

Hadronic decays of the highly excited $2D$ D_s resonances

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Abstract Hadronic decays of the highly excited $2D$ D_s resonances have been studied in the 3P_0 model. The widths of all possible hadronic decay channels of the $2D$ D_s have been computed. $D_{s1}^*(2700)$, $D_{s1}^*(2860)$, $D_{s3}^*(2860)$, $D(2600)$, and $D(2750)$ can be produced from hadronic decays of the $2D$ D_s , and the relevant hadronic decay widths have been particularly paid attention to. The hadronic decay widths of $2D$ D_s to $D(2600)$ or $D(2750)$ may be large, and the numerical results are different in different assignments of $D(2600)$ and $D(2750)$. The hadronic decay widths of $2D$ D_s to $D_{s1}^*(2860)$, $D_{s3}^*(2860)$ or $D_{s1}^*(2700)$ are very small, and different in different assignments of $D_{s1}^*(2700)$.

1 Introduction

The properties of highly excited heavy–light meson states have been studied for a long time; the S-wave and P-wave D and D_s resonances are believed to be established. In recent years, more and more heavy–light resonances such as the higher excited D_s have been observed. However, some candidates of these highly excited heavy–light resonances have not been definitely pinned down.

$D_{s1}^*(2700)$ was first observed by Belle [1] in $B^+ \rightarrow \bar{D}^0 D_{s1} \rightarrow \bar{D}^0 D^0 K^+$. It was also observed by BaBar in $D^* K$ channel [2]. This state is included in PDG [3] with $M = 2709 \pm 4$ MeV, $J^P = 1^-$, and $\Gamma = 117 \pm 13$ MeV.

$D_{sJ}^*(2860)$ was first reported by BaBar [2] in $D_{s1}(2860) \rightarrow D^0 K^+, D^+ K^0$ with a mass $M = 2856.6 \pm 1.5$ (stat) ± 5.0 (syst) MeV and a width $\Gamma = 48 \pm 7$ (stat) ± 10 (syst) MeV. It was observed once again in the $D^* K$ channel [2]. This state is included in PDG [3] with $M = 2863.2_{-2.6}^{+4.0}$ MeV, $\Gamma = 58 \pm 11$ MeV, and unknown J^P . Recently, the LHCb Collaboration reported that the resonance $m(\bar{D}^0 K^- \approx 2.86)$ GeV contain both spin-1 and spin-3 components [4,5]. These two states, $D_{s1}^*(2860)^-$ and

$D_{s3}^*(2860)^-$, are suggested to be the $J^P = 1^-$ and $J^P = 3^-$ members of the $1D$ family. In addition, two new charmed states, $D(2600)$ and $D(2750)$, were observed by the BaBar Collaboration [3,6].

$D_{s1}^*(2700)$ was supposed to be the first radially excited S-wave state $D_s(2^3S_1)$ [7–10], the orbitally excited $D_s(1^3D_1)$ [11,12] or a mixture of them [7,9,13]. Similarly, $D_{sJ}^*(2860)^\pm$ was once suggested as the $J^P = 0^+$ [7,14], $J^P = 3^-$ [9,12,15,16] excited D_s or the orthogonal partner of $D_{s1}^*(2700)$ [7,12,13,17]. The 0^+ possibility has subsequently been excluded by the observation of $D_{s1}(2860) \rightarrow D^* K$ channel. A recent experiment suggests that there are in fact two possibilities, $D_{s1}^*(2860)^-$ and $D_{s3}^*(2860)^-$ close to $D_{sJ}^*(2860)^\pm$ [4,5], where $D_{s1}^*(2860)^-$ and $D_{s3}^*(2860)^-$ are suggested to be the $J^P = 1^-$ and $J^P = 3^-$ members of the $1D$ family [4,5,12].

$D(2600)$ is observed and suggested to be the first radially excited S-wave states $D(2^3S_1)$, based on an analysis of their masses and helicity–angle distribution, while $D(2750)$ is observed and suggested the orbitally excited $1D$ state [6]. In Ref. [18,19], $D(2600)$ is suggested as an admixture of 2^3S_1 and 1^3D_1 with $J^P = 1^-$, and $D(2750)$ is interpreted as an orthogonal partner of $D(2600)$ or 1^3D_3 . In Ref. [10], $D(2600)$ was interpreted as a pure 2^3S_1 state from its hadronic decays in the heavy quark symmetry theory. In Ref. [20], an analysis of the assignment of $D(2600)$ to the first radial excitation of D^* has been done in detail in an effective Lagrangian approach.

There are different interpretations of these resonances. Obviously, the nature of these resonances is not understood clearly. In the literature, the arrangements of these resonances are mainly based on the study of their J^P quantum numbers, masses, and strong decay modes.

It is well known that the study of the productions of these resonances is also an important way to understand them. $D_{s1}^*(2700)$, $D_{s1}^*(2860)$, $D_{s3}^*(2860)$, $D(2600)$, and $D(2750)$ can be produced from the strong decays of highly excited resonances. It will be interesting to study the hadronic pro-

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duction of $D_{s1}^*(2700)$, $D_{s1}^*(2860)$, $D_{s3}^*(2860)$, $D(2600)$, and $D(2750)$ from higher excited resonances. In fact, some highly excited D_s resonances have been observed by BaBar, LHCb, etc., more and more highly excited D_s resonances are expected to be observed by these Collaborations. For kinematical reasons, these resonances can be produced from hadronic decays of $2D D_s$. Unfortunately, the strong decays of the highly excited $2D D_s$ resonances have seldom been studied before. In this paper, the hadronic decays of these $2D D_s$ resonances will be studied in the 3P_0 model.

The paper is organized as follows. In Sect. 2, we give a brief review of the 3P_0 model and possible decay modes of the $2D$ resonances. In Sect. 3, we present the formula and numerical results of the hadronic decay of the $2D D_s$ resonances, and the decays with $D_{s1}^*(2700)$, $D_{s1}^*(2860)$, $D_{s3}^*(2860)$, $D(2600)$ or $D(2750)$ involved in the final states are particularly paid attention to. Finally, the conclusions and discussions are given in Sect. 4.

2 3P_0 model and possible decay modes of the $2D D_s$ resonances

The 3P_0 model is popularly known as a quark-pair creation (QPC) model. It has been extensively applied to the calculation of the OZI-allowed strong decay of meson A to meson B and C . The model was first proposed by Micu [21], and then developed by Yaouanc et al. [22–27]. The decay process is shown in Fig. 1 [28, 29], where a pair of quarks $q_3\bar{q}_4$ with $J^{PC} = 0^{++}$ are created from the vacuum and regroup with the $q_1\bar{q}_2$ within the initial meson A into two outgoing mesons B and C .

In 3P_0 model, the hadronic decay width of $A \rightarrow BC$ is

$$\Gamma = \pi^2 \frac{|\mathbf{K}|}{M_A^2} \sum_{JL} |\mathcal{M}^{JL}|^2 \tag{1}$$

where the momentum of the daughter meson in the initial meson A 's center of mass frame is

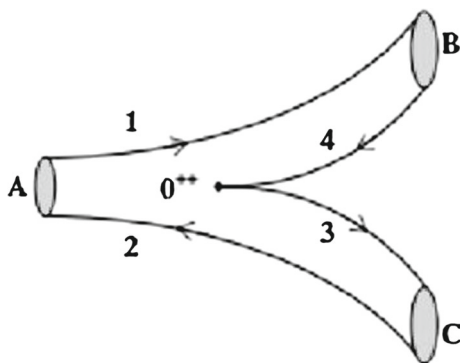


Fig. 1 Decay process of $A \rightarrow B + C$ in the 3P_0 model

$$|\mathbf{K}| = \frac{\sqrt{[m_A^2 - (m_B - m_C)^2][m_A^2 - (m_B + m_C)^2]}}{2m_A} \tag{2}$$

and M^{JL} is the partial wave amplitude of $A \rightarrow BC$. In terms of the Jacob–Wick formula [30, 31], the partial wave amplitude can be obtained from the helicity amplitude $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$

$$\begin{aligned} \mathcal{M}^{JL}(A \rightarrow BC) &= \frac{\sqrt{2L+1}}{2J_A+1} \\ &\times \sum_{M_{J_B}, M_{J_C}} \langle LOJ M_{J_A} | J_A M_{J_A} \rangle \\ &\times \langle J_B M_{J_B} J_C M_{J_C} | J, J M_{J_A} \rangle \\ &\times \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{K}) \end{aligned} \tag{3}$$

with $\mathbf{J} = \mathbf{J}_B + \mathbf{J}_C$, $\mathbf{J}_A = \mathbf{J}_B + \mathbf{J}_C + \mathbf{L}$, and $M_{J_A} = M_{J_B} + M_{J_C}$. In this equation, the helicity amplitude is

$$\begin{aligned} \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}} &= \sqrt{8E_A E_B E_C \gamma} \sum_{M_{L_A}, M_{S_A}, M_{L_B}, M_{S_B}, M_{L_C}, M_{S_C}, m} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle \\ &\times \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B} \rangle \langle L_C M_{L_C} S_C M_{S_C} | J_C M_{J_C} \rangle \\ &\times \langle 1m; 1-m | 00 \rangle \langle \chi_{S_B M_{S_B}}^{13} \chi_{S_C M_{S_C}}^{24} | \chi_{S_A M_{S_A}}^{12} \chi_{1-m}^{34} \rangle \\ &\times \langle \varphi_B^{13} \varphi_C^{24} | \varphi_A^{12} \varphi_0^{34} \rangle I_{M_{L_B}, M_{L_C}}^{M_{L_A}, m}(\mathbf{K}) \end{aligned}$$

while the spatial integral $I_{M_{L_B}, M_{L_C}}^{M_{L_A}, m}(\mathbf{K})$ is

$$\begin{aligned} I_{M_{L_B}, M_{L_C}}^{M_{L_A}, m}(\mathbf{K}) &= \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 d\mathbf{k}_4 \\ &\times \delta^3(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}_A) \delta^3(\mathbf{k}_3 + \mathbf{k}_4) \\ &\times \delta^3(\mathbf{p}_B - \mathbf{k}_1 - \mathbf{k}_3) \delta^3(\mathbf{p}_C - \mathbf{k}_2 - \mathbf{k}_4) \\ &\times \Psi_{n_B L_B M_{L_B}}^*(\mathbf{k}_1, \mathbf{k}_3) \Psi_{n_C L_C M_{L_C}}^*(\mathbf{k}_2, \mathbf{k}_4) \\ &\times \Psi_{n_A L_A M_{L_A}}(k_1, k_2) Y_{1m} \left(\frac{\mathbf{k}_3 - \mathbf{k}_4}{2} \right). \end{aligned}$$

The details of the indices, matrix elements, and other indications are given in Ref. [29]

With these formulas in hand, we proceed with our calculation. In the calculation, the simple harmonic oscillator (SHO) wave function is employed to represent the meson wave function. The meson flavor functions follow the convention in Ref. [32]: $D^0 = c\bar{u}$, $D^+ = -c\bar{d}$, $D_s^+ = -c\bar{s}$, $K^+ = -u\bar{s}$, $K^0 = s\bar{u}$, $\phi = -s\bar{s}$, $\eta = (u\bar{u} - d\bar{d})/2 - s\bar{s}/\sqrt{2}$, and $\eta' = (u\bar{u} - d\bar{d})/2 + s\bar{s}/\sqrt{2}$.

For the parameters involved in the 3P_0 model, the light nonstrange quark-pair creation strength γ and the strange

Table 1 Meson masses used in our calculation (MeV)

States	Mass	States	Mass
K^\pm	493.677	$K_1(1270)^{0(\pm)}$	1272
K^0	497.614	$K_1(1400)^{0(\pm)}$	1403
$K^{*\pm}$	891.66	$K^*(1410)^{0(\pm)}$	1410
K^{*0}	896	$K_2^*(1430)^0$	1432.4
η	547.853	$K_2^*(1430)^\pm$	1425.6
η'	957.78	$K_0^*(1430)^{0(\pm)}$	1425
ϕ	1019.455	$D_1(2430)^{0(\pm)}$	2427
D^\pm	1869.2	$D_1(2420)^0$	2422.3
D^0	1864.84	$D_1(2420)^\pm$	2423.4
$D^{*\pm}$	2010.27	$D_0(2400)^0$	2308
D^{*0}	2006.97	$D_0(2400)^\pm$	2403
D_s	1968.49	$D_2(2460)^\pm$	2460.1
D_s^*	2112.3	$D_2(2460)^0$	2461.1
$D_{s0}(2317)$	2317.8	$D(2550)^{0(\pm)}$	2539.4
$D_{s1}(2460)$	2459.6	$D(2600)^0$	2608.7
$D_{s1}(2536)$	2535.35	$D(2600)^\pm$	2621.3
$D_{s2}(2573)$	2571.9	$D(2750)^0$	2763.3
$D_{s1}^*(2700)$	2709	$D(2750)^\pm$	2769.7
		$D_{s1}^*(2860)$	2859
		$D_{s3}^*(2860)$	2860.5

quark-pair creation strength $\gamma_{s\bar{s}}$ are correlated by $\gamma_{s\bar{s}} \approx \gamma/\sqrt{3}$ [25,26] with $\gamma = 7.85$ [33]. The constituent quarks masses are taken to be $m_c = 1.43$ GeV, $m_u = m_d = 0.45$ GeV, and $m_s = 0.55$ GeV [17]. The resonance masses and the effective scale parameters β for different resonances used in our calculation are listed in Table 1 [3,17] and Table 2 [17], respectively. There is not a $2D D_s$ observed, and the masses of these resonances are unknown. In our calculation, theoretical predicted masses of the $1^- 2^3 D_1 D_s$ (3383 MeV) and the $3^- 2^3 D_3 D_s$ (3469 MeV) [34] are employed, respectively. For the $2^- D_s$ resonance, $2^3 D_2$ may mix with $2^1 D_2$, which may result in a complicated mixing. Only when the details of the mixing are clear can we give the hadronic decay widths of each 2^- resonances. In this paper, we give only the results of pure $2^3 D_2$ and $2^1 D_2$. As an approximation, the average mass (3429.5 MeV) of the two predicted $2^- D_s$ in Ref. [34] is taken as the mass input of $2^3 D_2$ and $2^1 D_2$.

The possible kinematically allowed decay modes of these four $2D D_s$ resonances are presented in Tables 3 and 4.

3 Hadronic decays of $2D D_s$ resonances

Possible hadronic decay modes and their numerical decay widths of $D_s(2^3 D_1)$ and $D_s(2^3 D_3)$ except for final states

Table 2 Different β values for the S-wave, P-wave and D-wave resonances in MeV

$n^{2S+1}L_J$	$u\bar{u}$	$u\bar{s}$	$s\bar{s}$	$c\bar{u}$	$c\bar{s}$
$1^1 S_0$	470	466	470	453	484
$2^1 S_0$	294	301	310	325	343
$1^3 S_1$	308	322	338	379	406
$2^3 S_1$	258	267	279	306	324
$1^3 P_J$	280	290	302	328	348
$2^3 P_J$	247	255	265	287	303
$1^1 P_1$	284	294	306	332	352
$2^1 P_1$	250	259	269	290	306
$1^3 D_J$	261	270	281	304	321
$2^3 D_J$	238	246	255	275	290
$1^1 D_2$	261	270	281	304	321
$2^1 D_2$	238	246	255	275	290

including $D_{s1}^*(2700)$, $D_{*s1}(2860)$, $D_{*s3}(2860)$, $D(2600)$ or $D(2750)$ are shown in Table 5. Similar results of the strong decays of $D_s(2^3 D_2)$ and $D_s(2^1 D_2)$ are shown in Table 6.

In our calculation, $D_1(2430)$ and $D_{s1}(2460)$ are assigned as the $1^+(j^P = \frac{1}{2}^+)$ D and D_s , respectively. $D_1(2420)$ and $D_{s1}(2536)$ are assigned as the excited $1^+(j^P = \frac{3}{2}^+)$ D and D_s . Through the relation between the j^P eigenstates and the $2^{S+1}L_J$ eigenstates, these two 1^+ resonances are regarded as a mixture of $1^1 P_1$ and $1^3 P_1$ resonances

$$\begin{pmatrix} |1^+, j^P = \frac{1}{2}^+ \rangle \\ |1^+, j^P = \frac{3}{2}^+ \rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |1^1 P_1 \rangle \\ |1^3 P_1 \rangle \end{pmatrix}$$

where the mixing angle $\theta = -\tan^{-1} \sqrt{2} = -54.7^\circ$ [33,34].

For a particular J , the larger the angular momentum (L) between the two final states, the smaller the corresponding M^{JL} . However, for a particular decay channel, both J and L could vary, and there is not an one-to-one relation between the decay width and the M^{JL} . From the numerical results in Table 5, the dominant decay modes of $D_s(2^3 D_1)$ are $D(2430)K$, $D^*K_1(1270)$ and $DK^*(1410)$ etc., while the dominant decay modes of $D_s(2^3 D_3)$ are $DK_1(1270)$, $D^*K^*(1410)$, and $D^*K_2(1430)$ etc. From the numerical results in Table 6, the dominant decay modes of $D_s(2^1 D_2)$ are D^*K , $D^*K_1(1270)$, and $DK^*(1410)$ etc., while the dominant decay modes of $D_s(2^3 D_2)$ are $DK^*(1410)$, $D^*K_1(1400)$, and D^*K etc. In forthcoming experiments, the $2D D_s$ resonances are expected to be observed in these dominant hadronic decay channels.

Since there are different assignments to $D_{s1}^*(2700)$, $D(2600)$ and $D(2750)$, their hadronic productions (together with the production of $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$) from $2D D_s$ resonances are studied independently in the following subsection.

Table 3 OZI-allowed hadronic decay modes of $D_s(2^3D_1)$ and $D_s(2^3D_3)$

$D_s(2^3D_1)$		$D_s(2^3D_3)$	
Mode	Channels	Mode	Channels
$0^+ + 1^-$	$D(2400)^0 K^{*+}, D(2400)^+ K^{*0}, D_s(2317)\phi$	$0^+ + 1^-$	$D(2400)^0 K^{*+}, D(2400)^+ K^{*0}, D_s(2317)\phi$
$0^- + 1^+$	$D^0 K_1(1270)^+, D^+ K_1(1270)^0, D^0 K_1(1400)^+, D^+ K_1(1400)^0$	$0^- + 1^+$	$D^0 K_1(1270)^+, D^+ K_1(1270)^0, D^0 K_1(1400)^+, D^+ K_1(1400)^0$
$1^+ + 0^-$	$D(2430)^0 K^+, D(2430)^+ K^0, D(2420)^0 K^+, D(2420)^+ K^0, D_{s1}(2460)\eta, D_{s1}(2536)\eta$	$1^- + 0^+$	$D^{*0} K_0^*(1430)^+, D^{*+} K_0^*(1430)^0$
$1^+ + 1^-$	$D(2430)^0 K^{*+}, D(2430)^+ K^{*0}, D(2420)^0 K^{*+}, D(2420)^+ K^{*0}$	$1^+ + 0^-$	$D(2430)^0 K^+, D(2430)^+ K^0, D(2420)^0 K^+, D(2420)^+ K^0, D_{s1}(2460)\eta, D_{s1}(2460)\eta', D_{s1}(2536)\eta$
$1^- + 1^+$	$D^{*0} K_1(1270)^+, D^{*+} K_1(1270)^0$	$1^+ + 1^-$	$D(2430)^0 K^{*+}, D(2430)^+ K^{*0}, D(2420)^0 K^{*+}, D(2420)^+ K^{*0}$
$2^+ + 1^-$	$D_2(2460)^0 K^{*+}, D_2(2460)^+ K^{*0}$	$1^- + 1^+$	$D^{*0} K_1(1270)^+, D^{*+} K_1(1270)^0, D^{*0} K_1(1400)^+, D^{*+} K_1(1400)^0$
$0^- + 0^-$	$D^0 K^+, D^+ K^0, D_s \eta, D(2550)^0 K^+, D(2550)^+ K^0$	$2^+ + 1^-$	$D_2(2460)^0 K^{*+}, D_2(2460)^+ K^{*0}$
$1^- + 0^-$	$D^{*0} K^+, D^{*+} K^0, D_s^* \eta, D_s^* \eta', D_s(2^3S_1)\eta, D(2^3S_1)^0 K^+, D(2^3S_1)^+ K^0, D(1^3D_1)^0 K^+, D(1^3D_1)^+ K^0$	$1^- + 2^+$	$D^{*0} K_2(1430)^+, D^{*+} K_2(1430)^0$
$0^- + 1^-$	$D^0 K^{*+}, D^+ K^{*0}, D_s \phi, D^0 K^*(1410)^+, D^+ K^*(1410)^0$	$0^- + 0^-$	$D^0 K^+, D_s \eta, D(2550)^0 K^+, D^+ K^0, D(2550)^+ K^0$
$1^- + 1^-$	$D^{*0} K^{*+}, D^{*+} K^{*0}, D_s^* \phi$	$1^- + 0^-$	$D^{*0} K^+, D^{*+} K^0, D_s^* \eta, D_s^* \eta', D_s(2^3S_1)\eta, D_s(1^3D_1)\eta, D(2^3S_1)^0 K^+, D(2^3S_1)^+ K^0, D(1^3D_1)^0 K^+, D(1^3D_1)^+ K^0$
$2^+ + 0^-$	$D_2(2460)^0 K^+, D_2(2460)^+ K^0, D_{s2}(2573)\eta$	$0^- + 1^-$	$D^0 K^{*+}, D^+ K^{*0}, D_s \phi, D^0 K^*(1410)^+, D^+ K^*(1410)^0$
$0^- + 2^+$	$D^0 K_2^*(1430)^+, D^+ K_2^*(1430)^0$	$1^- + 1^-$	$D^{*0} K^{*+}, D^{*+} K^{*0}, D_s^* \phi, D^{*0} K^*(1410)^+, D^{*+} K^*(1410)^0$
		$2^+ + 0^-$	$D_2(2460)^0 K^+, D_2(2460)^+ K^0, D_{s2}(2573)\eta$
		$0^- + 2^+$	$D^0 K_2^*(1430)^+, D^+ K_2^*(1430)^0$
		$3^- + 0^-$	$D(1^3D_3)^0 K^+, D(1^3D_3)^+ K^0, D_s(1^3D_3)\eta$

The masses of $D_s(2^3D_1)$ and $D_s(2^3D_3)$ are 3383 and 3469 MeV, respectively [34]

3.1 $D_{s1}^*(2700)$, $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$

For kinematical reason and conservation of some quantum numbers in hadronic decay, $D_{s1}^*(2700)$ can only be produced through $2D D_s \rightarrow D_{s1}^*(2700)\eta$. The numerical results are presented in Table 7. The first column in the table indicates three possible assignments of $D_{s1}^*(2700)$, where the mixture possibility is from Ref. [12] with a mixing angle $\theta = 88^\circ$. The mixing of $D_{s1}^*(2700)$ has also been studied in other references [9, 17, 35]. In the table, all the decay widths are very small though they are different in different assignments of $D_{s1}^*(2700)$.

Once $D_{s1}^*(2860)^-$ and $D_{s3}^*(2860)^-$ are assigned as the $J^P = 1^-$ and $J^P = 3^-$ members of the $1D$ family, the hadronic decay widths of $2D D_s \rightarrow D_{sJ}(2860)\eta$ can also be calculated. However, predicted masses of $2D$

3D_1 , 3D_2 , and $^1D_2 D_s$ [34] are close to the threshold of $D_{s1}^*(2860)/D_{s3}^*(2860)\eta$, their hadronic decays may be complicated. Therefore, we give only the results of the hadronic decay channels $D_s(2^3D_3) \rightarrow D_{s1}^*(2860)/D_{s3}^*(2860)\eta$. The decay widths for $D_s(2^3D_3) \rightarrow D_{s1}^*(2860)\eta$ and $D_s(2^3D_3) \rightarrow D_{s3}^*(2860)\eta$ are 0 and 0.37 MeV, respectively.

3.2 $D(2600)$ and $D(2750)$

$D(2600)$ is possibly a 2^3S_1 , or a $1^3D_1 D$, or an orthogonal partner of the mixtures of 2^3S_1 and 1^3D_1 with $J^P = 1^-$ [9, 17, 28],

$$|(SD)_{1L}\rangle = \cos \theta |2^3S_1\rangle - \sin \theta |1^3D_1\rangle,$$

$$|(SD)_{1R}\rangle = \sin \theta |2^3S_1\rangle + \cos \theta |1^3D_1\rangle.$$

Table 4 OZI-allowed hadronic decay modes of $D_s(2^1D_2)$ and $D_s(2^3D_2)$

$D_s(2^1D_2)$		$D_s(2^3D_2)$	
Mode	Channels	Mode	Channels
$0^+ + 1^-$	$D(2400)^0 K^{*+}, D(2400)^+ K^{*0}, D_s(2317)\phi$	$0^+ + 1^-$	$D(2400)^0 K^{*+}, D(2400)^+ K^{*0}, D_s(2317)\phi$
$0^- + 1^+$	$D^0 K_1(1270)^+, D^+ K_1(1270)^0, D^0 K_1(1400)^+, D^+ K_1(1400)^0$	$0^- + 1^+$	$D^0 K_1(1270)^+, D^+ K_1(1270)^0, D^0 K_1(1400)^+, D^+ K_1(1400)^0$
$1^+ + 0^-$	$D(2430)^0 K^+, D(2430)^+ K^0, D(2420)^0 K^+, D(2420)^+ K^0, D_{s1}(2460)\eta, D_{s1}(2536)\eta$	$1^+ + 0^-$	$D(2430)^0 K^+, D(2430)^+ K^0, D(2420)^0 K^+, D(2420)^+ K^0, D_{s1}(2460)\eta, D_{s1}(2460)\eta', D_{s1}(2536)\eta$
$1^+ + 1^-$	$D(2430)^0 K^{*+}, D(2430)^+ K^{*0}, D(2420)^0 K^{*+}, D(2420)^+ K^{*0}$	$1^+ + 1^-$	$D(2430)^0 K^{*+}, D(2430)^+ K^{*0}, D(2420)^0 K^{*+}, D(2420)^+ K^{*0}$
$1^- + 1^+$	$D^{*0} K_1(1270)^+, D^{*+} K_1(1270)^0, D^{*0} K_1(1400)^+, D^{*+} K_1(1400)^0$	$1^- + 1^+$	$D^{*0} K_1(1270)^+, D^{*+} K_1(1270)^0, D^{*0} K_1(1400)^+, D^{*+} K_1(1400)^0$
$2^+ + 1^-$	$D_2(2460)^0 K^{*+}, D_2(2460)^+ K^{*0}$	$2^+ + 1^-$	$D_2(2460)^0 K^{*+}, D_2(2460)^+ K^{*0}$
$0^+ + 0^-$	$D(2400)^0 K^+, D_s(2317)\eta, D(2400)^+ K^0, D_s(2317)\eta'$	$0^+ + 0^-$	$D(2400)^0 K^+, D_s(2317)\eta, D(2400)^+ K^0, D_s(2317)\eta'$
$0^- + 0^+$	$D^0 K_0^*(1430)^+, D^+ K_0^*(1430)^0$	$0^- + 0^+$	$D^0 K_0^*(1430)^+, D^+ K_0^*(1430)^0$
$1^- + 0^-$	$D^{*0} K^+, D^{*+} K^0, D_s^* \eta, D_s^* \eta', D_s(2^3S_1)\eta, D(2^3S_1)^0 K^+, D(2^3S_1)^+ K^0, D(1^3D_1)^0 K^+, D(1^3D_1)^+ K^0$	$1^- + 0^-$	$D^{*0} K^+, D^{*+} K^0, D_s^* \eta, D_s^* \eta', D_s(2^3S_1)\eta, D_s(1^3D_1)\eta, D(2^3S_1)^0 K^+, D(2^3S_1)^+ K^0, D(1^3D_1)^0 K^+, D(1^3D_1)^+ K^0$
$0^- + 1^-$	$D^0 K^{*+}, D^+ K^{*0}, D_s \phi, D^0 K^*(1410)^+, D^+ K^*(1410)^0$	$0^- + 1^-$	$D^0 K^{*+}, D^+ K^{*0}, D_s \phi, D^0 K^*(1410)^+, D^+ K^*(1410)^0$
$1^- + 1^-$	$D^{*0} K^{*+}, D^{*+} K^{*0}, D_s^* \phi$	$1^- + 1^-$	$D^{*0} K^{*+}, D^{*+} K^{*0}, D_s^* \phi$
$2^+ + 0^-$	$D_2(2460)^0 K^+, D_2(2460)^+ K^0, D_{s2}(2573)\eta$	$2^+ + 0^-$	$D_2(2460)^0 K^+, D_2(2460)^+ K^0, D_{s2}(2573)\eta$
$0^- + 2^+$	$D^0 K_2^*(1430)^+, D^+ K_2^*(1430)^0$	$0^- + 2^+$	$D^0 K_2^*(1430)^+, D^+ K_2^*(1430)^0$
$3^- + 0^-$	$D(1^3D_3)^0 K^+, D(1^3D_3)^+ K^0, D_s(1^3D_3)\eta$	$3^- + 0^-$	$D(1^3D_3)^0 K^+, D(1^3D_3)^+ K^0, D_s(1^3D_3)\eta$

The mass of $D_s(2^1D_2)$ and $D_s(2^3D_2)$ is 3429.5 MeV [34]

The hadronic decay widths of $2D D_s \rightarrow D(2600)K$ in different assignments of $D(2600)$ are given in Table 8. Obviously, the decay widths have large difference in different assignments of $D(2600)$, therefore the final decay widths depend heavily on the mixing angle. In the table, results corresponding to two different mixing angles are given, where we have the mixing angles $0.9 \leq \theta \leq 1.5$ [17] and $0.36 \leq \theta \leq 0.40$ [28].

$D(2750)$ may be a 1^3D_1 or a 1^3D_3 D , numerical results of relevant channels are given in Table 9, where the “-” indicates an impossible channel. As indicated in Ref. [10], there are possibly two D resonances close to 2750 MeV, which requires careful study of this energy region. Therefore, other possible assignments of $D(2750)$ are not studied here.

4 Conclusions and discussions

In this work, the hadronic decay properties of the highly excited $2D D_s$ resonances are studied in 3P_0 model. The hadronic decay widths of all possible OZI-allowed channels of the highly excited $2D D_s$ resonances have been calculated. The dominant decay modes of $D_s(2^3D_1)$ are $D(2430)K, D^*K_1(1270)$, and $DK^*(1410)$ etc., while the dominant decay modes of $D_s(2^3D_3)$ are $DK_1(1270), D^*K^*(1410)$, and $D^*K_2(1430)$ etc. The dominant decay modes of $D_s(2^1D_2)$ are $D^*K, D^*K_1(1270)$, and $DK^*(1410)$ etc., while the dominant decay modes of $D_s(2^3D_2)$ are $DK^*(1410), D^*K_1(1400)$, and D^*K etc. The $2D D_s$ resonances are suggested to be observed in these dominant hadronic decay channels in forthcoming experiments.

Table 5 Hadronic decay widths of $D_s(2^3D_1)$ and $D_s(2^3D_3)$ in MeV

$D_s(2^3D_1)$				$D_s(2^3D_3)$			
Channels	Width	Channels	Width	Channels	Width	Channels	Width
$D(2400)^0 K^{*+}$	13.53	$D^+ K^0$	0.24	$D(2400)^0 K^{*+}$	6.10	$D^{*+} K_2(1430)^0$	53.04
$D(2400)^+ K^{*0}$	7.28	$D_s \eta$	0.05	$D(2400)^+ K^{*0}$	7.48	$D^0 K^+$	0.00
$D_s(2317) \phi$	2.83	$D_s \eta'$	1.03	$D_s(2317) \phi$	2.19	$D^+ K^0$	0.00
$D^0 K_1(1270)^+$	14.88	$D(2550)^0 K^+$	12.34	$D^0 K_1(1270)^+$	81.61	$D_s \eta$	0.00
$D^+ K_1(1270)^0$	14.58	$D(2550)^+ K^0$	12.55	$D^+ K_1(1270)^0$	75.06	$D_s \eta'$	0.16
$D^0 K_1(1400)^+$	15.29	$D^{*0} K^+$	3.91	$D^0 K_1(1400)^+$	33.16	$D(2550)^0 K^+$	0.06
$D^+ K_1(1400)^0$	13.32	$D^{*+} K^0$	4.08	$D^+ K_1(1400)^0$	36.30	$D(2550)^+ K^0$	0.06
$D(2430)^0 K^+$	20.88	$D_s^* \eta$	1.60	$D^{*0} K_0^*(1430)^+$	1.03	$D^{*0} K^+$	0.00
$D(2430)^+ K^0$	21.18	$D_s^* \eta'$	0.48	$D^{*+} K_0^*(1430)^0$	0.84	$D^{*+} K^0$	0.00
$D(2420)^0 K^+$	13.91	$D^0 K^{*+}$	6.31	$D(2430)^0 K^+$	14.00	$D_s^* \eta$	0.00
$D(2420)^+ K^0$	14.04	$D^+ K^{*0}$	6.55	$D(2430)^+ K^0$	13.98	$D_s^* \eta'$	0.95
$D_{s1}(2460) \eta$	4.78	$D_s \phi$	2.20	$D(2420)^0 K^+$	1.93	$D^0 K^{*+}$	0.01
$D_{s1}(2536) \eta$	2.33	$D^0 K^*(1410)^+$	20.22	$D(2420)^+ K^0$	1.97	$D^+ K^{*0}$	0.01
$D(2430)^0 K^{*+}$	11.41	$D^+ K^*(1410)^0$	20.19	$D_{s1}(2460) \eta$	1.34	$D_s \phi$	0.00
$D(2430)^+ K^{*0}$	11.26	$D^{*0} K^{*+}$	9.82	$D_{s1}(2460) \eta'$	0.01	$D^0 K^*(1410)^+$	38.98
$D(2420)^0 K^{*+}$	11.33	$D^{*+} K^{*0}$	9.67	$D_{s1}(2536) \eta$	0.59	$D^+ K^*(1410)^0$	37.16
$D(2420)^+ K^{*0}$	10.01	$D_s^* \phi$	2.76	$D(2430)^0 K^{*+}$	15.27	$D^{*0} K^*(1410)^+$	68.97
$D^{*0} K_1(1270)^+$	22.02	$D_2(2460)^0 K^+$	0.05	$D(2430)^+ K^{*0}$	14.64	$D^{*+} K^*(1410)^0$	66.56
$D^{*+} K_1(1270)^0$	21.12	$D_2(2460)^+ K^0$	0.04	$D(2420)^0 K^{*+}$	19.17	$D^{*0} K^{*+}$	2.91
$D_2(2460)^0 K^{*+}$	10.20	$D_{s2}(2573) \eta$	0.21	$D(2420)^+ K^{*0}$	18.90	$D^{*+} K^{*0}$	2.93
$D_2(2460)^+ K^{*0}$	10.26	$D^0 K_2^*(1430)^+$	0.67	$D^{*0} K_1(1270)^+$	36.47	$D_s^* \phi$	0.37
$D^0 K^+$	0.23	$D^+ K_2^*(1430)^0$	0.68	$D^{*+} K_1(1270)^0$	36.51	$D_2(2460)^0 K^+$	0.14
				$D^{*0} K_1(1400)^+$	4.31	$D_2(2460)^+ K^0$	0.14
				$D^{*+} K_1(1400)^0$	3.87	$D_{s2}(2573) \eta$	0.01
				$D_2(2460)^0 K^{*+}$	25.44	$D^0 K_2^*(1430)^+$	0.61
				$D_2(2460)^+ K^{*0}$	25.54	$D^+ K_2^*(1430)^0$	0.62
				$D^{*0} K_2(1430)^+$	61.94		

$D_{s1}^*(2700)$, $D_{s1}^*(2860)$, and $D_{s3}^*(2860)$ can be produced from the hadronic decays of the highly excited $2D D_s$ resonances. Possible hadronic decay channels are $2D D_s \rightarrow D_{s1}^*(2700) \eta$ and $2D D_s \rightarrow D_{s1}^*(2860)/D_{s3}^*(2860) \eta$, respectively. In every possible assignments of $D_{s1}^*(2700)$, all the hadronic decay widths are very small. The hadronic decay widths of $D_s(2^3D_3) \rightarrow D_{s1}^*(2860) \eta$ and $D_s(2^3D_3) \rightarrow D_{s3}^*(2860) \eta$ are also very small. It is not appropriate to classify these resonances according to their hadronic production from the $2D D_s$. The thresholds of $D_{s1}^*(2860)/D_{s3}^*(2860) \eta$ are close to the theoretical predicted masses of $2D^3D_1$, 3D_2 , and 1D_2 , and the relevant hadronic decays may be complicated.

The hadronic decay widths of $2D D_s \rightarrow D(2600)K$ and $2D D_s \rightarrow D(2750)K$ in different assignments of $D(2600)$ and $D(2750)$ have also been calculated. The hadronic decay widths may be large, and the numerical results are different in different assignments of $D(2600)$ and $D(2750)$. If the $2D D_s$ resonances are observed in forthcoming experiments, the measure of these hadronic decay widths will help us to understand $D(2600)$ and $D(2750)$.

In our paper, the uncertainties of the input parameters and the model have not been studied, nor the details of the possible mixing of some resonances. More theoretical study of these highly excited resonances are required. Of course, the most important thing is to expect more

Table 6 Hadronic decay widths of $D_s(2^1D_2)$ and $D_s(2^3D_2)$ in MeV

$D_s(2^1D_2)$				$D_s(2^3D_2)$			
Channels	Width	Channels	Width	Channels	Width	Channels	Width
$D(2400)^0 K^{*+}$	0.12	$D(2400)^+ K^0$	6.76	$D(2400)^0 K^{*+}$	5.55	$D^+ K^*(1430)^0$	0.02
$D(2400)^+ K^{*0}$	0.23	$D_s(2317)\eta$	0.87	$D(2400)^+ K^{*0}$	7.68	$D(2400)^0 K^+$	3.58
$D_s(2317)\phi$	0.04	$D_s(2317)\eta'$	0.61	$D_s(2317)\phi$	1.80	$D(2400)^+ K^0$	1.84
$D^0 K_1(1270)^+$	0.25	$D^0 K_0^*(1430)^+$	10.87	$D^0 K_1(1270)^+$	30.32	$D_s(2317)\eta$	0.54
$D^+ K_1(1270)^0$	0.20	$D^+ K_0^*(1430)^0$	10.60	$D^+ K_1(1270)^0$	30.43	$D_s(2317)\eta'$	0.30
$D^0 K_1(1400)^+$	0.16	$D^{*0} K^+$	50.30	$D^0 K_1(1400)^+$	17.50	$D^{*0} K^+$	35.95
$D^+ K_1(1400)^0$	0.17	$D^{*+} K^0$	49.46	$D^+ K_1(1400)^0$	17.23	$D^{*+} K^0$	35.53
$D(2430)^0 K^+$	1.50	$D_s^* \eta$	21.32	$D(2430)^0 K^+$	1.21	$D_s^* \eta$	9.53
$D(2430)^+ K^0$	1.50	$D_s^* \eta'$	8.59	$D(2430)^+ K^0$	1.12	$D_s^* \eta'$	6.09
$D(2420)^0 K^+$	0.80	$D^0 K^{*+}$	23.57	$D(2420)^0 K^+$	14.26	$D^0 K^{*+}$	20.41
$D(2420)^+ K^0$	0.78	$D^+ K^{*0}$	22.58	$D(2420)^+ K^0$	14.12	$D^+ K^{*0}$	20.03
$D_{s1}(2460)\eta$	0.16	$D^0 K^*(1410)^+$	45.32	$D_{s1}(2460)\eta$	0.00	$D_s \phi$	2.32
$D_{s1}(2460)\eta'$	0.00	$D^+ K^*(1410)^0$	44.17	$D_{s1}(2460)\eta'$	1.40	$D^0 K^*(1410)^+$	47.75
$D_{s1}(2536)\eta$	0.03	$D_s \phi$	1.60	$D_{s1}(2536)\eta$	0.80	$D^+ K^*(1410)^0$	47.16
$D(2430)^0 K^{*+}$	21.33	$D^{*0} K^*(1410)^+$	5.83	$D(2430)^0 K^{*+}$	11.58	$D^{*0} K^*(1410)^+$	2.92
$D(2430)^+ K^{*0}$	20.13	$D^{*+} K^*(1410)^0$	2.93	$D(2430)^+ K^{*0}$	10.73	$D^{*+} K^*(1410)^0$	1.46
$D(2420)^0 K^{*+}$	22.74	$D^{*0} K^{*+}$	25.46	$D(2420)^0 K^{*+}$	19.74	$D^{*0} K^{*+}$	15.67
$D(2420)^+ K^{*0}$	21.62	$D^{*+} K^{*0}$	25.12	$D(2420)^+ K^{*0}$	18.75	$D^{*+} K^{*0}$	14.91
$D^{*0} K_1(1270)^+$	49.69	$D_s^* \phi$	3.92	$D^{*0} K_1(1270)^+$	34.58	$D_s^* \phi$	2.67
$D^{*+} K_1(1270)^0$	49.74	$D_2(2460)^0 K^+$	19.98	$D^{*+} K_1(1270)^0$	34.13	$D_2(2460)^0 K^+$	17.32
$D^{*0} K_1(1400)^+$	29.63	$D_2(2460)^+ K^0$	19.98	$D^{*0} K_1(1400)^+$	43.44	$D_2(2460)^+ K^0$	17.45
$D^{*+} K_1(1400)^0$	30.20	$D_{s2}(2573)\eta$	2.68	$D^{*+} K_1(1400)^0$	44.63	$D_{s2}(2573)\eta$	3.66
$D_2(2460)^0 K^{*+}$	14.08	$D^0 K_2^*(1430)^+$	23.23	$D_2(2460)^0 K^{*+}$	6.93	$D^0 K_2^*(1430)^+$	15.22
$D_2(2460)^+ K^{*0}$	13.84	$D^+ K_2^*(1430)^0$	19.93	$D_2(2460)^+ K^{*0}$	7.04	$D^+ K_2^*(1430)^0$	11.49
$D(2400)^0 K^+$	6.39			$D^0 K^*(1430)^+$	0.03		

Table 7 Hadronic decay widths (in MeV) of $2D D_s \rightarrow D_{s1}^*(2700)\eta$ in different assignments of $D_{s1}^*(2700)$, where the mixture is from Ref. [12]

State	$2^3D_1 \rightarrow D_{s1}^*(2700)\eta$	$2^3D_3 \rightarrow D_{s1}^*(2700)\eta$	$2^1D_2 \rightarrow D_{s1}^*(2700)\eta$	$2^3D_2 \rightarrow D_{s1}^*(2700)\eta$
$D_{s1}^*(2700)(2^3S_1)$	2.48	1.43	3.98	6.26
$D_{s1}^*(2700)(1^3D_1)$	0.78	0.03	8.39	3.19
$D_{s1}^*(2700)(\text{mixture})$	0.87	0.02	8.08	3.50

Table 8 Hadronic decay widths (in MeV) of $2D D_s \rightarrow D(2600)K$ in different assignments of $D(2600)$

Assignment\mode	$2^3D_1 \rightarrow D(2600)^+ K^0$	$2^3D_3 \rightarrow D(2600)^+ K^0$	$2^1D_2 \rightarrow D(2600)^+ K^0$	$2^3D_2 \rightarrow D(2600)^+ K^0$
$D(2600)^+(2^3S_1)$	9.85	22.27	28.80	26.35
$D(2600)^+(1^3D_1)$	36.20	0.87	138.02	52.26
$D(2600)^+(\text{mixture [17]})$	37.99–45.33	0.00–4.86	90.81–136.75	40.85–51.93
$D(2600)^+(\text{mixture [28]})$	25.53–27.14	15.87–16.63	39.22–41.67	28.68–29.25
Assignment\mode	$2^3D_1 \rightarrow D(2600)^0 K^+$	$2^3D_3 \rightarrow D(2600)^0 K^+$	$2^3D_3 \rightarrow D(2600)^0 K^+$	$2^3D_3 \rightarrow D(2600)^0 K^+$
$D(2600)^0(2^3S_1)$	8.89	22.96	29.10	25.76
$D(2600)^0(1^3D_1)$	42.35	0.82	145.99	56.35
$D(2600)^0(\text{mixture [17]})$	44.92–51.24	0.00–5.15	99.58–145.22	38.37–53.97
$D(2600)^0(\text{mixture [28]})$	26.03–27.88	16.50–17.27	43.07–45.91	25.45–25.92

Table 9 Hadronic decay widths (in MeV) of $2D D_s \rightarrow D(2750)K$ in different assignments of $D(2750)$

Assignment\mode	$2^3D_1 \rightarrow D(2750)^+K^0$	$2^3D_3 \rightarrow D(2750)^+K^0$	$2^1D_2 \rightarrow D(2750)^+K^0$	$2^3D_2 \rightarrow D(2750)^+K^0$
$D(2750)^+(1^3D_3)$	–	6.02	45.59	52.32
$D(2750)^+(1^3D_1)$	1.91	0.44	48.27	20.64
Assignment\mode	$2^3D_1 \rightarrow D(2750)^0K^+$	$2^3D_3 \rightarrow D(2750)^0K^+$	$2^3D_3 \rightarrow D(2750)^0K^+$	$2^3D_3 \rightarrow D(2750)^0K^+$
$D(2750)^0(1^3D_3)$	–	5.81	49.98	56.97
$D(2750)^0(1^3D_1)$	2.58	0.49	53.22	22.28

highly excited D_s resonances observed in forthcoming experiments.

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