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k-Inflation in noncommutative space–time

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Abstract The power spectra of the scalar and tensor perturbations in the noncommutative k-inflation model are calculated in this paper. In this model, all the modes created when the stringy space–time uncertainty relation is satisfied, and they are generated inside the sound/Hubble horizon during inflation for the scalar/tensor perturbations. It turns out that a linear term describing the noncommutative space–time effect contributes to the power spectra of the scalar and tensor perturbations. Confronting the general noncommutative k-inflation model with latest results from *Planck* and BICEP2, and taking c_S and λ as free parameters, we find that it is well consistent with observations. However, for the two specific models, i.e. the tachyon and DBI inflation models, it is found that the DBI model is not favored, while the tachyon model lies inside the 1σ contour, when the e-folding number is assumed to be around $50 \sim 60$.

1 Introduction

Inflation [1–3], which solves a number of cosmological conundrums, such as the horizon, monopole, and entropy problems, is now a crucial part of the standard model of our universe. Within the simplest inflation model, a field called inflaton plays an important role during the early time of the universe. It does not only drives the universe to expand nearly exponentially, but also generates small fluctuations, which are very important for large-scale structure formation in the later time. Inflation may never end, and we call this kind of phenomenon the eternal inflation [4–6]. From observations of the Cosmic Microwave Background (CMB), like the satellite-based Wilkinson Microwave Anisotropy Probe (WMAP) [7] and *Planck* [8] experiments, we can obtain lots of information about the early universe, such as the power spectra of the perturbations.

The observed CMB temperature fluctuations mainly generated by scalar perturbations have already constrained many inflation models, but there are still many models consistent with observations. Currently, a large number of CMB experiment efforts target B-*mode* polarization, which could be only generated by tensor perturbations. The ground-based "Background Imaging of Cosmic Extragalactic Polarization" experiment has reported their results (BICEP2). They show that the observed B-*mode* power spectrum at certain angular scales is well fitted by a lensed-ΛCDM + tensor theoretical model with tensor-to-scalar ratio $r = 0.20^{+0.07}_{-0.05}$, and $r = 0$ is disfavoured at 7.0σ [9].

On the other hand, general relativity might break down due to the high energy density during inflation. As a candidate for the theory of everything, string theory should tell us what is a successful theory of the cosmology at that time, and some corrections from string theory should be not omitted. In the non-perturbative string/M theory, any physical process at the very short distance takes an uncertainty relation, called the stringy space–time uncertainty relation (SSUR):

$$\Delta t_{\rm p} \Delta x_{\rm p} \geq l_{\rm s}^2, \tag{1}$$

where $l_{\rm s}$ is the string length scale, and $\Delta t_{\rm p}(= \Delta t)$, $\Delta x_{\rm p}$ are the uncertainties in the physical time and space coordinates. The SSUR may be a universal relation for strings and D-branes [10–12]. Brandenberger and Ho [13] have proposed a variation of space–time noncommutative field theory to realize the stringy space–time uncertainty relation without breaking any of the global symmetries of the homogeneous isotropic universe. If inflation is affected by the physics at a scale close to string scale, one expects that space–time uncertainty must leave vestiges in the CMB power spectrum [14–21]. Recently, Feng et al. [22] propose a new power-law inflation model, by choosing a new different kind of $\beta_k^{\pm}$ function (which will be defined below), which is equivalent

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to that proposed by Brandenberger and Ho [13] in the sense of integration. But it is much more clear to see the effect of noncommutative space–time and much easier to deal with the perturbation functions. For detail calculations and discussions on it, see the Appendix A in Ref. [22].

It has been shown that a large class of non-quadratic scalar kinetic terms can derive an inflationary evolution without the help of potential term, usually referred to as k-inflation [23,24], e.g. the tachyon [25,26] inflation and the Dirac–Born–Infeld (DBI) inflation [27]. In this paper, we shall study the k-inflation model in the noncommutative space–time following the method in Ref. [22]. A linear contribution to the power spectra of the scalar and tensor perturbations is given in this model. We also confront two specific k-inflation models, namely the tachyon and DBI models, with latest results from the *Planck* and BICEP2 experiments, and we find that the DBI model is not favored, while the tachyon model lies inside the 1σ contour, when the e-folding number is assumed to be around $50 \sim 60$. This paper is organized as follows. In next section, the power spectra of the k-inflation model in the noncommutative space–time shall be calculated; in Sect. 3 the parameters in tachyon and DBI models shall be constrained by using the latest observations. In the last section, we will draw our conclusions and give some discussions.

2 K-inflation in noncommutative space–time

The general Lagrangian for a single field model with second-order field equations is an arbitrary function $p(\varphi, X)$ of the scalar field φ and its kinetic energy $X = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi$. With the addition of gravity, the action takes the following form

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{R}{2} + p(\varphi, X)\right], \tag{2}$$

in the units of $M_{\mathrm{pl}}^{-2} = 8\pi G = 1$. Here, one may replace the term $\mathrm{d}x^4$ in the action by $\mathrm{d}x * \mathrm{d}t$ or $\mathrm{d}t * \mathrm{d}x$, but the result could not be changed, see Appendix A. The energy-momentum tensor of the inflaton reads

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_\varphi}{\delta g^{\mu\nu}} = p_{,X}\nabla_\mu\varphi\nabla_\nu\varphi + p g_{\mu\nu},$$
$$p_{,X} \equiv \frac{\partial p(\varphi, X)}{\partial X}, \tag{3}$$

which is equivalent to a perfect fluid, namely $T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}$ with pressure p, energy density

$$\rho = 2Xp_{,X} - p, \tag{4}$$

and four-velocity $U_\mu = -\nabla_\mu\varphi/\sqrt{2X}$. In the following, we will consider a spatially flat universe ($K = 0$), which is described by the Friedmann–Robertson–Walker metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x^2. \tag{5}$$

Then, we have $X = \dot{\varphi}^2/2$ and the SSUR relation (1)

$$\Delta t\Delta x \geq \frac{l_s^2}{a(t)}. \tag{6}$$

But this relation is not well defined when Δt is large, because the argument t for the scale factor on the r.h.s. of Eq. (6) changes over time interval Δt, and it is thus not clear what to use for $a(t)$ in Eq. (6). The problem is the same when one uses the conformal time η defined by $\mathrm{d}t = a\mathrm{d}\eta$. Therefore, for later use, a new time coordinate τ is introduced as

$$\mathrm{d}\tau = a(t)\mathrm{d}t, \tag{7}$$

such that the metric becomes

$$\mathrm{d}s^2 = -a^{-2}(\tau)\mathrm{d}\tau^2 + a^2(\tau)\mathrm{d}x^2, \tag{8}$$

and the SSUR relation is now well defined:

$$\Delta\tau\Delta x \geq l_s^2. \tag{9}$$

Therefore, the Friedmann equation becomes

$$H^2 = \frac{\rho}{3} = \frac{1}{3}(\dot{\varphi}^2 p_{,X} - p), \tag{10}$$

and the equation of motion for the inflaton is

$$\ddot{\varphi}\,(p_{,X} + \dot{\varphi}^2 p_{,XX}) + 3H\dot{\varphi}\,p_{,X} = p_{,\varphi} - \dot{\varphi}^2\,p_{,X\varphi}. \tag{11}$$

Also we have the relation $\dot{H} = -Xp_{,X} = -\dot{\varphi}^2 p_{,X}/2$. As a result of non-trivial kinetic terms in p, the dispersion relation for the inflaton is changed, and the fluctuations in the scalar field are no longer travelling at the speed of light. Instead, the sound speed of φ is given by

$$c_s^2 \equiv \frac{p_{,X}}{\rho_{,X}} = \frac{p_{,X}}{p_{,X} + \dot{\varphi}^2 p_{,XX}} \leq 1. \tag{12}$$

In the canonical case, $p(\varphi, X) = X - V(\varphi)$ with inflaton potential $V(\varphi)$, we have $c_s^2 = 1$. To use the slow-roll approximation we may define

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{Xp_{,X}}{H^2}, \quad \epsilon_2 = \frac{\dot{\epsilon}_1}{\epsilon_1 H}, \quad s = \frac{\dot{c}_s}{c_s H}, \tag{13}$$

which are all small parameters during inflation, i.e. $|\epsilon_1|$, $|\epsilon_2|$, $|s| \ll 1$.

It should be noticed that the stringy space–time uncertainty (SSUR) does not impose a restriction on quantities with only time dependence, then if we only look for solutions with a single variable, noncommutative field theories are equivalent to commutative field theories. The background equations (10) and (11) that describing a homogeneous and isotropic universe are only time-dependent, while the perturbations have both time and space dependences. Therefore, only perturbations are affected by noncommutative effects, see the following subsections.

2.1 Scalar perturbation

The action of the scalar perturbation could be written as

$$S=\frac{V}{2}\int_{k<k_0}\mathrm{d}\eta\mathrm{d}^3k\, z^2(\eta)(\zeta'_{-k}\zeta'_k-c_s^2k^2\zeta_{-k}\zeta_k), \tag{14}$$

where V is the total spatial coordinate volume and the prime denotes the derivatives with respect to a new time coordinate η defined as

$$\frac{\mathrm{d}\eta}{\mathrm{d}\tau}\equiv a_{\mathrm{eff}}^{-2}=\left(\frac{\beta_k^-}{\beta_k^+}\right)^{1/2}=a^{-2}(\tau+\Delta\tau). \tag{15}$$

Here we have defined

$$\beta_k^{\pm}(\tau)=a^{\pm 2}(\tau+\Delta\tau),\quad \Delta\tau=-l_s^2k, \tag{16}$$

where ζ_k is the curvature perturbation and $z\equiv a\sqrt{2\epsilon_1}/c_s$ is the so-called "Mukhanov variable". Here we have taken a different form of the $\beta_k^{\pm}$ functions, which is equivalent to that used in the literatures in the sense of integration. In the present form, it is much more easier to deal with the equations of motion for the field ζ_k:

$$u_k''+\left(c_s^2k^2-\frac{z''}{z}\right)u_k=0, \tag{17}$$

which is derived from the action (14). Here the mode function is defined by $u_k=z\zeta_k$. By using the definitions of slow-roll parameters, we get the coefficient of the third term in the perturbative equation (17) as

$$\frac{z''}{z}\approx\frac{1}{\eta^2}(1-\epsilon_1)^{-2}\Sigma^{-2}(\tau,\Delta\tau)\times\left[2\Sigma(\tau,\Delta\tau)\left(1+\frac{\epsilon_2}{2}-s\right)\epsilon_1+\frac{\epsilon_2}{2}-s\right], \tag{18}$$

where we have used

$$\eta\approx-[a(\tau+\Delta\tau)H(\tau+\Delta\tau)(1-\epsilon_1)]^{-1}, \tag{19}$$

which is derived from Eq. (15). Here, the Σ function is defined as

$$\Sigma(\tau,\Delta\tau)=\frac{a(\tau)}{a(\tau+\Delta\tau)}\frac{H(\tau+\Delta\tau)}{H(\tau)}. \tag{20}$$

By using the approximation $a(\tau+\Delta\tau)\approx a(\tau)+H(\tau)\Delta\tau$ and $H(\tau+\Delta\tau)\approx H(\tau)-\epsilon_1H^2(\tau)a^{-1}(\tau)\Delta\tau$, we get

$$\Sigma(\tau,\Delta\tau)\approx 1-(1+\epsilon_1)\frac{\lambda}{1-\lambda}+\mathcal{O}(\Delta\tau^2), \tag{21}$$

where we have defined the parameter

$$\lambda=\frac{\Delta\tau}{a(\tau)H^{-1}(\tau)+\Delta\tau}. \tag{22}$$

All the modes are created when the SSUR is saturated with the upper bound of the comoving wave number

$$k_0(\tau)=\frac{a_{\mathrm{eff}}}{l_s}=\frac{a(\tau+\Delta_\tau)}{l_s}\approx\frac{H(\tau)}{l_s}[a(\tau)H^{-1}(\tau)+\Delta\tau], \tag{23}$$

which means that at time τ, a mode with wave number k_0 is generated. The corresponding parameter λ during inflation at the mode creating time is given by

$$\lambda_0\equiv\left.\frac{\Delta\tau}{a(\tau)H^{-1}(\tau)+\Delta\tau}\right|_{k=k_0}\approx -l_sH_*, \tag{24}$$

where H_* is the value of Hubble parameter during inflation. Since the scale factor is increasing nearly exponentially ($a\sim e^{Ht}$) or with a large power ($a\sim t^n$, with a large n) and H_* is almost a constant, the absolute value of parameter λ will decrease with time, see Eq. (22). At the same time when a mode is created (23), the wave number cross the comoving sound horizon is given by

$$k_c=a(\tau)\frac{H(\tau)}{c_s}. \tag{25}$$

Therefore, during inflation, we have

$$\frac{k_c}{k_0}\approx\frac{a(\tau)}{a(\tau)+H(\tau)\Delta\tau}\frac{l_sH(\tau)}{c_s}\approx\frac{l_sH_*}{c_s}. \tag{26}$$

In the following, we will focus on the case of $l_sH_*/c_s\ll 1$, which means that all the modes are created inside the horizon ($k_0\gg k_c$). The other case $l_sH_*/c_s\geq 1$ will not be considered in this paper, since in this case it is hard to explain the flatness of the universe, see Ref. [22] for detail discussions. Thus, during inflation the parameter $|\lambda|\ll 1$, and we will treat it as a free small parameter in the model. All the calculations are taken up to the first order of λ. Furthermore, the scalar power spectrum will be calculated at the time when the mode crosses the sound horizon ($k=aH/c_s$). By using the approximation, Eq. (17) becomes

$$u_k''+\left(c_s^2k^2-\frac{\nu^2-1/4}{\eta^2}\right)u_k=0, \tag{27}$$

where

$$\nu \approx \frac{3}{2} + \epsilon_1 + \frac{\epsilon_2}{2} - s + \frac{2}{3}\lambda, \tag{28}$$

up to the first order of slow-roll parameters and λ. With the initial Bunch–Davies vacuum condition:

$$u_k = \frac{1}{\sqrt{2c_s k}} e^{-ic_s k\eta}, \tag{29}$$

we get the solution to Eq. (27)

$$u_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu+1/2)\pi/2} \sqrt{-\eta} H_\nu^{(1)}(-c_s k\eta), \tag{30}$$

where $H_\nu^{(1)}$ is the Hankel's function of the first kind. At the superhorizon scales the solution becomes

$$u_k(\eta) = 2^{\nu-3/2} e^{i(\nu-1/2)\pi/2} \frac{\Gamma(\nu)}{\Gamma(3/2)} \frac{1}{\sqrt{2c_s k}} (-c_s k\eta)^{1/2-\nu}. \tag{31}$$

Therefore, the power spectrum of the metric scalar perturbation is given by

$$\begin{aligned}\mathcal{P}_s &= \frac{k^3}{2\pi^2}|\zeta_k|^2 = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2 \\ &= \frac{2^{2\nu-4}}{\epsilon_1 c_s}\left[\frac{\Gamma(\nu)}{\Gamma(3/2)}\right]^2 \left(\frac{H}{2\pi}\right)^2 \left(\frac{c_s k}{aH}\right)^{3-2\nu}\Bigg|_{c_s k = aH} \\ &\approx \frac{1}{8\pi^2 \epsilon_1 c_s}\frac{H^2}{M_{pl}^2}\Bigg|_{c_s k = aH},\end{aligned} \tag{32}$$

and the spectrum index of the power spectrum for the scalar perturbation reads

$$n_s - 1 \equiv \frac{d\ln \mathcal{P}_s}{d\ln k} = 3 - 2\nu - s \approx -2\epsilon_1 - \epsilon_2 + s - \frac{4}{3}\lambda. \tag{33}$$

Also we obtain the running of the index

$$\begin{aligned}\alpha_s &\equiv \frac{dn_s}{d\ln k} = -2\epsilon_2\epsilon_1 - \epsilon_3\epsilon_2 + s_1 s + \frac{4}{3}\lambda(1-\lambda)(1+\epsilon_1) \\ &\approx -2\epsilon_2\epsilon_1 - \epsilon_3\epsilon_2 + s_1 s + \frac{4}{3}\lambda(1-\lambda+\epsilon_1),\end{aligned} \tag{34}$$

where $\epsilon_3 \equiv \dot{\epsilon}_2/(\epsilon_2 H)$ and $s_1 \equiv \dot{s}/(sH)$.

2.2 Tensor perturbation

The equation of motion for the tensor perturbation is almost the same as the one for the scalar perturbation, except that the Mukhanov variable becomes $z = a$. Then the equation of motion for the mode function is given by

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0, \tag{35}$$

where $v_k \equiv ah_k/2$. Here h_k denotes the independent degree of the tensor mode, i.e. h_+ and $h_\times$. By using the same approximation as that in the scalar perturbation, we get

$$\begin{aligned}\frac{a''}{a} &\approx \frac{1}{\eta^2}(1-\epsilon_1)^{-2}\Sigma^{-2}(\tau,\Delta\tau)[2\Sigma(\tau,\Delta\tau) - \epsilon_1] \\ &\approx \frac{\nu^2 - 1/4}{\eta^2},\end{aligned} \tag{36}$$

with

$$\nu \approx \frac{3}{2} + \epsilon_1 + \frac{2}{3}\lambda. \tag{37}$$

Then the solution to Eq. (35) is given by

$$v_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu+1/2)\pi/2} \sqrt{-\eta} H_\nu^{(1)}(-k\eta), \tag{38}$$

which also satisfies the Bunch–Davies vacuum initial condition. At the superhorizon scales the solution becomes

$$u_k(\eta) = 2^{\nu-3/2} e^{i(\nu-1/2)\pi/2} \frac{\Gamma(\nu)}{\Gamma(3/2)} \frac{1}{\sqrt{2k}} (-k\eta)^{1/2-\nu}. \tag{39}$$

Therefore, the power spectrum of the metric tensor perturbation is given by

$$\begin{aligned}\mathcal{P}_t &= 2 \times \frac{k^3}{2\pi^2}|h_k|^2 = \frac{k^3}{\pi^2}\left|\frac{2v_k}{z}\right|^2 \\ &= 2^{2\nu}\left[\frac{\Gamma(\nu)}{\Gamma(3/2)}\right]^2 \left(\frac{H}{2\pi}\right)^2 \times \left(\frac{k}{aH}\right)^{3-2\nu}\Bigg|_{k=aH} \\ &\approx \frac{2}{\pi^2}\frac{H^2}{M_{pl}^2}\Bigg|_{k=aH},\end{aligned} \tag{40}$$

and the spectrum index of the power spectrum for the tensor perturbation reads

$$n_t \equiv \frac{d\ln \mathcal{P}_t}{d\ln k} = 3 - 2\nu \approx -2\epsilon_1 - \frac{4}{3}\lambda. \tag{41}$$

We also get the tensor-to-scalar ratio

$$r \equiv \frac{\mathcal{P}_t}{\mathcal{P}_s} = 16 c_s \epsilon_1, \tag{42}$$

and the consistency relation

$$r = -8c_s\left(n_t + \frac{4}{3}\lambda\right). \tag{43}$$

When $\lambda \to 0$, it reduces to the one in the commutative case, i.e. $r = -8c_s n_t$. With the help of λ term in the above equation, one shall see that the power-law inflation in noncommutative space–time may be more consistent with observations than that in the commutative case.

3 Confront two specific k-inflation models with observations

3.1 Tachyon inflation models

The tachyon inflation model was introduced by Sen [25,26], and later studied in Refs. [19,28–35]. In the following, we will consider the so-called assisted tachyon inflation mode [31], which solves some difficulties with a single tachyonic field [29]. These difficulties could be also solved by taking account of non-minimal coupling to the gravity, see Ref. [36]. The Lagrangian for the assisted tachyon inflation reads

$$p(\varphi_i, X_i) = -\sum_{i=1}^{n} V(\varphi_i)\sqrt{1-2l_s^2 X_i}, \tag{44}$$

which is a simply the sum of n single-tachyonic fields without interactions. The 4-d effective theory is applicable when n is large enough, say $n \geq 10^7$ [31]. The Friedmann equation (10) becomes

$$H^2 = \sum_{i=1}^{n} \frac{V(\varphi_i)}{3\sqrt{1-2l_s^2 X_i}} = \sum_{i=1}^{n} \frac{V(\varphi_i)}{3\sqrt{1-l_s^2\dot{\varphi}_i^2}} \approx \frac{n}{3}V(\varphi_i), \tag{45}$$

and the equation of motion (11) for the inflaton is given by

$$\ddot{\varphi}_i + 3H\dot{\varphi}_i(1-l_s^2\dot{\varphi}_i^2) + \frac{V_{,\varphi_i}}{V l_s^2}(1-l_s^2\dot{\varphi}_i^2) = 0. \tag{46}$$

And also we have

$$\dot{H} = -\sum_{i=1}^{n} \frac{l_s^2\dot{\varphi}_i^2 V(\varphi_i)}{2\sqrt{1-l_s^2\dot{\varphi}_i^2}}, \quad \text{and} \quad c_s^2 = 1 - l_s^2\dot{\varphi}_i^2 \approx 1 - \frac{2\epsilon_1}{3n}. \tag{47}$$

Thus, the slow-roll parameters (13) are given by

$$\epsilon_1 = \frac{3}{2}\sum_{i=1}^{n} l_s^2\dot{\varphi}_i^2 \approx \frac{1}{2l_s^2 nV(\varphi_i)}\left(\frac{V_{\varphi_i}}{V}\right)^2 \approx \frac{1}{2l_s^2 nV(\varphi_i)}\left(\frac{V_{,\varphi_i}}{V}\right)^2, \tag{48}$$

$$\epsilon_2 \approx -\frac{2}{l_s^2 nV(\varphi_i)}\frac{V_{,\varphi_i\varphi_i}}{V} + 6\epsilon_1 + \mathcal{O}(\epsilon^2), \tag{49}$$

$$s = \frac{\dot{c}_s}{c_s H} = \frac{\dot{c}_s^2}{2c_s^2 H} \approx -\frac{\dot{\epsilon}_1}{3nc_s^2 H} \approx -\frac{\epsilon_2\epsilon_1}{3n}\left(1-\frac{1}{3n}\epsilon_1\right)^{-1} \sim \mathcal{O}(\epsilon^2). \tag{50}$$

Considering the power-law potential $V(\varphi_i) = \alpha\varphi_i^m$, we have

$$\epsilon_1 \approx \frac{m^2}{2l_s^2 nV(\varphi_i)\varphi_i^2} \approx \frac{m}{2(m+2)N}, \quad \epsilon_2 \approx \frac{m(m+2)}{l_s^2 nV(\varphi_i)\varphi_i^2} \approx \frac{1}{N}, \quad c_s \approx 1 - \frac{\epsilon_1}{3n}, \tag{51}$$

where N denotes the number of e-folds during inflation, which is defined by

$$N \equiv \int_t^{t_{\rm end}} H\,\mathrm{d}t \approx -\int_{\varphi_i}^{\varphi_i^{\rm end}} 3H^2 l_s^2 \frac{V}{V_{,\varphi_i}}\mathrm{d}\varphi_i = -\frac{nl_s^2\alpha}{m}\int_{\varphi_i}^{\varphi_i^{\rm end}} \varphi_i^{m+1}\mathrm{d}\varphi_i \approx \frac{l_s^2 n\alpha}{m(m+2)}\varphi_i^{m+2} \approx \frac{l_s^2 nV(\varphi_i)\varphi_i^2}{m(m+2)}. \tag{52}$$

Therefore, the spectrum indices of the power spectra for the scalar and tensor perturbations are given by

$$n_s - 1 = -\frac{2m+2}{(m+2)N} - \frac{4}{3}\lambda, \quad n_t = -\frac{m}{(m+2)N} - \frac{4}{3}\lambda, \tag{53}$$

and the tensor-to-scalar ratio is

$$r = \frac{8m}{(m+2)N}\left[1 - \frac{m}{6n(m+2)N}\right] \approx \frac{8m}{(m+2)N}. \tag{54}$$

In fact, the sound speed c_s is very closed to 1 for a positive value of m when $n \geq 10^7$.

3.2 DBI inflation models

The Lagrangeian of DBI inflation is given by [27] (see also [37]):

$$p(\varphi, X) = -\frac{1}{f(\varphi)}\sqrt{1-2f(\varphi)X} - V(\varphi), \tag{55}$$

where $f(\varphi)$ takes the form $f(\varphi) = \alpha/\varphi^4$. The Friedmann equation (10) becomes

$$H^2 = \frac{1}{3}\left[\frac{1}{f(\varphi)c_s} + V(\varphi)\right], \tag{56}$$

where the speed of sound from Eq. (12) is given by

$$c_s = \sqrt{1-2f(\varphi)X} = \sqrt{1-f(\varphi)\dot{\varphi}^2}. \tag{57}$$

While the equation of motion (11) for the inflaton is

$$\frac{\ddot{\varphi}}{c_s^2} + 3H\dot{\varphi} + \frac{2\varphi^3}{\alpha}\left(3 - \frac{1}{c_s^2}\right) + c_s V_{,\varphi} = 0. \tag{58}$$

And also we have

$$\dot{H} = -\frac{\dot{\varphi}^2}{2c_s}. \tag{59}$$

Thus, the slow-roll parameters (13) are given by

$$\epsilon_1 = \frac{\dot{\varphi}^2}{2c_s H^2} = \frac{1-c_s^2}{2c_s f(\varphi)H^2} = \frac{3(1-c_s^2)}{2} \times[1+c_s f(\varphi)V(\varphi)]^{-1} = \frac{3(1-c_s^2)}{2(1+\alpha' c_s)}, \tag{60}$$

$$\epsilon_2 = -\frac{2c_s^2 s}{1-c_s^2} - \frac{s c_s f v}{1+c_s f V} - \sqrt{\frac{3c_s(1-c_s^2)}{1+c_s f V}}\frac{c_s f V}{1+c_s f V} \times(\ln fV)_{,\varphi} = -\left(\frac{2c_s^2}{1-c_s^2} + \frac{\alpha' c_s}{1+\alpha' c_s}\right)s, \tag{61}$$

where we have consider the quartic potential $V(\varphi) \sim \varphi^4$ that makes $f(\varphi)V(\varphi) = \alpha'$. From Eq. (60), we get $c_s \approx 1-(1+\alpha')\epsilon_1/3$ and the rate of change of the sound speed $s \approx -(1+\alpha')\epsilon_2\epsilon_1/3 \sim \mathcal{O}(\epsilon^2)$. Then the Friedmann equation becomes

$$H^2 = \frac{\varphi^4}{3\alpha c_s}(1+\alpha' c_s) \approx \frac{\varphi^4}{3\alpha}(1+\alpha')\left(1+\frac{\epsilon_1}{3}\right). \tag{62}$$

And the slow-roll parameters could be approximated as

$$\epsilon_1 \approx \frac{8}{\varphi^2} \approx \frac{1}{N}, \quad \epsilon_2 \approx \frac{8}{\varphi^2} \approx \frac{1}{N}, \tag{63}$$

where

$$N = \int_t^{t_{\rm end}} H\mathrm{d}t = \int_\varphi^{\varphi^{\rm end}} \frac{H}{\dot{\varphi}}\mathrm{d}\varphi \approx \frac{\varphi^2}{8}. \tag{64}$$

Therefore, the spectrum indices of the power spectra for the scalar and tensor perturbations are given by

$$n_s - 1 = -\frac{3}{N} - \frac{4}{3}\lambda, \quad n_t = -\frac{2}{N} - \frac{4}{3}\lambda, \tag{65}$$

and the tensor-to-scalar ratio is

$$r = \frac{16}{N}\left[1 - \frac{(1+\alpha')}{3N}\right] \approx \frac{16}{N}. \tag{66}$$

With the help of λ term in the Eqs. (53) and (65), one shall see that the tachyon and DBI inflation models in noncommutative space–time may be more consistent with observations than that in the commutative case.

3.3 Confront models with *Planck* and BICEP2

In this subsection, we will constrain the noncommutative k-inflation by using the analysis results from data including the *Planck* CMB temperature likelihood supplemented by the WMAP large scale polarization likelihood (henceforth *Planck* + WP). Other CMB data extending the *Planck* data to higher-l, the *Planck* lensing power spectrum, and BAO data are also combined, see Ref. [8] for details. In Ref. [8], the index of scalar power spectrum is given by: 0.9583 ± 0.0081 (*Planck* + WP), 0.9633 ± 0.0072 (*Planck* + WP + lensing), 0.9570 ± 0.0075 (*Planck* + WP + highL), 0.9607 ± 0.0063 (*Planck* + WP + BAO). From the recent reports of BICEP2 experiment, we get the tensor-scalar-ratio as $r = 0.20^{+0.07}_{-0.05}$, see Ref. [9] for details. Also, adopting the data from BICEP2 together with *Planck* and WMAP polarization data, Cheng and Huang [38] got the constraints of $r = 0.23^{+0.05}_{-0.09}$, and $n_t = 0.03^{+0.13}_{-0.11}$. By using these results, we obtain the constraints on the parameters c_s and λ as

$$c_s = 0.65 \pm 0.19, \quad \lambda = -0.0527 \pm 0.041, \quad (68\,\%\mathrm{CL}). \tag{67}$$

We plot the contours from 1σ to 2σ confidence levels for the parameters, see Fig. 1, in which the n_s–r plane that based on Fig. 13 from Ref. [9] is also presented. From Fig. 1, one can see that the general noncommutative k-inflation with its best fitting parameters is well consistent with observations, However, for the two specific models, if we assume that the number of e-folds number is around $50 \sim 60$, the DBI model is not favored, while the tachyon model lies inside the 1σ contour.

4 Conclusions

In conclusion, we have studied the k-inflation model in the noncommutative space–time following the method in Ref. [22]. A linear contribution to the power spectra of the scalar and tensor perturbations is given in this model. We confront two specific k-inflation models, namely the tachyon and DBI models, with latest results from the *Planck* and BICEP2 experiments, and we find that the DBI model is not favored, while the tachyon model lies inside the 1σ contour, when the e-folding number is assumed to be around $50 \sim 60$. We also constrained the parameter c_s and γ for a generic k-inflation models, and find it is well-consistent with observations, see Fig. 1.

From Eq. (67), one can see that that the value of the λ indeed has a large error bar, which means that the present observational data can not constrain λ very well. This is mainly because the error bar of r is still too large (with an error $\sim 20\,\%$ at 1σ C.L.) to constraint the e-folding number, see Eq. (66). We hope that the future data could constrain the value of r much more tight, say with an error of only about $1\,\%$ or even smaller, then the value of λ could be constrained with an error less than $10\,\%$.

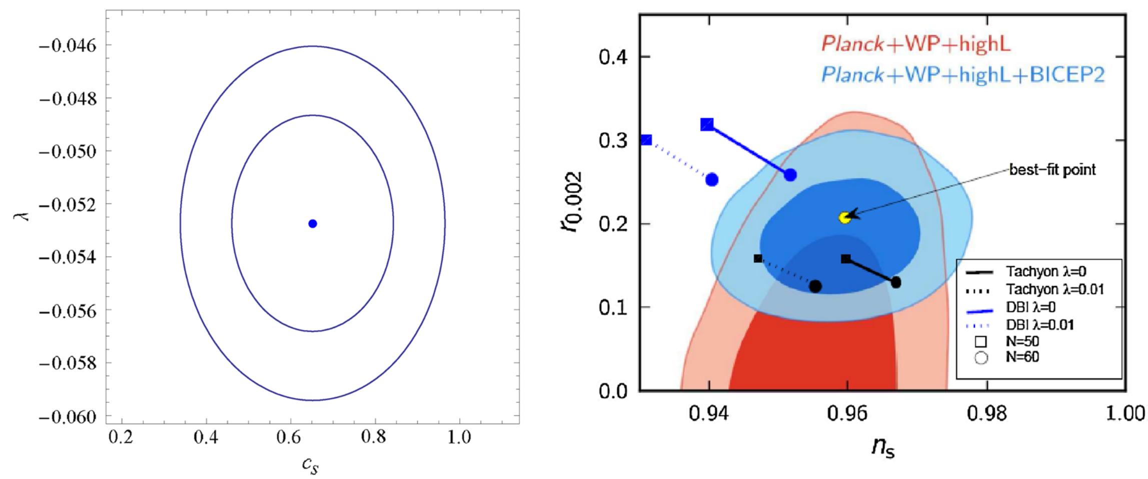

Fig. 1 *Left* Constraints on the values of c_s and λ. Two constraint contours are given at 68 and 95 % confidence level. The *central dot* corresponds to the best-fit point ($c_s = 0.65$, $\lambda = -0.0527$). *Right* The n_s–r plane based on Fig. 13 from Ref. [9], in which the *red contours* are simply the MCMC result provided with the *Planck* data release, while the *blue ones* are plotted when the BICEP2 data are added. The *dark lines* correspond to the tachyon inflation model with model parameter $m \to \infty$ and the *blue lines* correspond to DBI inflation model. The *solid lines* correspond to the model in usual commutative spacetime ($\lambda = 0$), while the *dotted ones* correspond to the model in noncommutative spacetime with $\lambda = 0.01$. The best-fit model for general k-inflation is also pointed out

By using the amplitude value of the power spectrum from *Planck*, $\mathcal{R}_s(k = 0.002\,\mathrm{Mpc}^{-1}) = 2.215 \times 10^{-19}$ [39], we can also estimate the value of Hubble parameter during inflation of

$$\frac{H_*}{M_{\rm pl}} = \pi\sqrt{r\mathcal{R}_s/2} \approx 4.67 \times 10^{-5}, \tag{68}$$

where $r = 0.20$ was used. Then, by using the fitting value of c_s and λ, we estimate the string scale as $l_s \approx 1.13 \times 10^3 l_p \approx 9.14 \times 10^{-30}$ cm.

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Appendix A: Action with noncommutative product

In fact, one should replace the term $\mathrm{d}x^4$ in the action (2) by $\mathrm{d}x * \mathrm{d}t$ or $\mathrm{d}t * \mathrm{d}x$, where $*$ indicates the noncommutative product. But the results could not be changed, since this replacement is equivalent to multiply a overall constant factor to the action. To see this, we need the explicit definition of the $*$ product as follows, see Refs. [13,15]:

$$(f * g)(\tau, x) = \exp\left[-\frac{i}{2} l_s^2 (\partial_x \partial_{\tau'} - \partial_\tau \partial_{x'})\right] \times f(\tau, x) g(\tau', x')\Big|_{\tau'=\tau, x'=x}. \tag{A1}$$

Then, one can obtain

$$\begin{aligned}\mathrm{d}x * \mathrm{d}t &= \exp\left[-\frac{i}{2} l_s^2 (\partial_x \partial_{t'} - \partial_t \partial_{x'})\right] \mathrm{d}x\mathrm{d}t'\Big|_{\tau'=\tau, x'=x} \\ &= \exp\left(-\frac{i}{2} l_s^2\right) \mathrm{d}x\mathrm{d}t'\Big|_{\tau'=\tau, x'=x} \\ &= \exp\left(-\frac{i}{2} l_s^2\right) \mathrm{d}x\mathrm{d}t,\end{aligned} \tag{A2}$$

or

$$\begin{aligned}\mathrm{d}t * \mathrm{d}x &= \exp\left[-\frac{i}{2} l_s^2 (\partial_x \partial_{t'} - \partial_t \partial_{x'})\right] \mathrm{d}t\mathrm{d}x'\Big|_{\tau'=\tau, x'=x} \\ &= \exp\left(\frac{i}{2} l_s^2\right) \mathrm{d}t\mathrm{d}x'\Big|_{\tau'=\tau, x'=x} = \exp\left(\frac{i}{2} l_s^2\right) \mathrm{d}x\mathrm{d}t,\end{aligned} \tag{A3}$$

Thus we have

$$\begin{aligned}\int \mathrm{d}x^4 \cdots &= \exp\left(\frac{i}{2} l_s^2\right) \int \mathrm{d}x * \mathrm{d}t \cdots \\ &= \exp\left(-\frac{i}{2} l_s^2\right) \int \mathrm{d}t * \mathrm{d}x \cdots .\end{aligned} \tag{A4}$$

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