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$D^*_{s3}(2860)$ and $D^*_{s1}(2860)$ as the 1D $c\bar{s}$ states

Zhi-Gang Wang[a]

Department of Physics, North China Electric Power University, Baoding 071003, People's Republic of China

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Abstract In this article, we take the $D^*_{s3}(2860)$ and $D^*_{s1}(2860)$ as the $1^3\mathrm{D}_3$ and $1^3\mathrm{D}_1$ $c\bar{s}$ states, respectively, and we study their strong decays with the heavy meson effective theory by including the chiral symmetry-breaking corrections. We can reproduce the experimental data $\mathrm{Br}\left(D^*_{sJ}(2860) \to D^*K\right)/\mathrm{Br}\left(D^*_{sJ}(2860) \to DK\right) = 1.10 \pm 0.15 \pm 0.19$ with suitable hadronic coupling constants; the assignment of the $D^*_{sJ}(2860)$ as the $D^*_{s3}(2860)$ is favored, the chiral symmetry-breaking corrections are large. Furthermore, we obtain the analytical expressions of the decay widths, which can be confronted with the experimental data in the future to fit the unknown coupling constants. The predictions of the ratios among the decay widths can be used to study the decay properties of the $D^*_{s3}(2860)$ and $D^*_{s1}(2860)$ so as to identify them unambiguously. On the other hand, if the chiral symmetry-breaking corrections are small, the large ratio $R = 1.10 \pm 0.15 \pm 0.19$ requires that the $D^*_{sJ}(2860)$ consists of at least the four resonances, $D^*_{s1}(2860)$, $D^*_{s2}(2860)$, $D^{*\prime}_{s2}(2860)$, $D^*_{s3}(2860)$.

1 Introduction

In 2006, the BaBar collaboration observed the $D^*_{sJ}(2860)$ meson with the mass $(2856.6 \pm 1.5 \pm 5.0)$ MeV and the width $(48 \pm 7 \pm 10)$ MeV in decays to the final states D^0K^+ and $D^+K^0_S$ [1]. There have been several possible assignments. Beveren and Rupp [2] assigned the $D^*_{sJ}(2860)$ to the first radial excitation of the $D^*_{s0}(2317)$ based on a coupled-channel model. Colangelo et al. [3] assigned the $D^*_{sJ}(2860)$ to the $1^3\mathrm{D}_3$ $c\bar{s}$ state using the heavy meson effective theory. Close et al. [4] assigned the $D^*_{sJ}(2860)$ to the $2^3\mathrm{P}_0$ state in a constituent quark model with novel spin-dependent interactions. Zhang et al. [5] assigned the $D^*_{sJ}(2860)$ to the $2^3\mathrm{P}_0$ or $1^3\mathrm{D}_3$ state based on the $^3\mathrm{P}_0$ model; Li et al. [6] share this interpretation based on the Regge phenomenology. However, Ebert et al. [7] observed that the $D^*_{sJ}(2860)$ does not fit well to the Regge trajectory $D^*_s(2112)$, $D^*_{s2}(2573)$, $D^*_{sJ}(2860)$, ... Later, Li and Ma [8] assigned the $D^*_{s1}(2700)$ to the $1^3\mathrm{D}_1$–$2^3\mathrm{S}_1$ mixing state and the $D^*_{sJ}(2860)$ to its orthogonal partner, or the $D^*_{sJ}(2860)$ to the $1^3\mathrm{D}_3$ state based on the $^3\mathrm{P}_0$ model. Zhong and Zhao [9,10] assigned the $D^*_{sJ}(2860)$ to the $1^3\mathrm{D}_3$ state with some $1^3\mathrm{D}_2$–$1^1\mathrm{D}_2$ mixing component using the chiral quark model, i.e. they assume that the $D^*_{sJ}(2860)$ arises from two overlapping resonances. Vijande et al. [11] assigned the $D^*_{sJ}(2860)$ to the $c\bar{s}$–$cn\bar{s}\bar{n}$ mixing state. Chen et al. [12] assigned the $D^*_{sJ}(2860)$ to the $1^3\mathrm{D}_3$ state based on a semi-classic flux tube model. Badalian and Bakker [13] assigned the $D^*_{sJ}(2860)$ to the $1^3\mathrm{D}_3$ state based on the QCD string model. Guo and Meissner [14] take the $D^*_{sJ}(2860)$ as the dynamically generated $D_1(2420)K$ bound state.

In 2009, the BaBar collaboration confirmed the $D^*_{sJ}(2860)$ in the D^*K channel, and they measured the ratio R among the branching fractions [15],

$$R = \frac{\mathrm{Br}\left(D^*_{sJ}(2860) \to D^*K\right)}{\mathrm{Br}\left(D^*_{sJ}(2860) \to DK\right)} = 1.10 \pm 0.15 \pm 0.19. \quad (1)$$

The observation of the decays $D^*_{sJ}(2860) \to D^*K$ rules out the $J^P = 0^+$ assignment [2,4–6]. On the other hand, if we take the $D^*_{sJ}(2860)$ as the $1^3\mathrm{D}_3$ state, Colangelo et al. [3] obtained the value $R = 0.39$ based on the heavy meson effective theory, while in the $^3\mathrm{P}_0$ model, Zhang et al. [5] obtained the value 0.59, Li and Ma [8] obtained the value 0.75, Song et al. [16] obtained the value 0.55–0.80. Recently, Godfrey and Jardine [17] obtained the value 0.43 based on the relativized quark model combined with the pseudoscalar emission decay model. The theoretical values differ from the experimental value greatly.

Recently, the LHCb collaboration observed a structure at 2.86 GeV with a significance of more than 10 standard deviations in the $\overline{D}^0K^-$ mass spectrum in the Dalitz plot analysis of the decays $B^0_s \to \overline{D}^0K^-\pi^+$; the structure contains

[a] e-mail: zgwang@aliyun.com

both spin-1 and spin-3 components [i.e. the $D_{s1}^{*-}(2860)$ and the $D_{s3}^{*-}(2860)$, respectively], which supports an interpretation of these states as the $J^P = 1^-$ and 3^- members of the 1D family [18,19]. The measured masses and widths are $M_{D_{s3}^*} = (2860.5 \pm 2.6 \pm 2.5 \pm 6.0)$ MeV, $M_{D_{s1}^*} = (2859 \pm 12 \pm 6 \pm 23)$ MeV, $\Gamma_{D_{s3}^*} = (53 \pm 7 \pm 4 \pm 6)$ MeV, and $\Gamma_{D_{s1}^*} = (159 \pm 23 \pm 27 \pm 72)$ MeV, respectively. Furthermore, the LHCb collaboration obtained the conclusion that the $D_{sJ}^*(2860)$ observed by the BaBar collaboration in the inclusive $e^+e^- \to \overline{D}^0 K^- X$ production and by the LHCb collaboration in the $pp \to \overline{D}^0 K^- X$ processes consists of at least these two resonances [15,20].

According to the predictions of the potential models [7,21,22], see Table 1, the masses of the 1D $c\bar{s}$ states is about 2.9 GeV. It is reasonable to assign the $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ to the $1^3\mathrm{D}_1$ and $1^3\mathrm{D}_3$ $c\bar{s}$ states, respectively [18,19]. However, the theoretical values R differ from the experimental value greatly in the case of the $D_{s3}^*(2860)$ or the $1^3\mathrm{D}_3$ assignment of the $D_{sJ}^*(2860)$. In Ref. [3], Colangelo et al. take the leading-order heavy meson effective Lagrangian. The two-body strong decays $D_{s3}^*(2860) \to D^*K, DK$ take place through the relative F-wave; the final K mesons have the three momenta $p_K = 584$ and 705 MeV, respectively. The decay widths

$$\Gamma(D_{s3}^*(2860) \to D^*K, DK) \propto p_K^7, \tag{2}$$

where $p_K^7 = 2.3 \times 10^{19}$ and $8.6 \times 10^{19}\,\mathrm{MeV}^7$ in the decays to the final states D^*K and DK, respectively. A small difference in p_K can lead to a large difference in p_K^7, so we have to take into account the heavy quark symmetry-breaking corrections and chiral symmetry-breaking corrections so as to make robust predictions.

In this article, we take into account the chiral symmetry-breaking corrections and study the two-body strong decays of the $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ with the heavy meson effective Lagrangian, and we try to reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$ by assigning the $D_{sJ}^*(2860)$ to the $D_{s1}^*(2860)$ and the $D_{s3}^*(2860)$, respectively. Recently, Wu and Huang studied the strong decays of the $D_{s0}^*(2317)$ and $D_{s1}'(2460)$ by including the chiral symmetry-breaking corrections [23]. Heavy meson effective theory has been applied to identify the charmed mesons and bottom mesons [3,24–30], and to calculate the radiative, vector-meson, two-pion decays of the heavy quarkonium states [31–37].

Table 1 The masses of the 1D $c\bar{s}$ mesons from the potential models compared to the experimental data

	Expt [18,19]	[7]	[21]	[22]
$1^3\mathrm{D}_1$	2859	2913	2899	2913
$1^1\mathrm{D}_2$	–	2931	2900	2900
$1^3\mathrm{D}_2$	–	2961	2926	2953
$1^3\mathrm{D}_3$	2860	2871	2917	2925

The article is arranged as follows: we derive the strong decay widths of the charmed mesons $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ with the heavy meson effective theory in Sect. 2; in Sect. 3, we present the numerical results and discussions; and Sect. 4 is reserved for our conclusions.

2 The strong decays with the heavy meson effective theory

In the heavy quark limit, the heavy-light mesons $Q\bar{q}$ can be classified in doublets according to the total angular momentum of the light antiquark $\mathbf{s}_\ell$, $\mathbf{s}_\ell = \mathbf{s}_{\bar{q}} + \mathbf{L}$, where the $\mathbf{s}_{\bar{q}}$ and $\mathbf{L}$ are the spin and orbital angular momentum of the light antiquark, respectively [38,39]. In this article, the relevant doublets are the $L = 0$ (S-wave) doublet (P, P^*) with $J_{s_\ell}^P = (0^-, 1^-)_{\frac{1}{2}}$, and the $L = 2$ (D-wave) doublets (P_1^*, P_2) and (P_2, P_3^*) with $J_{s_\ell}^P = (1^-, 2^-)_{\frac{3}{2}}$ and $(2^-, 3^-)_{\frac{5}{2}}$, respectively. In heavy meson effective theory, those doublets can be described by the effective super-fields H_a, X_a, and Y_a, respectively [40,41],

$$\begin{aligned}
H_a &= \frac{1+\not{v}}{2}\left\{P_{a\mu}^*\gamma^\mu - P_a\gamma_5\right\}, \\
X_a^\mu &= \frac{1+\not{v}}{2}\left\{P_{2a}^{\mu\nu}\gamma_5\gamma_\nu - P_{1a\nu}^*\sqrt{\frac{3}{2}}\left[g^{\mu\nu} - \frac{\gamma^\nu(\gamma^\mu+v^\mu)}{3}\right]\right\}, \\
Y_a^{\mu\nu} &= \frac{1+\not{v}}{2}\left\{P_{3a}^{*\mu\nu\sigma}\gamma_\sigma - P_{2a}^{\alpha\beta}\sqrt{\frac{5}{3}}\gamma_5\right. \\
&\quad \left.\times\left[g_\alpha^\mu g_\beta^\nu - \frac{g_\beta^\nu\gamma_\alpha(\gamma^\mu - v^\mu)}{5} - \frac{g_\alpha^\mu\gamma_\beta(\gamma^\nu - v^\nu)}{5}\right]\right\},
\end{aligned} \tag{3}$$

where the heavy meson fields $P^{(*)}$ contain a factor $\sqrt{M_{P^{(*)}}}$ and have dimension of mass $\frac{3}{2}$. The super-fields H_a contain the S-wave mesons (P, P^*); X_a, Y_a contain the D-wave mesons (P_1^*, P_2), (P_2, P_3^*), respectively.

The light pseudoscalar mesons are described by the fields $\xi = \mathrm{e}^{\frac{i\mathcal{M}}{f_\pi}}$, where

$$\mathcal{M} = \lambda^j\mathcal{P}^j = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix},$$

and the decay constant $f_\pi = 130$ MeV.

At the leading-order approximation, the heavy meson chiral Lagrangians $\mathcal{L}_X$ and $\mathcal{L}_Y$ for the strong decays to the light pseudoscalar mesons can be written as

$$\mathcal{L}_X = \frac{g_X}{\Lambda}\mathrm{Tr}\left\{\bar{H}_a X_b^{\mu}(i\mathcal{D}_\mu \not{\mathcal{A}} + i\not{\mathcal{D}}\mathcal{A}_\mu)_{ba}\gamma_5\right\} + h.c.,$$
$$\mathcal{L}_Y = \frac{1}{\Lambda^2}\mathrm{Tr}\left\{\bar{H}_a Y_b^{\mu\nu}\left[g_Y\{i\mathcal{D}_\mu, i\mathcal{D}_\nu\}\mathcal{A}_\lambda + \tilde{g}_Y\left(i\mathcal{D}_\mu i\mathcal{D}_\lambda\mathcal{A}_\nu + i\mathcal{D}_\nu i\mathcal{D}_\lambda\mathcal{A}_\mu\right)\right]_{ba}\gamma^\lambda\gamma_5\right\} + h.c., \quad (4)$$

where

$$\mathcal{D}_\mu = \partial_\mu + \mathcal{V}_\mu,$$
$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right),$$
$$\mathcal{A}_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right),$$
$$\{\mathcal{D}_\mu, \mathcal{D}_\nu\} = \mathcal{D}_\mu\mathcal{D}_\nu + \mathcal{D}_\nu\mathcal{D}_\mu, \quad (5)$$

the hadronic coupling constants g_X, g_Y, and $\tilde{g}_Y$ are parameters and can be fitted to the experimental data [42–46]; Λ is the chiral symmetry-breaking scale and is chosen as $\Lambda = 1$ GeV [28–30].

We construct the chiral symmetry-breaking Lagrangians $\mathcal{L}_X^\chi$ and $\mathcal{L}_Y^\chi$ according to Refs. [47–50],

$$\begin{aligned}\mathcal{L}_X^\chi =& \frac{k_X^1}{\Lambda^2}\mathrm{Tr}\left\{\bar{H}_a X_b^{\mu}(i\mathcal{D}_\mu \not{\mathcal{A}} + i\not{\mathcal{D}}\mathcal{A}_\mu)_{bc}\left(m_q^\xi\right)_{ca}\gamma_5\right\}\\
&+\frac{k_X^2}{\Lambda^2}\mathrm{Tr}\left\{\bar{H}_a X_c^{\mu}\left(m_q^\xi\right)_{cb}(i\mathcal{D}_\mu \not{\mathcal{A}} + i\not{\mathcal{D}}\mathcal{A}_\mu)_{ba}\gamma_5\right\}\\
&+\frac{k_X^3}{\Lambda^2}\mathrm{Tr}\left\{\bar{H}_a X_b^{\mu}(i\mathcal{D}_\mu \not{\mathcal{A}} + i\not{\mathcal{D}}\mathcal{A}_\mu)_{ba}\left(m_q^\xi\right)_{cc}\gamma_5\right\}\\
&+\frac{k_X^4}{\Lambda^2}\mathrm{Tr}\left\{\bar{H}_a X_a^{\mu}(i\mathcal{D}_\mu \not{\mathcal{A}} + i\not{\mathcal{D}}\mathcal{A}_\mu)_{bc}\left(m_q^\xi\right)_{cb}\gamma_5\right\}\\
&+\frac{1}{\Lambda^2}\mathrm{Tr}\left\{\bar{H}_a X_b^{\mu}\left[k_X^5\{i\mathcal{D}_\mu, iv\cdot\mathcal{D}\}\mathcal{A}_\lambda + \tilde{k}_X^5\right.\right.\\
&\left.\left.\times\{iv\cdot\mathcal{D}, i\mathcal{D}_\lambda\}\mathcal{A}_\mu + \tilde{\tilde{k}}_X^5\{i\mathcal{D}_\mu, i\mathcal{D}_\lambda\}v\cdot\mathcal{A}\right]_{ba}\gamma^\lambda\gamma_5\right\}\\
&+h.c.,\end{aligned} \quad (6)$$

$$\begin{aligned}\mathcal{L}_Y^\chi =& \frac{1}{\Lambda^3}\mathrm{Tr}\left\{\bar{H}_a Y_b^{\mu\nu}\left[k_Y^1\{i\mathcal{D}_\mu, i\mathcal{D}_\nu\}\mathcal{A}_\lambda + \tilde{k}_Y^1\left(i\mathcal{D}_\mu i\mathcal{D}_\lambda\mathcal{A}_\nu\right.\right.\right.\\
&\left.\left.\left.+i\mathcal{D}_\nu i\mathcal{D}_\lambda\mathcal{A}_\mu\right)\right]_{bc}\left(m_q^\xi\right)_{ca}\gamma^\lambda\gamma_5\right\}\\
&+\frac{1}{\Lambda^3}\mathrm{Tr}\left\{\bar{H}_a Y_b^{\mu\nu}\left(m_q^\xi\right)_{bc}\left[k_Y^2\{i\mathcal{D}_\mu, i\mathcal{D}_\nu\}\mathcal{A}_\lambda\right.\right.\\
&\left.\left.+\tilde{k}_Y^2\left(i\mathcal{D}_\mu i\mathcal{D}_\lambda\mathcal{A}_\nu + i\mathcal{D}_\nu i\mathcal{D}_\lambda\mathcal{A}_\mu\right)\right]_{ca}\gamma^\lambda\gamma_5\right\}\\
&+\frac{1}{\Lambda^3}\mathrm{Tr}\left\{\bar{H}_a Y_b^{\mu\nu}\left[k_Y^3\{i\mathcal{D}_\mu, i\mathcal{D}_\nu\}\mathcal{A}_\lambda\right.\right.\\
&\left.\left.+\tilde{k}_Y^3\left(i\mathcal{D}_\mu i\mathcal{D}_\lambda\mathcal{A}_\nu + i\mathcal{D}_\nu i\mathcal{D}_\lambda\mathcal{A}_\mu\right)\right]_{ba}\left(m_q^\xi\right)_{cc}\gamma^\lambda\gamma_5\right\}\\
&+\frac{1}{\Lambda^3}\mathrm{Tr}\left\{\bar{H}_a Y_a^{\mu\nu}\left[k_Y^4\{i\mathcal{D}_\mu, i\mathcal{D}_\nu\}\mathcal{A}_\lambda\right.\right.\\
&\left.\left.+\tilde{k}_Y^4\left(i\mathcal{D}_\mu i\mathcal{D}_\lambda\mathcal{A}_\nu + i\mathcal{D}_\nu i\mathcal{D}_\lambda\mathcal{A}_\mu\right)\right]_{bc}\left(m_q^\xi\right)_{cb}\gamma^\lambda\gamma_5\right\}\\
&+\frac{1}{\Lambda^3}\mathrm{Tr}\left\{\bar{H}_a Y_b^{\mu\nu}\left[k_Y^5\{i\mathcal{D}_\mu, i\mathcal{D}_\nu, iv\cdot\mathcal{D}\}\mathcal{A}_\lambda\right.\right.\\
&+\tilde{k}_Y^5\left(\{i\mathcal{D}_\mu, iv\cdot\mathcal{D}\}i\mathcal{D}_\lambda\mathcal{A}_\nu + \{i\mathcal{D}_\nu, iv\cdot\mathcal{D}\}i\mathcal{D}_\lambda\mathcal{A}_\mu\right.\\
&\left.\left.\left.+\{i\mathcal{D}_\mu, i\mathcal{D}_\nu\}i\mathcal{D}_\lambda v\cdot\mathcal{A}\right)\right]_{ba}\gamma^\lambda\gamma_5\right\} + h.c.,\end{aligned} \quad (7)$$

where

$$\{\mathcal{D}_\mu, \mathcal{D}_\nu, \mathcal{D}_\rho\} = \mathcal{D}_\mu\mathcal{D}_\nu\mathcal{D}_\rho + \mathcal{D}_\mu\mathcal{D}_\rho\mathcal{D}_\nu + \mathcal{D}_\nu\mathcal{D}_\mu\mathcal{D}_\rho + \mathcal{D}_\nu\mathcal{D}_\rho\mathcal{D}_\mu + \mathcal{D}_\rho\mathcal{D}_\mu\mathcal{D}_\nu + \mathcal{D}_\rho\mathcal{D}_\nu\mathcal{D}_\mu, \quad (8)$$

$m_q = \mathrm{diag}(m_u, m_d, m_s)$, $m_q^\xi = \xi m_q\xi + \xi^\dagger m_q\xi^\dagger$, $v^\mu = (1, 0, 0, 0)$; the hadronic coupling constants $k_{X/Y}^j$, $\tilde{k}_Y^j$, $\tilde{k}_X^5$, $\tilde{\tilde{k}}_X^5$ with $j = 1, 5$ can be fitted to the experimental data. The flavor and spin violation corrections of the order $\mathcal{O}(1/m_Q)$ are neglected, as there are too many unknown couplings to be determined, we expect that the corrections are not as large as the chiral symmetry-breaking corrections. At the hadronic level, the $1/m_Q$ corrections can be crudely estimated to be of the order $p_K/M_{D_{sJ}^*} \approx 0.20$–$0.25$ or $(M_{D_{sJ}^*} - \overline{M}_{D_s})/M_{D_{sJ}^*} \approx 0.27$ with $\overline{M}_{D_s} = \left(3M_{D_s^*} + M_{D_s}\right)/4$. We can also consistently take into account the $1/m_Q$ corrections by resorting to the covariant heavy meson chiral theory [51,52], however, we have no experimental data or lattice QCD data to fit the unknown hadronic constants, and it is beyond the present work.

From the heavy meson chiral Lagrangians $\mathcal{L}_X$, $\mathcal{L}_Y$, $\mathcal{L}_X^\chi$, and $\mathcal{L}_Y^\chi$, we can obtain the decay widths Γ of the strong decays to the light pseudoscalar mesons,

- $(1^-, 2^-)_{\frac{3}{2}} \to (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P}_j$,

$$\Gamma(2^- \to 1^- + \mathcal{P}_j) = \frac{M_f\left(p_f^2 + m_{\mathcal{P}_j}^2\right)p_f^3}{6\pi M_i}F_j^2, \quad (9)$$

$$\Gamma(1^- \to 1^- + \mathcal{P}_j) = \frac{M_f\left(p_f^2 + m_{\mathcal{P}_j}^2\right)p_f^3}{18\pi M_i}F_j^2, \quad (10)$$

$$\Gamma(1^- \to 0^- + \mathcal{P}_j) = \frac{M_f\left(p_f^2 + m_{\mathcal{P}_j}^2\right)p_f^3}{9\pi M_i}F_j^2, \quad (11)$$

where

$$\begin{aligned}F_j =& \frac{2g_X}{f_\pi\Lambda}\lambda_{ba}^j + \frac{4k_X^1}{f_\pi\Lambda^2}\lambda_{bc}^j(m_q)_{ca} + \frac{4k_X^2}{f_\pi\Lambda^2}(m_q)_{bc}\lambda_{ca}^j\\
&+\frac{4k_X^3}{f_\pi\Lambda^2}\lambda_{ba}^j(m_q)_{cc} + \frac{4k_X^4}{f_\pi\Lambda^2}\delta_{ba}\lambda_{cd}^j(m_q)_{dc}\\
&-\frac{2(k_X^5 + \tilde{k}_X^5 + \tilde{\tilde{k}}_X^5)\sqrt{p_f^2 + m_{\mathcal{P}_j}^2}}{f_\pi\Lambda^2}\lambda_{ba}^j,\\
p_f =& \frac{\sqrt{(M_i^2 - (M_f + m_{\mathcal{P}_j})^2)(M_i^2 - (M_f - m_{\mathcal{P}_j})^2)}}{2M_i},\end{aligned} \quad (12)$$

the i (or b) and f (or a) denote the initial and final state heavy mesons, respectively.

- $(2^-, 3^-)_{\frac{5}{2}} \to (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P}_j$,

$$\Gamma(3^- \to 1^- + \mathcal{P}_j) = \frac{4M_f p_f^7}{105\pi M_i} F_j^2, \quad (13)$$

$$\Gamma(3^- \to 0^- + \mathcal{P}_j) = \frac{M_f p_f^7}{35\pi M_i} F_j^2, \quad (14)$$

$$\Gamma(2^- \to 1^- + \mathcal{P}_j) = \frac{M_f p_f^7}{15\pi M_i} F_j^2, \quad (15)$$

where

$$F_j = \frac{2(g_Y + \tilde{g}_Y)}{f_\pi \Lambda^2} \lambda^j_{ba} + \frac{4(k_Y^1 + \tilde{k}_Y^1)}{f_\pi \Lambda^3} \lambda^j_{bc}(m_q)_{ca} + \frac{4(k_Y^2 + \tilde{k}_Y^2)}{f_\pi \Lambda^3}(m_q)_{bc}\lambda^j_{ca} + \frac{4(k_Y^3 + \tilde{k}_Y^3)}{f_\pi \Lambda^3}\lambda^j_{ba}(m_q)_{cc} + \frac{4(k_Y^4 + \tilde{k}_Y^4)}{f_\pi \Lambda^3}\delta_{ba}\lambda^j_{cd}(m_q)_{dc} - \frac{6(k_Y^5 + \tilde{k}_Y^5)\sqrt{p_f^2 + m^2_{\mathcal{P}_j}}}{f_\pi \Lambda^3}\lambda^j_{ba}. \quad (16)$$

In those decays, the energy release $E_{\mathcal{P}_j} = \sqrt{p_f^2 + m^2_{\mathcal{P}_j}}$ is rather large, the chiral expansion does not converge very quickly, the next-to-leading order chiral corrections may be manifest themselves.

3 Numerical results and discussions

The input parameters are taken as $M_{K^+} = 493.677\,\text{MeV}$, $M_{K^0} = 497.614\,\text{MeV}$, $M_\eta = 547.862\,\text{MeV}$, $M_{D^+} = 1869.5\,\text{MeV}$, $M_{D^0} = 1864.91\,\text{MeV}$, $M_{D_s^+} = 1969\,\text{MeV}$, $M_{D^{*+}} = 2010.29\,\text{MeV}$, $M_{D^{*0}} = 2006.99\,\text{MeV}$, $M_{D_s^{*+}} = 2112.3\,\text{MeV}$ from the Particle Data Group [53].

We redefine the hadronic coupling constants $\bar{g}_Y = g_Y + \tilde{g}_Y$, $\bar{k}_Y^j = \left(k_Y^j + \tilde{k}_Y^j\right)/\bar{g}_Y$, $j = 1$–5, and we write down the decay widths of the $D^*_{s3}(2860)$ explicitly from Eqs. (13) and (14). We have

$$\Gamma(D^{*+}_{s3} \to D^{*0} + K^+) = \frac{16\bar{g}_Y^2 M_{D^*} p_K^7}{105\pi f_\pi^2 \Lambda^4 M_{D^*_{s3}}} \times \left(1 + \frac{2m_u\bar{k}_Y^1}{\Lambda} + \frac{2m_s\bar{k}_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\bar{k}_Y^3}{\Lambda} - \frac{3\sqrt{p_K^2 + m_K^2}\bar{k}_Y^5}{\Lambda}\right)^2,$$

$$\Gamma(D^{*+}_{s3} \to D^0 + K^+) = \frac{4\bar{g}_Y^2 M_D p_K^7}{35\pi f_\pi^2 \Lambda^4 M_{D^*_{s3}}} \times \left(1 + \frac{2m_u\bar{k}_Y^1}{\Lambda} + \frac{2m_s\bar{k}_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\bar{k}_Y^3}{\Lambda} - \frac{3\sqrt{p_K^2 + m_K^2}\bar{k}_Y^5}{\Lambda}\right)^2, \quad (17)$$

$$\Gamma(D^{*+}_{s3} \to D^{*+} + K^0) = \frac{16\bar{g}_Y^2 M_{D^*} p_K^7}{105\pi f_\pi^2 \Lambda^4 M_{D^*_{s3}}} \times \left(1 + \frac{2m_d\bar{k}_Y^1}{\Lambda} + \frac{2m_s\bar{k}_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\bar{k}_Y^3}{\Lambda} - \frac{3\sqrt{p_K^2 + m_K^2}\bar{k}_Y^5}{\Lambda}\right)^2,$$

$$\Gamma(D^{*+}_{s3} \to D^+ + K^0) = \frac{4\bar{g}_Y^2 M_D p_K^7}{35\pi f_\pi^2 \Lambda^4 M_{D^*_{s3}}} \times \left(1 + \frac{2m_d\bar{k}_Y^1}{\Lambda} + \frac{2m_s\bar{k}_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\bar{k}_Y^3}{\Lambda} - \frac{3\sqrt{p_K^2 + m_K^2}\bar{k}_Y^5}{\Lambda}\right)^2, \quad (18)$$

$$\Gamma(D^{*+}_{s3} \to D_s^{*+} + \eta) = \frac{32\bar{g}_Y^2 M_{D_s^*} p_\eta^7}{315\pi f_\pi^2 \Lambda^4 M_{D^*_{s3}}} \times \left(1 + \frac{2m_s\bar{k}_Y^1}{\Lambda} + \frac{2m_s\bar{k}_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\bar{k}_Y^3}{\Lambda} - \frac{(m_u + m_d - 2m_s)\bar{k}_Y^4}{\Lambda} - \frac{3\sqrt{p_\eta^2 + m_\eta^2}\bar{k}_Y^5}{\Lambda}\right)^2,$$

$$\Gamma(D^{*+}_{s3} \to D_s^+ + \eta) = \frac{8\bar{g}_Y^2 M_D p_\eta^7}{105\pi f_\pi^2 \Lambda^4 M_{D^*_{s3}}} \times \left(1 + \frac{2m_s\bar{k}_Y^1}{\Lambda} + \frac{2m_s\bar{k}_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\bar{k}_Y^3}{\Lambda} - \frac{(m_u + m_d - 2m_s)\bar{k}_Y^4}{\Lambda} - \frac{3\sqrt{p_\eta^2 + m_\eta^2}\bar{k}_Y^5}{\Lambda}\right)^2. \quad (19)$$

In this article, we neglect the one-loop chiral corrections, which are estimated to be of the order $\frac{m^2_{\mathcal{P}_j}}{16\pi^2 f_\pi^2} < 10\,\%$. In the case of the hadronic coupling constants g and h in the heavy meson chiral theory, the one-loop chiral corrections are less than 10 % [50], which are consistent with the crude estimation.

We define the ratios R_{0+}, R_{+0}, R_s among the decay widths,

$$R_{0+} = \frac{\Gamma(D^{*+}_{s3} \to D^{*0} + K^+)}{\Gamma(D^{*+}_{s3} \to D^0 + K^+)}, \quad R_{+0} = \frac{\Gamma(D^{*+}_{s3} \to D^{*+} + K^0)}{\Gamma(D^{*+}_{s3} \to D^+ + K^0)}, \quad R_s = \frac{\Gamma(D^{*+}_{s3} \to D_s^{*+} + \eta)}{\Gamma(D^{*+}_{s3} \to D_s^+ + \eta)}. \quad (20)$$

The ratios R_{0+}, R_{+0}, R_s are independent on the hadronic coupling constants $\bar{g}_Y$, $\bar{k}_Y^1$, $\bar{k}_Y^2$, $\bar{k}_Y^3$, $\bar{k}_Y^4$, we can absorb the coupling constants $\bar{k}_Y^1$, $\bar{k}_Y^2$, $\bar{k}_Y^3$, $\bar{k}_Y^4$ into the effective coupling $\bar{g}_Y$ or set $\bar{k}_Y^1 = \bar{k}_Y^2 = \bar{k}_Y^3 = \bar{k}_Y^4 = 0$.

Firstly, let us assign the $D_{sJ}^*(2860)$ to the $D_{s3}^*(2860)$; then we can obtain the value

$$\bar{k}_Y^5 = 0.33223 \pm 0.01248, \tag{21}$$

by setting the $\frac{R_{0+}+R_{+0}}{2}$ to be the experimental data, $\frac{R_{0+}+R_{+0}}{2} = R = 1.10 \pm 0.15 \pm 0.19$ [15]. On the other hand, if we retain only the leading-order coupling constant $\bar{g}_Y$, then $\frac{R_{0+}+R_{+0}}{2} = 0.3866$, which is consistent with the value 0.39 obtained by Colangelo et al. [3]. The value of the hadronic coupling constant $k_Y^5 + \tilde{k}_Y^5$ in the chiral symmetry-breaking Lagrangian is about $\frac{1}{3}$ of that of the hadronic coupling constant $g_Y + \tilde{g}_Y$ in the leading-order Lagrangian according to the relation $\bar{k}_Y^5 = (k_Y^5 + \tilde{k}_Y^5)/(g_Y + \tilde{g}_Y)$. However, taking into account such a chiral symmetry-breaking term can enlarge the ratio R about 2.8 times.

Next we write down the prediction of the ratio R_s,

$$R_s = 0.42 \pm 0.06(0.18), \tag{22}$$

the value 0.18 in the brackets comes from the leading-order heavy meson effective Lagrangian $\mathcal{L}_Y$, i.e. only the $\bar{g}_Y$ is retained. The chiral symmetry-breaking corrections are rather large, the present predictions can be confronted with the experimental data in the future to study the chiral symmetry-breaking corrections.

We can also define the ratios R_+^0 and $\widetilde{R}_+^0$,

$$R_+^0 = \frac{\Gamma(D_{s3}^{*+} \to D^{*0} + K^+)}{\Gamma(D_{s3}^{*+} \to D^{*+} + K^0)}, \quad \widetilde{R}_+^0 = \frac{\Gamma(D_{s3}^{*+} \to D^0 + K^+)}{\Gamma(D_{s3}^{*+} \to D^+ + K^0)}, \tag{23}$$

which are sensitive to the chiral symmetry-breaking corrections associated with $\bar{k}_Y^1$. We can estimate the $\bar{k}_Y^1$ by confronting the ratios R_+^0 and $\widetilde{R}_+^0$ with the experimental data in the future.

Now we assign the $D_{sJ}^*(2860)$ to the $D_{s1}^*(2860)$ and study the strong decays of the $D_{s1}^*(2860)$ as the $1^3\mathrm{D}_1$ $c\bar{s}$ state. Firstly, let us redefine the hadronic coupling constants $\bar{k}_X^j = k_X^j/g_X$, $j = 1$–4, $\bar{k}_X^5 = (k_X^5 + \tilde{k}_X^5 + \tilde{\tilde{k}}_X^5)/g_X$, and we write down the decay widths explicitly from Eqs. (10) and (11),

$$\Gamma(D_{s1}^{*+} \to D^{*0} + K^+) = \frac{2g_X^2 M_{D^*}\left(p_K^2 + m_K^2\right) p_K^3}{9\pi f_\pi^2 \Lambda^2 M_{D_{s1}^*}} \times \left(1 + \frac{2m_u\bar{k}_X^1}{\Lambda} + \frac{2m_s\bar{k}_X^2}{\Lambda} + \frac{2(m_u+m_d+m_s)\bar{k}_X^3}{\Lambda} - \frac{\sqrt{p_K^2+m_K^2}\,\bar{k}_X^5}{\Lambda}\right)^2,$$
$$\Gamma(D_{s1}^{*+} \to D^0 + K^+) = \frac{4g_X^2 M_D\left(p_K^2 + m_K^2\right) p_K^3}{9\pi f_\pi^2 \Lambda^2 M_{D_{s1}^*}} \times \left(1 + \frac{2m_u\bar{k}_X^1}{\Lambda} + \frac{2m_s\bar{k}_X^2}{\Lambda} + \frac{2(m_u+m_d+m_s)\bar{k}_X^3}{\Lambda} - \frac{\sqrt{p_K^2+m_K^2}\,\bar{k}_X^5}{\Lambda}\right)^2, \tag{24}$$

$$\Gamma(D_{s1}^{*+} \to D^{*+} + K^0) = \frac{2g_X^2 M_{D^*}\left(p_K^2 + m_K^2\right) p_K^3}{9\pi f_\pi^2 \Lambda^2 M_{D_{s1}^*}} \times \left(1 + \frac{2m_d\bar{k}_X^1}{\Lambda} + \frac{2m_s\bar{k}_X^2}{\Lambda} + \frac{2(m_u+m_d+m_s)\bar{k}_X^3}{\Lambda} - \frac{\sqrt{p_K^2+m_K^2}\,\bar{k}_X^5}{\Lambda}\right)^2,$$
$$\Gamma(D_{s1}^{*+} \to D^+ + K^0) = \frac{4g_X^2 M_D\left(p_K^2 + m_K^2\right) p_K^3}{9\pi f_\pi^2 \Lambda^2 M_{D_{s1}^*}} \times \left(1 + \frac{2m_d\bar{k}_X^1}{\Lambda} + \frac{2m_s\bar{k}_X^2}{\Lambda} + \frac{2(m_u+m_d+m_s)\bar{k}_X^3}{\Lambda} - \frac{\sqrt{p_K^2+m_K^2}\,\bar{k}_X^5}{\Lambda}\right)^2, \tag{25}$$

$$\Gamma(D_{s1}^{*+} \to D_s^{*+} + \eta) = \frac{4g_X^2 M_{D_s^*}\left(p_\eta^2 + m_\eta^2\right) p_\eta^3}{27\pi f_\pi^2 \Lambda^2 M_{D_{s1}^*}} \times \left(1 + \frac{2m_s\bar{k}_X^1}{\Lambda} + \frac{2m_s\bar{k}_X^2}{\Lambda} + \frac{2(m_u+m_d+m_s)\bar{k}_X^3}{\Lambda} - \frac{(m_u+m_d-2m_s)\bar{k}_X^4}{\Lambda} - \frac{\sqrt{p_\eta^2+m_\eta^2}\,\bar{k}_X^5}{\Lambda}\right)^2,$$
$$\Gamma(D_{s1}^{*+} \to D_s^+ + \eta) = \frac{8g_X^2 M_{D_s}\left(p_\eta^2 + m_\eta^2\right) p_\eta^3}{27\pi f_\pi^2 \Lambda^2 M_{D_{s1}^*}} \times \left(1 + \frac{2m_s\bar{k}_X^1}{\Lambda} + \frac{2m_s\bar{k}_X^2}{\Lambda} + \frac{2(m_u+m_d+m_s)\bar{k}_X^3}{\Lambda} - \frac{(m_u+m_d-2m_s)\bar{k}_X^4}{\Lambda} - \frac{\sqrt{p_\eta^2+m_\eta^2}\,\bar{k}_X^5}{\Lambda}\right)^2. \tag{26}$$

Then we define the ratio $\overline{R}$,

$$\overline{R} = \frac{1}{2}\left\{\frac{\Gamma(D_{s1}^{*+} \to D^{*0} + K^+)}{\Gamma(D_{s1}^{*+} \to D^0 + K^+)} + \frac{\Gamma(D_{s1}^{*+} \to D^{*+} + K^0)}{\Gamma(D_{s1}^{*+} \to D^+ + K^0)}\right\}, \tag{27}$$

which is independent on the hadronic coupling constants g_X, $\bar{k}_X^1$, $\bar{k}_X^2$, $\bar{k}_X^3$, $\bar{k}_X^4$. We can also absorb the coupling constants $\bar{k}_X^1$, $\bar{k}_X^2$, $\bar{k}_X^3$, $\bar{k}_X^4$ into the effective coupling g_X or set $\bar{k}_X^1 = \bar{k}_X^2 = \bar{k}_X^3 = \bar{k}_X^4 = 0$. By setting $\overline{R} = R = 1.10 \pm 0.15 \pm 0.19$ [15], we can obtain the value

$$\bar{k}_X^5 = 1.0555 \pm 0.01953. \tag{28}$$

The value of the hadronic coupling constant $k_X^5 + \tilde{k}_X^5 + \tilde{\tilde{k}}_X^5$ in the chiral symmetry-breaking Lagrangian is as large as that of the hadronic coupling constant g_X in the leading-order Lagrangian according to the relation $\bar{k}_X^5 = (k_X^5 + \tilde{k}_X^5 + \tilde{\tilde{k}}_X^5)/g_X$. The dimensionless coupling constants $\bar{k}_X^5$ and $\bar{k}_Y^5$

are normalized in the same way, and $\bar{k}_X^5 \gg \bar{k}_Y^5$; the convergent behavior is much better in the chiral expansion in the case of assigning the $D_{sJ}^*(2860)$ to the $D_{s3}^*(2860)$. The assignment of the $D_{sJ}^*(2860)$ as the $D_{s1}^*(2860)$ is not favored nor excluded, because a larger coupling constant $\bar{k}_X^5$ does not mean that the chiral expansion breaks down.

Those strong decays of the $D_{s1}^*(2860)$ take place through the relative P-wave, the decay widths are proportional to p_f^3, while the strong decays of the $D_{s3}^*(2860)$ take place through the relative F-wave, and the decay widths are proportional to p_f^7. The decay widths of the $D_{s1}^*(2860)$ are very insensitive to the p_f compared to that of the $D_{s3}^*(2860)$. At present, there is no experimental data to fit the hadronic coupling constants. In the leading-order approximation, i.e. we neglect the chiral symmetry-breaking corrections, the ratios $\overline{R}$, $\widetilde{R}$, and $\overline{R}_s$ among the decay widths are

$$\overline{R} = 0.24 \ \ (0.46 \sim 0.70), \tag{29}$$

$$\widetilde{R} = \frac{\Gamma(D_{s1}^{*+} \to D_s^+ + \eta)}{\Gamma(D_{s1}^{*+} \to D^0 + K^+) + \Gamma(D_{s1}^{*+} \to D^+ + K^0)} = 0.177 \ \ (0.10 \sim 0.14), \tag{30}$$

$$\overline{R}_s = \frac{\Gamma(D_{s1}^{*+} \to D_s^{*+} + \eta)}{\Gamma(D_{s1}^{*+} \to D_s^+ + \eta)} = 0.17, \tag{31}$$

where in the bracket we present the values from the recent studies based on the $^3\mathrm{P}_0$ model [16]. Also in the $^3\mathrm{P}_0$ model, Zhang et al. obtained the value $\overline{R} = 0.16$ [5]. The present value $\overline{R}$ differs greatly from that obtained in Ref. [16], while it is compatible with that obtained in Ref. [5]. In Ref. [17], Godfrey and Jardine obtained the value 0.34 based on the relativized quark model combined with the pseudoscalar emission decay model, which is larger than the present calculation. The present predictions can be confronted with the experimental data in the future to study the strong decays of the $D_{s1}^*(2860)$.

In the leading-order approximation, we obtain the values $R = 0.39$ and $\overline{R} = 0.24$ in the cases of assigning the $D_{sJ}^*(2860)$ to the $D_{s3}^*(2860)$ and $D_{s1}^*(2860)$ respectively, which differ from the experimental value $1.10 \pm 0.15 \pm 0.19$ greatly [15]. If the $D_{sJ}^*(2860)$ observed by the BaBar collaboration in the inclusive $e^+e^- \to \overline{D}^0 K^- X$ production and by the LHCb collaboration in the $pp \to \overline{D}^0 K^- X$ processes consists of two resonances $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ [15,20], we expect to obtain an even smaller ratio R in the case of the chiral symmetry-breaking corrections are small. On the other hand, if the $D_{sJ}^*(2860)$ consists of at least the four resonances $D_{s1}^*(2860)$, $D_{s2}^*(2860)$, $D_{s2}^{*\prime}(2860)$, $D_{s3}^*(2860)$, the large ratio $R = 1.10 \pm 0.15 \pm 0.19$ is easy to account for, as the $J^P = 2^-$ mesons $D_{s2}^*(2860)$ and $D_{s2}^{*\prime}(2860)$ only decay to the final states D^*K; see Eqs. (9) and (15).

In the decays $(1^-, 2^-)_{\frac{3}{2}} \to (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P}_j$, the ratio R_{21}

$$R_{21} = \frac{\Gamma(2^- \to 1^- + \mathcal{P}_j)}{\Gamma(1^- \to 1^- + \mathcal{P}_j)} = 3, \tag{32}$$

while in the decays $(2^-, 3^-)_{\frac{5}{2}} \to (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P}_j$, the ratio R_{23} is

$$R_{23} = \frac{\Gamma(2^- \to 1^- + \mathcal{P}_j)}{\Gamma(3^- \to 1^- + \mathcal{P}_j)} = \frac{7}{4}. \tag{33}$$

According to the ratios R_{21} and R_{23}, the 2^- state in a special doublet, irrespective of the $(1^-, 2^-)_{\frac{3}{2}}$ doublet and the $(2^-, 3^-)_{\frac{5}{2}}$ doublet, has a much larger decay width to the final state $1^- + \mathcal{P}_j$ compared to its partner (the 1^- state or the 3^- state). The 2^- states in the $D_{sJ}^*(2860)$ can enhance the ratio R significantly and account for the large ratio $R = 1.10 \pm 0.15 \pm 0.19$ naturally. The ratios R_{21} and R_{23} are independent on the hadronic coupling constants and determined by the heavy quark symmetry and chiral symmetry. We can confront the present predictions to the experimental data in the future to examine the nature of the $D_{sJ}^*(2860)$ or identify the $D_{s2}^*(2860)$ and $D_{s2}^{*\prime}(2860)$ states. Furthermore, the chiral symmetry-breaking Lagrangians $\mathcal{L}_X^\chi$ and $\mathcal{L}_Y^\chi$ have other phenomenological applications in the heavy-light meson systems; for example, we can study the strong decays of the D-wave $Q\bar{q}$ mesons and calculate the scattering amplitudes of the S-wave and D-wave $Q\bar{q}$ mesons.

4 Conclusion

In this article, we take the $D_{s3}^*(2860)$ and $D_{s1}^*(2860)$ as the $1^3\mathrm{D}_3$ and $1^3\mathrm{D}_1$ $c\bar{s}$ states, respectively, and we study their strong decays with heavy meson effective theory by including the chiral symmetry-breaking corrections. We can reproduce the experimental value of the ratio R, $R = \mathrm{Br}\left(D_{sJ}^*(2860) \to D^*K\right)/\mathrm{Br}\left(D_{sJ}^*(2860) \to DK\right) = 1.10 \pm 0.15 \pm 0.19$, with suitable hadronic coupling constants, the assignment of the $D_{sJ}^*(2860)$ as the $D_{s3}^*(2860)$ is favored. The chiral symmetry-breaking corrections are large, we should take them into account. Furthermore, we obtain the analytical expressions of the decay widths, which can be confronted with the experimental data in the future from the LHCb, CDF, D0, and KEK-B collaborations to fit the unknown coupling constants. The present predictions of the ratios among the decay widths can be used to study the decay properties of the $D_{s3}^*(2860)$ and $D_{s1}^*(2860)$ so as to identify them unambiguously. On the other hand, if the chiral symmetry-breaking corrections are small, the large ratio $R = 1.10 \pm 0.15 \pm 0.19$ requires that the $D_{sJ}^*(2860)$ consists of at least the four resonances $D_{s1}^*(2860)$, $D_{s2}^*(2860)$, $D_{s2}^{*\prime}(2860)$, $D_{s3}^*(2860)$.

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