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Stability of regular energy density in Palatini $f(R)$ gravity

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Abstract The present work explores the effects of the three-parametric $f(R)$ model on the stability of the regular energy density of planar fluid configurations with the Palatini $f(R)$ formalism. For this purpose, we develop a link between the Weyl scalar and structural properties of the system by evaluating a couple of differential equations. We also see the effects of Palatini $f(R)$ terms in the formulation of structure scalars obtained by orthogonal splitting of the Riemann tensor in general relativity. We then identify the parameters which produce energy density irregularities in expansive and expansion-free dissipative as well as non-dissipative matter distributions. It is found that particular combinations of the matter variables lead to irregularities in an initially homogeneous fluid distribution. We conclude that Palatini $f(R)$ extra corrections tend to decrease the inhomogeneity, thereby imparting stability to the self-gravitating system.

1 Introduction

The phenomenon of accelerated cosmic expansion has been considered as a fundamental theme of relativistic astrophysics and modern cosmology. It is suggested that a mysterious energy characterizing the hidden features known as dark energy (DE) is responsible for this puzzling mechanism in the cosmos [1], whose vague nature is explored by modified gravity theories. The $f(R)$ gravity [2,3] obtained by replacing R with a non-linear function is the consequence of such an attempt, which is simple enough to understand many cosmic puzzles due to its high-energy corrections. The most challenging constituent of this theory is to present an $f(R)$ model that could explain cosmic expansion at both early- and late-time epochs without invoking a dark component.

The Palatini approach offers metric and connections as independent geometric quantities in the variational principle. The metric and Palatini formulations lead to the same gravitational dynamics in Einstein gravity but this is not the case if one considers such variations in modified gravitational theories. This formalism provides a platform to study novel phenomenological general relativity (GR) extensions to explain cosmic large scale structures and DE aspects [4–8]. Palatini $f(R)$ gravity leads to singularity-free field equations of second order [9], thus yielding a gravitational alternative for DE.

Among the well-known criteria for a viable theory of gravitation, one is the property of having a well-posed initial value problem (IVP). This leads to the problem of how the theory can have predictive powers for discussing gravitational dynamics. Like GR, $f(R)$ gravity is a gauge theory in which the initial value formulation relies on some acceptable conditions as well as on satisfactory gauge choices. This signifies that coordinates should be selected in such a way that it gives a well-formulated and possibly well-posed Cauchy problem. Some researchers [10,11] found well-posed IVPs by taking into account quadratic corrections in metric $f(R)$ gravity.

Trembley and Faraoni [12] used the dynamical equivalence between Palatini and metric generic $f(R)$ gravity theories and concluded that, in a metric formalism, IVPs are well posed in vacuo and well formulated in relativistic matter distributions. However, they raised an objection against well-formulated Cauchy problems in Palatini $f(R)$ gravity. Capozziello and Vignolo [13] commented on the question raised by [12] as regards the viability of metric-affine $f(R)$ gravity. They related this issue with the well posedness of the Cauchy problem and concluded that $f(R)$ gravity reduces to GR and thus the Cauchy problem is well posed and well formulated, indicating consistent $f(R)$ gravity. Faraoni [14] favored these arguments [13] and concluded that the statement that Palatini $f(R)$ gravity is ill formulated is incorrect.

Capozziello and Vignolo [15] identified $f(R)$ theory as a viable extension of GR through a debate on the Cauchy problem being well formulated and well posed. Moreover, Capozziello and Vignolo [16] found the vacuo Cauchy prob-

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lem in metric-affine $f(R)$ gravity to be well posed and well formulated by adopting Gaussian normal coordinates. They argued that it is possible to achieve well-formulated IVPs for several kinds of matter fields, like Klein–Gordon, perfect fluid, and Yang–Mills fields. Olmo and Alepuz [17] proved that one cannot get extra order time derivatives in evolution equations from Palatini $f(R)$ gravity. This yields well-formulated as well as well-posed Cauchy problems in this theory, thus presenting the Palatini formalism as a viable theory. Capozziello and Laurentis [18] reviewed several aspects of viability constraints on gravitation theories and proposed that the exploration of being well posed and well formulated is still a burning issue for any modified gravity theory.

There has been growing interest in the dynamical evolution of the collapsing self-gravitating systems modeled with homogeneous and inhomogeneous backgrounds in modified gravity theories [19–21]. It is well known that an extremely inhomogeneous initial state of stellar systems may trigger a collapsing mechanism. Asencio et al. [22] obtained exact analytical models for spherical self-gravitating systems in quadratic Palatini $f(R)$ gravity. Reverberi [23] studied the impact of a DE $f(R)$ model on a collapsing dust matter configuration through numerical techniques and concluded that the final outcome of the system depends upon the values of the energy/mass density ratios. Alavirad and Weller [24] studied the role of logarithmic $f(R)$ corrections on the structural evolution of stellar systems and found results consistent with binary star observations. We have investigated the effect of DE and matter $f(R)$ models on the relativistic compact objects with both metric [25–27] as well as Palatini [28,29] formalisms and found relatively stable matter distributions due to higher derivative dark source terms.

Arbuzova et al. [30] obtained models for a self-gravitating celestial matter distribution in $f(R)$ gravity and found a repulsive character of $f(R)$ corrections for huge relativistic systems. Roshan and Abbassi [31] explored the effects of extra curvature terms on the dynamical properties of compact objects coupled with a perfect fluid and claimed that modified gravity theories modify the structure-formation phenomenon at large scales. Guo et al. [32] discussed a spherical collapsing matter distribution in an Einstein frame with $f(R)$ models and found that during evolution most of the scalar field energy moves towards the system's central point. Astashenok et al. [33] studied the effects of $f(R)$ dark source terms on collapsing compact objects and obtained relatively more stable celestial distributions due to cubic $f(R)$ corrections.

The modeling of relativistic compact objects under various assumptions has received substantial interest in relativistic astrophysics. In this perspective, Skripkin [34] innovated the existence of a central core within adiabatic matter configurations. It so happened that if a system evolves by incorporating a zero expansion scalar then the innermost shell of the celestial body moves away from its central point, which leaves the system with a central vacuum core [35]. Herrera et al. [36] obtained zero expansion dust solutions with a spherical inhomogeneous background. Di Prisco et al. [37] found a Minkowskian cavity within the stellar collapsing self-gravitating systems due to the zero expansion scalar. We have found a vacuum core in non-adiabatic expansion-free celestial anisotropic configurations and concluded that the $f(R)$ high-energy degree of freedom is likely to host more massive stellar configurations [38–40].

Herrera et al. [41] used an orthogonal splitting of the Riemann tensor [42] to study the dynamical evolution of spherical collapse and presented a set of four structure scalars, i.e., Y_T, X_T, Y_{TF}, and X_{TF}. Furthermore, Herrera et al. [43] explored the consequences of the cosmological constant for radiating spherical relativistic collapse by evaluating the shear and expansion evolution equations. Sharif and Bhatti [44,45] extended their results for planar and cylindrical compact objects. Herrera [46] investigated the causes of density irregularities in radiating spherical stellar systems and explored some inhomogeneity factors. Recently, we have studied the dynamical properties of an adiabatic spherical self-gravitating collapsing system through modified structure scalars with positive and negative powers of Ricci scalar corrections [47,48].

This paper is devoted to a study of the stability of the regular energy density and explores irregularity factors for expansive and expansion-free planar radiating/non-radiating matter configurations with the Palatini $f(R)$ gravity model. The paper is organized as follows. We start our analysis by presenting the Palatini $f(R)$ formulation for planar spacetime in the next section. Section 3 provides a formulation of the modified structure scalars equipped with viable Palatini $f(R)$ degrees of freedom to present the conservation as well as the Ellis equations. In Sect. 4, we discuss a variety of collapsing compact objects to explore the density irregularity factors. Finally, we conclude and present our results in the last section.

2 $f(R)$ formalism

$f(R)$ gravity can be obtained through an extension in the gravitational component of the Einstein–Hilbert action [49]:

$$S_{f(R)} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_M,$$

where κ, $f(R)$ and S_M are the coupling constant, a non-linear Ricci function, and the matter action, respectively. Varying this action with respect to $g_{\alpha\beta}$ (metric) and $\Gamma^{\rho}_{\alpha\beta}$ (connection), we establish the following equations of motion:

$$f_R(\breve{R})\breve{R}_{\alpha\beta} - [g_{\alpha\beta}f(\breve{R})]/2 = \kappa T_{\alpha\beta}, \quad (1)$$

$$\breve{\nabla}_\mu(g^{\alpha\beta}\sqrt{-g}f_R(\breve{R})) = 0. \quad (2)$$

The correspondence between $R \equiv R(\Gamma)$ and $T \equiv g^{\alpha\beta}T_{\alpha\beta}$ can be obtained by taking the trace of Eq. (1) as follows:

$$Rf_R(R) - 2f(R) = \kappa T, \tag{3}$$

which also shows the Ricci algebraic expression as a function of T. To represent Palatini $f(R)$ gravity as a consistent classical theory of gravity, we restrict our study only to those instances for which roots of the above equation exist. Under the current constant cosmological value of the Ricci invariant, i.e., $R = \tilde{R}$, the above equation yields covariant conservation of metric and thereby fixes $\Gamma^{\rho}_{\alpha\beta}$ to the Levi-Civita quantity. Thus, for the vacuum case, Eq. (1) provides

$$\breve{R}_{\alpha\beta} - \Lambda(\tilde{R})g_{\alpha\beta} = 0, \tag{4}$$

where $\breve{R}_{\alpha\beta}$ is now the metric Ricci tensor of $g_{\alpha\beta}$ and $\Lambda(\tilde{R}) = \tilde{R}/4$. This theory would reduce to GR with and without the cosmological constant determined by the chosen $f(R)$ model. Solving Eq. (1) for $\Gamma^{\sigma}_{\alpha\beta}$, substituting it in Eq. (1), and expressing it by means of $g_{\alpha\beta}$, we have the following single formulation of the Palatini $f(R)$ field equation:

$$\frac{1}{f_R}\left(\breve{\nabla}_\alpha\breve{\nabla}_\beta - g_{\alpha\beta}\breve{\Box}\right)f_R + \frac{1}{2}g_{\alpha\beta}\breve{R} + \frac{\kappa}{f_R}T_{\alpha\beta} + \frac{1}{2}g_{\alpha\beta}\left(\frac{f}{f_R} - R\right) + \frac{3}{2f_R^2}\left[\frac{1}{2}g_{\alpha\beta}(\breve{\nabla}f_R)^2 - \breve{\nabla}_\mu f_R\breve{\nabla}_\beta f_R\right] - \breve{R}_{\alpha\beta} = 0, \tag{5}$$

which in an Einstein-like configuration can be written as

$$\breve{G}_{\alpha\beta} = \frac{\kappa}{f_R}(T_{\alpha\beta} + \mathcal{T}_{\alpha\beta}), \tag{6}$$

where

$$\mathcal{T}_{\alpha\beta} = \frac{1}{\kappa}\left(\breve{\nabla}_\alpha\breve{\nabla}_\beta - g_{\alpha\beta}\breve{\Box}\right)f_R + \frac{f_R}{2\kappa}g_{\alpha\beta}\left(\frac{f}{f_R} - R\right) + \frac{3}{2\kappa f_R}\left[\frac{1}{2}g_{\alpha\beta}(\breve{\nabla}f_R)^2 - \breve{\nabla}_\alpha f_R\breve{\nabla}_\beta f_R\right]$$

is the effective stress-energy tensor indicating the gravitational contribution due to Palatini $f(R)$ terms, $\breve{\Box} = \breve{\nabla}_\alpha\breve{\nabla}_\beta g^{\alpha\beta}$, $\breve{G}_{\alpha\beta} \equiv \breve{R}_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}\breve{R}$, while $\breve{\nabla}_\alpha$ represents the covariant derivative with respect to the Levi-Civita connection of the metric tensor. It is worth stressing that f_R and f are functions of $R(\Gamma) \equiv g^{\alpha\beta}R_{\alpha\beta}(\Gamma)$.

We consider a non-static plane symmetric spacetime [50,51],

$$ds_-^2 = -A^2(t,z)dt^2 + B^2(t,z)\left(dx^2 + dy^2\right) + C^2(t,z)dz^2, \tag{7}$$

filled with a radiating matter distribution and locally anisotropic pressure. Here the fluid configuration is dissipating as described by means of diffusion (heat) as well as free-streaming (null radiation) approximations. The corresponding stress-energy tensor is

$$T_{\alpha\beta} = (P_\perp + \mu)V_\alpha V_\beta + \varepsilon l_\alpha l_\beta + q_\beta V_\alpha + P_\perp g_{\alpha\beta} + q_\alpha V_\beta + (P_z - P_\perp)\chi_\alpha\chi_\beta, \tag{8}$$

where μ, ε, $P_\perp$, P_r and q_β are the energy density, radiation density, tangential as well as radial pressure components, and heat conducting vector, respectively. The radial four-vector, $\chi^\beta = \frac{1}{C}\delta^\beta_3$, the unit four-vector $l^\beta = \frac{1}{A}\delta^\beta_0 + \frac{1}{C}\delta^\beta_3$ and the fluid four-velocity $V^\beta = \frac{1}{A}\delta^\beta_0$ under comoving coordinates obey the following relations:

$$V^\alpha V_\alpha = -1, \quad \chi^\alpha\chi_\alpha = 1, \quad \chi^\alpha V_\alpha = 0,$$
$$V^\alpha q_\alpha = 0, \quad l^\alpha V_\alpha = -1, \quad l^\alpha l_\alpha = 0.$$

The scalar describing expansion rate of the matter distribution with Palatini $f(R)$ background is

$$\Theta_P = V_{\alpha;\beta}V^\beta = \frac{2}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{f}_R}{f_R} + \frac{\dot{C}}{2C}\right), \tag{9}$$

where a dot denotes the operator $\frac{\partial}{\partial t}$. The shear scalar for plane symmetric spacetime in GR becomes [50,51]

$$9\sigma^2 = \frac{9}{2}\sigma^{ab}\sigma_{ab} = W_{GR}^2, \quad \text{with} \quad W_{GR} = \frac{1}{A}\left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right). \tag{10}$$

Using Eqs. (9) and (10), we obtain

$$W_{GR} = \Theta_P - \frac{3\dot{C}}{AC} - \frac{2\dot{f}_R}{Af_R}. \tag{11}$$

The Palatini $f(R)$ field equations for Eq. (7) can be expressed as

$$\frac{\kappa}{f_R}\left[A^2(\mu+\varepsilon) - \frac{A^2}{\kappa}\left\{\frac{f_R'}{C^2}\left(\frac{C'}{C} + \frac{f_R'}{4f_R} - \frac{2B'}{B}\right) - \frac{f_R}{2}\times\left(R - \frac{f}{f_R}\right) - \frac{f_R''}{C^2} + \left(\frac{\dot{C}}{C} + \frac{9\dot{f}_R}{4f_R} + \frac{2\dot{B}}{B}\right)\frac{\dot{f}_R}{A^2}\right\}\right] = \left(\frac{\dot{B}}{B}\right)^2 + \frac{2\dot{C}\dot{B}}{CB} + \left\{\frac{B'}{B}\left(\frac{2C'}{C} - \frac{B'}{B}\right) - \frac{2C''}{C}\right\}\left(\frac{A}{C}\right)^2, \tag{12}$$

$$\frac{\kappa}{f_R}\left[CA(q+\varepsilon) - \frac{1}{\kappa}\left(\dot{f}_R' - \frac{5}{2}\frac{\dot{f}_R f_R'}{f_R} - \frac{\dot{C}f_R'}{C} - \frac{A'\dot{f}_R}{A}\right)\right] = 2\left(\frac{\dot{B}'}{B} - \frac{A'\dot{B}}{BA} - \frac{B'\dot{C}}{BC}\right), \tag{13}$$

$$\frac{\kappa}{f_R}\left[P_\perp B^2+\frac{B^2}{\kappa}\left\{\frac{\ddot{f}_R}{A^2}-\frac{f''_R}{C^2}+\left(\frac{\dot{B}}{B}-\frac{\dot{f}_R}{4f_R}-\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\right.\right.$$
$$\left.\left.\times\frac{\dot{f}_R}{A^2}-\frac{f_R}{2}\left(R-\frac{f}{f_R}\right)+\left(\frac{C'}{C}+\frac{f'_R}{4f_R}-\frac{B'}{B}-\frac{A'}{A}\right)\frac{f'_R}{C^2}\right\}\right]$$
$$=\left\{\frac{\dot{B}}{B}\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)-\frac{\ddot{C}}{C}+\frac{\dot{C}\dot{A}}{CA}-\frac{\ddot{B}}{B}\right\}$$
$$\times\frac{C^2}{A^2}+\left\{\frac{A'}{A}\left(\frac{B'}{B}-\frac{C'}{C}\right)+\frac{B''}{B}-\frac{B'C'}{BC}+\frac{A''}{A}\right\}\frac{B^2}{C^2}, \tag{14}$$

$$\frac{\kappa}{f_R}\left[C^2(P_z+\varepsilon)+\frac{C^2}{\kappa}\left\{\frac{\ddot{f}_R}{A^2}-\frac{f'_R}{C^2}\left(\frac{A'}{A}+\frac{9f'_R}{4f_R}+\frac{2B'}{B}\right)\right.\right.$$
$$\left.\left.-\frac{f_R}{2}\left(R-\frac{f}{f_R}\right)+\frac{\dot{f}_R}{A^2}+\left(\frac{2\dot{B}}{B}-\frac{\dot{f}_R}{4f_R}-\frac{\dot{A}}{A}\right)\right\}\right]$$
$$=\left\{\left(\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{B}}{B}-\frac{2\ddot{B}}{B}\right\}\frac{C^2}{A^2}+\frac{B'}{B}\left(\frac{B'}{B}+\frac{2A'}{A}\right), \tag{15}$$

where a prime represents the operator $\frac{\partial}{\partial r}$. The Taub mass function for a plane symmetric spacetime can be given as [52]

$$m(t,r)=\frac{(g)^{\frac{3}{2}}}{2}R_{12}{}^{12}=\frac{B}{2}\left(\frac{\dot{B}^2}{A^2}-\frac{B'^2}{C^2}\right), \tag{16}$$

whose temporal and radial mass variations, after using Eqs. (12)–(14), turn out to be

$$D_T m=-\frac{\kappa}{2f_R}\left\{U\left(\hat{P}_r+\frac{\mathcal{T}_{11}}{B^2}\right)+E\left(\hat{q}-\frac{\mathcal{T}_{01}}{AB}\right)\right\}B^2, \tag{17}$$

$$D_B m=\frac{\kappa}{2f_R}\left\{\hat{\mu}+\frac{\mathcal{T}_{00}}{A^2}+\frac{U}{E}\left(\hat{q}-\frac{\mathcal{T}_{01}}{CA}\right)\right\}B^2, \tag{18}$$

where U is the collapsing matter velocity defined by means of proper derivative operators ($D_T=\frac{1}{A}\frac{\partial}{\partial t}$) as $U=D_T B$, $\hat{P}_r=P_r+\varepsilon$, $\hat{q}=q+\varepsilon$, $\hat{\mu}=\mu+\varepsilon$, while $D_B=\frac{1}{B'}\frac{\partial}{\partial r}$ indicates the radial derivative operator. Here we take U to be less than unity.

In view of the fluid velocity, the mass function can be expressed as

$$E\equiv\frac{B'}{C}=\sqrt{U^2-\frac{2m(t,r)}{B}}. \tag{19}$$

The correspondence between the mass function and the matter parameters with Palatini $f(R)$ extra curvature terms can be found through integration of Eq. (18) as

$$\frac{3m}{B^3}=\frac{3\kappa}{2B^3}\int_0^z\left[\frac{1}{f_R}\left\{\hat{\mu}+\frac{\mathcal{T}_{00}}{A^2}+\left(\hat{q}-\frac{\mathcal{T}_{01}}{CA}\right)\frac{U}{E}\right\}B^2B'\right]dz. \tag{20}$$

The electric portion of the Weyl tensor by means of radial four-vector and unit four-velocity is given as

$$E_{\alpha\beta}=\mathcal{E}\left[\chi_\alpha\chi_\beta-\frac{1}{3}(g_{\alpha\beta}+V_\alpha V_\beta)\right],$$

where

$$\mathcal{E}=\left[\frac{\ddot{B}}{B}+\left(\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right)-\frac{\ddot{C}}{C}\right]\frac{1}{A^2}$$
$$-\left[\frac{C''}{C}-\left(\frac{C'}{C}+\frac{B'}{B}\right)\left(\frac{B'}{B}-\frac{A'}{A}\right)-\frac{A''}{A}\right]\frac{1}{C^2}, \tag{21}$$

is the Weyl scalar encapsulating the spacetime curvature effects. It can be written, after using Eqs. (12) and (14)–(16), as

$$\frac{3m}{B^3}=\frac{\kappa}{2f_R}\left(\hat{\mu}-\hat{\Pi}+\frac{\mathcal{T}_{00}}{A^2}-\frac{\mathcal{T}_{33}}{C^2}+\frac{\mathcal{T}_{11}}{B^2}\right)-\mathcal{E}, \tag{22}$$

where $\hat{\Pi}=\hat{P}_z-P_\perp$. This equation expresses the gravitational contribution of the plane symmetric line element with its mass function, fluid variables, and $f(R)$ higher curvature terms.

3 Structure scalars and Ellis equations

In this section, we construct modified structure functions after discussing a consistent $f(R)$ model. We then study the correspondence between Weyl scalar and other fluid parameters with Palatini $f(R)$ corrections by evaluating the modified Ellis equations. We take the three-parametric form of the $f(R)$ model [53,54],

$$f(R)=R+\lambda R_c\left[1-\left(1+\frac{R^2}{R_c^2}\right)^{-n}\right], \tag{23}$$

where λ and n belong to the set of positive real numbers, while R_c is also a constant entity with values of the order of the present effective Ricci scalar. This model provides a null contribution at $R=0$, which indicates the absence of the cosmological constant in flat spacetime. For $R=\text{constant}=\tilde{R}=R_0x_1$ with $x_1>0$, one can attain the de Sitter model by specifying the value of λ as

$$\lambda=\frac{(x_1^2+1)^{n+1}x_1}{2[(x_1^2+1)^{n+1}-(n+1)x_1^2-1]}.$$

Moreover, instead of assuming a specific choice of λ, one can get this value by first taking a particular value of x_1 and then calculating the value of λ. It is observed from the above equation that $x_1<2\lambda$ which leads to $\Lambda(R_1)=\frac{R_1}{4}<\Lambda(\infty)$ at the de Sitter point. Also, by keeping a fixed value of x_1 with $n \gg 1$ and by fixing n with $x_1 \gg 1$, one can have $x_1 \to 2\lambda$. In this scenario, cosmic evolution favors the results of the ΛCDM model. The dynamics induced by Einstein gravity can be achieved by taking the limit $f(R) \to R$.

To see effects of $f(R)$ higher curvature terms in the formulation of structure scalars, we take GR explicit expressions of $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ (developed by orthogonal splitting of Riemann tensor) [41,42]:

$$X_{\alpha\beta} = {}^*R^*_{\alpha\mu\beta\nu}V^\mu V^\nu = \frac{1}{2}\eta^{\epsilon\rho}{}_{\alpha\mu}R^*_{\epsilon\rho\beta\nu}V^\mu V^\nu,$$
$$Y_{\alpha\beta} = R_{\alpha\mu\beta\nu}V^\mu V^\nu,$$

where right, left and double dual of the Riemann tensor can, respectively, be written in a standard form as

$$R^*_{\alpha\beta\gamma\delta} \equiv \frac{1}{2}\eta_{\varepsilon\rho\gamma\delta}R^{\epsilon\rho}{}_{\alpha\beta}, \quad {}^*R_{\alpha\beta\gamma\delta} \equiv \frac{1}{2}\eta_{\alpha\beta\varepsilon\rho}R^{\varepsilon\rho}{}_{\gamma\delta},$$
$${}^*R^*_{\alpha\beta\gamma\delta} \equiv \frac{1}{2}\eta_{\alpha\beta}{}^{\varepsilon\rho}R^*_{\varepsilon\rho\gamma\delta}.$$

Using trace and trace-less components, the above relations can be recast as

$$X_{\alpha\beta} = \frac{1}{3}X_T h_{\alpha\beta} + X_{TF}\left(\chi_\alpha\chi_\beta - \frac{1}{3}h_{\alpha\beta}\right), \quad (24)$$
$$Y_{\alpha\beta} = \frac{1}{3}Y_T h_{\alpha\beta} + Y_{TF}\left(\chi_\alpha\chi_\beta - \frac{1}{3}h_{\alpha\beta}\right). \quad (25)$$

We can obtain the following set of scalar functions after using Eqs. (12), (14), (15), and (23)–(24):

$$X_T = \frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{R_c(R_c^2+\tilde{R}^2)^{(n+1)} - 2n\lambda\tilde{R}R_c^{(2n+2)}} \times\left(\hat{\mu}+\frac{\delta_\mu}{A^2}\right), \quad (26)$$

$$X_{TF} = -\mathcal{E} - \frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)} - 2n\lambda\tilde{R}R_c^{(2n+2)}]} \times\left(\hat{\Pi} - 2W\eta + \frac{\delta_{P_z}}{C^2} - \frac{\delta_{P_\perp}}{B^2}\right), \quad (27)$$

$$Y_T = \frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)} - 2n\lambda\tilde{R}R_c^{(2n+2)}]} \times\left(\hat{\mu}+\frac{\delta_\mu}{A^2}+\frac{\delta_{P_z}}{C^2}+\frac{2\delta_{P_\perp}}{B^2}+3\hat{P}_r - 2\hat{\Pi}\right), \quad (28)$$

$$Y_{TF} = \mathcal{E} - \frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)} - 2n\lambda\tilde{R}R_c^{(2n+2)}]} \times\left(\hat{\Pi} - 2\eta W + \frac{\delta_{P_z}}{C^2} - \frac{\delta_{P_\perp}}{B^2}\right), \quad (29)$$

where δ_μ, δ_{P_z} and $\delta_{P_\perp}$ are given in Appendix A. It is seen from the above relations that the scalar function X_T is involved in defining the energy density of the planar system with the extra degrees of freedom due to $f(R)$. The rest of the structure scalars do have a close relevance in the investigation of the structure and evolution of relativistic self-gravitating planar dissipative systems. It has been shown that Y_{TF} controls the stability of the shear-free condition in the geodesic case [55].

The independent components of the contracted Bianchi identities for a non-static planar system with effective and ordinary stress-energy tensors are

$$\left(T^{\alpha\beta} + \overset{(D)}{T^{\alpha\beta}}\right)_{;\beta} = 0, \quad \left(T^{\alpha\beta} + \overset{(D)}{T^{\alpha\beta}}\right)_{;\beta} = 0,$$

which yield

$$\frac{\dot{\hat{\mu}}}{A} + \frac{\hat{q}'}{C} + \frac{1}{A}\left(\frac{\dot{C}}{C}+\frac{\dot{f}_R}{2f_R}\right)(\hat{P}_z+\mu) + \frac{1}{A}(\mu+P_\perp) \times\left(\frac{2\dot{B}}{B}+\frac{\dot{f}_R}{f_R}\right) + \frac{\mu\dot{f}_R}{Af_R} + \frac{\hat{q}}{C}\left(\frac{2A'}{A}+\frac{3f_R'}{f_R}+\frac{2B'}{B}\right) + \frac{D_0(t,r)}{\kappa} = 0, \quad (30)$$

$$\frac{\dot{\hat{q}}}{C} + \frac{\hat{P}_z{}'}{C} + \frac{1}{C}\left(\frac{A'}{A}+\frac{f_R'}{2f_R}\right)(\mu+\hat{P}_z) + \left(\frac{2B'}{B}+\frac{f_R'}{f_R}\right)(\hat{P}_z - P_\perp)\frac{1}{C} + \frac{\hat{P}_z f_R'}{Cf_R} + \frac{\hat{q}}{A}\left(\frac{2\dot{C}}{C}+\frac{3\dot{f}_R}{f_R}+\frac{2\dot{B}}{B}\right) + \frac{D_1(t,r)}{\kappa} = 0, \quad (31)$$

where the terms D_0 and D_1 arise due to Palatini $f(R)$ gravity and are mentioned in Appendix A. Now we evaluate the set of modified equations after the pioneering work of Ellis [56] which provide a peculiar link between the Weyl tensor and the fluid parameters along with Palatini $f(R)$ extra curvature terms. These equations are found by using Eqs. (12)–(15), (17), (18), and (23) as

$$\left[\mathcal{E} - \frac{\kappa R_c(R_c^2+R^2)^{(n+1)}}{2[R_c(R_c^2+R^2)^{(n+1)} - 2n\lambda RR_c^{(2n+2)}]} \times\left(\hat{\mu} - \hat{\Pi} + \frac{\delta_\mu}{A^2} - \frac{\delta_{P_z}}{C^2} + \frac{\delta_{P_\perp}}{B^2}\right)\right]_{,0} = \frac{3\dot{B}}{B}\left[\frac{\kappa R_c(R_c^2+R^2)^{(n+1)}}{2[R_c(R_c^2+R^2)^{(n+1)} - 2n\lambda RR_c^{(2n+2)}]} \times\left(\hat{\mu} + P_\perp + \frac{\delta_\mu}{A^2} + \frac{\delta_{P_\perp}}{B^2}\right) - \mathcal{E}\right] + \frac{6\kappa\epsilon R}{4\epsilon R(1+2\epsilon R)+\lambda_n(2\epsilon R)^n}\left(\frac{AC'}{BC}\right)\left(\hat{q} - \frac{\varphi_q}{AB}\right), \quad (32)$$

$$\left[\mathcal{E} - \frac{\kappa R_c(R_c^2+R^2)^{(n+1)}}{2[R_c(R_c^2+R^2)^{(n+1)} - 2n\lambda RR_c^{(2n+2)}]} \times\left(\hat{\mu} - \hat{\Pi} + \frac{\delta_\mu}{A^2} - \frac{\delta_{P_z}}{C^2} + \frac{\delta_{P_\perp}}{B^2}\right)\right]_{,1}$$

$$= -\frac{3B'}{B}\left[\frac{\kappa R_c(R_c^2+R^2)^{(n+1)}}{2[R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{(2n+2)}]}\right.$$
$$\left.\times\left(\hat{\mu}+\frac{\delta_\mu}{A^2}\right)-\frac{3m}{B^3}\right]$$
$$-\frac{3\kappa R_c(R_c^2+R^2)^{(n+1)}C\dot{B}}{2[R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{(2n+2)}]AB}$$
$$\times\left(\hat{q}-\frac{\delta_q}{CA}\right), \tag{33}$$

where δ_q is presented in Appendix A. The limit $\lambda \to 0$ in the above equations leads to the GR Ellis equations.

4 Stability of homogeneous energy density

Here, we explore various matter factors that make the system's energy density irregular. We investigate this issue by taking some specific cases with Palatini $f(R)$ extra curvature corrections and discuss the inhomogeneity factors in the initial homogeneous planar celestial body. In order to solve this system of equations, we confine our attention on the presently observed value of the cosmological Ricci scalar, i.e., $R=\tilde{R}$. We also discuss our corresponding results as regards the null expansion scalar.

4.1 Non-dissipative fluids

We consider some particular streams of non-dissipative systems like dust, isotropic, and anisotropic fluid distributions with Palatini $f(R)$ gravity.

4.1.1 Dust cloud

In this case, we take $P_\perp=\hat{q}=\hat{P}_z=0$ and $A=1$ which shows the geodesic motion of non-radiating dust matter. Consequently, the two equations of the Weyl tensor, i.e., Eqs. (32) and (33), reduce to

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right.$$
$$\times\left\{\mu-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\right.$$
$$\left.\left.\times\left(R_c+\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]_{,0}$$
$$=\left[\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\mu}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}-\mathcal{E}\right]\frac{3\dot{B}}{B}, \tag{34}$$

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right.$$
$$\times\left\{\mu-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\right.$$
$$\left.\left.\times\left(R_c+\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]'=-\frac{3B'}{B}\mathcal{E}. \tag{35}$$

Using Eqs. (11) and (30) in Eqs. (34) and (35), we obtain

$$\dot{\mathcal{E}}+\frac{3\dot{B}}{B}\mathcal{E}=\frac{-\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\mu W_{GR}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}, \tag{36}$$

$$\mathcal{E}'+\frac{3B'}{B}\mathcal{E}=\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\mu'}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}. \tag{37}$$

The general solution of the differential equation (36) can be written as

$$\mathcal{E}=\frac{-\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^t W_{GR}\mu B^3 dt}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}. \tag{38}$$

One can easily identify the geometric entity which is involved in controlling the inhomogeneity in a dust cloud to be the Weyl scalar. Further, Eq. (36) relates the Weyl scalar with the shear scalar, thereby showing the importance of shearing motion of a dust celestial object in the appearance of irregularities. It also indicates that the system will be conformally flat if and only if it is shear-free in Palatini $f(R)$ gravity. In order to explore the role of the expansion scalar in the evolution of collapsing dust matter, we consider the expansion-free scenario, i.e., $\Theta_P=0$; then the above expression can be written as

$$\mathcal{E}=\frac{3\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^t \mu B^2\dot{B}dt}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}. \tag{39}$$

This provides an inhomogeneity factor for those non-radiating dust clouds which evolve with zero expansion. It is well known that if the system evolves with the expansion-free condition then an implosion of the matter distribution towards its central point causes blowing up of the shear scalar, which eventually yields a naked singularity [38–40].

4.1.2 Isotropic fluid

Here we investigate the causes of inhomogeneities by increasing complexity order of a planar collapsing system evolving with isotropic pressure components. The Ellis equations (32) and (33) become

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right.\\ \times\left\{\mu-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\right.\\ \left.\left.\times\left(R_c+\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]_{,0}\\ +\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}(\mu+P)}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right]\\ \times\frac{3\dot{B}}{B}=0, \tag{40}$$

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right.\\ \times\left\{\mu-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\right.\\ \left.\left.\times\left(R_c\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]'+\frac{3B'}{B}\mathcal{E}=0. \tag{41}$$

We see that the first Ellis equation for the isotropic case turns out to be same as given in the previous case [see Eq. (34)], indicating that the Weyl scalar is a key factor for the emergence of an inhomogeneous energy density. Using Eqs. (11) and (30), Eq. (40) turns out to be

$$\dot{\mathcal{E}}+\frac{3\dot{B}}{B}\mathcal{E}=\frac{-\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}(\mu+P)W_{GR}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}, \tag{42}$$

whose integration leads to

$$\mathcal{E}=\frac{-\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^t W_{GR}(\mu+P)B^3\mathrm{d}t}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}. \tag{43}$$

This also highlights the role of the shear scalar in the stability of the regular energy density over the planar isotropic celestial body. It indicates that besides the matter variables, $f(R)$ corrections are also involved in the maintenance of a homogeneous energy density.

Let us consider a shear-free motion of the fluid; then Eq. (42) reduces to

$$\mathcal{E}=\frac{\varrho(r)}{C^3},$$

where $\varrho(r)$ is an integration function. If a system embodies a regular energy density at $t=0$ ($\mathcal{E}(0,r)=0$), then we obtain $\varrho=0$ which eventually leads to a null value of the Weyl scalar for all time even in Palatini $f(R)$ gravity. This complies with the usual homogeneity criterion of the energy density that if a system has a zero Weyl scalar, then its energy density will be homogeneously distributed over the celestial surface and vice versa. If expansion of the system occurs in such a way that the fluid keeps small non-zero $\mathcal{E}$ at $t=0$, then the system will retain this value in all of its dynamical stages. For contracting systems, the Weyl scalar $\mathcal{E}$ does not vanish for all t. Under the expansion-free condition, Eq. (42) yields

$$\mathcal{E}=\frac{3\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^t(\mu+P)B^2\dot{B}\mathrm{d}t}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}, \tag{44}$$

which gives an irregularity factor for the perfect matter configuration evolving by making an internal Minkowkian cavity due to the expansion-free condition.

4.1.3 Anisotropic fluid

Now we take an anisotropic non-radiating matter configuration of the collapsing planar symmetry with Palatini $f(R)$ dark source terms. In this case, we need to take $\Pi\neq0$ and $\hat{q}=0$ for which Eqs. (32) and (33) yield

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right.\\ \times\left\{\mu-\Pi-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\right.\\ \left.\left.\times\left(R_c+\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]_{,0}\\ =\left[\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}(\mu+P_\perp)}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}-\mathcal{E}\right]\frac{3\dot{B}}{B}, \tag{45}$$

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right.\\ \times\left\{\mu-\Pi-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\right.\\ \left.\left.\times\left(R_c+\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]'\\ +\left[\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\Pi}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}+\mathcal{E}\right]$$

$$\times\frac{3B'}{B}=0. \tag{46}$$

By making use of Eqs. (11) and (30) in Eqs. (45) and (46) with some manipulations, we obtain the following equations:

$$\left[\mathcal{E}+\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\Pi}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right]_{,0}+\left[\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\Pi}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}+\mathcal{E}\right]\times\frac{3\dot{B}}{B}=\frac{-\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}A}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\times\left\{W_{GR}(\mu+P_z)-\Pi\left(\Theta_P-\frac{\dot{C}}{AC}\right)\right\},$$

$$\times\left[\mathcal{E}+\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\Pi}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right]'-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\mu'}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}=\frac{-3B'}{B}\left[\mathcal{E}+\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\Pi}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right],$$

which by means of the modified structure function (26) can be recast as

$$\dot{X}_{TF}+\frac{3X_{TF}\dot{B}}{B}=\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}A}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\times\left\{W_{GR}(\mu+P_z)-\Pi\left(\Theta_P-\frac{\dot{C}}{AC}\right)\right\},$$

$$X'_{TF}+\frac{3X_{TF}B'}{B}=\frac{-\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\mu'}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}.$$

Their solutions can be expressed, respectively, as

$$X_{TF}=-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^t[2\Pi\dot{B}-ABW_{GR}(\mu+P_z)]B^2\mathrm{d}t}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]B^3}, \tag{47}$$

$$X_{TF}=-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^z B^3\mu'\mathrm{d}z}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]B^3}. \tag{48}$$

The entity causing the emergence of irregularities in an anisotropic matter configuration is found to be a $f(R)$ scalar function which was initially obtained through orthogonal splitting of the Riemann tensor. Thus the trace-free part of the tensor, $X_{\alpha\beta}$, does have some significance in the structure and evolution of self-gravitating stars. Equation (48) indicates that if $\mu=\mu(z)$, then $X_{TF}=0$ and vice versa, thereby representing X_{TF} as a parameter of controlling the irregularities in anisotropic stellar systems, which supports the analysis of [41], [43–48]. Moreover, the gravitational contribution due to Palatini $f(R)$ terms does not disrupt the significance of the modified structure scalars X_{TF}. The above relations reduce to GR in the $\epsilon\to 0$ limit. However, Eq. (47) relates the modified structure scalar X_{TF} with the system's pressure anisotropy, $f(R)$ corrections, and shearing scalar, thus indicating the importance of these variables in the study of the inhomogeneity of the energy density of the planar compact objects. For an expansion-free collapsing anisotropic matter distribution, Eq. (45) gives

$$X_{TF}=\frac{3\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^t[2\Pi-(\mu+P_z)]B^2\dot{B}\mathrm{d}t}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]B^3}. \tag{49}$$

This indicates that the structure scalar can be used in the modeling of expansion-free collapsing stars with Palatini $f(R)$ corrections. It is well known that Θ_P controls the volume expansion rate of the matter. Since zero expansion provides zero compression, the above expression governs the energy density irregularity for anisotropic matter evolving without being compressed and holding an inner core.

4.2 Dissipative dust cloud

Finally, we assume the system to be a dust cloud undergoing dissipation by means of diffusion and free-streaming approximations with geodesic motion. We need to impose the $P_z=P_\perp=0,\ A=1$ constraints to determine the role of the radiating parameters in the emergence of energy density irregularities. The motivation for considering geodesic motion in a dust collapse scenario can be well justified in the light of several recent works [50,51,57–60] in relativistic astrophysics. Equations (32) and (33) provide

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\times\left\{\mu-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\times\left(R_c+\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]_{,0}=\left[\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\hat{\mu}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}-\mathcal{E}\right]\times\frac{3\dot{B}}{B}+\frac{3\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}$$

$$\times\left(\frac{AB'}{BC}\right)\hat{q}=0, \tag{50}$$

$$\left[\mathcal{E}-\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\right.$$
$$\times\left\{\mu-\frac{\lambda R_c}{2\kappa}+\frac{\lambda}{2\kappa}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-n}\right.$$
$$\left.\left.\times\left(R_c+\frac{2n\tilde{R}^2}{R_c}\left(1+\frac{\tilde{R}^2}{R_c^2}\right)^{-1}\right)\right\}\right]'$$
$$=-\frac{3B'}{B}\mathcal{E}-\frac{3\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}$$
$$\times\left(\frac{C\dot{B}}{AB}\right)\hat{q}=0. \tag{51}$$

If we take $\mu'=0$, then we obtain the inhomogeneity factor from Eq. (51) as

$$\Phi\equiv\mathcal{E}+\frac{3\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}\int_0^z CB^2\hat{q}\dot{B}dz}{2B^3[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}, \tag{52}$$

which consequently gives $\Phi=0\Leftrightarrow\mu'=0$. The equation representing evolution of Φ can be found by using Eqs. (11) and (30) in Eq. (50) as follows:

$$\dot{\Phi}-\frac{\dot{\Psi}}{B^3}=\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}$$
$$\times\left(\frac{\hat{q}B'}{BC}-\hat{\mu}W_{GR}-\frac{\hat{q}'}{C}\right)-\frac{3\dot{B}}{B}\Phi, \tag{53}$$

where $\Psi=\frac{-3\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\int_0^z CB^2\hat{q}\dot{B}dz$. The general solution of the above differential equation can be obtained as follows:

$$\Phi=\frac{\int_0^t\left[\dot{\Psi}+\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\left(\frac{\tilde{q}B'}{C}-\tilde{\mu}CW_{GR}-\frac{\tilde{q}'B}{C}\right)\right]\mathrm{d}t}{B^3}. \tag{54}$$

This shows that the energy density irregularity of the radiating compact object is directly related with the shear scalar and the radiating parameters. Under expansion-free collapse, we obtain

$$\Phi=\frac{\int_0^t\left[\dot{\Psi}+\frac{\kappa R_c(R_c^2+\tilde{R}^2)^{(n+1)}}{2[R_c(R_c^2+\tilde{R}^2)^{(n+1)}-2n\lambda\tilde{R}R_c^{(2n+2)}]}\left(\frac{\tilde{q}B'}{C}+\tilde{\mu}\dot{B}-\frac{\tilde{q}'B}{C}\right)\right]\mathrm{d}t}{B^3}, \tag{55}$$

which provides the inhomogeneity parameter for dust radiating collapse.

5 Final remarks

In this paper, we have explored factors affecting the homogeneity of the energy density for expansive and expansion-free planar self-gravitating systems with Palatini $f(R)$ background corrections. For this purpose, we have explored conservation laws through the Bianchi identities of the usual as well as effective stress-energy tensors and we have evaluated the so-called modified Ellis equations linking the Weyl scalar with the matter variables. We have constructed Palatini $f(R)$ structure scalars with three-parametric high-energy degrees of freedom which are explored for radiating as well as non-radiating systems. We have also explored the irregularity factors for those cosmological structures which have vacuum cores in the cosmos, like voids.

For adiabatic dust and ideal non-radiating self-gravitating objects, it is found that the density irregularities are determined by curvature terms originating from the Weyl scalar, which in turn is directly related to the shearing motion of the collapsing system. Thus the shear scalar determines the extent of the regular energy density in both non-dissipative dust and ideal fluid configurations. It is also seen that for a geodesic dust cloud, zero shear motion and conformal flatness imply each other through a one-one correspondence. However, for a perfect celestial body all conformal non-radiating flat metrics are shear-free. Equations (38) and (43) suggest that Palatini $f(R)$ DE terms tend to relax the conformal flatness constraints, thus favoring the maintenance of the regular energy density. If one considers expansion-free planar dust and ideal matter configurations, then the shear scalar plays no role in the emergence of energy density irregularities due to the existence of vacuum cores as implied by Eqs. (39) and (44).

For an anisotropic adiabatic planar self-gravitating system, the entity responsible for the appearance of energy density irregularities is found to be one of the structure scalars, X_{TF}. This further depends on particular combinations of the components of an anisotropic pressure. Thus, we have found X_{TF} as an irregularity factor for both expansive and expansion-free systems. It is also found that in the $f(R)$ case three-parametric corrections tend to decrease the effects of X_{TF}, thus resisting the energy density inhomogeneities. The factor Φ is evaluated from the modified Ellis equations for dust radiating geodesic expansive and expansion-free fluid configurations. We conclude that this factor depends upon higher order $f(R)$ dark source terms, the heat conducting vector, and the shear scalar. The invoking of an expansion-free condition in the evolution of a radiating dust cloud makes the inhomogeneity factor independent of the shear scalar as implied by Eq. (55).

Appendix A

The higher curvature terms D_0 and D_1 of Eqs. (30) and (31) are given as

$$
\begin{aligned}
D_0 =\ & \frac{(-1)}{A^2}\Big\{\Big(\frac{f}{R}-f_R\Big)\frac{R}{2}-\frac{f''_R}{C^2}+\frac{\dot f_R}{A^2}\Big(\frac{\dot C}{C}+\frac{9\dot f_R}{4f_R}+\frac{2\dot B}{B}\Big)\\
& -\frac{f'_R}{C^2}\Big(\frac{C'}{C}+\frac{f'_R}{4f_R}-\frac{2B'}{B}\Big)\Big\}_{,0}+\frac{\dot f_R}{f_R A}\Big\{\frac{3\ddot f_R}{2A^2}-\frac{R}{2}\\
& \times\Big(f_R-\frac{f}{R}\Big)+\frac{3f''_R}{2C^2}-\frac{\dot f_R}{A^2}\Big(\frac{3\dot C}{2C}+\frac{3\dot A}{2A}+\frac{5\dot B}{B}+\frac{6\dot f_R}{f_R}\Big)\\
& -\frac{f'_R}{C^2}\Big(\frac{3A'}{2A}+\frac{3C'}{2C}-\frac{3B'}{B}+\frac{3f'_R}{2f_R}\Big)\Big\}+\frac{\dot C}{AC}\Big\{\frac{f''_R}{C^2}\\
& +\frac{\ddot f_R}{A^2}-\frac{\dot f_R}{A^2}\Big(\frac{5\dot f_R}{2f_R}+\frac{\dot A}{A}+\frac{4\dot B}{B}+\frac{\dot C}{C}\Big)-\frac{f'_R}{C^2}\\
& \times\Big(\frac{A'}{A}+\frac{5f'_R}{2f_R}+\frac{C'}{C}\Big)\Big\}+\frac{(-1)}{C^2A}\Big(\dot f'_R-\frac{5}{2}\frac{\dot f_R f'_R}{f_R}\\
& -\frac{A'}{A}\dot f_R-\frac{\dot C}{C}f'_R\Big)\Big(\frac{3A'}{A}+\frac{C'}{C}+\frac{3f'_R}{f_R}+\frac{2B'}{B}\Big)\Big]\\
& +A\Big[\frac{1}{A^2C^2}\Big\{\dot f'_R-\frac{A'}{A}\dot f_R-\frac{\dot C}{C}f'_R-\frac{5}{2}\frac{\dot f_R f'_R}{f_R}\Big\}\Big]_{,1},
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
D_1 =\ & C\Big\{\frac{-1}{(CA)^2}\Big(\dot f'_R-\frac{5\dot f_R f'_R}{2f_R}-\frac{A'}{A}\dot f_R-\frac{\dot C}{C}f'_R\Big)\Big\}_{,0}\\
& +\frac{1}{C}\Big\{\frac{\ddot f_R}{A^2}-\frac{R}{2}\Big(f_R-\frac{f}{R}\Big)-\frac{\dot f_R}{A^2}\Big(\frac{\dot A}{A}+\frac{\dot f_R}{4f_R}+\frac{2\dot B}{B}\Big)\\
& -\frac{f'_R}{C^2}\Big(\frac{2B'}{B}+\frac{9f'_R}{4f_R}+\frac{A'}{A}\Big)\Big\}_{,1}+\frac{A'}{CA}\Big\{\frac{f''_R}{C^2}+\frac{\ddot f_R}{A^2}\\
& -\frac{\dot f_R}{A^2}\Big(\frac{5\dot f_R}{2f_R}+\frac{\dot A}{A}+\frac{4\dot B}{B}+\frac{\dot C}{C}\Big)-\frac{f'_R}{C^2}\Big(\frac{5f'_R}{2f_R}+\frac{A'}{A}+\frac{C'}{C}\Big)\Big\}\\
& +\frac{f'_R}{f_R C}\Big\{\Big(\frac{f}{R}-f_R\Big)\frac{R}{2}+\frac{3\ddot f_R}{2A^2}+\frac{3f''_R}{2C^2}-\frac{3\dot f_R}{2A^2}\\
& \times\Big(\frac{\dot A}{A}+\frac{\dot C}{C}+\frac{\dot f_R}{f_R}+\frac{14\dot B}{3B}\Big)-\frac{f'_R}{C^2}\Big(\frac{3B'}{B}+\frac{3A'}{2A}+\frac{3C'}{2C}\\
& +\frac{6f'_R}{f_R}\Big)\Big\}+\frac{2B'}{CB}\Big\{\frac{f''_R}{C^2}-\frac{\dot f_R}{A^2}\Big(\frac{3\dot B}{B}+\frac{\dot C}{C}\Big)-\frac{f'_R}{C^2}\\
& \times\Big(\frac{B'}{B}+\frac{5f'_R}{2f_R}+\frac{C'}{C}\Big)\Big\}+\frac{(-1)}{CA^2}\Big(-\frac{A'}{A}\dot f_R+\dot f'_R\\
& -\frac{5\dot f_R f'_R}{2f_R}-\frac{\dot C}{C}f'_R\Big)\Big(\frac{\dot A}{A}+\frac{3\dot C}{C}+\frac{3\dot f_R}{f_R}\Big).
\end{aligned}
\tag{A2}
$$

The quantities δ_μ, δ_{P_z}, $\delta_{P_\perp}$, and δ_q are

$$
\begin{aligned}
\delta_\mu =\ & \frac{-A^2}{\kappa}\Big[-\frac{\lambda R_c}{2}+\frac{\lambda}{2}\Big(1+\frac{R^2}{R_c^2}\Big)^{-n}\\
& \times\Big\{R_c+\frac{2nR^2}{R_c^2}\Big(1+\frac{R^2}{R_c^2}\Big)^{-1}\Big\}-\frac{2n\lambda R''}{C^2R_c}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{-(n+1)}+\frac{4n(n+1)\lambda R}{C^2R_c^3}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{-(n+2)}\Big\{3R'^2+RR''-\frac{2}{R_c^2}\\
& \times(n+2)R^2R'^2\Big(1+\frac{R^2}{R_c^2}\Big)^{(-n+1)}\Big\}\\
& +\frac{2n\lambda}{A^2R_c}\Big(1+\frac{R^2}{R_c^2}\Big)^{-(n+1)}\Big\{-\dot R+\frac{2}{R_c^2}\\
& \times R^2(n+1)\dot R\Big(1+\frac{R^2}{R_c^2}\Big)^{-1}\Big\}\\
& \times\Big\{\frac{9n\lambda(R_c^2+R^2)^{(n+1)}}{2\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{(n+1)}\Big\{-\dot R+\frac{2R^2(n+1)\dot R}{R_c^2}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{-1}\Big\}+\frac{\dot C}{C}+2\frac{\dot B}{B}\Big\}\\
& +\frac{2n\lambda}{C^2R_c}\Big(1+\frac{R^2}{R_c^2}\Big)^{-(n+1)}\Big\{-R'\\
& +\frac{2R^2(n+1)R'}{R_c^2}\Big(1+\frac{R^2}{R_c^2}\Big)^{-1}\Big\}\Big\{\frac{C'}{C}\\
& -\frac{2B'}{B}\frac{n\lambda(R_c^2+R^2)^{(n+1)}}{2\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{(n+1)}\Big\{-R'+\frac{2}{R_c^2}\\
& \times R^2(n+1)R'\Big(1+\frac{R^2}{R_c^2}\Big)^{-1}\Big\}\Big\}\Big\}\Big],
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
\delta_{P_\perp} =\ & \frac{B^2}{\kappa}\Big[-\frac{\lambda R_c}{2}+\frac{\lambda}{2}\Big(1+\frac{R^2}{R_c^2}\Big)^{-n}\\
& \times\Big\{R_c+\frac{2nR^2}{R_c}\Big(1+\frac{R^2}{R_c^2}\Big)^{-1}\Big\}-\frac{2n\lambda\ddot R}{A^2R_c}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{-(n+1)}+\frac{4n(n+1)R\lambda}{A^2R_c^3}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{-(n+2)}\Big\{-\frac{2(n+2)R^2\dot R^2}{R_c^2}\\
& \times\Big(1+\frac{R^2}{R_c^2}\Big)^{-(n+1)}+3\dot R^2+R\ddot R\Big\}
\end{aligned}
$$

$$-\frac{2n\lambda R''}{C^2R_c}\left(1+\frac{R^2}{R_c^2}\right)^{-(n+1)}-\frac{4nR^2\dot{R}}{R_c^2}$$

$$\times(n+1)\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\{3R'^2+RR''$$

$$-\frac{2(n+2)R^2R'^2}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-(n+1)}\Bigg\}$$

$$+\frac{2n\lambda}{A^2R_c}\left(1+\frac{R^2}{R_c^2}\right)^{-(n+1)}\Bigg\{-\dot{R}$$

$$+\frac{2R^2(n+1)\dot{R}}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}\Bigg\{\frac{\dot{B}}{B}+\frac{\dot{C}}{C}$$

$$-\frac{\dot{A}}{A}-\frac{n\lambda(R_c^2+R^2)^{(n+1)}}{2\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}\Bigg\{-\dot{R}+\frac{2R^2}{R_c^2}$$

$$+(n+1)\dot{R}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}\Bigg\}$$

$$+\frac{2n\lambda}{C^2R_c}\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}$$

$$\times\Bigg\{-R'+\frac{2R^2(n+1)R'}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}$$

$$\times\Bigg\{\frac{C'}{C}+\frac{n\lambda(R_c^2+R^2)^{(n+1)}}{2\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}$$

$$\times\Bigg\{-R'+\frac{2R^2(n+1)R'}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}$$

$$\times-\frac{B'}{B}-\frac{A'}{A}\Bigg\}\Bigg], \qquad \text{(A4)}$$

$$\delta_{P_z}=\frac{C^2}{\kappa}\Bigg[-\frac{\lambda R_c}{2}+\frac{\lambda}{2}\left(1+\frac{R^2}{R_c^2}\right)^{-n}$$

$$\times\Bigg\{R_c+\frac{2nR^2}{R_c}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}-\frac{2n\lambda\ddot{R}}{A^2R_c}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{-(n+1)}+\frac{4n(n+1)R\lambda}{A^2R_c^3}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{-(n+2)}\Bigg\{-\frac{2(n+2)R^2\dot{R}^2}{R_c^2}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{-(n+1)}+3\dot{R}^2+R\ddot{R}\Bigg\}$$

$$+\frac{2n\lambda}{A^2R_c}\left(1+\frac{R^2}{R_c^2}\right)^{-(n+1)}\Bigg\{-\dot{R}+2\frac{R^2}{R_c^2}$$

$$\times(n+1)\dot{R}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}\Bigg\{\frac{2\dot{B}}{B}-\frac{\dot{A}}{A}$$

$$-\frac{n\lambda(R_c^2+R^2)^{(n+1)}}{2\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}\Bigg\{-\dot{R}+\frac{2R^2}{R_c^2}+(n+1)\dot{R}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}\Bigg\}+\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}$$

$$+\frac{2n\lambda(R_c^2+R^2)^{(n+1)}}{\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}$$

$$\times\Bigg\{-R'+\frac{2R^2(n+1)R'}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}$$

$$\times\Bigg\{\frac{A'}{A}+\frac{2B'}{B}$$

$$+\frac{9n\lambda(R_c^2+R^2)^{(n+1)}}{2\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}$$

$$\times\Bigg\{-R'+\frac{2R^2(n+1)R'}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}\Bigg\}\Bigg], \qquad \text{(A5)}$$

$$\delta_q=\frac{1}{\kappa}\Bigg[-\frac{2n\lambda\dot{R}'}{R}\left(1+\frac{R^2}{R_c^2}\right)^{-(n+1)}$$

$$+\frac{4n(n+1)\lambda R}{R_c^3}\left(1+\frac{R^2}{R_c^2}\right)^{-(n+2)}$$

$$\times\Bigg\{-R'+\frac{2R^2}{R_c^2}(n+1)R'\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}$$

$$-\frac{2n\lambda A'}{AR_c}\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}$$

$$\times\Bigg\{-\dot{R}+\frac{2R^2(n+1)\dot{R}}{R_c^2}$$

$$\times\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg\}-\frac{2n\lambda\dot{C}}{CR_c}\left(1+\frac{R^2}{R_c^2}\right)^{(n+1)}$$

$$\times\Bigg\{\frac{2R^2(n+1)R'}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-1}-R'\Bigg\}$$

$$-\left(1+\frac{R^2}{R_c^2}\right)^{-(2n+2)}$$

$$\times\frac{10n^2\lambda^2}{R_c\{R_c(R_c^2+R^2)^{(n+1)}-2n\lambda RR_c^{2n+2}\}}$$

$$\times\Bigg\{\dot{R}R'-\frac{4(n+1)^2R^4R'\dot{R}}{R_c^2}\left(1+\frac{R^2}{R_c^2}\right)^{-1}$$

$$+\frac{4(n+1)^2R^4R'\dot{R}}{R_c^4}\left(1+\frac{R^2}{R_c^2}\right)^{-2}\Bigg\}\Bigg]. \qquad \text{(A6)}$$

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