s^2 m_c \tilde{I}_{m_s m_s m_c}^{151} \right. \\
 & - 6m_s I_{m_s m_s m_c}^{141} - 8m_s^3 I_{m_s m_s m_c}^{151} \Big] + \frac{1}{6} \left[(m_c - m_s) \tilde{I}_{m_s m_s m_c}^{222} \right. \\
 & \left. \left. - m_s N_{m_s m_s m_c}^{222} \right] \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{53}
 \end{aligned}$$

Appendix C: Full expressions about the condensate terms for D_s^* off-shell case

$$\begin{aligned}
 \Pi_{\mu\nu}^{(\bar{s}s)(D_s^*)} = & \frac{\langle \bar{s}s \rangle}{2} \left\{ \frac{m_s^2}{(p^2 - m_s^2)^2(p^2 - m_c^2)} \right. \\
 & \left. - \frac{m_c m_s}{(p^2 - m_s^2)(p^2 - m_c^2)^2} + \frac{2}{(p^2 - m_s^2)(p^2 - m_c^2)} \right\} \\
 & p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{54}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{\mu\nu}^{(\bar{s}s)^2(D_s^*)} = & \left\{ \frac{2 \langle \bar{s}s \rangle^2}{27(p^2 - m_c^2)(p^2 - m_s^2)} \left[\frac{m_c}{(p^2 - m_c^2)^2} \right. \right. \\
 & + \frac{m_s}{(p^2 - m_s^2)^2} \Big] - \frac{\langle \bar{s}s \rangle^2}{162(p^2 - m_c^2)(p^2 - m_s^2)} \\
 & \times \left[\frac{3m_c}{(p^2 - m_c^2)^2} + \frac{2m_c^3}{(p^2 - m_c^2)^3} \right. \\
 & - \frac{m_s}{(p^2 - m_s^2)(p^2 - m_c^2)} - \frac{2m_s m_c^2}{(p^2 - m_s^2)(p^2 - m_c^2)^2} \\
 & + \frac{m_c}{(p^2 - m_s^2)(p^2 - m_c^2)} + \frac{2m_c m_s^2}{(p^2 - m_s^2)^2(p^2 - m_c^2)} \\
 & \left. \left. - \frac{3m_s}{(p^2 - m_s^2)^2} - \frac{2m_s^3}{(p^2 - m_s^2)^3} \right] \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \tag{55}
 \end{aligned}$$

$$\begin{aligned} \Pi_{\mu\nu}^{(\bar{s}g\sigma.Gs)(D_s^*)} &= \langle \bar{s}g\sigma.Gs \rangle \\ &\times \left\{ \frac{1}{12} \left[\frac{1}{(p^2 - m_s^2)^2(p^2 - m_c^2)} - \frac{3}{(p^2 - m_s^2)(p^2 - m_c^2)^2} \right] \right. \\ &- \frac{1}{24} \left[-m_s^2 \left(\frac{3}{(p^2 - m_s^2)^3(p^2 - m_c^2)} \right. \right. \\ &+ \frac{2m_s^2}{(p^2 - m_s^2)^4(p^2 - m_c^2)} + \frac{1}{(p^2 - m_s^2)^2(p^2 - m_c^2)^2} \\ &+ \left. \left. \frac{2m_c^2}{(p^2 - m_s^2)^2(p^2 - m_c^2)^3} \right) + m_c m_s \left(\frac{3}{(p^2 - m_s^2)(p^2 - m_c^2)^3} \right. \right. \\ &+ \frac{2m_c^2}{(p^2 - m_s^2)(p^2 - m_c^2)^4} + \frac{1}{(p^2 - m_s^2)^2(p^2 - m_c^2)^2} \\ &+ \left. \left. \frac{2m_s^2}{(p^2 - m_s^2)^3(p^2 - m_c^2)^2} \right) + 3 \left(\frac{1}{(p^2 - m_s^2)(p^2 - m_c^2)^2} \right. \right. \\ &+ \frac{2m_c^2}{(p^2 - m_s^2)^2(p^2 - m_c^2)^3} + \frac{1}{(p^2 - m_s^2)^2(p^2 - m_c^2)} \\ &+ \left. \left. \frac{2m_s^2}{(p^2 - m_s^2)^3(p^2 - m_c^2)} \right) \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \end{aligned} \quad (56)$$

$$\begin{aligned} \Pi_{\mu\nu}^{(\bar{s}sGG)(D_s^*)} &= \frac{\langle g^2 \bar{s}sGG \rangle}{72} \left\{ -\frac{m_s^2}{2(p^2 - m_s^2)^3(p^2 - m_c^2)^2} \right. \\ &+ \frac{m_c m_s}{2(p^2 - m_s^2)^2(p^2 - m_c^2)^3} + \frac{1}{(p^2 - m_s^2)^2(p^2 - m_c^2)^2} \\ &+ \frac{2}{(p^2 - m_s^2)^2(p^2 - m_c^2)^2} + \frac{4m_s^2}{(p^2 - m_s^2)^2(p^2 - m_c^2)^3} \\ &+ \frac{4m_c^2}{(p^2 - m_s^2)^3(p^2 - m_c^2)^2} + \frac{4m_s^2 m_c^2}{(p^2 - m_s^2)^3(p^2 - m_c^2)^3} \\ &+ \frac{3}{(p^2 - m_s^2)(p^2 - m_c^2)^3} + \frac{12m_s^2}{(p^2 - m_s^2)(p^2 - m_c^2)^4} \\ &+ \frac{4m_s^4}{(p^2 - m_s^2)(p^2 - m_c^2)^5} + \frac{3}{(p^2 - m_s^2)^3(p^2 - m_c^2)} \\ &+ \frac{12m_c^2}{(p^2 - m_s^2)^4(p^2 - m_c^2)} + \frac{4m_c^4}{(p^2 - m_s^2)^5(p^2 - m_c^2)} \\ &\times \frac{m_c^2}{(p^2 - m_s^2)(p^2 - m_c^2)^4} - \frac{3m_c m_s}{2(p^2 - m_s^2)(p^2 - m_c^2)^4} \\ &- \frac{m_s m_c^3}{2(p^2 - m_s^2)(p^2 - m_c^2)^5} + \frac{m_s^2 m_c^2}{2(p^2 - m_s^2)^2(p^2 - m_c^2)^4} \\ &- \frac{m_c m_s^3}{2(p^2 - m_s^2)^4(p^2 - m_c^2)^2} + \frac{m_s^4}{2(p^2 - m_s^2)^5(p^2 - m_c^2)} \\ &+ \left. \frac{5m_s^2}{2(p^2 - m_s^2)^4(p^2 - m_c^2)} \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \end{aligned} \quad (57)$$

$$\begin{aligned} \Pi_{\mu\nu}^{(g^2G^2)(D_s^*)} &= \frac{i \langle \frac{g^2 G^2}{4\pi^2} \rangle}{4\pi^2} \left\{ -m_s \left[(m_s^2 - m_s m_c) I_{m_s m_c m_s}^{411} \right. \right. \\ &+ I_{m_s m_c m_s}^{311} + N_{m_s m_c m_s}^{311} + m_s^2 N_{m_s m_c m_s}^{411} + \tilde{I}_{m_s m_c m_s}^{311} \left. \right] \\ &+ m_c \left[m_c(m_c - m_s) I_{m_s m_c m_s}^{141} - m_s m_c N_{m_s m_c m_s}^{141} \right. \end{aligned}$$

$$\begin{aligned} &+ I_{m_s m_c m_s}^{131} \left. \right] + m_s \left[\tilde{I}_{m_s m_c m_s}^{113} + m_s(m_c - m_s) I_{m_s m_c m_s}^{114} \right. \\ &- m_s^2 N_{m_s m_c m_s}^{114} \left. \right] + \frac{1}{6} \left[(m_s - m_c) I_{m_s m_c m_s}^{122} + m_s N_{m_s m_c m_s}^{122} \right] \\ &+ \frac{1}{6} \left[3(m_c - m_s) I_{m_s m_c m_s}^{221} - 2m_s \tilde{I}_{m_s m_c m_s}^{221} - 3m_s N_{m_s m_c m_s}^{221} \right] \\ &+ \frac{1}{6} \left[(m_s - m_c) I_{m_s m_c m_s}^{212} + m_s N_{m_s m_c m_s}^{212} \right] \left. \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \end{aligned} \quad (58)$$

$$\begin{aligned} \Pi_{\mu\nu}^{(f^3G^3)(D_s^*)} &= \frac{i \langle f^3 G^3 \rangle m_c}{32\pi^4} \left\{ -I_{m_s m_c m_s}^{321} + \frac{1}{6} I_{m_s m_c m_s}^{312} \right. \\ &+ \frac{1}{2} I_{m_s m_c m_s}^{132} + \frac{1}{3} \left[\tilde{I}_{m_s m_c m_s}^{231} + I_{m_s m_c m_s}^{231} \right] \\ &+ \frac{1}{3} I_{m_s m_c m_s}^{123} + \frac{1}{6} I_{m_s m_c m_s}^{213} + \frac{1}{2} I_{m_s m_c m_s}^{114} \\ &+ \frac{1}{2} I_{m_s m_c m_s}^{411} + 3I_{m_s m_c m_s}^{141} - 2m_c^2 I_{m_s m_c m_s}^{151} \\ &+ \left. \frac{1}{6} I_{m_s m_c m_s}^{222} \right\} p_\alpha p'_\beta \varepsilon^{\mu\nu\alpha\beta} \end{aligned} \quad (59)$$

References

1. T. Matsui, H. Satz, Phys. Lett. B **178**, 416 (1986)
2. S.G. Matinyan, B. Muller, Phys. Rev. C **58**, 2994 (1998)
3. K.L. Haglin, Phys. Rev. C **61**, 031902 (2000)
4. Z.W. Lin, C.M. Ko, Phys. Rev. C **62**, 034903 (2000)
5. A. Sibirtsev, K. Tsushima, A.W. Thomas, Phys. Rev. C **63**, 044906 (2001)
6. Z.W. Lin, C.M. Ko, Phys. Lett. B **503**, 104 (2001)
7. R. Casalbuoni, A. Deandrea, N. Bartolomeo, R. Gatto, F. Feruglio, G. Nardulli, Phys. Rep. **281**, 145 (1997)
8. X. Liu, B. Zhang, S.L. Zhu, Phys. Lett. B **645**, 185 (2007)
9. C. Meng, K.T. Chao, Phys. Rev. D **78**, 074001 (2008)
10. F.K. Guo, C. Hanhart, G. Li, U.G. Meissner, Q. Zhao, Phys. Rev. D **83**, 034013 (2011)
11. B. Aubert et al., BaBar Collaboration, Phys. Rev. Lett. **95**, 142001 (2005)
12. T. Aaltonen et al., CDF, Phys. Rev. Lett. **102**, 242002 (2009)
13. C. Shen et al., Belle, Phys. Rev. Lett. **104**, 112004 (2010)
14. N. Mahajan, Phys. Lett. B **679**, 228 (2009)
15. T. Branz, T. Gutsche, V.E. Lyubovitskij, Phys. Rev. D **80**, 054019 (2009)
16. X. Liu, Phys. Lett. B **680**, 137 (2009)
17. X. Liu, Z.G. Luo, Y.R. Liu, S.L. Zhu, Eur. Phys. J. C **61**, 411 (2009)
18. J.M. Dias, F.S. Navarra, M. Nielsen, C. Zanetti, [arXiv:1311.7591](https://arxiv.org/abs/1311.7591) [hep-ph]
19. Z.G. Wang, Eur. Phys. J. C **74**, 2963 (2014)
20. Z.G. Wang, T. Huang, Nucl. Phys. A **930**, 63 (2014)
21. Z.-J. Zhao, D.-M. Pan, [arXiv:1104.1838](https://arxiv.org/abs/1104.1838) [hep-ph]
22. S.J. Brodsky, D.S. Hwang, R.F. Lebed, Phys. Rev. Lett. **113**, 112001 (2014)
23. C. Aydin, A.H. Yilmaz, Mod. Phys. Lett. A **19**, 2129 (2004)
24. C. Aydin, M. Bayar, A.H. Yilmaz, Phys. Rev. D **73**, 074020 (2006)
25. V.V. Braguta, A.I. Onishchenko, Phys. Lett. B **591**, 267 (2004)
26. V.V. Braguta, A.I. Onishchenko, Phys. Rev. D **70**, 033001 (2004)
27. M.E. Bracco, M. Chiapparini, F.S. Navarra, M. Nielsen, Phys. Lett. B **659**, 559 (2008)
28. M.E. Bracco, M. Nielsen, Phys. Rev. D **82**, 034012 (2010)
29. Z.G. Wang, Phys. Rev. D **89**, 034017 (2014)

30. K. Azizi, H. Sundu, J. Phys. G **38**, 045005 (2011)
31. K. Azizi, Y. Sarac, H. Sundu, Phys. Rev. D **90**, 114011 (2014)
32. A. Khodjamirian, Th Mannel, N. Offen, Y.M. Wang, Phys. Rev. D **83**, 094031 (2011)
33. A. Khodjamirian, Ch. Klein, Th. Mannel, Y.M. Wang, [arXiv:1108.2971](https://arxiv.org/abs/1108.2971) [hep-ph]
34. T.M. Aliev, M. Savci, [arXiv:1308.3142](https://arxiv.org/abs/1308.3142) [hep-ph]
35. T.M. Aliev, M. Savci, [arXiv:1409.5250](https://arxiv.org/abs/1409.5250) [hep-ph]
36. T. Doi, Y. Kondo, M. Oka, Phys. Rep. **398**, 253 (2004)
37. R. Altmeyer, M. Goeckeler, R. Horsley et al., Nucl. Phys. Proc. Suppl. **34**, 373 (1994)
38. Z.G. Wang, Phys. Rev. D **74**, 014017 (2006)
39. A. Cerqueira Jr, B.O. Rodrigues, M.E. Bracco, Nucl. Phys. A **874**, 130 (2012)
40. B.O. Rodrigues, M.E. Bracco, M. Chiapparini, Nucl. Phys. A **929**, 143 (2014)
41. E. Yazici et al., Eur. Phys. J. Plus. **128**(10), 113 (2013)
42. R. Khosravi, M. Janbazi, Phys. Rev. D **87**, 016003 (2013)
43. R. Khosravi, M. Janbazi, Phys. Rev. D **89**, 016001 (2014)
44. B. Aubert et al., BaBar Collaboration, Phys. Rev. D **72**, 091101 (2005)
45. L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. **127**, 1 (1985)
46. P. Pascual, R. Tarrach, Lect. Notes Phys. **194**, 1 (1984)
47. Z.G. Wang, Z.Y. Di, Eur. Phys. J. A **50**, 143 (2014)
48. P.Colangelo, A. Khodjamirian, At the frontier of particle physics. in *Handbook of QCD*, vol. 3. (World Scientific, Singapore, 2000), p. 1495. [arXiv:hep-ph/0010175](https://arxiv.org/abs/hep-ph/0010175)
49. M.E. Bracco, M. Chiapparini, A. Lozea, F.S. Navarra, M. Nielsen, Phys. Lett. B **521**, 1 (2001)
50. Z.G. Wang, Phys. Rev. D **81**, 036002 (2010)
51. Z.G. Wang, Eur. Phys. J. A **44**, 105 (2010)
52. K.A. Olive et al., Particle Data Group, Chin. Phys. C **38**(9), 090001 (2014)
53. B.L. Ioffe, Prog. Part. Nucl. Phys. **56**, 232 (2006)
54. S. Narison, Phys. Lett. B **693**, 559 (2010)
55. S. Narison, Phys. Lett. B **706**, 412 (2012)
56. S. Narison, Phys. Lett. B **707**, 259 (2012)
57. Z.G. Wang, Nucl. Phys. A **796**, 61 (2007)
58. G.C. Donald, C.T.H. Davies, J. Koponen, G.P. Lepage, Phys. Rev. Lett. **112**, 212002 (2014)