

Twelve-dimensional effective action and T -duality

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Abstract We propose a 12-dimensional supergravity action which describes low-energy dynamics of F -theory. Dimensional reduction leads the theory to become 11-dimensional, IIA, and IIB supergravities. Self-duality of the four-form field in IIB supergravity is understood. It is necessary to abandon 12-dimensional Poincaré symmetry by making one dimension compact, which is to be decompactified in some region of parameter space, such that the physical degrees of freedom are the same as those of 11-dimensional supergravity. This makes T -duality explicit as a relation between different compactification schemes.

1 Introduction

The ideas of Kaluza and Klein (KK) [1,2], generalized to higher dimensions, are beautiful ones that translate the known field degrees of freedom and their interactions into geometry of extra dimensions. Most of the supergravity theories, which one hopes to have an intimate connection to our world, can be obtained by dimensional reduction of an 11-dimensional one [3]. However, it does not directly give type IIB supergravity in ten dimensions, although their relation is well understood in the context of string theory.

Eleven-dimensional supergravity is a low-energy description of M -theory [4,5]. It has also been suggested that type IIB superstring theory is obtained by a reduction of F -theory on a torus, with its complex structure identified by the axion-dilaton, and the latter is shown to be T -dual to M -theory [6]. Thus, the effective field theory of F -theory should be 12-dimensional; however, it is not easy to write down the action. One crucial difficulty might be that the 12-dimensional minimal fermion with Lorentzian signature (11, 1), which must be the case for F -theory, should have superpartner components with spin higher than two in the four-dimensional language, whose interacting theory would be inconsistent [7]. Another

obstacle is that, if F -theory is dual to M -theory, there should be no surplus field degrees of freedom, although the former is a higher-dimensional theory.

An important hint comes from a careful look at the derivation of F -theory [6,8,9]. Although it is T -dual to M -theory, F -theory has one more dimension than the latter. Now, this extra dimension is a *dual* dimension to one of the dimension shared by the two theories. In other words, F -theory has *two redundant* dimensions whose radii are inverse to each other. Although we cannot maintain 12-dimensional Poincaré symmetry fully, each of the 10- and 11-dimensional theories can be symmetric on its own. There is no contradiction if we cannot see both at once. Therefore, it is natural to keep *both dimensions*. In this picture, M -theory looks like a compactification F -theory on a circle, as schematically shown in Fig. 1.

We propose for the bosonic part a desired 12-dimensional effective action, whose dimensional reductions lead to those of all known supergravities in 11 and 10 dimensions, found in the standard textbooks [10]. Since we follow and make use of the duality relation between M - and F -theory from the 11-dimensional supergravity, this theory shall provide the effective field theory for F -theory.

Supergravity is powerful enough in the sense that many of the new results here, like the existence of a three-brane and generalized T -duality, are obtained *without* referring to string theory. Of course, the effective field description of F -theory is timely in realistic model building, because we have so far borrowed descriptions, for instance of the gauge fields, from M -theory [11–13].

2 The bosonic action of 12-dimensional supergravity

We start with the fundamental bosonic degrees of freedom of 11-dimensional supergravity: the graviton G_{mn} and the rank three antisymmetric tensor field C_{mnp} . The latter is promoted to a four-form field,

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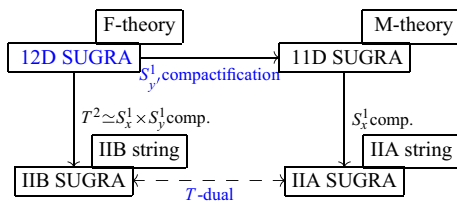


Fig. 1 Relation among superstrings and supergravities (SUGRA). In 12 dimensions, we make T -duality explicit in terms of compactification, by taking the other routes. The diagonal direction is the zero size limit

$$C_{mnp}(x^m) \rightarrow C_{mnp y'}(x^m, y'),$$

with total antisymmetrization, for instance $C_{mny'p} \equiv -C_{mnp y'}$. Here y' denotes the twelfth direction. Although this field is 12-dimensional, we do not introduce any more degrees of freedom if one of the indices is forced to be on y' and the others are 11-dimensional. The graviton is also regarded as a part of the 12-dimensional one,

$$ds^2 = G_{mn} dx^m dx^n + r^2 dy'^2. \tag{1}$$

We suggest a formally 12-dimensional action

$$S = \frac{1}{2\kappa_{12}^2} \int \left(\mathcal{R} * 1 - \frac{1}{2} \mathcal{G}_5 \wedge * \mathcal{G}_5 + \frac{1}{6} \mathcal{C}_4 \wedge G_4 \wedge G_4 \right), \tag{2}$$

with the 12-dimensional Hodge star operator. The Ricci scalar \mathcal{R} is made of the 12-dimensional metric (1). We will define κ_{12} shortly. The presence of the last term was noticed in Refs. [14,15]. We have defined

$$\mathcal{C}_4 = \frac{1}{3!} C_{mnp y'} dx^m \wedge dx^n \wedge dx^p \wedge dy' \tag{3}$$

$$\equiv C_3 \wedge r dy',$$

$$\mathcal{G}_5 \equiv dC_3 \wedge r dy' \equiv G_4 \wedge r dy'. \tag{4}$$

It is important to note that the indices correspond only to 11-dimensional coordinates. Therefore the action (2) has at best 11-dimensional Poincaré invariance. Nevertheless this form is useful, since we may also have ten-dimensional invariance in which we include y' and exclude some of the other directions. There is another loop correction term, having the form $\mathcal{C}_4 \wedge I_8$, where I_8 is again dependent on the 11-dimensional metric only, given in Ref. [16].

The equation of motion and the Bianchi identity of \mathcal{C}_4 follow:

$$dG_4 = 0, \quad d*\mathcal{G}_5 = -\frac{1}{2} G_4 \wedge G_4. \tag{5}$$

Exchanging the role of the two, we also have a dual field strength

$$*\mathcal{G}_5 \equiv dC_6 - \frac{1}{2} C_3 \wedge G_4, \tag{6}$$

which defines a six-form C_6 . In components, the dual field strength to \mathcal{G}_5 is defined as

$$(*\mathcal{G}_5)_{lmnpqrs} = \frac{1}{4!} \sqrt{-G} \epsilon_{lmnpqrs}{}^{tuvvy'} \mathcal{G}_{tuvvy'}, \tag{7}$$

where the indices are raised by the metric (1).

Note that we have converted the 11-dimensional field C_3 to the 12-dimensional field

$$C_{mnp y'}(x^m, y') = r(x^m, y') C_{mnp}(x^m), \tag{8}$$

using the metric (1). They should not be treated as independent degrees of freedom, otherwise we cannot match the equation of motion with the 11-dimensional one.

The four-form structure (8) suggests that there is a sourcing three-brane wrapped on the y' -direction, becoming the $M2$ -brane of M -theory [17]. When a dimension is compact, this wrapping behavior should not be strange, since in the decompactification limit it becomes a $D3$ -brane along the y' -direction, which we are familiar with. Also we may understand that its magnetic five-brane transverse to the y' -direction becomes the $M5$ -brane of M -theory.

We consider in this letter only the bosonic degrees of freedom. The fermionic part will be dealt with elsewhere [17].

3 Reduction to 11-dimensional supergravity

The action (2) is meaningful only if we take the y' -direction as a circle with a radius $2\pi r$, measured in a length unit ℓ . Dimensional reduction gives us the KK tower of the fields C_4, \mathcal{G}_{mn}, r with masses

$$M_k^2 = k^2 \ell^{-2} \langle r \rangle^{-2}. \tag{9}$$

All of them shall play an important role later in decompactification, but we keep the zero modes only for the moment. We can show that the kinetic terms of the graviton and the three-form field become the standard form of 11-dimensional supergravity. The last term in (2) is

$$\int \mathcal{C}_4 \wedge G_4 \wedge G_4 = - \int_{S^1} r dy' \wedge \int_{M^{10,1}} C_3 \wedge G_4 \wedge G_4.$$

The 11-dimensional coupling κ_{11} may reversely define the coupling κ_{12}

$$\frac{2\pi \ell \langle r \rangle}{2\kappa_{12}^2} = \frac{1}{2\kappa_{11}^2}, \tag{10}$$

with the scale r is to be fixed shortly.

Table 1 Identification of ten-dimensional fields. Two subtables respectively show IIA and IIB fields. Indices are nine-dimensional and y' denotes the twelfth direction. Componentwise $C_{mnp y'} = r C_{mnp}$ as in (3). After decompactifying the y' or y directions ten-dimensional Poincaré covariance is recovered

10D Field	Type	(9+1)D Components	12D Components
A_I	RR	$\{A_\mu, A_y\}$	$\{a_\mu, \tau_1\}$
A_3	RR	$\{A_{\mu\nu\rho}, A_{\mu\nu y}\}$	$\{r^{-1}C_{\mu\nu\rho y'}, r^{-1}C_{\mu\nu y y'}\}$
B_2	NSNS	$\{B_{\nu\mu}, B_{\mu y}\}$	$\{r^{-1}C_{\mu\nu x y'}, r^{-1}C_{\mu x y y'}\}$
b_I	KK	b_μ	b_μ
A_4	RR	$A_{\mu\nu\rho y'}$	$r^{-1}C_{\mu\nu\rho y'}$
A_2	RR	$\{A_{\mu\nu}, A_{\mu y'} = -A_{y'\mu}\}$	$\{r^{-1}C_{\mu\nu y y'}, a_\mu\}$
A_0	RR	A	τ_1
B_2	NSNS	$\{B_{\nu\mu}, B_{\mu y'} = -B_{y'\mu}\}$	$\{r^{-1}C_{\mu\nu x y'}, b_\mu\}$
K_I	KK	K_μ	$r^{-1}C_{\mu x y y'}$

4 Reduction to IIB supergravity

Next, we compactify two more dimensions on a torus. It has a complex structure, $\tau = \tau_1 + i\tau_2$, and we take the coordinate x, y such that we identify $x + \tau y \sim x + \tau y + 2\pi\ell \sim x + \tau y + 2\pi\tau\ell$. Still we keep the y -direction orthogonal to the other directions, as in (1). The most general metric is

$$ds^2 = L^2(dx + \tau_1 dy + (a_\mu - \tau_1 b_\mu)dx^\mu)^2 + L^2\tau_2^2(dy - b_\mu dx^\mu)^2 + r^2 dy'^2 + g'_{\mu\nu} dx^\mu dx^\nu. \tag{11}$$

From now on, fields and their Greek indices are nine-dimensional. Here, $\{a_\mu, \tau_1\}, b_\mu$ are ten- and nine-dimensional Lorentz vectors promoting the S^1 isometries of x - and y -directions, respectively, to $U(1)$ gauge symmetries.

We identify the fields of IIB supergravity as in Table 1. They have either all indices nine-dimensional or one component fixed to be y' . Consider, for example, the reduction from $\mathcal{G}_{\alpha\beta\gamma y y'}$ to $H_{\alpha\beta\gamma} = 3\partial_{[\alpha} B_{\beta\gamma]}$, given in (31) in the appendix.¹ Neglecting the normalization, there are two possible expressions,

$$H_{\alpha\beta\gamma} + 3b_{[\alpha} H_{\beta\gamma]} = H_{\alpha\beta\gamma} + 6K_{[\alpha} \partial_{\beta} b_{\gamma]} \tag{12}$$

up to a total derivative which is a gauge transformation of B_2 . The left-hand side is the result of dimensional reduction of the ten-dimensional IIA field $\{H_{\mu\nu\rho}^{(10)}, H_{\mu\nu}^{(10)} \equiv H_{\mu\nu y}^{(10)}\}$ coupled to the KK field b_μ , whereas the right-hand side looks as *dimensional reduction* of the IIB field $\{H_{\mu\nu\rho}^{(10)}, (db)_{\mu\nu}^{(10)} \equiv H_{\mu\nu y'}^{(10)}\}$ coupled to the KK field $K_\mu = r^{-1}C_{\mu x y y'}$ under the metric [19]

$$ds_{10}^2 = r^2(dy' + K_\mu dx^\mu)^2 + g'_{\mu\nu} dx^\mu dx^\nu. \tag{13}$$

¹ We are using the standard antisymmetric tensor notation [18].

In the latter picture, the vectors a_μ and b_μ become components $A_{\mu y'}$ and $B_{\mu y'}$, respectively, of rank two Neveu–Schwarz Neveu–Schwarz (NSNS) and Ramond–Ramond (RR) tensors. We already have the KK tower of the fields (9) completing the fields $B_{\mu\nu}, b_\mu, K_\mu, g'_{\mu\nu}, r$ to be ten-dimensional. This is a crucial necessary condition to recover ten-dimensional Poincaré symmetry and fully covariant interactions. In low-energy theory, this is a possible way to see the presence of extra dimensions, if we admit that the gravitational interactions are not observable.

The RR four-form is obtained as

$$A_{\mu\nu\rho y'} \equiv r^{-1}C_{\mu\nu\rho y'}, \quad F_{\mu\nu\rho\sigma y'} \equiv r^{-1}\mathcal{G}_{\mu\nu\rho\sigma y'} \tag{14}$$

with one of the indices fixed to be y' . We can perform dimensional reduction, as in (30) in the appendix (with a different nine-dimensional metric) and perform decompactification in the y' direction with the help of the one-form K_I as above. The corresponding part of the second term in (2) gives the kinetic term for

$$F_5^{(10)} - \frac{1}{2}A_2^{(10)} \wedge H_3^{(10)} + \frac{1}{2}B_2^{(10)} \wedge F_3^{(10)}. \tag{15}$$

Remember that one of the indices is *fixed to be* y' for every term in (15). To avoid confusion later, we name this $\tilde{F}_5^{w(10)}$.

Due to the fixing of the component in (15) we do not have complete ten-dimensional four-form. The other part may come from another 12-dimensional field (6). The only possible nine-dimensionally covariant four-form can be

$$A_{\mu\nu\rho\sigma} \equiv C_{\mu\nu\rho\sigma x y}, \quad F_{\mu\nu\rho\sigma\tau} \equiv (dA)_{\mu\nu\rho\sigma\tau}. \tag{16}$$

Due to the 12-dimensional structure in (7), the fields in (16) cannot have any index on y' . The left-hand side of the duality relation (7) becomes the $dC_6 - \frac{1}{2}C_3 \wedge G_4$ as in (6), with two components fixed to be x and y . By expansion and decompactification, we obtain precisely the same expression as (15), up to normalization, with *all the indices nine-dimensional*. Hence we may call the result as $L^{-2}\tau_2^{-1}\tilde{F}_5^{wo(10)}$.

Expressing the duality relation (7) in a local Lorentz frame by ten-dimensional fields, we have

$$\tilde{F}_5^{wo(10)} L^{-2}\tau_2^{-1} = r*_{10}\tilde{F}_5^{w(10)}, \tag{17}$$

with the ten-dimensional Hodge operation $*_{10}$. The different components of F_5 have the different origins, therefore the Lorentz symmetry is not trivial. For the covariance we need the same coefficient

$$r = L^{-2}\tau_2^{-1}. \tag{18}$$

This means that the three radii in the x, y, y' directions are inverse among themselves, so there is no point in the moduli space where we have all the 12 dimensions noncompact.

Moreover, this graviton component $r(x^m, y')$, which was the only surplus component compared to the gravity multiplet in the 11-dimensional supergravity, now turns out to be redundant and expressed in terms of the other existing fields $L(x^m, y')$ and $\tau_2(x^m, y')$ in this multiplet. Therefore we have the *identical* degrees of freedom to those of 11-dimensional supergravity. This explains how the difficulty of the maximal dimensional problem in the beginning is circumvented.

Therefore we have arrived at the self-duality condition for the fully covariant ten-dimensional four-form field via its modified field strength $\tilde{F}_5^{(10)}$, re-expressed using Eq. (17),

$$\tilde{F}_5^{(10)} \equiv \tilde{F}_5^{w(10)} + \tilde{F}_5^{wo(10)} = *_10 \tilde{F}_5^{(10)}. \tag{19}$$

We emphasize that this self-duality condition (19) is the *defining* relation of half the components of the four-form field in (16).

The ten-dimensional Einstein–Hilbert term is obtained as

$$\int_{T^2} \mathcal{R} * 1 = (2\pi\ell)^2 \sqrt{-G'} r^{-1} R_{(10)} d^9x \wedge dy' + \dots, \tag{20}$$

where $G' = g'r^2$ is the determinant of the ten-dimensional metric (13), with which the Ricci scalar $R_{(10)}$ is calculated. Note that $\tau_2 = g_{\text{IIB}}^{-1}$, if we require that L should be absent from the IIB supergravity action. Careful investigation shows that rescaling $g'_{\mu\nu} \equiv L^{-1} g_{\mu\nu}$ and $g_{y'y'} \equiv L g'_{y'y'} = L^3 \tau_2$ can pull out the overall factor r , which should be absorbed by the coupling

$$\frac{1}{2\kappa_{\text{IIB}}^2} = \frac{(2\pi\ell)^2 \langle r \rangle}{2\kappa_{12}^2} = \frac{2\pi\ell}{2\kappa_{11}^2}. \tag{21}$$

The rescaling should also rescale the coordinate periodicity as

$$\ell \rightarrow L^{-1/2} \ell \equiv \ell_s. \tag{22}$$

Finally, dimensional reduction of the last term in (2) gives

$$\frac{1}{2\kappa_{\text{IIB}}^2} \int F_5^{(10)} \wedge B_2 \wedge F_3 = \frac{1}{2\kappa_{\text{IIB}}^2} \int \tilde{F}_5^{w(10)} \wedge B_2^{(10)} \wedge F_3^{(10)}, \tag{23}$$

again with one index fixed to be y' for $F_5^{(10)}$ and $\tilde{F}_5^{(10)}$. For the equality, we used the relation $F_3 \wedge F_3 = 0$ and performed integration by parts. Again $\tilde{F}_5^{w(10)}$ can be exchanged by the covariant one (19). The remaining expansions give the kinetic terms for the IIB supergravity action in the standard form [10, 17].

5 Reduction to IIA supergravity

We may decompactify the y -direction in (11) using the KK field b_μ . Decompactification takes place in the same way. For example, Eq. (32) in the appendix, after the decompactification, gives the reduction of G_{mnpq} to ten-dimensional fields,

$$L^2(F_4 - A_1 \wedge H_3), \tag{24}$$

with one of the indices fixed on y , whereas Eq. (30) provides the remaining components. The A_1 is again the KK gauge field decompactifying the x -direction. This gives IIA supergravity. We identify ten-dimensional couplings $\langle L \rangle^3 = g_{\text{IIA}}^2 \equiv \langle e^{2\Phi} \rangle$. It is straightforward to have the type IIA supergravity action, because we know it is also obtained by further compactification of the 11-dimensional supergravity action along the x -circle.

In the unit (22) we can naturally convert between IIA and IIB theories in ten dimensions. The relation between the two radii from (11) now becomes the familiar T -duality relation,

$$R_y = L^{3/2} \tau_2 \ell_s, \quad R_{y'} = L^{-3/2} \tau_2^{-1} \ell_s = \ell_s^2 / R_y. \tag{25}$$

Without referring to string theory, we can perform T -duality via two different compactifications, as in Fig. 1. In particular, Eq. (18) also allows us to interpret the KK tower of fields above Eq. (9) as ones arisen by *wrapped* $M2$ -branes on the torus, whose mass is proportional to the dimensionless volume of the torus $L^2 \tau_2$ [19–21]. This will also be useful in describing physics around the self-dual radius where the two theories are not so much distinct, or in a strong coupling regime of one theory. Gauge theories coming from ‘seven-brane’ loci, on which the ‘fibered’ torus becomes singular [6, 11, 22], cannot be described by this action because they are phenomena of $M2$ -branes wrapped on the singularities. Rather, this action describes the bulk physics in terms newly defined native F -theory fields and potentials.

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Appendix

We briefly summarize the technical details of dimensional reduction taking into account the metric. We have tensors in components in a local Lorentz frame, after the rescaling (22):

$$\begin{aligned} C_{\alpha\beta\gamma y'} &= L^{3/2}(A_{\alpha\beta\gamma y'} - 3a_{[\alpha}B_{\beta\gamma]} + 3b_{[\alpha}A_{\beta\gamma]} \\ &\quad - 6a_{[\alpha}b_{\beta}K_{\gamma]}), \end{aligned} \tag{26}$$

$$C_{\alpha\beta xy'} = B_{\alpha\beta} + 2b_{[\alpha}K_{\beta]}, \tag{27}$$

$$\begin{aligned} C_{\alpha\beta yy'} &= \tau_2^{-1}(A_{\alpha\beta} - \tau_1 B_{\alpha\beta} + 2a_{[\alpha}K_{\beta]} \\ &\quad - 2\tau_1 b_{[\alpha}K_{\beta]}), \end{aligned} \tag{28}$$

$$C_{\alpha xyy'} = L^{-3/2}\tau_2^{-1}K_{\alpha}. \tag{29}$$

For convenience we have fixed some of the coordinates. We have the corresponding field strengths

$$\begin{aligned} \mathcal{G}_{\alpha\beta\gamma\delta y'} &= L^2(F_{\alpha\beta\gamma\delta y'} - 4a_{[\alpha}H_{\beta\gamma\delta]} \\ &\quad + 4b_{[\alpha}F_{\beta\gamma\delta]} + 12a_{[\alpha}b_{\beta}H_{\gamma\delta]}), \end{aligned} \tag{30}$$

$$\mathcal{G}_{\alpha\beta\gamma xy'} = L^{1/2}(H_{\alpha\beta\gamma} + 3b_{[\alpha}H_{\beta\gamma]}), \tag{31}$$

$$\begin{aligned} \mathcal{G}_{\alpha\beta\gamma yy'} &= L^{1/2}\tau_2^{-1}(F_{\alpha\beta\gamma} - \tau_1 H_{\alpha\beta\gamma} + 3a_{[\alpha}H_{\beta\gamma]} \\ &\quad - 3\tau_1 b_{[\alpha}H_{\beta\gamma]}), \end{aligned} \tag{32}$$

$$\mathcal{G}_{\alpha\beta xyy'} = L^{-1}\tau_2^{-1}H_{\alpha\beta}. \tag{33}$$

Here $H_{\alpha\beta} \equiv 2\partial_{[\alpha}K_{\beta]}$ and the derivative operator acts all the fields on the right.

The above relations are nine-dimensional and are to be lifted to ten-dimensional ones. For instance, we may rewrite the right-hand side of (30) in terms of differential forms,

$$L^2 \left(F_5 - \frac{1}{2} (A_2 \wedge H_3 - B_2 \wedge F_3 + A_3 \wedge H_2 - K_I \wedge F_4) \right). \tag{34}$$

This is a nine-dimensional relation with one component fixed to be on y' . After decompactification, we have a ten-dimensional relation,

$$L^{1/2}\tau_2^{-1} \left(F_5^{(10)} - \frac{1}{2} A_2^{(10)} \wedge H_3^{(10)} + \frac{1}{2} B_2^{(10)} \wedge F_3^{(10)} \right), \tag{35}$$

with the y' -component still fixed. There were two changes: The overall normalization has changed by the metric factor

$L^{-3/2}\tau_2^{-1}$ from the y' -dependence, and the couplings with K_I completed the covariant ten-dimensional fields.

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