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Kerr geometry in $f(T)$ gravity

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Abstract Null tetrads are shown to be a valuable tool in teleparallel theories of modified gravity. We use them to prove that Kerr geometry remains a solution for a wide family of $f(T)$ theories of gravity.

1 Introduction

Many models of modified gravity have been proposed in order to tackle the shortcomings of General Relativity (GR). Deformations of the Einstein theory at small or large scales, i.e. ultraviolet or infrared gravity, could provide a better handling of singularities and the cosmic acceleration. In particular, a proper deformation of GR in the ultraviolet regime could play the role of describing the transition between GR and quantum gravity. This frontier could be better understood by resorting to a *teleparallel* formulation of gravity. As a matter of fact, although with a different purpose in mind, it was Einstein himself who proposed in the 30's the reformulation of GR in a teleparallel framework, by taking the field of orthonormal frames or *tetrads* as the dynamical variable instead of the metric tensor [1–4]. Certainly, teleparallelism is part of a bigger picture. Gravity can be described by means of a connection having both curvature (like GR, through the Levi-Civita torsionless connection) and torsion (like teleparallelism, through the Weitzenböck curvatureless connection) leading to Einstein–Cartan theory [5–7].

In this article we will focus on the so-called $f(T)$ gravity, a theory of modified gravity based on a spacetime possessing absolute parallelism [8–20]. The first results of this alternative theory of gravity were very encouraging. It has been shown that the teleparallelism à la Born–Infeld cures the primordial singularity of flat Friedman–Lemaître–Robertson–Walker (FLRW) universes, providing a natural inflationary early stage without invoking a new field [8,9]. An extension of $f(T)$ gravity [21] changed the conical singularity of a 3D cosmic string into a geodesically complete smooth curved spacetime [22]. Like other theories of modified gravity, $f(T)$ theories display additional degrees of freedom; in this case they come from the loss of local Lorentz invariance and their physical nature is not well understood yet [17,18,23–25].

A remarkable feature of $f(T)$ theories is that the dynamics of tetrads is described by second order equations, which is not usual in the context of modified gravity. This property is guaranteed by the teleparallel Lagrangian, which is a function of the square of the first derivatives of the tetrad field (differing from GR, whose Lagrangian contains second derivatives of the metric). The teleparallel Lagrangian is built in a Weitzenböck spacetime, i.e., a spacetime endowed with a curvatureless connection proportional to first derivatives of the tetrad. The central piece of a teleparallel Lagrangian is the Weitzenböck torsion. The simplest teleparallel theory is the teleparallel equivalent of GR (TEGR), which is just Einstein's gravity in the language of tetrads. $f(T)$ theories are defined by deformations of the TEGR Lagrangian.

A main point to consider in modified gravity is the possibility of smoothing black hole singularities. The search for solutions of spacetimes displaying spherical or axial symmetry is not trivial in $f(T)$ gravity. In fact, usually one uses the symmetry for choosing coordinates such that the metric looks simple. Even so, there are many tetrads for a given metric. Since $f(T)$ theories are not invariant under local Lorentz transformations, the knowledge of the metric symmetry gives no idea of the ansatz for the tetrad field. In this article we will study vacuum axially symmetric rotating solutions of $f(T)$ gravity. We are going to prove that Kerr geometry [26] remains a solution for $f(T)$ gravity. We will employ null tetrads as a useful tool for straightforwardly getting the result. We remark that different aspects of rotating black holes have been discussed in the context of TEGR [27–31]. Among the

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papers about Kerr spacetime in modified gravity, we can mention those referred to $f(R)$ gravity [32–35]. Other rotating geometries, like the Gödel universe [36], have also been studied in TEGR [37,38], as well as in $f(R)$ theories of gravity in both metric and Palatini approaches [39–41]. Recently, the rotating cosmology was also explored in $f(T)$ gravity [42].

The paper is organized as follows. In Sect. 2 we introduce teleparallel gravity. In Sect. 3 we explain the equivalence between TEGR and GR, and the loss of local Lorentz invariance in $f(T)$ theories. We also analyze the survival of some TEGR solutions in $f(T)$ theories. In Sect. 4 we show that teleparallelism can be formulated in terms of null tetrads; we exploit this fact to search for surviving solutions. In Sect. 5 we show that Kerr geometry is one of this kind of solutions. In Sect. 6 we present the conclusions.

2 Teleparallel gravity

Teleparallelism is a name for theories of gravity where the dynamical variable is not the metric but the tetrad or *vierbein*. The tetrad field $\{\mathbf{e}_a(\mathbf{x})\}$ is a set of four orthonormal vectors at each point p of the manifold M that constitutes a basis of the tangent space T_pM. The dual co-frame $\{\mathbf{e}^a(\mathbf{x})\}$ is a basis of the co-tangent space T_p^*M. They can be decomposed in a coordinate basis as

$$\mathbf{e}^a = e^a_\mu \, dx^\mu \quad \text{and} \quad \mathbf{e}_a = e^\mu_a \, \partial_\mu, \tag{1}$$

where e^a_μ and e^μ_a are the respective components which fulfill

$$e^a_\mu \, e^\mu_b = \delta^a_b \quad \text{and} \quad e^a_\mu \, e^\nu_a = \delta^\nu_\mu. \tag{2}$$

Greek indices, $\mu, \nu, \ldots = 0, 1, 2, 3$, indicate spacetime coordinates. Latin indices, $a, b, \ldots = 0, 1, 2, 3$, are associated with the tangent space; we will call them Lorentzian indices.

The orthonormality condition is the link between the tetrad and the metric

$$\eta_{ab} = g_{\mu\nu} \, e^\mu_a \, e^\nu_b, \tag{3}$$

where $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$. Equation (2) is used for inverting this relation and getting the metric from the tetrad,

$$g_{\mu\nu} = \eta_{ab} \, e^a_\mu \, e^b_\nu \quad \text{or} \quad g^{\mu\nu} = \eta^{ab} \, e^\mu_a \, e^\nu_b, \tag{4}$$

then it is

$$\sqrt{-g} = \det[e^a_\mu] = e. \tag{5}$$

Noticeably, the metric–tetrad relation is invariant under local Lorentz transformations of the tetrad. In other words, there are many tetrad fields for the same metric field. This also means that a dynamical theory for the tetrad will determine the dynamic of the metric.

Teleparallelism is a dynamical theory for the tetrad whose Lagrangian is built from the torsion tensor $T^\mu{}_{\nu\rho}$ associated with the Weitzenböck connection [43]

$$\overset{\mathrm{W}}{\Gamma}{}^{\mu}_{\rho\nu} = e^\mu_a \, \partial_\nu e^a_\rho = -e^a_\rho \, \partial_\nu e^\mu_a, \tag{6}$$

$$T^\mu{}_{\nu\rho} = e^\mu_a \, (\partial_\nu e^a_\rho - \partial_\rho e^a_\nu). \tag{7}$$

Weitzenböck connection $\overset{\mathrm{W}}{\Gamma}{}^{\mu}_{\rho\nu}$ is curvatureless. Therefore, teleparallelism encodes gravity in the torsion instead of the Riemann tensor. The Weitzenböck connection has the nice property that parallel-transported vectors keep constant their projections on the vectors $\mathbf{e}_a$; in fact, we have $\overset{\mathrm{W}}{\nabla}_\nu V^\mu = e^\mu_a \, \partial_\nu (e^a_\lambda \, V^\lambda)$. In particular, $\overset{\mathrm{W}}{\nabla}_\nu e^\mu_a \equiv 0$, which means that the Weitzenböck connection is *metric*. Since the curvature vanishes and the tetrad is parallel-transported, one concludes that the tetrad field is a global frame.

In the teleparallel equivalent of GR, the tetrad is governed by the action [44–46]

$$S_{\mathrm{TEGR}}[\mathbf{e}_a] = \frac{1}{2\kappa} \int d^4x \; e \; S_\rho{}^{\mu\nu} \, T^\rho{}_{\mu\nu}, \tag{8}$$

where $\kappa = 8\pi G$ and $S_\rho{}^{\mu\nu}$ is defined as

$$2\, S_\rho{}^{\mu\nu} \equiv \underbrace{\frac{1}{2}\,(T_\rho{}^{\mu\nu} - T^{\mu\nu}{}_\rho + T^{\nu\mu}{}_\rho)}_{\text{contorsion } K^{\mu\nu}{}_\rho} + T_\lambda{}^{\lambda\mu} \, \delta^\nu_\rho - T_\lambda{}^{\lambda\nu} \, \delta^\mu_\rho. \tag{9}$$

The *contorsion* $K^{\mu\nu}{}_\rho$ equals the difference between Weitzenböck and Levi-Civita connections.

Analogously to $f(R)$ gravity [47,48], where the GR Lagrangian is changed to an arbitrary function f of the scalar curvature R, $f(T)$ theories constitute a teleparallel version of modified gravity obtained by deforming the TEGR Lagrangian in terms of an arbitrary function of the *Weitzenböck invariant* $T \equiv S_\rho{}^{\mu\nu} \, T^\rho{}_{\mu\nu}$ [8,9]. Its action reads

$$S[\mathbf{e}_a] = \frac{1}{2\kappa} \int d^4x \; e \; f(T). \tag{10}$$

For minimally coupled matter, the dynamical equations of $f(T)$ gravity are

$$4\, e^{-1} \, \partial_\mu (e \;\, e^\lambda_a \; S_\lambda{}^{\mu\nu} \; f'(T)) + 4\, e^\lambda_a \; T^\rho{}_{\mu\lambda} \; S_\rho{}^{\mu\nu} \; f'(T) - e^\nu_a \; f(T) = -2\,\kappa \; e^\lambda_a \; \mathcal{T}_\lambda{}^\nu, \tag{11}$$

where $\mathcal{T}_\lambda{}^\nu$ is the energy-momentum tensor. As seen, the dynamical equations are second order, which is a distinctive feature regarding other theories of modified gravity. TEGR dynamics corresponds to the particular case $f(T) = T$, $f'(T) = 1$.

3 Comparing TEGR and $f(T)$ gravity

The equivalence between GR and TEGR emanates from the following property: Einstein–Hilbert Lagrangian differs from TEGR Lagrangian in a four-divergence. In fact, by computing the Levi-Civita scalar curvature R for the metric (4) one gets the result

$$-\,e\ R[\mathbf{e}_a]\ =\ e\ T -\ 2\,\partial_\rho(e\,T^{\mu}{}_{\mu}{}^{\rho}). \qquad (12)$$

Therefore TEGR and Einstein–Hilbert actions are equivalent. This also means that GR and TEGR harbor the same number of degrees of freedom. Even though the tetrad field contains 16 components (six more than the metric field), TEGR is invariant under local Lorentz transformations of the tetrad. This gauge freedom cancels out the excess of degrees of freedom. At the level of the TEGR Lagrangian, a local Lorentz transformation is a local change to another orthonormal basis,

$$\mathbf{e}_{a'} = \Lambda^{a}_{a'}(\mathbf{x})\ \mathbf{e}_a, \quad \mathbf{e}^{a'} = \Lambda^{a'}_{a}(\mathbf{x})\ \mathbf{e}^a, \qquad (13)$$

that adds a four-divergence to the TEGR Lagrangian $e\,T$. This behavior is evident in Eq. (12) because $e\,R$ is invariant under local Lorentz transformations.

Instead $f(T)$ gravity, like other theories of modified gravity, possesses extra degrees of freedom. In fact, except for the case $f(T) = T$ (i.e., TEGR) the dynamical equations (11) are sensitive to local Lorentz transformations of the tetrad. This implies that the dynamical equations not only contain information about the evolution of the metric but also about some extra degrees of freedom exclusively associated with the tetrad that are not present in the undeformed theory [17,18,23–25]. At the level of the $f(T)$ Lagrangian, under a local Lorentz transformation the Lagrangian changes as

$$e\ \ f(T) \longrightarrow e\ \ f(T + \text{four-divergence}). \qquad (14)$$

In this case the four-divergence term remains encapsulated inside the function f spoiling the local Lorentz invariance. The loss of the local Lorentz invariance implies the existence of a preferential global reference frame defined by the autoparallel curves of the manifold that consistently solve the dynamical equations. That is, Eq. (11) not only determine the metric but they also choose some other characteristics of the tetrad field, so endowing the spacetime with an absolute parallelization. The tetrads connected by local Lorentz transformations lead to the same metric but they are different with respect to the proper parallel framework. Due to this essential feature of $f(T)$ theories, when looking for solutions of a given symmetry it is quite complicated to make an ansatz for the tetrad field. Actually, the symmetry helps us to choose suitable coordinates to write the metric in a simple way. But this does not say much about the ansatz for the tetrad. Indeed, a very common mistake is to force e^a_μ to be diagonal in the chosen coordinates. Frequently this choice does not work as an ansatz for solving Eq. (11); it is not consistent. For instance, in Ref. [15] it was shown that a diagonal choice for FLRW universes only works in the flat case; open and closed universes require non-trivial tetrads for solving the $f(T)$ dynamical equations.

In Ref. [16] it was proved that a naive diagonal tetrad does not properly parallelize a static spherically symmetric geometry in $f(T)$ gravity. Therefore, we emphasize that the symmetries of the geometry are not enough to visualize the absolute parallelization of the manifold, being those quite futile in order to obtain the right answer. Certainly, in the context of $f(T)$ theories, the proper frame which parallelizes the spacetime for a given symmetry of the geometry must be independent of the function f [49].

This article is aimed to find the parallelization for axially symmetric solutions in $f(T)$ theories. In particular, we want to know whether Kerr geometry survives or not in $f(T)$ gravity. To answer the question we should find the correct ansatz to solve the Eq. (11). This search is greatly facilitated by invoking the following argument concerning the survival of certain TEGR solutions [16]: if a vacuum solution of $f(T)$ gravity has $T = 0$, then it will be a solution of TEGR as well (a cosmological constant might be necessary). In fact, the replacement of $T = 0$ in Eq. (11) leads to

$$4\ e^{-1}\partial_\mu(e\ e^{\lambda}_a\ S_\lambda{}^{\mu\nu}) + 4\ e^{\lambda}_a\ T^{\rho}{}_{\mu\lambda}\ S_\rho{}^{\mu\nu} - e^{\nu}_a\ \frac{f(0)}{f'(0)} = 0, \quad (15)$$

which is a TEGR vacuum equation with cosmological constant $2\Lambda\ =\ f(0)/f'(0)$. We can avoid the cosmological constant term by restricting the family of functions f to those having $f(0)\ =\ 0$, $f'(0)\ \neq\ 0$. In other words, we can exploit the freedom to do local Lorentz transformations in TEGR to look for a tetrad having $T\ =\ 0$; if we success, then we will state that such solution survives in $f(T)$ gravity. Notice that TEGR vacuum solutions does not compels T to vanish; R must vanish. Thus Eq. (12) says that T is a four-divergence. So, the former argument is based on the sensitivity of T to local Lorentz transformations. The above argument means that TEGR vacuum solutions having $T = 0$ (or T = constant) cannot be deformed by $f(T)$ gravity. We are going to show that this is the case for Kerr geometry, what means that $f(T)$ gravity is unable to smooth the singularity of a black hole [16].

4 Null tetrad approach

The search for a tetrad having $T\ =\ 0$, if it exists, is easier by working with a null tetrad. Any orthonormal tetrad $\{\mathbf{e}^a\}$

defines a null tetrad $\{\mathbf{n}^a\} = \{\mathbf{l}, \mathbf{n}, \mathbf{m}, \overline{\mathbf{m}}\}$

$$\mathbf{l} = \frac{(\mathbf{e}^0 + \mathbf{e}^1)}{\sqrt{2}}, \quad \mathbf{n} = \frac{(\mathbf{e}^0 - \mathbf{e}^1)}{\sqrt{2}},$$
$$\mathbf{m} = \frac{(\mathbf{e}^2 + i\,\mathbf{e}^3)}{\sqrt{2}}, \quad \overline{\mathbf{m}} = \frac{(\mathbf{e}^2 - i\,\mathbf{e}^3)}{\sqrt{2}}. \tag{16}$$

This tetrad form a null basis

$$\mathbf{l}\cdot\mathbf{l} = 0, \quad \mathbf{n}\cdot\mathbf{n} = 0, \quad \mathbf{m}\cdot\mathbf{m} = 0, \quad \overline{\mathbf{m}}\cdot\overline{\mathbf{m}} = 0 \tag{17}$$

but it is not orthonormal

$$\mathbf{l}\cdot\mathbf{n} = 1, \quad \mathbf{m}\cdot\overline{\mathbf{m}} = -1, \quad \mathbf{l}\cdot\mathbf{m} = 0, \quad \mathbf{n}\cdot\mathbf{m} = 0. \tag{18}$$

We can solve $\{\mathbf{e}^a\}$ in Eq. (16) and replace in Eq. (4) to get the metric in terms of a null tetrad

$$g_{\mu\nu} = \eta_{ab}\, n^a_\mu\, n^b_\nu, \tag{19}$$

where η_{ab} is now

$$\eta_{ab} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{20}$$

Therefore, the metric reads

$$\mathbf{g} = \mathbf{n}\otimes\mathbf{l} + \mathbf{l}\otimes\mathbf{n} - \mathbf{m}\otimes\overline{\mathbf{m}} - \overline{\mathbf{m}}\otimes\mathbf{m}. \tag{21}$$

We are going to transform the tetrad $\{\mathbf{e}^a\}$ of a given TEGR vacuum solution to look for a tetrad having $T = 0$. This procedure is equivalent to change $\{\mathbf{n}^a\}$. Since the geometry (21) and the relations (17), (18) cannot be changed, a simple try is

$$\mathbf{l} \longrightarrow \exp[\lambda(\mathbf{x})]\,\mathbf{l}, \quad \mathbf{n} \longrightarrow \exp[-\lambda(\mathbf{x})]\,\mathbf{n}. \tag{22}$$

This change implies a local Lorentz boost along the direction of $\mathbf{e}^1$ with parameter $\gamma(\mathbf{x}) = \cosh[\lambda(\mathbf{x})]$.

Since $R = 0$ for vacuum solutions, and we are looking for solutions having $T = 0$, then Eq. (12) states that the four-divergence will vanish as well

$$\partial_\rho (e\, T^{\mu}{}_{\mu}{}^{\rho}) = 0. \tag{23}$$

Remarkably, Weitzenböck torsion (7) does not change under global linear transformations of the basis. This implies that even the null tetrad can be used to compute $T^{\mu}{}_{\nu\rho}$

$$T^{\mu}{}_{\nu\rho} = n^\mu_a\, (\partial_\nu n^a_\rho - \partial_\rho n^a_\nu). \tag{24}$$

Under the transformation (22), the vector sector of torsion (the one appearing in the four-divergence) changes as

$$T^{\mu}{}_{\mu}{}^{\rho} \longrightarrow T^{\mu}{}_{\mu}{}^{\rho} + (l^\mu l^\rho - n^\mu n^\rho)\,\partial_\mu\lambda(\mathbf{x}). \tag{25}$$

5 Kerr geometry with vanishing T

In a proper chart, Kerr geometry [50,51] reads

$$\begin{aligned} ds^2 = {} & \left(1 - \frac{2\,m\,r}{\Sigma}\right) dt^2 + \frac{4\,m\,r}{\Sigma}\, dt\, dr \\ & + \frac{4\,a\,m\,r\,\sin^2\theta}{\Sigma}\, dt\, d\varphi - \left(1 + \frac{2\,m\,r}{\Sigma}\right) dr^2 \\ & - 2\,a\,\sin^2\theta \left(1 + \frac{2\,m\,r}{\Sigma}\right) dr\, d\varphi - \Sigma\, d\theta^2 \\ & - \left[\Sigma\,\sin^2\theta + \left(1 + \frac{2\,m\,r}{\Sigma}\right) a^2 \sin^4\theta\right] d\varphi^2, \end{aligned} \tag{26}$$

where m is the mass of the black hole and $\Sigma = r^2 + a^2\cos^2\theta$, a being the angular momentum per unit of mass. In this expression, coordinates $x^\mu = (t, r, \theta, \phi)$ are linked to the usual Boyer–Lindquist coordinates $\tilde{x}^\mu = (\tilde{t}, r, \theta, \tilde{\phi})$ through the relations

$$d\tilde{t} = dt + \frac{2\,m\,r}{r^2 + a^2 - 2\,m\,r}\, dr \quad \text{and}$$
$$d\tilde{\phi} = d\phi + \frac{a}{r^2 + a^2 - 2\,m\,r}\, dr. \tag{27}$$

The geometry (26) can be written in the way (21) by using the null tetrad

$$\mathbf{n}^a{}_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \exp[\lambda]\left(1 - \frac{2\,m\,r}{\Sigma}\right) & \exp[\lambda]\left(1 + \frac{2\,m\,r}{\Sigma}\right) & 0 & \exp[\lambda]\left(1 + \frac{2\,m\,r}{\Sigma}\right) a\,\sin^2\theta \\ \exp[-\lambda] & -\exp[-\lambda] & 0 & -\exp[-\lambda]\,a\,\sin^2\theta \\ 0 & 0 & r + i\,a\cos\theta & (r + i a\cos\theta)\, i\,\sin\theta \\ 0 & 0 & r - i\,a\cos\theta & -(r - i a\cos\theta)\, i\,\sin\theta \end{pmatrix}, \tag{28}$$

where $\lambda = \lambda(t, r, \theta)$. Then, the Weitzenböck invariant becomes

$$T = \frac{2}{\Sigma^3}\,(\Sigma^2 - 4\,a^2\cos^2\theta\,(\Sigma + m\,r) - 2\,r\,\Sigma^2\,\partial_t\lambda). \tag{29}$$

So, there is a family of functions $\lambda(t, r, \theta)$ that realize the vanishing of T,

$$\lambda(t, r, \theta) = \frac{t}{2\,r}\left(1 - 4\,a^2\cos^2\theta\,\frac{\Sigma + m\,r}{\Sigma^2}\right) + \lambda_1(r, \theta). \tag{30}$$

Therefore, the Kerr geometry is a solution of $f(T)$ gravity. In Ref. [52], an axially symmetric tetrad having null T was also found by directly solving the vacuum $f(T)$ equations.

That Kerr tetrad not necessarily coincides with the one here obtained due to remnant symmetries characterizing $f(T)$ solutions [53].

Notice that the freedom to choose the function λ_1 can be exploited for replacing t with $\tilde{t}$ in Eqs. (30) and (31). It should also be pointed out that the function λ is not well defined at $r = 0$. In Kerr geometry, the region $r = 0$ is a circle where θ plays the role of radial coordinate. The edge of the circle ($r = 0$ and $\theta = \pi/2$) is the Kerr ring singularity, but its inner region is not singular. The solution in this region should be reelaborated according to the radial meaning of θ coordinate. Of course, the Schwarzschild tetrad is free from this problem because the singularity at $r = 0$ is just a point. In this case, the function λ becomes

$$\lambda(t, r) = \frac{t}{2\,r} + \lambda_1(r). \tag{31}$$

6 Final comments

We have proved that the Kerr geometry survives as a solution of $f(T)$ gravity whenever the function f satisfies $f(0) = 0$ and $f'(0) \neq 0$. We invoked the argument that any GR vacuum solution will remain a solution of $f(T)$ gravity if it admits a tetrad for which the Weitzenböck invariant T vanishes. This argument was used in Ref. [16] for showing that Schwarzschild geometry survives in $f(T)$ theories. Here we showed that the use of null tetrads can help to easily prove the existence of a tetrad with vanishing T even if the symmetry is not spherical but axial. We remark that T remains null when passing from the null tetrad to an orthonormal tetrad by means of the relations (16). This is because the torsion $T^{\mu}{}_{\nu\rho}$ is invariant under global linear transformations of the basis.

It should be emphasized that the simplicity of the demonstration given in Sect. 5 relies on a good choice of the null tetrad. We started from the null tetrad associated with the Kerr–Schild form of Kerr metric [50,51]

$$\begin{aligned}\mathbf{g} = \mathbf{n}_{\rm o} \otimes (\mathbf{l}_{\rm o} + f\ \mathbf{n}_{\rm o}) + (\mathbf{l}_{\rm o} + f\ \mathbf{n}_{\rm o}) \otimes \mathbf{n}_{\rm o} - \mathbf{m}_{\rm o} \\ \otimes \overline{\mathbf{m}}_{\rm o} - \overline{\mathbf{m}}_{\rm o} \otimes \mathbf{m}_{\rm o} \ = \ \mathbf{g}_{\rm o} + 2\, f\ \mathbf{n}_{\rm o} \otimes \mathbf{n}_{\rm o}\end{aligned} \tag{32}$$

(i.e., $\mathbf{l} = \mathbf{l}_{\rm o} + f\ \mathbf{n}_{\rm o}$), where $\{\mathbf{n}_0^a\}$ is a suitable null tetrad for Minkowski metric $\mathbf{g}_{\rm o}$ that leaves Kerr gravity completely encoded in the function f,

$$f(r, \theta) \ = \ -\frac{2\,m\,r}{\Sigma}. \tag{33}$$

Then we applied the ansatz (22), and obtained a differential equation for λ by demanding the vanishing of the Weitzenböck invariant (29). This differential equation was particularly simple because of a good choice of coordinates: we used the chart employed in the Newman–Janis algorithm for passing from a Schwarzschild solution to a Kerr solution [54].

The obtained results show that $f(T)$ theories are unable to smooth black hole singularities. It would be of major interest to know whether this conclusion can be extended to more general geometries. On the other hand, it is well known that this kind of singularities can be smoothed by means of a different scheme of modified teleparallel gravity [15].

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References

1. A. Einstein, Pruess. Akad. Wiss., 414 (1925)
2. A. Einstein, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl., 217 (1928)
3. A. Einstein, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl., 401 (1930)
4. A. Einstein, Math. Ann. **102**, 685 (1930)
5. E. Cartan, C. R. Acad. Sci. Paris **174**, 593 (1922)
6. E. Cartan, C. R. Acad. Sci. Paris **174**, 734 (1922)
7. F.W. Hehl, P. von der Heyde, G.D. Kerlick, J.M. Nester, Rev. Mod. Phys. **48**, 393 (1976)
8. R. Ferraro, F. Fiorini, Phys. Rev. D **75**, 084031 (2007)
9. R. Ferraro, F. Fiorini, Phys. Rev. D **78**, 124019 (2008)
10. G.R. Bengochea, R. Ferraro, Phys. Rev. D **79**, 124019 (2009)
11. K. Bamba, C.Q. Geng, C.C. Lee, L.W. Luo, JCAP **1101**, 021 (2011)
12. E.V. Linder, Phys. Rev. D **81**, 127301 (2010)
13. P. Wu, H.W. Yu, Phys. Lett. B **692**, 176 (2010)
14. G.R. Bengochea, Phys. Lett. B **695**, 405 (2011)
15. R. Ferraro, F. Fiorini, Phys. Lett. B **702**, 75 (2011a)
16. R. Ferraro, F. Fiorini, Phys. Rev. D **84**, 083518 (2011b)
17. B. Li, T.P. Sotiriou, J.D. Barrow, Phys. Rev. D **83**, 064035 (2011)
18. T.P. Sotiriou, B. Li, J.D. Barrow, Phys. Rev. D **83**, 104030 (2011)
19. H. Wei, X.P. Ma, H.Y. Qi, Phys. Lett. B **703**, 74 (2011)
20. R. Zheng, Q.G. Huang, JCAP **1103**, 002 (2011)
21. F. Fiorini, Phys. Rev. Lett. **111**, 041104 (2014)
22. R. Ferraro, F. Fiorini, Phys. Lett. B **692**, 206 (2010)
23. R.J. Yang, Europhys. Lett. **93**, 60001 (2011)
24. B. Li, T. Sotiriou, J.D. Barrow, Phys. Rev. D **83**, 104017 (2011)
25. M. Li, R.X. Miao, Y.G. Miao, JHEP **1107**, 108 (2011)
26. R.P. Kerr, Phys. Rev. Lett. **11**, 237 (1963)
27. J.G. Pereira, T. Vargas, C.M. Zhang, Class. Quantum Grav. **18**, 833 (2001)
28. J.W. Maluf, J.F. da Rocha-Neto, T.M.L. Toribio, K.H. Castello-Branco, Phys. Rev. D **65**, 124001 (2002)
29. J.F. da Rocha-Neto, K.H. Castello-Branco, JHEP, 002 (2003)
30. J.W. Maluf, M.V.O. Veiga, J.F. da Rocha-Neto, Gen. Rel. Grav. **39**, 227 (2007)
31. L. Tiago Gribl, Y.N. Obukhov, J.G. Pereira, Phys. Rev. D **80**, 064043 (2009)
32. S. Capozziello, M. De Laurentis, A. Stabile, Class. Quantum Grav. **27**, 165008 (2010)
33. Y.S. Myung, Phys. Rev. D **84**, 024048 (2011)
34. Y.S. Myung, Phys. Rev. D **88**, 104017 (2013)

35. D. Pérez, G.E. Romero, S.E. Perez Bergliaffa. Astron. Astrophys. **551**, A4 (2013)
36. K. Gödel, Rev. Mod. Phys. **21**, 447 (1949)
37. Y.N. Obukhov, T. Vargas, Phys. A Stat. Mech. Appl. **327**, 365 (2004)
38. A.A. Sousa, R.B. Pereira, A.C. Silva, Grav. Cosmol. **16**, 25 (2010)
39. T. Clifton, J.D. Barrow, Phy. Rev. D **72**, 123003 (2005)
40. M.J. Rebouças, J. Santos, Phy. Rev. D **80**, 063009 (2009)
41. J. Santos, M.J.J. Rebouças, T.B.R.F. Oliveira, Phys. Rev. D **81**, 123017 (2010)
42. D. Liu, P. Wu, H. Yu, Int. J. Mod. Phys. D **21**, 1250074 (2012)
43. R. Weitzenböck, *Invarianten Theorie* (Noordhoff, Groningen, 1923)
44. K. Hayashi, T. Shirafuji, Phys. Rev. D **19**, 3524 (1979)
45. H.I. Arcos, J.G. Pereira, Int. J. Mod. Phys. D **13**, 2193 (2004)
46. J.W. Maluf, Ann. Phys. **525**, 339 (2013)
47. H.A. Buchdahl, Mon. Not. R. Astron. Soc. **150**, 1 (1970)
48. T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010)
49. N. Tamanini, C.G. Boehmer, Phys. Rev. D **86**, 044009 (2012)
50. R.P. Kerr, A. Schild, A new class of vacuum solutions of the Einstein field equations. In: Atti del Convegno sulla Relativita Generale: Problemi dell' Energia e Onde Gravitazionali (Fourth Centenary of Galileo's Birth), ed. by G.Barbéra (Firenze, 1965) (republished in Gen. Rel. Grav. 41, 2485, 2009)
51. G.C. Debney, R.P. Kerr, A. Schild, J. Math. Phys. **10**, 1842 (1969)
52. G.G.L. Nashed, Adv. High Energy Phys. **2014**, 857936 (2014)
53. R. Ferraro, F. Fiorini, The remnant group of local Lorentz transformations in $f(T)$ theories (2014). arXiv:1412.3424
54. R. Ferraro, Gen. Rel. Gravit. **46**, 1705 (2014)