

S_3 discrete group as a source of the quark mass and mixing pattern in 331 models

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Abstract We propose a model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry with an extra $S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12}$ discrete group, which successfully accounts for the SM quark mass and mixing pattern. The observed hierarchy of the SM quark masses and quark mixing matrix elements arises from the Z_4 and Z_{12} symmetries, which are broken at a very high scale by the $SU(3)_L$ scalar singlets (σ, ζ) and τ , charged under these symmetries, respectively. The Cabibbo mixing arises from the down-type quark sector whereas the up quark sector generates the remaining quark mixing angles. The obtained magnitudes of the CKM matrix elements, the CP violating phase, and the Jarlskog invariant are in agreement with the experimental data.

1 Introduction

The discovery of a scalar field with a mass of 125 GeV by LHC experiments [1–4] confirms that the standard model (SM) is the right theory of electroweak interactions and may provide an explanation for the origin of mass of fundamental particles and for the spontaneous symmetry breaking. Despite the success of the LHC experiments, there are many aspects not yet explained, such as the fermion mass hierarchy. This discovery of the Higgs scalar field opens the possibility to formulate theories beyond the SM that include additional scalar fields that can be useful to explain the existence of dark matter [5–8].

One of the outstanding unresolved issues in Particle Physics is the origin of the masses of fundamental fermions. The current theory of strong and electroweak interactions, the SM, has proven to be remarkably successful in passing all experimental tests. Despite its great success, the SM based on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry is unlikely

to be a truly fundamental theory due to unexplained features [9–12]. Most of them are linked to the existence of three families of fermions as well as the fermion mass and mixing hierarchy; problems presented in its quark and lepton sectors. Neutrino oscillation experiments provide a clear indication that neutrinos are massive particles, but these experiments do not explain neither the neutrino mass squared splittings nor the Dirac or Majorana identity of neutrinos. While in the quark sector the mixing angles are small, in the lepton sector two of the mixing angles are large, and one mixing angle is small. This suggests different mechanisms for the generation of mass in the quark and lepton sectors. Experiments with solar, atmospheric and reactor neutrinos provide evidence of neutrino oscillations from the measured non-vanishing neutrino mass squared splittings.

One clear and outstanding feature in the pattern of quark masses is that they increase from one generation to the next spreading over a range of five orders of magnitude [12–20]. From the phenomenological point of view, it is possible to describe some features of the mass hierarchy by assuming zero-texture Yukawa matrices [21–35]. Recently, discrete groups have been considered to explain the observed pattern of fermion masses and mixing [36–71]. Other models with horizontal symmetries have been proposed in the literature [72–77].

On the other hand, the origin of the structure of fermions can be addressed in family dependent models. Alternatively, an explanation to this issue can also be provided by the models based on the gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, also called 3-3-1 models, which introduce a family non-universal $U(1)_X$ symmetry [78–91]. Models based on the gauge symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$ are very interesting since they predict the existence of three families from the quiral anomaly cancelation [92–97]. In these models, two families of quarks have the same quantum numbers, which are associated to the two families of light quarks to correctly predict the Cabibbo mixing angle. The third family has dif-

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ferent $U(1)_X$ values and thus it is associated to the heavy quarks. Thus, the fact that the third family is treated under a different representation, can explain the large mass difference between the heaviest quark family and the two lighter ones [98]. These models include a Peccei–Quinn symmetry that sheds light into the strong CP problem [99–101]. The 331 models with sterile neutrinos have weakly interacting massive fermionic dark matter candidates [102].

In this paper we propose a version of the $SU(3)_C \times SU(3)_L \times U(1)_X$ model with an additional discrete symmetry group $S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12}$ and an extended scalar sector needed in order to reproduce the specific patterns of mass matrices in the quark sector that successfully account for the quark mass and mixing hierarchy. The particular role of each additional scalar field and the corresponding particle assignments under the symmetry group of the model are explained in details in Sect. 2. Our model successfully describes the prevailing pattern of the SM quark masses and mixing.

This paper is organized as follows. In Sect. 2 we outline the proposed model. In Sect. 3 we present our results in terms of quark masses and mixing, which is followed by a numerical analysis. In Sect. 4, we discuss the scalar mass spectrum resulting from the low energy scalar potential. Finally in Sect. 5, we state our conclusions. In the appendices we present several technical details. Appendix A gives a brief description of the S_3 group. Appendix B presents a discussion of the stability conditions of the low energy scalar potential.

2 The model

We consider an extension of the minimal $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (331) model with the full symmetry \mathcal{G} experiencing a three-step spontaneous breaking:

$$\begin{aligned} \mathcal{G} &= SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12} \\ &\quad \downarrow \Lambda_{int} \\ &SU(3)_C \otimes SU(3)_L \otimes U(1)_X \\ &\quad \downarrow v_\chi \\ &SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ &\quad \downarrow v_\eta, v_\rho \\ &SU(3)_C \otimes U(1)_Q \end{aligned} \quad (2.1)$$

where the different symmetry breaking scales satisfy the following hierarchy: $\Lambda_{int} \gg v_\chi \gg v_\eta, v_\rho$.

In our model 331 model, the electric charge is defined in terms of the $SU(3)$ generators and the identity by

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + XI, \quad (2.2)$$

with $I = \text{Diag}(1, 1, 1)$, $T_3 = \frac{1}{2}\text{Diag}(1, -1, 0)$ and $T_8 = (\frac{1}{2\sqrt{3}})\text{Diag}(1, 1, -2)$.

The anomaly cancellation of $SU(3)_L$ requires that the two families of quarks be accommodated in 3^* irreducible representations (irreps). From the quark colors, it follows that the number of 3^* irreducible representations is six. The other family of quarks is accommodated with its three colors, into a 3 irreducible representation. When including the three families of leptons, we have six 3 irreps. Consequently, the $SU(3)_L$ representations are vector like and anomaly free. In order to have anomaly free $U(1)_X$ representations, one needs to assign quantum numbers to the fermion families in such a way that the combination of the $U(1)_X$ representations with other gauge sectors be anomaly free. Therefore, from the requirement of anomaly cancellation we get the following ($SU(3)_C, SU(3)_L, U(1)_X$) left-handed fermionic representations:

$$\begin{aligned} Q_L^{1,2} &= \begin{pmatrix} D^{1,2} \\ -U^{1,2} \\ J^{1,2} \end{pmatrix}_L : (3, 3^*, 0), \\ Q_L^3 &= \begin{pmatrix} U^3 \\ D^3 \\ T \end{pmatrix}_L : (3, 3, 1/3), \\ L_L^{1,2,3} &= \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1, 3, -1/3). \end{aligned} \quad (2.3)$$

Let us note that the right-handed sector transforms as singlets under $SU(3)_L$. The right-handed up- and down-type SM quarks transform under ($SU(3)_C, SU(3)_L, U(1)_X$) as $U_R^{1,2,3} : (3^*, 1, 2/3)$ and $D_R^{1,2,3} : (3^*, 1, -1/3)$, respectively. In addition, we see that the model has the following heavy fermions: a single flavor quark T with electric charge $2/3$, two flavor quarks $J^{1,2}$ with charge $-1/3$. The right-handed sector of the exotic quarks transforms as $T_R : (3^*, 1, 2/3)$ and $J_R^{1,2} : (3^*, 1, -1/3)$. In the concerning to the lepton sector, we have three right-handed charged leptons $e_R^{1,2,3} : (1, 1, -1)$ and three right-handed Majorana leptons $N_R^{1,2,3} : (1, 1, 0)$ (recently, a discussion as regards neutrino masses via double and inverse see-saw mechanism was perform in Refs. [103, 104]).

The scalar sector of the 331 model includes three 3's irreps of $SU(3)_L$, where one triplet χ acquires a vacuum expectation value (VEV) at high energy scale, v_χ , responsible for the breaking of the $SU(3)_L \times U(1)_X$ symmetry down to the $SU(2)_L \times U(1)_Y$ electroweak group of the SM; and two light triplet fields η and ρ get VEVs v_η and v_ρ , respectively, at the electroweak scale and give mass to the fermion and gauge sector. In addition to the aforementioned scalar spectrum, we introduce six $SU(3)_L$ scalar singlets, namely, $\xi_1, \xi_2, \zeta_1,$

χ_2 , σ , and τ . Their role and importance will be explained later in this section.

The $[SU(3)_L, U(1)_X]$ group structure of the scalar fields of our model is

$$\begin{aligned} \chi &= \left(\begin{array}{c} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\nu_\chi + \xi_\chi \pm i\zeta_\chi) \end{array} \right) : (3, -1/3), \xi_1 : (1, 0), \\ \rho &= \left(\begin{array}{c} \rho_1^+ \\ \frac{1}{\sqrt{2}}(\nu_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{array} \right) : (3, 2/3), \xi_2 : (1, 0), \\ \eta &= \left(\begin{array}{c} \frac{1}{\sqrt{2}}(\nu_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{array} \right) : (3, -1/3), \sigma : (1, 0), \\ \tau &: (1, 0), \zeta_1 : (1, 0), \zeta_2 : (1, 0). \end{aligned} \tag{2.4}$$

We group the scalar fields into doublet and singlet representations of S_3 . The $S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12}$ assignments of the scalar fields are

$$\begin{aligned} \Phi &= (\eta, \chi) \sim (\mathbf{2}, 1, 1, 1), \quad \rho \sim (\mathbf{1}', 1, 1, 1), \\ \sigma &\sim (\mathbf{1}, -1, i, 1), \quad \tau \sim (\mathbf{1}, 1, 1, \omega^{-\frac{1}{4}}), \\ \xi &= (\xi_1, \xi_2) \sim (\mathbf{2}, -1, 1, 1), \end{aligned} \tag{2.5}$$

$$\zeta = (\zeta_1, \zeta_2) \sim (\mathbf{2}, -1, i, 1). \tag{2.6}$$

where $\omega = e^{2\pi i/3}$.

Regarding the quark sector, we assign the quark fields in trivial and non-trivial singlet representations of S_3 . We assumed that all left-handed quarks and right-handed quarks are assigned to S_3 trivial singlets excepting, $U_R^1, U_R^2, U_R^3, T_R, D_R^3, J_R^1$, and J_R^2 , which are assumed to be non-trivial singlets. The quark assignments under $S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12}$ are

$$\begin{aligned} Q_L^1 &\sim (\mathbf{1}, 1, 1, \omega^{-\frac{1}{2}}), \quad Q_L^2 \sim (\mathbf{1}, 1, 1, \omega^{-\frac{1}{4}}), \\ Q_L^3 &\sim (\mathbf{1}, 1, 1, 1), \quad U_R^1 \sim (\mathbf{1}', 1, -1, \omega), \\ U_R^2 &\sim (\mathbf{1}', 1, -1, \omega^{\frac{1}{4}}), \quad U_R^3 \sim (\mathbf{1}', -1, -i, 1), \\ D_R^1 &\sim (\mathbf{1}, -1, 1, \omega), \quad D_R^2 \sim (\mathbf{1}, -1, 1, i), \\ D_R^3 &\sim (\mathbf{1}', 1, 1, i), \quad T_R \sim (\mathbf{1}', -1, 1, 1), \\ J_R^1 &\sim (\mathbf{1}', -1, 1, \omega^{-\frac{1}{2}}), \quad J_R^2 \sim (\mathbf{1}', -1, 1, \omega^{-\frac{1}{4}}). \end{aligned} \tag{2.7}$$

With the above particle content, the following relevant Yukawa terms for the quark sector arise:

$$\begin{aligned} -\mathcal{L}_Y &= y_{11}^{(U)} \bar{Q}_L \rho^* U_R^1 \frac{\sigma^2 \tau^6}{\Lambda^8} + y_{22}^{(U)} \bar{Q}_L \rho^* U_R^2 \frac{\sigma^2 \tau^2}{\Lambda^4} \\ &+ y_{13}^{(U)} \bar{Q}_L \rho^* U_R^3 \frac{\sigma \tau^2}{\Lambda^3} + y_{23}^{(U)} \bar{Q}_L \rho^* U_R^3 \frac{\sigma \tau}{\Lambda^2} \\ &+ y_{33}^{(U)} \bar{Q}_L \Phi U_R^3 \frac{\zeta}{\Lambda} + y^{(T)} \bar{Q}_L \Phi T_R \frac{\xi}{\Lambda} \end{aligned}$$

$$\begin{aligned} &+ y_{11}^{(D)} \bar{Q}_L \Phi^* D_R^1 \frac{\xi \tau^6}{\Lambda^7} + y_{12}^{(D)} \bar{Q}_L \Phi^* D_R^2 \frac{\xi \tau^5}{\Lambda^6} \\ &+ y_{22}^{(D)} \bar{Q}_L \Phi^* D_R^2 \frac{\xi \tau^4}{\Lambda^5} + y_{21}^{(D)} \bar{Q}_L \Phi^* D_R^1 \frac{\xi \tau^5}{\Lambda^6} \\ &+ y_{33}^{(D)} \bar{Q}_L \rho D_R^3 \frac{\tau^3}{\Lambda^3} + y_1^{(J)} \bar{Q}_L \Phi^* J_R^1 \frac{\xi}{\Lambda} \\ &+ y_2^{(J)} \bar{Q}_L \Phi^* J_R^2 \frac{\xi}{\Lambda} \end{aligned} \tag{2.8}$$

where the dimensionless couplings $y_{ij}^{(U,D)}$ ($i, j = 1, 2, 3$), $y^{(T)}$, $y_{1,2}^{(J)}$ are $\mathcal{O}(1)$ parameters.

To explain the fermion mass hierarchy it is necessary to assume an ansatz for the Yukawa matrices. A candidate for generating specific Yukawa textures is the $S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12}$ discrete group that can explain the prevailing pattern of fermion masses and mixing. The S_3 discrete symmetry is the smallest non-Abelian discrete symmetry group having three irreducible representations (irreps), explicitly two singlets and one doublet irreps. The Z_2 symmetry determines the allowed Yukawa terms for the quark sector, thus resulting in a reduction of model parameters and allowing one to decouple the bottom quark from the light down and strange quarks. The Z_4 and Z_{12} symmetries shape the hierarchical structure of the quark mass matrices that yields a realistic pattern of quark masses and mixing. It is noteworthy that the properties of the Z_N groups imply that the Z_4 and Z_{12} symmetries are the lowest cyclic symmetries that allow one to build Yukawa terms of dimensions six and ten, respectively. Consequently, the $Z_4 \otimes Z_{12}$ symmetry is the lowest cyclic symmetry from which a 12 dimensional Yukawa term can be built, crucial to get the required λ^8 suppression in the 11 entry of the up-type quark mass matrix, where $\lambda = 0.225$ is one of the Wolfenstein parameters. Furthermore, thanks to the $Z_4 \otimes Z_{12}$ symmetry, the lowest down-type quark Yukawa term contributing to the 11 entry of the down-type quark mass matrix has dimension 11. Thus, the Z_4 and Z_{12} symmetries are crucial to explain the smallness of the up and down quark masses.

We assume the following VEV pattern for the $SU(3)_L$ singlet scalar fields:

$$\begin{aligned} \langle \xi \rangle &= v_\xi (1, 0), \quad \langle \tau \rangle = v_\tau \\ \langle \zeta \rangle &= v_\zeta (0, 1), \quad \langle \sigma \rangle = v_\sigma, \end{aligned} \tag{2.9}$$

i.e. the VEVs of ξ and ζ are aligned as $(1, 0)$ and $(0, 1)$ in the S_3 directions, respectively. Besides that, the $SU(3)_L$ scalar singlets, ξ_1, ζ_2, σ , and τ , are assumed to acquire VEVs at a scale Λ_{int} much larger than v_χ in order to break the $\mathcal{G} = SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12}$ symmetry group down to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$. Let us note that the S_3 doublet $SU(3)_L$ singlet scalars ξ and ζ are the only scalar fields odd under the Z_2 symmetry. Furthermore the only scalar fields charged under the Z_4 and

Z_{12} symmetries are the $SU(3)_L$ singlet scalars σ , ζ , and τ , respectively. Thus, the breaking of the Z_2 , Z_4 , and Z_{12} symmetries is caused by the scalar fields ξ , (σ, ζ) , and τ , respectively, acquiring VEVs at a very high scale. It is worth mentioning that we have chosen a VEV patterns for the S_3 doublets $SU(3)_L$ singlet scalar ξ and ζ , in the $(1, 0)$ and $(0, 1)$ S_3 directions, respectively, as indicated by Eq. (2.9), in order to decouple the heavy exotic quarks from the SM quarks. Due to the aforementioned choice of the VEV pattern of ξ , only the $SU(3)_L$ scalar triplet χ participates in the Yukawa interactions giving masses to the exotic T , J^1 , and J^2 quarks. Furthermore, the masses of the SM quarks will arise from the Yukawa terms involving the $SU(3)_L$ scalar triplets η and ρ .

Considering that the quark mass and mixing pattern arises from the Z_4 and Z_{12} symmetries, and in order to relate the quark masses with the quark mixing parameters, we set the VEVs of the $SU(3)_L$ singlet scalar fields excepting v_ζ as follows:

$$v_\xi = v_\tau = v_\sigma = \Lambda_{\text{int}} = \lambda\Lambda, \tag{2.10}$$

where $\lambda = 0.225$ is one of the parameters of the Wolfenstein parametrization and Λ is the cutoff of our model.

To reproduce the right value of the top quark mass while keeping $y_{33}^{(U)} \sim \mathcal{O}(1) \lesssim \sqrt{4\pi}$ as required by perturbativity, we set v_ζ in the following range:

$$\frac{\sqrt{2}m_t}{\sqrt{4\pi}v_\eta}\Lambda < v_\zeta < \Lambda. \tag{2.11}$$

3 Quark masses and mixing

Using Eq. (2.8) and considering that the VEV pattern of the $SU(3)_L$ singlet scalar fields satisfies Eq. (2.9) with the non-vanishing VEVs set to be equal to $\lambda\Lambda$ (Λ being the cutoff of our model) as indicated by Eq. (2.10), we find that the SM quarks do not mix with the heavy exotic quarks and that the mass matrices for up- and down-type SM quarks are

$$M_U = \begin{pmatrix} a_{11}^{(U)}\lambda^8 & 0 & a_{13}^{(U)}\lambda^3 \\ 0 & a_{22}^{(U)}\lambda^4 & a_{23}^{(U)}\lambda^2 \\ 0 & 0 & a_{33}^{(U)} \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{3.1}$$

$$M_D = \begin{pmatrix} a_{11}^{(D)}\lambda^7 & a_{12}^{(D)}\lambda^6 & 0 \\ a_{21}^{(D)}\lambda^6 & a_{22}^{(D)}\lambda^5 & 0 \\ 0 & 0 & a_{33}^{(D)}\lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}},$$

where $\lambda = 0.225$ is one of the Wolfenstein parameters, $v = 246$ GeV the symmetry breaking scale, and $a_{ij}^{(U,D)}$ ($i, j = 1, 2, 3$) are $\mathcal{O}(1)$ parameters. From the SM quark mass matrix textures given by Eq. (3.1), it follows that the Cabibbo mixing arises from the down-type quark sector

whereas the up quark sector generates the remaining quark mixing angles. The $\mathcal{O}(1)$ dimensionless couplings $a_{ij}^{(U,D)}$ ($i, j = 1, 2, 3$) in Eq. (3.1) are given by the following relations:

$$a_{11}^{(U)} = y_{11}^{(U)} \frac{v_\rho}{v}, \quad a_{22}^{(U)} = y_{11}^{(U)} \frac{v_\rho}{v}, \quad a_{13}^{(U)} = y_{13}^{(U)} \frac{v_\rho}{v},$$

$$a_{23}^{(U)} = y_{23}^{(U)} \frac{v_\rho}{v}, \quad a_{33}^{(U)} = y_{33}^{(U)} \frac{v_\zeta v_\eta}{v\Lambda},$$

$$a_{11}^{(D)} = y_{11}^{(D)} \frac{v_\eta}{v}, \quad a_{12}^{(D)} = y_{12}^{(D)} \frac{v_\eta}{v}, \quad a_{21}^{(D)} = y_{21}^{(D)} \frac{v_\eta}{v},$$

$$a_{22}^{(D)} = y_{22}^{(D)} \frac{v_\eta}{v}, \quad a_{33}^{(D)} = y_{33}^{(D)} \frac{v_\rho}{v}. \tag{3.2}$$

Furthermore, we find that the exotic quark masses are

$$m_T = \lambda y^{(T)} \frac{v_\chi}{\sqrt{2}},$$

$$m_{J^{1,2}} = \lambda y_{1,2}^{(J)} \frac{v_\chi}{\sqrt{2}} = \frac{y_{1,2}^{(J)}}{y^{(T)}} m_T. \tag{3.3}$$

From Eq. (3.1) we find that the up- and down-type SM quark masses are approximately given by

$$m_u \simeq a_{11}^{(U)} \lambda^8 \frac{v}{\sqrt{2}}, \quad m_c \simeq a_{22}^{(U)} \lambda^4 \frac{v}{\sqrt{2}},$$

$$m_t \simeq a_{33}^{(U)} \frac{v}{\sqrt{2}},$$

$$m_d \simeq \left| a_{11}^{(D)} a_{22}^{(D)} - a_{12}^{(D)} a_{21}^{(D)} \right| \lambda^7 \frac{v}{\sqrt{2}},$$

$$m_s \simeq a_{22}^{(D)} \lambda^5 \frac{v}{\sqrt{2}}, \quad m_b \simeq a_{33}^{(D)} \lambda^3 \frac{v}{\sqrt{2}}. \tag{3.4}$$

We also find that the CKM quark mixing matrix is approximately given by

$$V_{\text{CKM}} \simeq \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{i\delta}s_{13} \\ e^{-i\delta}c_{12}s_{13}s_{23} - c_{23}s_{12} & c_{12}c_{23} + e^{-i\delta}s_{12}s_{13}s_{23} & -c_{13}s_{23} \\ -s_{12}s_{23} - e^{-i\delta}c_{12}c_{23}s_{13} & c_{12}s_{23} - e^{-i\delta}c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}, \tag{3.5}$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ (with $i \neq j$ and $i, j = 1, 2, 3$), θ_{ij} and δ being the quark mixing angles and the CP violating phase, respectively. The quark mixing angles and the CP violating phase are given by

$$\tan \theta_{12} \simeq \frac{a_{12}^{(D)}}{a_{22}^{(D)}} \lambda, \quad \tan \theta_{23} \simeq \frac{a_{23}^{(U)}}{a_{33}^{(U)}} \lambda^2,$$

$$\tan \theta_{13} \simeq \frac{|a_{13}^{(U)}|}{a_{33}^{(U)}} \lambda^3, \quad \delta = -\arg \left(a_{13}^{(U)} \right). \tag{3.6}$$

Here we assume that the $\mathcal{O}(1)$ dimensionless couplings $a_{ij}^{(U,D)}$ ($i, j = 1, 2, 3$) in Eq. (3.1) are real except for $a_{13}^{(U)}$. It is noteworthy that Eqs. (3.4)–(3.6) give an elegant description of the SM quark masses and mixing angles in terms of the Wolfenstein parameter $\lambda = 0.225$ and of parameters of order unity. It is worth commenting that the observables in the

quark sector are connected with the electroweak symmetry breaking scale $v = 246$ GeV through their power dependence on the Wolfenstein parameter $\lambda = 0.225$, with $\mathcal{O}(1)$ coefficients.

The Wolfenstein parameterization [105] of the CKM matrix is given by

$$V_W \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (3.7)$$

with

$$\lambda = 0.22535 \pm 0.00065, \quad A = 0.811_{-0.012}^{+0.022}, \quad (3.8)$$

$$\bar{\rho} = 0.131_{-0.013}^{+0.026}, \quad \bar{\eta} = 0.345_{-0.014}^{+0.013}, \quad (3.9)$$

$$\bar{\rho} \simeq \rho \left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} \simeq \eta \left(1 - \frac{\lambda^2}{2}\right). \quad (3.10)$$

The comparison with Eq. (3.7) leads to the following relations:

$$\begin{aligned} a_{33}^{(U)} &\simeq 1, & a_{23}^{(U)} &\simeq 0.81, & a_{13}^{(U)} &\simeq -0.3e^{i\delta}, & \delta &= 67^\circ, \\ a_{22}^{(U)} &\simeq \frac{m_c}{\lambda^4 m_t} \simeq 1.43, & a_{11}^{(U)} &\simeq \frac{m_u}{\lambda^8 m_t} \simeq 1.27, \end{aligned} \quad (3.11)$$

then it follows that $a_{13}^{(U)}$ is required to be complex, as previously assumed and its magnitude is a bit smaller than the remaining $\mathcal{O}(1)$ coefficients.

Assuming that the hierarchy of the SM quark masses and quark mixing matrix elements arises from the Z_4 and Z_{12} symmetries, we set $a_{21}^{(D)} = a_{22}^{(D)}$. We fit the parameters $a_{ij}^{(D)}$ ($i \neq j$) in Eq. (3.1) to reproduce the down-type quark masses and quark mixing parameters. The results are shown in Table 1 for the following best-fit values:

$$a_{11}^{(D)} \simeq 0.84, \quad a_{12}^{(D)} \simeq 0.4, \quad a_{22}^{(D)} \simeq 0.57, \quad a_{33}^{(D)} \simeq 1.42. \quad (3.12)$$

The obtained quark masses and CKM parameters are consistent with the experimental data. The values of these observables as well as the quark masses together with the experimental data are shown in Table 1. The experimental values of the quark masses, which are given at the M_Z scale, have been taken from Ref. [106] (which are similar to those in [107]), whereas the experimental values of the CKM matrix elements and the Jarlskog invariant J are taken from Ref. [12]. As seen from Table 1, all observables in the quark sector are in excellent agreement with the experimental data, excepting $|V_{td}|$, which turns out to be larger by a factor ~ 1.3 than its corresponding experimental value, and naively deviated 8 sigma away from it.

Table 1 Model and experimental values of the quark masses and CKM parameters

Observable	Model value	Experimental value
m_u (MeV)	1.47	$1.45_{-0.45}^{+0.56}$
m_c (MeV)	641	635 ± 86
m_t (GeV)	172.2	$172.1 \pm 0.6 \pm 0.9$
m_d (MeV)	2.2	$2.9_{-0.4}^{+0.5}$
m_s (MeV)	60.0	$57.7_{-15.7}^{+16.8}$
m_b (GeV)	2.82	$2.82_{-0.04}^{+0.09}$
$ V_{ud} $	0.97419	0.97427 ± 0.00015
$ V_{us} $	0.22572	0.22534 ± 0.00065
$ V_{ub} $	0.00351	$0.00351_{-0.00014}^{+0.00015}$
$ V_{cd} $	0.22548	0.22520 ± 0.00065
$ V_{cs} $	0.97338	0.97344 ± 0.00016
$ V_{cb} $	0.0411	$0.0412_{-0.0005}^{+0.0011}$
$ V_{td} $	0.0110	$0.00867_{-0.00031}^{+0.00029}$
$ V_{ts} $	0.0398	$0.0404_{-0.0005}^{+0.0011}$
$ V_{tb} $	0.999147	$0.999146_{-0.000046}^{+0.000021}$
J	2.96×10^{-5}	$(2.96_{-0.16}^{+0.20}) \times 10^{-5}$
δ	68°	68°

4 The scalar potential

To build a $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3$ invariant scalar potential it is necessary to decompose the direct product of S_3 representations into irreducible S_3 representations. The S_3 group has three irreducible representations that can be characterized by their dimension, i.e., **2**, **1** and **1'**. With the multiplication rules of the S_3 group given in the appendix, we have to assign to the scalar fields in the S_3 irreps and build the corresponding scalar potential invariant under the symmetry group.

Since all singlet scalars acquire VEVs at a scale much larger than v_χ , they are very heavy and thus the mixing between these scalar singlets and the $SU(3)_L$ scalar triplets can be neglected. For the sake of simplicity we assume a CP scalar potential with only real couplings as done in Refs. [85–91, 108]. Then the renormalizable low energy scalar potential of the model is constructed with the S_3 doublet $\Phi = (\eta, \chi)$ and the non-trivial S_3 singlet ρ fields, in the way invariant under the group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3$. The renormalizable low energy scalar potential is given by

$$\begin{aligned} V_H &= \mu_\rho^2 (\rho^\dagger \rho) + \mu_\Phi^2 (\Phi^\dagger \Phi)_1 + \lambda_1 (\rho^\dagger \rho) (\rho^\dagger \rho) \\ &+ \lambda_2 (\Phi^\dagger \Phi)_1 (\Phi^\dagger \Phi)_1 + \lambda_3 (\Phi^\dagger \Phi)_{1'} (\Phi^\dagger \Phi)_{1'} \end{aligned}$$

$$\begin{aligned}
 & + \lambda_4 (\Phi^\dagger \Phi)_2 (\Phi^\dagger \Phi)_2 + \lambda_5 (\rho^\dagger \rho) (\Phi^\dagger \Phi)_1 \\
 & + \lambda_6 ((\rho^\dagger \Phi) (\Phi^\dagger \rho))_1 + f \left[\varepsilon^{ijk} (\Phi_i \Phi_j)_1 \rho_k + h.c. \right],
 \end{aligned}
 \tag{4.1}$$

where $\Phi_i = (\eta_i, \chi_i)$ is a S_3 doublet with $i = 1, 2, 3$.

The S_3 symmetry in the quadratic term of the scalar potential is softly broken because the vacuum expectation values of the scalar fields η and χ contained in the S_3 doublet Φ satisfy the hierarchy $v_\chi \gg v_\eta$.

Then we include the quadratic S_3 soft-breaking terms $(\mu_\eta^2 - \mu_\chi^2) (\eta^\dagger \eta)$ and $\mu_{\eta\chi}^2 (\chi^\dagger \eta) + h.c.$ as done in Refs. [103,104], and we use the S_3 multiplication rules to rewrite the low energy scalar potential as follows:

$$\begin{aligned}
 V_H = & \mu_\rho^2 (\rho^\dagger \rho) + \mu_\eta^2 (\eta^\dagger \eta) + \mu_\chi^2 (\chi^\dagger \chi) + \mu_{\eta\chi}^2 \left[(\chi^\dagger \eta) \right. \\
 & \left. + (\eta^\dagger \chi) \right] + \lambda_1 (\rho^\dagger \rho)^2 + (\lambda_2 + \lambda_4) \left[(\chi^\dagger \chi)^2 + (\eta^\dagger \eta)^2 \right] \\
 & + \lambda_5 \left[(\rho^\dagger \rho)(\chi^\dagger \chi) + (\rho^\dagger \rho)(\eta^\dagger \eta) \right] \\
 & + 2(\lambda_2 - \lambda_4) (\chi^\dagger \chi) (\eta^\dagger \eta) + 2(\lambda_4 - \lambda_3) (\chi^\dagger \eta) \\
 & \times (\eta^\dagger \chi) + \lambda_6 \left[(\chi^\dagger \rho) (\rho^\dagger \chi) + (\eta^\dagger \rho) (\rho^\dagger \eta) \right] \\
 & + (\lambda_3 + \lambda_4) \left[(\chi^\dagger \eta)^2 + (\eta^\dagger \chi)^2 \right] \\
 & + 2f \left(\varepsilon^{ijk} \eta_i \chi_j \rho_k + h.c. \right).
 \end{aligned}
 \tag{4.2}$$

It is noteworthy that the S_3 soft-breaking term $\mu_{\eta\chi}^2 (\chi^\dagger \eta) + h.c.$ does not play an important role for the minimization of the scalar potential, nor for the generation of the physical scalar masses Refs. [103,104].

From the previous expressions and from the scalar potential minimization conditions, the following relations are obtained:

$$\begin{aligned}
 -\mu_\chi^2 = & (\lambda_2 + \lambda_4) v_\chi^2 + \frac{\lambda_5}{2} v_\rho^2 + (\lambda_2 - \lambda_4) v_\eta^2 - \sqrt{2} \frac{f v_\rho v_\eta}{v_\chi}, \\
 -\mu_\eta^2 = & (\lambda_2 + \lambda_4) v_\eta^2 + \frac{\lambda_5}{2} v_\rho^2 + (\lambda_2 - \lambda_4) v_\chi^2 - \sqrt{2} \frac{f v_\chi v_\rho}{v_\eta}, \\
 -\mu_\rho^2 = & \lambda_1 v_\rho^2 + \frac{\lambda_5}{2} (v_\chi^2 + v_\eta^2) - \sqrt{2} \frac{f v_\chi v_\eta}{v_\rho}.
 \end{aligned}
 \tag{4.3}$$

Considering the quartic scalar couplings of the same order of magnitude, we find from the previous relations that the trilinear scalar coupling f has to be of the order of v_χ . Furthermore, from Eq. 4.3, we get the following relation:

$$\mu_\chi^2 - \mu_\eta^2 + \left(2\lambda_4 + \sqrt{2} \frac{v_\rho}{v_\eta} \right) (v_\chi^2 - v_\eta^2) = 0.
 \tag{4.4}$$

The previous relations imply that the negative quadratic couplings should satisfy $\mu_\chi^2 \sim -v_\chi^2$ and $\mu_\rho^2 \sim \mu_\eta^2 \sim -v_\rho^2 \sim -v_\eta^2 \sim -v^2$, being $v = 246$ GeV. Therefore, the negative

quadratic coupling for the $SU(3)_L$ scalar triplet χ is of the order of its squared VEV. The remaining negative quadratic couplings are of the order of the squared VEVs of the $SU(3)_L$ scalar triplets η and ρ .

From the low energy scalar potential given by Eq. (4.2), we find that the physical scalar fields at low energies have the following masses:

$$\begin{aligned}
 m_{h^0}^2 \simeq & \frac{[8\lambda_4 \lambda_5 v_\eta^2 v_\rho^2 + 16\lambda_2 \lambda_4 v_\eta^4 + (4\lambda_1 (\lambda_2 + \lambda_4) - \lambda_5^2) v_\rho^4]}{4(\lambda_2 + \lambda_4) (v_\eta^2 + v_\rho^2)}, \\
 m_{H_1^0}^2 \simeq & \frac{f v_\chi}{\sqrt{2}} \left(\frac{v_\rho}{v_\eta} + \frac{v_\eta}{v_\rho} \right), \\
 m_{A^0}^2 \simeq & \frac{f v_\chi}{\sqrt{2}} \left(\frac{v_\eta}{v_\rho} + \frac{v_\rho}{v_\eta} \right), \\
 m_{H_2^0}^2 = m_{\bar{H}_2^0}^2 \simeq & 2\lambda_4 v_\chi^2 + \sqrt{2} f v_\chi \frac{v_\rho}{v_\eta}, \\
 m_{H_3^0}^2 \simeq & (\lambda_2 + \lambda_4) v_\chi^2, \\
 m_{H_1^\pm}^2 \simeq & \sqrt{2} \left(\frac{v_\rho}{v_\eta} + \frac{v_\eta}{v_\rho} \right) f v_\chi, \\
 m_{H_2^\pm}^2 \simeq & \frac{\lambda_6}{2} v_\chi^2 + \sqrt{2} f v_\chi \frac{v_\eta}{v_\rho}, \\
 m_{G_1^0}^2 = m_{G_2^0}^2 = m_{\bar{G}_2^0}^2 = m_{G_3^0}^2 = m_{G_1^\pm}^2 = m_{G_2^\pm}^2 = & 0.
 \end{aligned}
 \tag{4.5}$$

It is noteworthy that the physical scalar spectrum at low energies of our model includes: four massive charged Higgs (H_1^\pm, H_2^\pm), one CP-odd Higgs (A^0), three neutral CP-even Higgs (h^0, H_1^0, H_3^0), and two neutral Higgs (H_2^0, \bar{H}_2^0) bosons. Here we identify the scalar h^0 with the SM-like 126 GeV Higgs boson observed at the LHC. Let us note that the neutral Goldstone bosons $G_1^0, G_3^0, G_2^0, \bar{G}_2^0$ are associated to the longitudinal components of the Z, Z', K^0 , and \bar{K}^0 gauge bosons, respectively. Besides that, the charged Goldstone bosons G_1^\pm and G_2^\pm are associated to the longitudinal components of the W^\pm and K^\pm gauge bosons, respectively [78,79,85–91].

In Appendix B we employ the method of Ref. [109] to show that the low energy scalar potential is stable when the following conditions are fulfilled:

$$\begin{aligned}
 & \lambda_1 > 0, \quad \lambda_2 > 0, \lambda_4 > 0, \lambda_6 > 0, f > 0, \\
 & \lambda_2 > \lambda_3, \quad \lambda_2 + \lambda_4 > 0, \lambda_5 + \lambda_6 > 0, \\
 & \lambda_1 (\lambda_2 + \lambda_4) > \lambda_5^2, \quad \lambda_5 + \lambda_6 > 2\sqrt{\lambda_1 (\lambda_2 + \lambda_4)}.
 \end{aligned}
 \tag{4.6}$$

5 Conclusions

In this paper we proposed a model based on the symmetry group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes S_3 \otimes Z_2 \otimes Z_4 \otimes Z_{12}$, which is an extension of the 331 model with $\beta = -\frac{1}{\sqrt{3}}$ of

Refs. [103,104]. Our model successfully accounts for the observed SM quark mass and mixing pattern. The S_3 and Z_2 symmetries are crucial for reducing the number of parameters in the Yukawa terms for the quark sector and decoupling the bottom quark from the light down and strange quarks. The observed hierarchy of the SM quark masses and quark mixing matrix elements arises from the Z_4 and Z_{12} symmetries, which are broken at a very high scale by the $SU(3)_L$ scalar singlets (σ, ζ) and τ , charged under these symmetries, respectively. The Cabibbo mixing arises from the down-type quark sector whereas the up quark sector generates the remaining mixing angles. The SM quark masses are generated from Yukawa terms involving the $SU(3)_L$ scalar triplets η and ρ , which acquire VEVs at the electroweak scale $v = 246$ GeV. On the other hand, the exotic quark masses arise from Yukawa terms involving the $SU(3)_L$ scalar triplet χ , which acquires a VEV at the TeV scale. The obtained values of the quark masses, the magnitudes of the CKM matrix elements, the CP violating phase, and the Jarlskog invariant are consistent with the experimental data. The complex phase responsible for CP violation in the quark sector has been assumed to come from a seven dimensional up-type quark Yukawa term.

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Appendix A: The product rules for S_3

The S_3 group has three irreducible representations that can be characterized by their dimension, i.e., $\mathbf{2}$, $\mathbf{1}$ and $\mathbf{1}'$. Considering two S_3 doublet representations (x_1, x_2) and (y_1, y_2) , the direct product can be decomposed as follows [38]:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 = (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} + \begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_2, \tag{A.1}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes (y)_{\mathbf{1}'} = \begin{pmatrix} -x_2 y \\ x_1 y \end{pmatrix}_2, \tag{A.2}$$

$$(x)_{\mathbf{1}'} \otimes (y)_{\mathbf{1}'} = (xy)_{\mathbf{1}}.$$

With these multiplication rules we have to assign to the scalar fields in the S_3 irreps and build the corresponding scalar potential invariant under the symmetry group.

Appendix B: Stability conditions of the low energy scalar potential

In this subsection we are going to determine the conditions required to have a stable scalar potential by following the method described in Ref. [109]. The gauge invariant and renormalizable low energy scalar potential as a function of the fields $\phi_1 = \chi$, $\phi_2 = \eta$, and $\phi_3 = \rho$ is a linear hermitian combination of the following terms:

$$\phi_i \phi_j, \quad \phi_i \phi_j \phi_k \phi_l \tag{B.1}$$

where $i, j, k, l = \phi_1, \phi_2,$ and ϕ_3 . To discuss the stability of the potential, its minimum, and its gauge invariance one can make the arrangement of the scalar fields by using 2×2 hermitian matrices as follows:

$$\begin{aligned} \tilde{K}_{(\phi_i \phi_j)} &= \begin{pmatrix} \phi_i^\dagger \phi_i & \phi_i^\dagger \phi_j \\ \phi_j^\dagger \phi_i & \phi_j^\dagger \phi_j \end{pmatrix}, \\ &= \frac{1}{2} \left(K_{0(\phi_i \phi_j)} 1_{2 \times 2} + K_{a(\phi_i \phi_j)} \sigma^a \right) \end{aligned} \tag{B.2}$$

where $(\phi_i \phi_j) = \rho \eta, \rho \chi, \eta \chi, \sigma^a$ ($a = 1, 2, 3$) are the Pauli matrices and $1_{2 \times 2}$ is the identity matrix. From the previous expressions one can build the following bilinear terms as functions of the scalar fields:

$$\begin{aligned} K_{0(\phi_i \phi_j)} &= \phi_i^\dagger \phi_i + \phi_j^\dagger \phi_j, \\ K_{a(\phi_i \phi_j)} &= \sum_{i,j} \left(\phi_i^\dagger \phi_j \right) \sigma_{ij}^a. \end{aligned} \tag{B.3}$$

The properties of the potential can be analyzed in terms of $K_{0(\phi_i \phi_j)}$ and $\mathbf{K}_{(\phi_i \phi_j)}$ with $\phi_i \phi_j = \rho \eta, \rho \chi, \eta \chi$ in the domain $K_0 \geq 0$ y $K_0^2 \geq \mathbf{K}^2$. Defining $\kappa = \mathbf{K}/K_0$ the potential can be written as

$$\begin{aligned} V &= V_2 + V_4, \\ V_2 &= \sum_{(\phi_i \phi_j)} K_{0(\phi_i \phi_j)} \mathbf{J}_{2(\phi_i \phi_j)}(\kappa), \\ \mathbf{J}_{2(\phi_i \phi_j)}(\kappa) &= \xi_{0(\phi_i \phi_j)} + \xi_{(\phi_i \phi_j)}^T \kappa_{(\phi_i \phi_j)}, \\ V_4 &= \sum_{(\phi_i \phi_j)} K_{0(\phi_i \phi_j)}^2 \mathbf{J}_{4(\phi_i \phi_j)}(\kappa), \\ \mathbf{J}_{4(\phi_i \phi_j)}(\kappa) &= \eta_{00(\phi_i \phi_j)} + 2\eta_{(\phi_i \phi_j)}^T \kappa_{(\phi_i \phi_j)} + \kappa_{(\phi_i \phi_j)}^T E_{(\phi_i \phi_j)} \kappa_{(\phi_i \phi_j)}, \end{aligned} \tag{B.4}$$

where $E_{(\phi_i \phi_j)}$ is a 3×3 matrix and the functions $J_{2(\phi_i \phi_j)}(\kappa)$ and $J_{4(\phi_i \phi_j)}(\kappa)$ are defined in the domain $|\kappa| \leq 1$. The stability of the scalar potential requires that it has to be bounded from below. The stability is determined from the behavior of V in the limit $K_0 \rightarrow \infty$, i.e.,

$$J_{4(\phi_i \phi_j)}(\kappa) \geq 0, \tag{B.5}$$

for all $|\kappa| \leq 1$. To impose $J_{4(\phi_i \phi_j)}(\kappa)$ to be positively defined it is enough to consider the values of all stationary points in

the domain $|\kappa| < 1$ and $|\kappa| = 1$. This results in a bound for $\eta_{00(\phi_i\phi_j)}$, $\eta_{0(\phi_i\phi_j)}$, and $E(\phi_i\phi_j)$, which parametrize the quartic terms of the potential included in V_4 .

For $|\kappa| < 1$ the stationary points should satisfy

$$E\kappa(\phi_i\phi_j) = -\eta(\phi_i\phi_j), \quad |\kappa| < 1. \quad (\text{B.6})$$

For the case where $\det E \neq 0$, the following relation is obtained:

$$J_{4(\phi_i\phi_j)}(\kappa)|_{est} = \eta_{00(\phi_i\phi_j)} - \eta_{(\phi_i\phi_j)}^T E^{-1} \eta(\phi_i\phi_j). \quad (\text{B.7})$$

For $|\kappa| = 1$ the stationary points are obtained from the function:

$$F_{4(\phi_i\phi_j)}(\kappa) = J_{4(\phi_i\phi_j)}(\kappa) + u(1 - \kappa^2), \quad (\text{B.8})$$

where u is a Lagrange multiplier that satisfies the following condition:

$$\begin{aligned} (E(\phi_i\phi_j) - u)\kappa &= -\eta(\phi_i\phi_j), \\ J_{4(\phi_i\phi_j)}(\kappa)|_{est} &= u + \eta_{00(\phi_i\phi_j)} \\ &\quad - \eta_{(\phi_i\phi_j)}^T (E(\phi_i\phi_j) - u)^{-1} \eta(\phi_i\phi_j). \end{aligned} \quad (\text{B.9})$$

The stationary points of $J_{4(\phi_i\phi_j)}(\kappa)$ for $|\kappa| \leq 1$ can be obtained from:

$$\begin{aligned} f_{(\phi_i\phi_j)}(u) &= J_{4(\phi_i\phi_j)}(\kappa)|_{est} > 0, \\ f'_{(\phi_i\phi_j)}(u) &> 0. \end{aligned} \quad (\text{B.10})$$

Considering that the quartic terms of the scalar potential are dominant when the vacuum expectation values of the scalar fields take large values, these terms will be the most relevant to analyze the stability of the scalar potential. Following the method described in Ref. [109], we proceed to rewrite the quartic terms of the scalar potential in terms of bilinear combinations of the scalar fields. To this end, the bilinear combinations of the scalar fields are included in the following matrices:

$$\begin{aligned} \tilde{K}_{\rho\eta} &= \begin{pmatrix} \rho^\dagger \rho & \eta^\dagger \rho \\ \rho^\dagger \eta & \eta^\dagger \eta \end{pmatrix} = \frac{1}{2} (K_{0(\rho\eta)} 1_{2 \times 2} + K_{a(\rho\eta)} \sigma^a), \\ \tilde{K}_{\rho\chi} &= \begin{pmatrix} \rho^\dagger \rho & \chi^\dagger \rho \\ \rho^\dagger \chi & \chi^\dagger \chi \end{pmatrix} = \frac{1}{2} (K_{0(\rho\chi)} 1_{2 \times 2} + K_{a(\rho\chi)} \sigma^a), \\ \tilde{K}_{\eta\chi} &= \begin{pmatrix} \eta^\dagger \eta & \chi^\dagger \eta \\ \eta^\dagger \chi & \chi^\dagger \chi \end{pmatrix} = \frac{1}{2} (K_{0(\eta\chi)} 1_{2 \times 2} + K_{a(\eta\chi)} \sigma^a), \end{aligned} \quad (\text{B.11})$$

where σ^a ($a = 1, 2, 3$) are the Pauli matrices and $1_{2 \times 2}$ is the 2×2 identity matrix. From the previous expressions, we find that the bilinear combinations of the scalar fields appearing in Eq. (B.11) are given by

$$\begin{aligned} K_{0(\rho\eta)} &= \rho^\dagger \rho + \eta^\dagger \eta, \quad K_{0(\rho\chi)} = \rho^\dagger \rho + \chi^\dagger \chi, \\ K_{0(\eta\chi)} &= \eta^\dagger \eta + \chi^\dagger \chi, \\ K_{a(\rho\eta)} &= (\rho^\dagger \rho) \sigma_{11}^a + (\eta^\dagger \eta) \sigma_{22}^a + (\rho^\dagger \eta) \sigma_{12}^a + (\eta^\dagger \rho) \sigma_{21}^a, \\ K_{a(\rho\chi)} &= (\rho^\dagger \rho) \sigma_{11}^a + (\chi^\dagger \chi) \sigma_{22}^a + (\rho^\dagger \chi) \sigma_{12}^a + (\chi^\dagger \rho) \sigma_{21}^a, \\ K_{a(\eta\chi)} &= (\eta^\dagger \eta) \sigma_{11}^a + (\chi^\dagger \chi) \sigma_{22}^a + (\eta^\dagger \chi) \sigma_{12}^a + (\chi^\dagger \eta) \sigma_{21}^a. \end{aligned} \quad (\text{B.12})$$

Since the stability of the scalar potential is determined from its quartic terms, the stationary solutions consistent with a stable scalar potential are described by the following functions:

$$\begin{aligned} f_{\rho\eta}(u) &= u + E_{00(\rho\eta)} - E_{a(\rho\eta)} (E_{(\rho\eta)} - u 1_{3 \times 3})_{ab}^{-1} E_{b(\rho\eta)}, \\ f_{\rho\chi}(u) &= u + E_{00(\rho\chi)} - E_{a(\rho\chi)} (E_{(\rho\chi)} - u 1_{3 \times 3})_{ab}^{-1} E_{b(\rho\chi)}, \\ f_{\eta\chi}(u) &= u + E_{00(\eta\chi)} - E_{a(\eta\chi)} (E_{(\eta\chi)} - u 1_{3 \times 3})_{ab}^{-1} E_{b(\eta\chi)}, \end{aligned} \quad (\text{B.13})$$

where, for the ρ and η fields, we have

$$\begin{aligned} E_{00(\rho\eta)} &= \frac{\lambda_1 + \lambda_2 + \lambda_4 + \lambda_5}{4}, \\ E_{a(\rho\eta)} &= \frac{\lambda_1 - \lambda_2 - \lambda_4}{4} \delta_{a3}, \\ E_{(\rho\eta)} &= \frac{1}{4} \begin{pmatrix} \lambda_6 & 0 & 0 \\ 0 & \lambda_6 & 0 \\ 0 & 0 & \lambda_1 + \lambda_2 + \lambda_4 - \lambda_5 \end{pmatrix}. \end{aligned} \quad (\text{B.14})$$

In the same manner, for the multiplets ρ and χ , the expressions are

$$\begin{aligned} E_{00(\rho\chi)} &= \frac{\lambda_1 + \lambda_2 + \lambda_4 + \lambda_5}{4}, \\ E_{a(\rho\chi)} &= \frac{\lambda_1 - \lambda_2 - \lambda_4}{4} \delta_{a3}, \\ E_{(\rho\chi)} &= \frac{1}{4} \begin{pmatrix} \lambda_6 & 0 & 0 \\ 0 & \lambda_6 & 0 \\ 0 & 0 & \lambda_1 + \lambda_2 + \lambda_4 - \lambda_5 \end{pmatrix}. \end{aligned} \quad (\text{B.15})$$

Similarly, for the η and χ fields, we find

$$E_{00(\eta\chi)} = \lambda_2, \quad E_{a(\eta\chi)} = 0, \quad E_{(\eta\chi)} = \begin{pmatrix} \lambda_4 & 0 & 0 \\ 0 & -\lambda_3 & 0 \\ 0 & 0 & \lambda_4 \end{pmatrix}. \quad (\text{B.16})$$

Following Ref. [109], we determine the stability of the scalar potential from the conditions:

$$f_{\rho\eta}(u) > 0, \quad f_{\rho\chi}(u) > 0, \quad f_{\eta\chi}(u) > 0. \quad (\text{B.17})$$

We use the theorem of stability of the scalar potential of Ref. [109] to determine the stability conditions of the scalar potential. To this end, the condition $f_{\rho\eta}(u) > 0$ is analyzed for the set of values of u which include the 0, (since $f'_{\rho\eta}(0) > 0$) the roots $u_{\rho\eta}^{(1)}$ and $u_{\rho\eta}^{(2)}$ of the equation $f'_{\rho\eta}(u) = 0$ and the eigenvalues $\tilde{E}_{(\rho\eta)}^{(a)}$ of the matrix $E_{(\rho\eta)}$ where $f_{\rho\eta}(\tilde{E}_{(\rho\eta)}^{(a)})$

is finite and $f'_{\rho\eta}(\tilde{E}_{(\rho\eta)}^{(a)}) \geq 0$. We proceed in a similar way when analyzing the conditions $f_{\rho\chi}(u) > 0$ and $f_{\eta\chi}(u) > 0$.

Therefore, the scalar potential is stable when the following conditions are fulfilled:

$$\begin{aligned} \lambda_1 > 0, \lambda_2 > 0, \lambda_6 > 0, \lambda_2 > \lambda_3, \\ \lambda_2 + \lambda_4 > 0, \lambda_5 + \lambda_6 > 2\sqrt{\lambda_1(\lambda_2 + \lambda_4)}. \end{aligned} \quad (\text{B.18})$$

From the minimization conditions of the scalar potential, the stability of the scalar potential and the Higgs masses we can find other restrictions for the quartic couplings of the scalar potentials. Having masses $m_{H_1^\pm}^2$, $m_{H_1^0}^2$, and $m_{H_1^0}^2$ to be positively defined requires the following condition:

$$f > 0. \quad (\text{B.19})$$

In the same manner, the conditions $\lambda_6 > 0$ y $\lambda_2 + \lambda_6 > 0$ guarantee that $m_{H_3^0}^2$ and $m_{H_2^\pm}^2$ are positively defined, respectively. From the expressions corresponding to the masses of the fields h^0 , H_2^0 y \tilde{H}_2^0 , it is necessary to impose additional conditions that guarantee that they are positively defined, i.e.,

$$\lambda_4 > 0, \quad \lambda_1(\lambda_2 + \lambda_4) > \lambda_5^2. \quad (\text{B.20})$$

Then we get

$$\lambda_5 + \lambda_6 > 0. \quad (\text{B.21})$$

Finally the stability conditions of the low energy scalar potential can be summarized in the following form:

$$\begin{aligned} \lambda_1 > 0, \lambda_2 > 0, \lambda_4 > 0, \lambda_6 > 0, f > 0, \\ \lambda_2 > \lambda_3, \lambda_2 + \lambda_4 > 0, \lambda_5 + \lambda_6 > 0, \\ \lambda_1(\lambda_2 + \lambda_4) > \lambda_5^2, \lambda_5 + \lambda_6 > 2\sqrt{\lambda_1(\lambda_2 + \lambda_4)}. \end{aligned} \quad (\text{B.22})$$

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