

Determination of $f_+^K(0)$ and extraction of $|V_{cs}|$ from semileptonic D decays

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Abstract By globally analyzing all existing measured branching fractions and partial rates in different four momentum transfer-squared q^2 bins of $D \rightarrow Ke^+\nu_e$ decays, we obtain the product of the form factor and magnitude of Cabibbo–Kobayashi–Maskawa matrix element V_{cs} to be $f_+^K(0)|V_{cs}| = 0.717 \pm 0.004$. With this product, we determine the $D \rightarrow K$ semileptonic form factor $f_+^K(0) = 0.737 \pm 0.004 \pm 0.000$ in conjunction with the value of $|V_{cs}|$ determined from the standard model global fit. Alternately, with the product together with the input of the form factor $f_+^K(0)$ calculated in lattice quantum chromodynamics (LQCD) recently, we extract $|V_{cs}|^{D \rightarrow Ke^+\nu_e} = 0.962 \pm 0.005 \pm 0.014$, where the error is still dominated by the uncertainty of the form factor calculated in LQCD. Combining the $|V_{cs}|^{D_s^+ \rightarrow \ell^+\nu_\ell} = 1.012 \pm 0.015 \pm 0.009$ extracted from all existing measurements of $D_s^+ \rightarrow \ell^+\nu_\ell$ decays and $|V_{cs}|^{D \rightarrow Ke^+\nu_e} = 0.962 \pm 0.005 \pm 0.014$ together, we find the most precisely determined $|V_{cs}|$ to be $|V_{cs}| = 0.983 \pm 0.011$, which improves the accuracy of the PDG'2014 value $|V_{cs}|^{\text{PDG}'2014} = 0.986 \pm 0.016$ by 45 %.

1 Introduction

In the standard model (SM) of particle physics, the mixing between the quark flavours in weak interaction is parameterized by the Cabibbo–Kobayashi–Maskawa (CKM) matrix, which is a 3×3 unitary matrix. Since the CKM matrix elements are fundamental parameters of the SM, precise determinations of these elements are necessary and very important in testing the SM and searching for new physics (NP).

Since the effects of strong interactions and weak interaction can be well separated in semileptonic D decays, these decays are excellent processes from which we can determine the magnitude of CKM matrix element $V_{cs(d)}$.

In the SM, neglecting the lepton mass, the tree-level differential decay rate in absence of radiative correction for $D \rightarrow Ke^+\nu_e$ process is given by

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{cs}|^2 \mathbf{p}^3 |f_+^K(q^2)|^2, \quad (1)$$

where G_F is the Fermi constant, \mathbf{p} is the three momentum of the K meson in the rest frame of the D meson, q^2 is the four momentum transfer-squared, i.e. the invariant mass of the lepton and neutrino system, and $f_+^K(q^2)$ is the form factor which parameterizes the effect of strong interaction.

In addition to extraction of $|V_{cs}|$, the precise measurements of the $D \rightarrow K$ semileptonic form factor is also very important to validate the lattice quantum chromodynamics (LQCD) calculation of the form factor. If the LQCD calculation of the form factor pass the test with the precisely measured form factor for $D \rightarrow Ke^+\nu_e$ decay, the uncertainty of the semileptonic B decay form factor calculated in LQCD would be reduced. This would help in reducing the uncertainty of the measured $|V_{ub}|$ from semileptonic B decays [1]. The improved measurement of $|V_{ub}|$ from semileptonic B decay will improve the determination of the B_d unitarity triangle, with which one can more precisely test the SM and search for NP.

In the past decades, copious measurements of branching fractions and/or decay rates for $D \rightarrow Ke^+\nu_e$ decays were performed at more than ten experiments. By comprehensive analysis of these existing measurements together with $|V_{cs}|$ from SM global fit or together with form factor $f_+^K(0)$ calculated in LQCD, one can precisely determine the form factor $f_+^K(0)$ or extract $|V_{cs}|$.

In this article, we report the determination of $f_+^K(0)$ or extraction of $|V_{cs}|$ by analyzing all of these existing measurements of the semileptonic $D \rightarrow Ke^+\nu_e$ decays in conjunction with $|V_{cs}|$ from SM global fit or with the form factor $f_+^K(0)$ calculated in LQCD.

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In the following sections, we first review the experimental measurements of branching fractions and decay rates for $D \rightarrow Ke^+\nu_e$ decays in Sect. 2. We then describe our comprehensive analysis procedure for dealing with these measurements to obtain the product of $f_+^K(0)$ and $|V_{cs}|$ in Sect. 3. In Sect. 4, we present the final results of our comprehensive analysis of these measurements. We finally give a summary for the determination of $f_+^K(0)$ and the extraction of $|V_{cs}|$ in Sect. 5.

2 Experiments

2.1 Relative measurements

In 1989, the Tagged Photon Spectrometer Collaboration studied the $D^0 \rightarrow K^-e^+\nu_e$ decays and found 250 signal events for $D^0 \rightarrow K^-e^+\nu_e$ decays at the E691 experiment. Based on these events, they measured the ratio of decay rates $R_0 \equiv \Gamma(D^0 \rightarrow K^-e^+\nu_e)/\Gamma(D^0 \rightarrow K^-\pi^+) = 0.91 \pm 0.07 \pm 0.11$ [2].

In 1991, by analyzing 490 pb⁻¹ data collected with the CLEO detector at the Cornell Electron Storage Ring (CESR), the CLEO Collaboration made a measurement of the branching ratio of D^0 semileptonic decays. They observed $584 \pm 37 \pm 39$ signal events from $D^0 \rightarrow K^-e^+\nu_e$ decays and obtained the ratio of branching fractions $R_0 \equiv B(D^0 \rightarrow K^-e^+\nu_e)/B(D^0 \rightarrow K^-\pi^+) = 0.90 \pm 0.06 \pm 0.06$ [3].

In 1993, the CLEO Collaboration measured the branching ratios of the semileptonic D decay modes using 1.68 fb⁻¹ data collected with the CLEO-II detector at the CESR. They selected the semileptonic D decays from $e^+e^- \rightarrow c\bar{c}$ events and measured the ratios $R_0 \equiv B(D^0 \rightarrow K^-e^+\nu_e)/B(D^0 \rightarrow K^-\pi^+) = 0.978 \pm 0.027 \pm 0.044$ and $R_+ \equiv B(D^+ \rightarrow \bar{K}^0e^+\nu_e)/B(D^+ \rightarrow \bar{K}^0\pi^+) = 2.60 \pm 0.35 \pm 0.26$ [4].

In 2007, the BaBar Collaboration studied the $D^0 \rightarrow K^-e^+\nu_e$ decays by analyzing 75 fb⁻¹ data collected at 10.6 GeV [5]. They selected $D^0 \rightarrow K^-e^+\nu_e$ decays from $e^+e^- \rightarrow c\bar{c}$ events and divide the candidate events into ten q^2 bins. In each q^2 bin, the partial decay rate is measured relative to the normalization mode, $D^0 \rightarrow K^-\pi^+$.

All above mentioned measurements are relative measurements which could not be used directly to determine the form factor $f_+^K(0)$ or $|V_{cs}|$. To use these measurements to determine $f_+^K(0)$ or $|V_{cs}|$, we should first transfer these measurements into absolute decay rates in certain q^2 range. The absolute decay rate $\Delta\Gamma$ can be obtained from the measured relative decay branching ratio R by

$$\Delta\Gamma = R \times B(D \rightarrow K\pi) \times \frac{1}{\tau_D}, \tag{2}$$

where $B(D \rightarrow K\pi)$ is the branching fraction for $D^0 \rightarrow K^-\pi^+$ or $D^+ \rightarrow \bar{K}^0\pi^+$ decays, and τ_D is the lifetime of D meson.

Table 1 The partial rates $\Delta\Gamma$ of the $D^0 \rightarrow K^-e^+\nu_e$ decays in q^2 ranges obtained from different experiments. q_{\max}^2 is the maximum value of q^2

Experiment	q^2 (GeV ² /c ²)	$\Delta\Gamma$ (ns ⁻¹)
E691 [2]	(0.0, q_{\max}^2)	86.76 ± 12.48
CLEO [3]	(0.0, q_{\max}^2)	85.81 ± 8.17
CLEO-II [4]	(0.0, q_{\max}^2)	93.25 ± 5.08
BaBar [5]	(0.0, 0.2)	17.75 ± 0.48
	(0.2, 0.4)	16.26 ± 0.49
	(0.4, 0.6)	14.42 ± 0.42
	(0.6, 0.8)	12.39 ± 0.38
	(0.8, 1.0)	9.92 ± 0.31
	(1.0, 1.2)	7.72 ± 0.26
	(1.2, 1.4)	5.32 ± 0.21
	(1.4, 1.6)	3.24 ± 0.14
	(1.6, 1.8)	1.29 ± 0.09
	(1.8, q_{\max}^2)	0.06 ± 0.01
Mark-III [15]	(0.0, q_{\max}^2)	82.91 ± 15.62
BES-II [16]	(0.0, q_{\max}^2)	93.15 ± 11.77
BES-III [18–20]	(0.0, q_{\max}^2)	85.47 ± 0.93
CLEO-c [21]	(0.0, 0.2)	17.82 ± 0.43
	(0.2, 0.4)	15.83 ± 0.39
	(0.4, 0.6)	13.91 ± 0.36
	(0.6, 0.8)	11.69 ± 0.32
	(0.8, 1.0)	9.36 ± 0.28
	(1.0, 1.2)	7.08 ± 0.24
	(1.2, 1.4)	5.34 ± 0.21
	(1.4, 1.6)	3.09 ± 0.16
	(1.6, q_{\max}^2)	1.28 ± 0.11

To avoid the possible correlations, here we use the value of the branching fraction of $D^0 \rightarrow K^-\pi^+$ decay, $B(D^0 \rightarrow K^-\pi^+) = (3.91 \pm 0.05)\%$, which is the average of the measurements from BaBar [6], CLEO-c [7], CLEO-II [8], ALEPH [9,10], and ARGUS [11]. For the branching fraction of $D^+ \rightarrow \bar{K}^0\pi^+$ decay, we use the value of $B(D^+ \rightarrow \bar{K}^0\pi^+) = (2.986 \pm 0.069)\%$, which is the sum of CLEO-c's measurements $B(D^+ \rightarrow K_S^0\pi^+) = (1.526 \pm 0.022 \pm 0.038)\%$ [7] and $B(D^+ \rightarrow K_L^0\pi^+) = (1.460 \pm 0.040 \pm 0.035)\%$ [12].

Using the lifetime of D meson, $\tau_{D^0} = (410.1 \pm 1.5) \times 10^{-15}$ s, and $\tau_{D^+} = (1040 \pm 7) \times 10^{-15}$ s from PDG [13], the branching fractions of $B(D^0 \rightarrow K^-\pi^+) = (3.91 \pm 0.05)\%$ and $B(D^+ \rightarrow \bar{K}^0\pi^+) = (2.986 \pm 0.069)\%$, we translate these measurements of relative branching fractions and relative partial decay rates into absolute partial decay rates as shown in Tables 1 and 2.

In addition to the measurements of relative branching fractions and relative partial rates, the FOCUS Collaboration measured the non-parametric relative form factors

Table 2 The partial rates of the $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ decays in q^2 ranges obtained from different experiments. q_{\max}^2 is the maximum value of q^2

Experiment	q^2 (GeV $^2/c^2$)	$\Delta\Gamma$ (ns $^{-1}$)
CLEO-II [4]	(0.0, q_{\max}^2)	74.65 ± 12.65
BES-II [17]	(0.0, q_{\max}^2)	86.06 ± 16.60
CLEO-c [21]	(0.0, 0.2)	17.79 ± 0.65
	(0.2, 0.4)	15.62 ± 0.59
	(0.4, 0.6)	14.02 ± 0.54
	(0.6, 0.8)	12.28 ± 0.49
	(0.8, 1.0)	8.92 ± 0.41
	(1.0, 1.2)	8.17 ± 0.37
	(1.2, 1.4)	4.96 ± 0.27
	(1.4, 1.6)	2.67 ± 0.19
	(1.6, q_{\max}^2)	1.19 ± 0.13

Table 3 Measurements of normalized form factors $f_+^K(q_i^2)/f_+^K(0)$ at the FOCUS experiment

i	q_i^2 (GeV $^2/c^2$)	$f_+^K(q_i^2)/f_+^K(0)$
1	0.09	1.01 ± 0.03
2	0.27	1.11 ± 0.05
3	0.45	1.15 ± 0.07
4	0.63	1.17 ± 0.08
5	0.81	1.24 ± 0.09
6	0.99	1.45 ± 0.09
7	1.17	1.47 ± 0.11
8	1.35	1.48 ± 0.16
9	1.53	1.84 ± 0.19

$f_+^K(q^2)/f_+^K(0)$ at the central values of nine q^2 bins by analyzing the $D^0 \rightarrow K^- \mu^+ \nu_\mu$ decays in 2005 [14]. These measured variations of $f_+^K(q^2)/f_+^K(0)$ at FOCUS experiment also provide useful information about the semileptonic decay form factor and are helpful to determine the product $f_+^K(0)|V_{cs}|$ and the shape parameters of the form factor. These measurements are listed in Table 3 and are used in the further analysis.

2.2 Absolute measurements

In 1989, the Mark III Collaboration performed a measurement of absolute branching fraction for semileptonic $D^0 \rightarrow K^- e^+ \nu_e$ decay by analyzing data taken at the peak of $\psi(3770)$ resonance with the Mark III detector. They tagged $3636 \pm 54 \pm 195 \bar{D}^0$ mesons and found $55 D^0 \rightarrow K^- e^+ \nu_e$ signal events in the system recoiling against the \bar{D}^0 tags. With these events, they measured the absolute decay branch-

ing fraction $B(D^0 \rightarrow K^- e^+ \nu_e) = (3.4 \pm 0.5 \pm 0.4) \%$ [15].

Using the similar method as the one used in Mark III, the BES-II Collaboration measured the branching fractions of $D \rightarrow K e^+ \nu_e$ decays by analyzing about 33 pb^{-1} data taken near 3.773 GeV with the BES-II detector at the BEPC collider. Their results are $B(D^0 \rightarrow K^- e^+ \nu_e) = (3.82 \pm 0.40 \pm 0.27) \%$ [16] and $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = (8.95 \pm 1.59 \pm 0.67) \%$ [17].

Recently, the BESIII Collaboration reported preliminary results of $D^0 \rightarrow K^- e^+ \nu_e$ decays obtained by analyzing 2.92 fb^{-1} data taken at 3.773 GeV. They accumulated $(279.3 \pm 0.4) \times 10^4 \bar{D}^0$ tags from five hadronic decay modes. In this sample of \bar{D}^0 tags, they observed 70727 ± 278 signal events for $D^0 \rightarrow K^- e^+ \nu_e$ decays and measured the branching fraction $B(D^0 \rightarrow K^- e^+ \nu_e) = (3.505 \pm 0.014 \pm 0.033) \%$ [18–20].

The partial decay rate is related to the decay branching fraction by

$$\Delta\Gamma = B(D \rightarrow K e^+ \nu_e) \times \frac{1}{\tau_D}. \tag{3}$$

Using the lifetimes of D^0 and D^+ mesons quoted from PDG [13], $\tau_{D^0} = (410.1 \pm 1.5) \times 10^{-15} \text{ s}$ and $\tau_{D^+} = (1040 \pm 7) \times 10^{-15} \text{ s}$, we translate these absolute measurements of branching fractions for $D \rightarrow K e^+ \nu_e$ decays into the partial decay rates, which are shown in Tables 1 and 2.

In 2009, the CLEO Collaboration studied the semileptonic decays of $D^0 \rightarrow K^- e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ by analyzing 818 pb^{-1} data collected at 3.773 GeV with the CLEO-c detector. Using double tag method, they measured the decay rates for semileptonic $D^0 \rightarrow K^- e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ decays in nine q^2 bins [21]. These measurements of decay rates are summarized in Tables 1 and 2.

In 2006, the Belle Collaboration published the results on the $D^0 \rightarrow K^- \ell^+ \nu_\ell$ decays. They accumulated $56461 \pm 309 \pm 830$ inclusive D^0 mesons and found $1318 \pm 37 \pm 7$ signal events for $D^0 \rightarrow K^- e^+ \nu_e$ decays and $1249 \pm 37 \pm 25$ signal events for $D^0 \rightarrow K^- \mu^+ \nu_\mu$ decays from a 282 fb^{-1} data set collected around 10.58 GeV with the Belle detector [22]. Using these selected events from semileptonic D^0 decays, they obtained the form factors $f_+^K(q^2)$ in 27 q^2 bins with the bin size of $0.067 \text{ GeV}^2/c^4$. To obtain the product $f_+^K(q_i^2)|V_{cs}|$ which will be used in our comprehensive analysis in Sect. 3, we extrapolate these measurements of form factors at the Belle experiment to the product $f_+^K(q_i^2)|V_{cs}|$ using the PDG’2006 value of $|V_{cs}| = 0.97296 \pm 0.00024$ [23] which was originally used in the Belle’s paper published. Table 4 lists the form factors $f_+^K(q_i^2)$ measured at the Belle experiment and our translated products $f_+^K(q_i^2)|V_{cs}|$. These products will be used in our further analysis described in Sect. 3.

Table 4 Measurements of form factors $f_+^K(q_i^2)$ at the Belle experiment and the products $f_+^K(q_i^2)|V_{cs}|$

i	q_i^2 (GeV $^2/c^2$)	$f_+^K(q_i^2)$	$f_+^K(q_i^2) V_{cs} $
1	0.100	0.707 ± 0.030	0.688 ± 0.029
2	0.167	0.783 ± 0.030	0.762 ± 0.029
3	0.233	0.763 ± 0.030	0.743 ± 0.029
4	0.300	0.833 ± 0.033	0.811 ± 0.032
5	0.367	0.783 ± 0.033	0.762 ± 0.032
6	0.433	0.840 ± 0.037	0.817 ± 0.036
7	0.500	0.880 ± 0.040	0.856 ± 0.039
8	0.567	0.940 ± 0.040	0.915 ± 0.039
9	0.633	0.907 ± 0.040	0.882 ± 0.039
10	0.700	0.820 ± 0.040	0.798 ± 0.039
11	0.767	1.023 ± 0.043	0.996 ± 0.042
12	0.833	0.997 ± 0.047	0.970 ± 0.045
13	0.900	0.947 ± 0.047	0.921 ± 0.045
14	0.967	1.043 ± 0.053	1.015 ± 0.052
15	1.033	1.100 ± 0.053	1.070 ± 0.052
16	1.100	0.937 ± 0.057	0.911 ± 0.055
17	1.167	1.113 ± 0.067	1.083 ± 0.065
18	1.233	1.097 ± 0.070	1.067 ± 0.068
19	1.300	1.253 ± 0.080	1.219 ± 0.078
20	1.367	1.380 ± 0.087	1.343 ± 0.084
21	1.433	1.313 ± 0.103	1.278 ± 0.101
22	1.500	1.190 ± 0.110	1.158 ± 0.107
23	1.567	1.417 ± 0.123	1.378 ± 0.120
24	1.633	1.473 ± 0.173	1.433 ± 0.169
25	1.700	1.413 ± 0.220	1.375 ± 0.214
26	1.767	1.147 ± 0.340	1.116 ± 0.331
27	1.833	1.450 ± 0.917	1.411 ± 0.892

3 Analysis

To obtain the product of the hadronic form factor at four momentum transfer $q = 0$, $f_+^K(0)$, and the magnitude of CKM matrix element $|V_{cs}|$, we perform a comprehensive χ^2 fit to these experimental measurements of the partial decay rates. The object function to be minimized in the fit is defined as

$$\chi^2 = \chi_R^2 + \chi_P^2 + \chi_F^2, \tag{4}$$

where χ_R^2 is for these measurements of decay branching fraction and/or partial decay rates in different q^2 ranges, χ_P^2 corresponds to the products of $f_+^K(q_i^2)|V_{cs}|$ measured at the Belle experiment, and χ_F^2 is built for the measurements of $f_+^K(q_i^2)/f_+^K(0)$ measured at the FOCUS experiment.

Taking into account the correlations between these measurements, the quantity χ_R^2 is given by

$$\chi_R^2 = \sum_{i=1}^{36} \sum_{j=1}^{36} (\Delta\Gamma_i^{\text{ex}} - \Delta\Gamma_i^{\text{th}})(C_R^{-1})_{ij}(\Delta\Gamma_j^{\text{ex}} - \Delta\Gamma_j^{\text{th}}), \tag{5}$$

where $\Delta\Gamma^{\text{ex}}$ denotes the experimentally measured partial decay rate, $\Delta\Gamma^{\text{th}}$ is the theoretical expectation of the decay rate, and C_R^{-1} is the inverse of the covariance matrix C_R , which is a 36×36 matrix containing the correlations between the measured partial decay rates. The construction of C_R is discussed in Sect. 3.2.

With the parametrization of the form factor, the theoretically predicted partial decay rate in a given q^2 bin is obtained by integrating Eq. (1) from the low boundary q_{low}^2 to the up boundary q_{up}^2 of the q^2 bin,

$$\Delta\Gamma^{\text{th}} = \int_{q_{\text{low}}^2}^{q_{\text{up}}^2} \frac{G_F^2}{24\pi^3} |V_{cs}|^2 p^3 |f_+^K(q^2)|^2 dq^2. \tag{6}$$

In this analysis, we used several forms of the form factor parameterizations which are discussed in Sect. 3.1.

Ignoring some possible correlations of the measurements of the product $f_+^K(q_i^2)|V_{cs}|$ measured at the Belle experiment, the function χ_P^2 in Eq. (4) is defined as

$$\chi_P^2 = \sum_{i=1}^{27} \left(\frac{\tilde{f}_i^{\text{ex}} - \tilde{f}_i^{\text{th}}}{\sigma_i} \right)^2, \tag{7}$$

where \tilde{f}_i^{ex} is the measured product $f_+^K(q_i^2)|V_{cs}|$ at the center of i th q^2 bin q_i^2 with the standard deviation σ_i , and \tilde{f}_i^{th} is the theoretical expectation of the product $f_+^K(q_i^2)|V_{cs}|$ at q_i^2 .

Considering the correlations of the non-parametric form factors measured at the FOCUS experiment, the χ_F^2 is constructed as

$$\chi_F^2 = \sum_{i=1}^9 \sum_{j=1}^9 (F_i^{\text{ex}} - F_i^{\text{th}})(C_F^{-1})_{ij}(F_j^{\text{ex}} - F_j^{\text{th}}), \tag{8}$$

where F_i^{ex} is the measured relative form factor $f_+^K(q_i^2)/f_+^K(0)$ at q_i^2 from the FOCUS experiment, F_i^{th} is the theoretically expected value of $f_+^K(q_i^2)/f_+^K(0)$, and C_F^{-1} is the inverse of the covariance matrix C_F . The construction of C_F is described later in the Sect. 3.2.

3.1 Form factor parameterizations

In general, the single pole model is the simplest approach to describe the q^2 dependent behavior of form factor. The single pole model is expressed as

$$f_+^K(q^2) = \frac{f_+^K(0)}{1 - q^2/m_{\text{pole}}^2}, \tag{9}$$

where $f_+^K(0)$ is the value of form factor at $q^2 = 0$, m_{pole} is the pole mass which is predicted to be the mass of the D_s^{*+} meson for semileptonic $D \rightarrow Ke^+\nu_e$ decays.

The so-called BK parameterization [24] is also widely used in LQCD calculations and experimental studies of this decay. In the BK parameterization, the form factor of the semileptonic $D \rightarrow Ke^+\nu_e$ decays is written as

$$f_+^K(q^2) = \frac{f_+^K(0)}{(1 - q^2/m_{D_s^{*+}}^2)(1 - \alpha q^2/m_{D_s^{*+}}^2)}, \tag{10}$$

where $m_{D_s^{*+}}$ is the mass of the D_s^{*+} meson, and α is a free parameter to be fitted. The value of α is assumed to be around 1.75 for $D \rightarrow K\ell^+\nu_\ell$ in the BK parameterization.

The ISGW2 model [25] assumes

$$f_+^K(q^2) = f_+^K(q_{\text{max}}^2) \left(1 + \frac{r^2}{12}(q_{\text{max}}^2 - q^2)\right)^{-2}, \tag{11}$$

where q_{max}^2 is the kinematical limit of q^2 , and r is the conventional radius of the meson. In this model, the predictions of $f_+^K(q_{\text{max}}^2)$ and r for $D \rightarrow K\ell^+\nu_\ell$ decays are 1.23 and $1.12 \text{ GeV}^{-1}c^2$, respectively.

The most general parameterization of the form factor is the series expansion [26], which is based on analyticity and unitarity. In this parametrization, the variable q^2 is mapped to a new variable z through

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \tag{12}$$

with $t_{\pm} = (m_D \pm m_K)^2$ and $t_0 = t_+(1 - \sqrt{1 - t_-/t_+})$. The form factor is then expressed in terms of the new variable z as

$$f_+^K(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2, t_0)]^k, \tag{13}$$

where $P(q^2) = z(q^2, m_{D_s^{*+}}^2)$ which accounts for the presence of the pole, $\phi(q^2, t_0)$ is an arbitrary function, and $a_k(t_0)$ are real coefficients. In this analysis, the choice of $\phi(q^2, t_0)$ is taken to be

$$\begin{aligned} \phi(q^2, t_0) &= \left(\frac{\pi m_c^2}{3}\right)^{\frac{1}{2}} \left(\frac{z(q^2, 0)}{-q^2}\right)^{\frac{5}{2}} \left(\frac{z(q^2, t_0)}{t_0 - q^2}\right)^{-\frac{1}{2}} \\ &\times \left(\frac{z(q^2, t_-)}{t_- - q^2}\right)^{-\frac{3}{4}} \frac{(t_+ - q^2)}{(t_+ - t_0)^{\frac{1}{4}}}, \end{aligned} \tag{14}$$

where m_c is the mass of charm quark, which is taken to be $1.2 \text{ GeV}/c^2$.

In practical use, one usually make a truncation on the above series. Actually, it is found that the current experimental data can be adequately described by only the first three terms in Eq. (13).

In this analysis we will fit the measured decay rates to the three-parameter series expansion. After optimizing the form factor parameters, we obtain the form for the three-parameter series expansion:

$$f_+^K(q^2) = \frac{f_+^K(0)P(0)\phi(0, t_0)(1 + \sum_{k=1}^2 r_k [z(q^2, t_0)]^k)}{P(q^2)\phi(q^2, t_0)(1 + \sum_{k=1}^2 r_k [z(0, t_0)]^k)}, \tag{15}$$

where $r_k \equiv a_k(t_0)/a_0(t_0)$ ($k = 1, 2$).

3.2 Covariance matrix

It's a little complicated to compute the covariances of these 36 partial decay rates measured in different q^2 ranges and at different experiments. To be clear, we separate the correlations among these $\Delta\Gamma$ measurements into two case: the one associated with the experimental status of each independent experiment, and the other related to the external inputs of parameters such as the lifetime of the D meson.

The statistical uncertainties in the $\Delta\Gamma$ measurements from the same experiment are correlated to some extent, while these are independent for the measurements from different experiments. The systematic uncertainties from tracking, particle identification, etc. are usually independent between different experiments. In this analysis, we treat the systematic uncertainties except the ones from D lifetimes and branching fractions as fully uncorrelated between the measurements performed at different experiments. We consider these below:

- The covariances of the $\Delta\Gamma$ measured at the same experiment are computed using the statistical errors, the systematic errors, and the correlation coefficients, which are presented in their original papers published.
- For the measurements of $D^0 \rightarrow K^-e^+\nu_e$ decay, the lifetime of D^0 meson is used to obtain the partial decay rates in particular q^2 ranges. The systematic uncertainties due to imperfect knowledge of D^0 lifetime are fully correlated among all these measurements of the partial rates of $D^0 \rightarrow K^-e^+\nu_e$ decay. Similarly, the systematic uncertainties related to D^+ lifetime are fully correlated among all of the $\Delta\Gamma$ measurements for $D^+ \rightarrow \bar{K}^0e^+\nu_e$ decay.
- An additional systematic uncertainty from $B(D^0 \rightarrow K^-\pi^+)$ is fully correlated between these relative measurements of $D^0 \rightarrow K^-e^+\nu_e$ decay at the E691, CLEO, CLEO-II and BaBar experiments. Since we only use

Table 5 Fitted parameters corresponding to different form factor parameterizations and $\chi^2/\text{d.o.f.}$ of the fit

Parameterization	$f_+^K(0) V_{cs} $	Shape parameters	$\chi^2/\text{d.o.f.}$
Single pole	0.720 ± 0.003	$m_{\text{pole}} = (1.909 \pm 0.011) \text{ GeV}/c^2$	106.0/70
BK	0.716 ± 0.003	$\alpha = 0.327 \pm 0.021$	101.0/70
ISGW2	0.714 ± 0.003	$r = (1.610 \pm 0.015) \text{ GeV}^{-1}c^2$	101.9/70
Series expansion	0.717 ± 0.004	$r_1 = -2.34 \pm 0.17$ $r_2 = 0.43 \pm 3.82$	101.1/69

one relative measurement of $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ decay which is from the CLEO-II experiment, there are no correlations due to the normalization branching fraction $B(D^+ \rightarrow \bar{K}^0 \pi^+)$ between this measurement and other measurements.

With these considerations mentioned above, we then construct a 36×36 covariance matrix C_R which is necessary in the form factor fit.

The entry in i th row and j th column of the covariance matrix C_R in Eq. (8) is given by $(C_R)_{ij} = \sigma_i \sigma_j \rho_{ij}$, where σ_i and σ_j are the errors of the $f_+^K(q^2)/f_+^K(0)$ at q_i^2 and q_j^2 measured at the FOCUS experiment, respectively, and ρ_{ij} is the correlation coefficient of these two measurements of $f_+^K(q^2)/f_+^K(0)$. The values of the errors and correlation coefficients are directly quoted from Ref. [14].

3.3 Fits to experimental data

Four fits are applied to the experimental data with the form factor hypothesis of single pole model, modified pole model, ISGW2 model and series expansion. The fit to experimental data returns the normalization $f_+^K(0)|V_{cs}|$ and the shape parameters of the form factor which govern the behavior of form factor in high q^2 range.

The numerical results of the fit corresponding to each form of the form factor parameterization are summarized in Table 5. As an example, Fig. 1 presents the result of the fit in the case of using the form factor parameterization of series expansion. In Fig. 1a and b, we compare the measured branching fractions of $D^0 \rightarrow K^- e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ decays from different experiments. Figure 1c and d show the measured differential decay rates for $D^0 \rightarrow K^- e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, respectively. Figure 1e depicts the measurements of $f_+^K(q^2)|V_{cs}|$ at different q^2 from the Belle experiments. The FOCUS measurements of the normalized form factor $f_+^K(q^2)/f_+^K(0)$ are illustrated in Fig. 1f. In these figures, the lines show the best fit to these measurements.

To check the fit quality and also the isospin invariance, the experimentally measured decay branching fractions and/or partial rates are mapped into the product $f_+^K(q_i^2)|V_{cs}|$ via

$$f_+^K(0)|V_{cs}| = \sqrt{\frac{B}{\tau_D} \frac{1}{N}} \tag{16}$$

and

$$f_+^K(q_i^2)|V_{cs}| = \sqrt{\left(\frac{d\Gamma}{dq^2}\right)_i \frac{24\pi^3}{G_F^2 p_i^3}} \tag{17}$$

where B denotes the measured branching fraction, the differential decay rate $(d\Gamma/dq^2)_i$ is obtained by dividing measured decay rate in q^2 bin i by the corresponding bin size. The normalization N is given by

$$N = \frac{G_F^2}{24\pi^3 |f_+^K(0)|^2} \int_0^{q_{\text{max}}^2} p^3 |f_+^K(q^2)|^2 dq^2. \tag{18}$$

The effective p_i^3 in q^2 bin i is given by

$$p_i^3 = \frac{\int_{q_{\text{low}}^2}^{q_{\text{up}}^2} p^3 |f_+^K(q^2)|^2 dq^2}{|f_+^K(q_i^2)|^2 (q_{\text{up}}^2 - q_{\text{low}}^2)}. \tag{19}$$

To calculate the integral in Eqs. (18) and (19), we use the shape parameters of form factor, which is obtained from the series expansion fit to the data.

Figure 2 shows the product $f_+^K(q^2)|V_{cs}|$ as a function of q^2 , where the blue curve corresponds to the best series expansion fit to the experimental data. In this fit, eight measurements of $f_+^K(0)|V_{cs}|$ locate at $q^2 = 0$, which overlap each other. To be clear, these $f_+^K(0)|V_{cs}|$ translated from the decay branching fractions measured at different experiments are also displayed in the insert plot in Fig. 2.

4 Results

In this analysis, we choose the result from the fit using series expansion as our primary results and use this to extract the form factor $f_+^K(0)$ and the magnitude of the CKM matrix element V_{cs} .

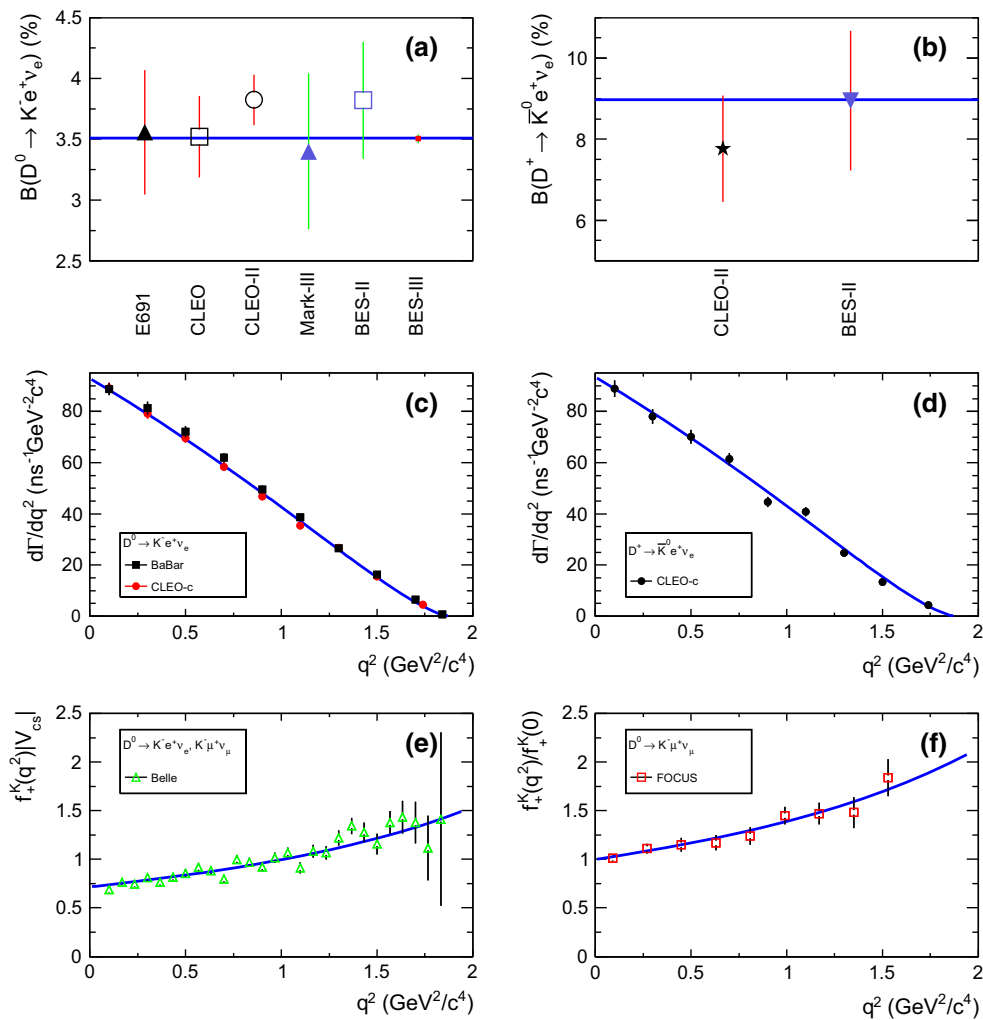


Fig. 1 Comparisons of branching fraction measurements for **a** $D^0 \rightarrow K^- e^+ \nu_e$, **b** $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, **c** measurements of differential decay rates for $D^0 \rightarrow K^- e^+ \nu_e$ measured at the BaBar and CLEO-c experiments, **d** differential decay rates for $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ measured at the CLEO-c

experiment, **e** the product of form factor and $|V_{cs}|$ measured at the Belle experiment, and **f** the normalized form factor measured at the FOCUS experiment. The *blue lines* show the fit to these measurements using the series expansion for the form factor

4.1 Form factor $f_+^K(0)$

Dividing the value of $f_+^K(0)|V_{cs}| = 0.717 \pm 0.004$ shown in Table 5 from the series expansion fit by the $|V_{cs}| = 0.97343 \pm 0.00015$ obtained using unitarity constraints [13] yields the form factor

$$f_+^K(0) = 0.737 \pm 0.004 \pm 0.000, \tag{20}$$

where the first uncertainty is from the combined statistical and systematic uncertainties in the partial decay rate measurements, and the second is due to the uncertainty in the $|V_{cs}|$.

The result for the form factor determined in this analysis is compared with the theoretical calculations of the form factor from the LQCD [27–29] and from QCD light-cone sum

rules [30] in Fig. 3. Our result of the form factor extracted by analyzing all existing experimental measurements is consistent with these values predicted by theory, but is with higher precision than the most accurate one from LQCD calculation by a factor of 2.8.

4.2 Parameters of form factor

When these shape parameters of the form factor parameterization are left free in the fit, the form factor parameterizations of the single pole model, BK model, the ISGW2 model, and the series expansion model are all capable of describing the experimental data with almost identical χ^2 probability. However, for the physical interpretation of the shape parameters in the single pole model, BK model, the ISGW2 model, the values of the parameters obtained from the fits are largely

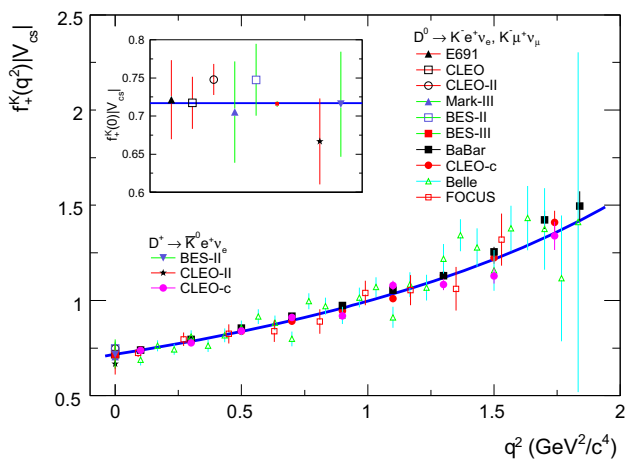


Fig. 2 The product $f_+^K(q^2)|V_{cs}|$ measured at different experiments as a function of q^2 . The blue curve represents the series expansion fit to these $f_+^K(q^2)|V_{cs}|$. The insert plot shows the comparison of the products $f_+^K(0)|V_{cs}|$ which are obtained using the branching fractions measured at different experiments

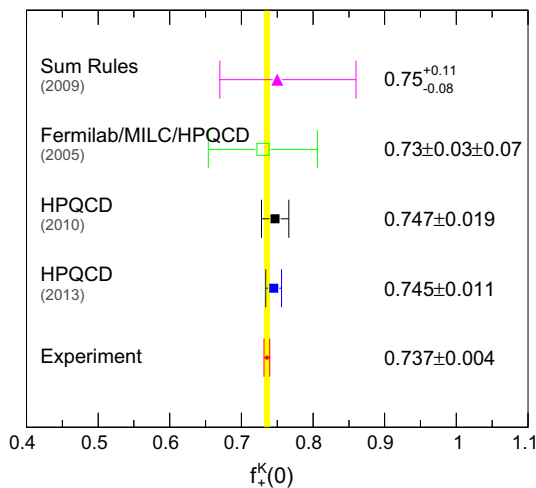


Fig. 3 Comparison of our determined form factor from experimental measurements with the theoretical calculations of the form factor

deviated from those expected values by these models. This indicates that the experimental data do not support the physical interpretation of the shape parameters in these parameterizations. Figure 4a–c show the comparisons between the measured values and the theoretically expected values for the pole mass m_{pole} in single pole model, α in BK model, and r in ISGW2 model. These measured parameters do not agree with the values predicted by these form factor models.

4.3 CKM matrix element $|V_{cs}|$

Using the product $f_+^K(0)|V_{cs}| = 0.717 \pm 0.004$ obtained from the comprehensive series expansion fit in conjunction with the form factor $f_+^K(0) = 0.745 \pm 0.011$ [27] calculated in LQCD for the $D \rightarrow K$ transition, we determine the

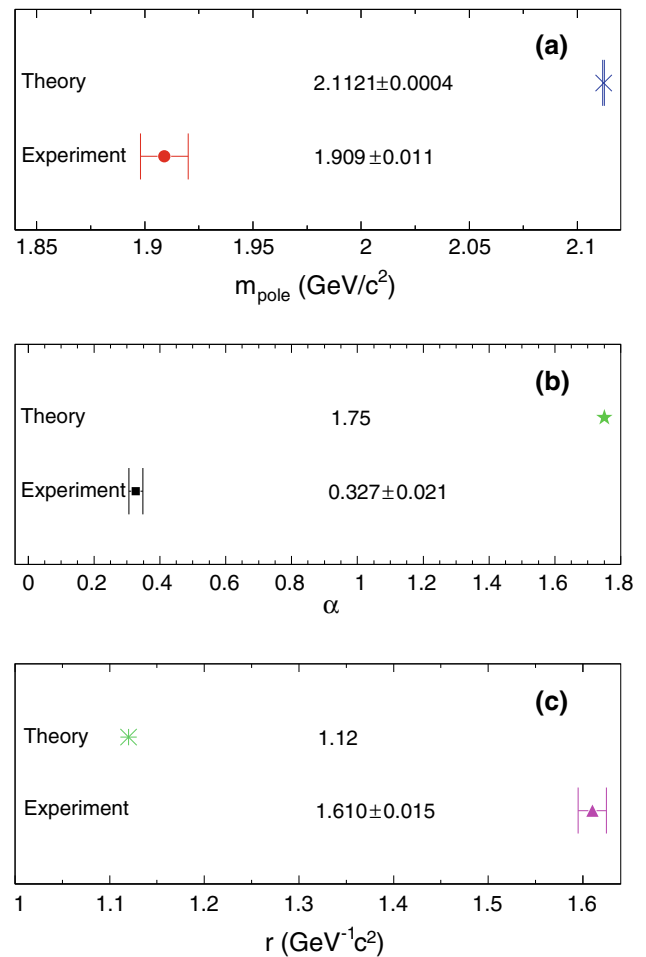


Fig. 4 Comparisons of the form factor parameters determined from experimental measurements and the theoretical expectations: **a** the pole mass m_{pole} in single pole model, **b** α in the BK model, and **c** r in the ISGW2 model

magnitude of the CKM matrix element V_{cs} to be

$$|V_{cs}|^{D \rightarrow Ke^+ \nu_e} = 0.962 \pm 0.005 \pm 0.014, \quad (21)$$

where the last uncertainty corresponds to the accuracy of the form factor $f_+^K(0)$ calculated in LQCD.

Combining with the value $|V_{cs}|^{D_s^+ \rightarrow \ell^+ \nu_\ell} = 1.012 \pm 0.015 \pm 0.009$, which is extracted from the measurements of leptonic D_s^+ decays (see Appendix A), we obtain the magnitude of the CKM matrix element V_{cs} to be

$$|V_{cs}| = 0.983 \pm 0.011. \quad (22)$$

Figure 5 shows the comparisons of the value of $|V_{cs}|$ which is determined with the $|V_{cs}|^{D \rightarrow Ke^+ \nu_e}$ in this analysis together with the $|V_{cs}|^{D_s^+ \rightarrow \ell^+ \nu_\ell}$ determined from leptonic D_s^+ decays, and the value from a SM global fit [13].

Figure 6 shows a comparison of our extracted $|V_{cs}|$ from all existing measurements of $D \rightarrow Ke^+ \nu_e$ and $D_s^+ \rightarrow \ell^+ \nu_\ell$

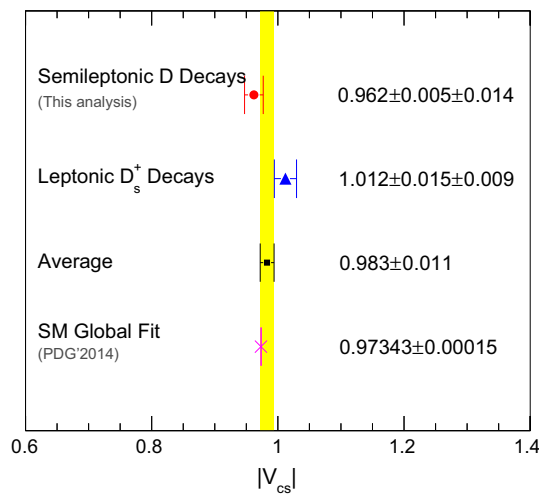


Fig. 5 Comparison of $|V_{cs}|$ extracted from semileptonic D decays in this analysis with the one extracted from leptonic D_s^+ decays and the one from SM global fit

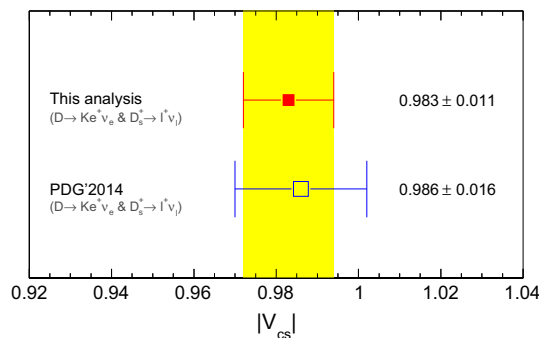


Fig. 6 Comparison of $|V_{cs}|$ extracted from semileptonic D decays and leptonic D_s^+ decays in this analysis with the PDG value

decays along with the PDG'2014 value of the $|V_{cs}|$ determined with CLEO-c, BaBar and Belle's measurements of $D \rightarrow Ke^+ \nu_e$ and $D_s^+ \rightarrow \ell^+ \nu_\ell$ decays [13].

The $|V_{cs}|^{D \rightarrow Ke^+ \nu_e}$ extracted from semileptonic D decays deviates from the $|V_{cs}|^{D_s^+ \rightarrow \ell^+ \nu_\ell}$ extracted from leptonic D_s^+ decays by 2.2σ . This discrepancy may arise from three sources: (1) some new physic effects involved in leptonic D_s^+ decays, which modify the decay rate; (2) underestimated decay constant $f_{D_s^+}$ in LQCD; (3) overestimated form factor $f_+^K(0)$ in LQCD. Any of these would modify these decay rates resulting in shifts of the $|V_{cs}|^{D \rightarrow Ke^+ \nu_e}$ and $|V_{cs}|^{D_s^+ \rightarrow \ell^+ \nu_\ell}$, which are extracted from semileptonic D and leptonic D_s^+ decays, respectively.

4.4 Effects of radiative correction on decay rate

From experimental aspect, it's difficult to exclude the soft photon emission in the final state in the procedure of event selection. As a consequence, the experimentally measured

branching fractions or partial decay rates usually include the contribution from the soft photon emission in the final states more or less. The theoretical prediction for the decay rate, Eq. (1), used in the extractions of form factor $f_+^K(0)$ and CKM matrix element $|V_{cs}|$ is for the tree-level semileptonic D decay process. As the experimental precision of the decay rate measurement has already achieved an accuracy level of 0.5 %, in addition to improve the precision of the LQCD calculation for $f_+^K(0)$, it should be also necessary to take into account the radiative correction in further precise determination of $|V_{cs}|$. However, unlike the situation in $K_{\ell 3}$ decays, due to the lack of an universally valid effective theory, the theoretical or phenomenological estimation of radiative corrections in the semileptonic D decays is absent at present stage. So we ignore this radiative correction in the analysis at present.

5 Summary

By globally analyzing all existing branching fractions of the $D \rightarrow Ke^+ \nu_e$ decays measured at earlier experiments and recent BESIII experiment as well as the precise measurements of partial decay rates in q^2 bins performed at the BaBar and CLEO-c experiments together, we obtain the most precise product of form factor and the magnitude of CKM matrix element V_{cs} from a comprehensive χ^2 fit. This obtained product reflects all of measurements for $D \rightarrow Ke^+ \nu_e$ decays in the world in the last 25 years. With the obtained $f_+^K(0)|V_{cs}|$ in conjunction with $|V_{cs}|$ from SM global fit, we determine the form factor

$$f_+^K(0) = 0.737 \pm 0.004 \pm 0.000,$$

which is in good agreement within error with LQCD calculations, but more precise than the most accurate LQCD calculation of the form factor by 2.8 factors. Alternately, with the recent most precise semileptonic $D \rightarrow Ke^+ \nu_e$ decay form factor calculated in LQCD, we obtain the $|V_{cs}|^{D \rightarrow Ke^+ \nu_e} = 0.962 \pm 0.005 \pm 0.014$, where the error is dominated by the uncertainties in LQCD calculation of the hadronic form factor. This determined $|V_{cs}|$ is in good agreement within error with the one from SM global fit, which indicates that no evidence of new physic effects involved in the semileptonic $D \rightarrow Ke^+ \nu_e$ decays is observed at present experimental accuracy level.

If combining the $|V_{cs}|^{D \rightarrow Ke^+ \nu_e} = 0.962 \pm 0.005 \pm 0.014$ determined from semileptonic D decays and $|V_{cs}|^{D_s^+ \rightarrow \ell^+ \nu_\ell} = 1.012 \pm 0.015 \pm 0.009$ determined from leptonic D_s^+ decays together, we find

$$|V_{cs}| = 0.983 \pm 0.011,$$

which improves the accuracy of the PDG'2014 value $|V_{cs}|^{\text{PDG}'2014} = 0.986 \pm 0.016$ by 45 %, and is the most precisely extracted $|V_{cs}|$ from all existing measurements of semileptonic D decays and leptonic D_s^+ decays up to date.

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Appendix A: Extraction of $|V_{cs}|$ from leptonic D_s^+ decays

In this appendix, we present the determination of $|V_{cs}|$ by analyzing the existing measurements of leptonic $D_s^+ \rightarrow \ell^+ \nu_\ell$ ($\ell = \mu, \tau$) decays.

In SM of particle physics, the decay width of $D_s^+ \rightarrow \ell^+ \nu_\ell$ is given by

$$\Gamma(D_s^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} m_\ell^2 m_{D_s^+} \left(1 - \frac{m_\ell^2}{m_{D_s^+}^2}\right)^2 f_{D_s^+}^2 |V_{cs}|^2, \tag{A1}$$

where m_ℓ is the mass of lepton and $m_{D_s^+}$ is the mass of D_s^+ meson. The parameter $f_{D_s^+}$ is the decay constant, which is associated with the strong interaction effects between the two initial-state quarks.

In the past two decades, many measurements of leptonic $D_s^+ \rightarrow \ell^+ \nu_\ell$ decays were performed at e^+e^- experiments and fixed-target experiments. These measured branching fractions are summarized in Table 6.

To extract the magnitude of CKM matrix element V_{cs} , we globally analyze all of these existing measurements of leptonic D_s^+ decay branching fractions shown in Table 6. Assuming lepton universality, we construct a object function χ^2 :

$$\chi^2 = \sum_{i=1}^5 \left(\frac{B_{\mu,i}^{\text{ex}} - B_\mu^{\text{th}}}{\sigma_i}\right)^2 + \sum_{j=1}^3 \left(\frac{R_{\mu,j}^{\text{ex}} - R_\mu^{\text{th}}}{\sigma_j}\right)^2 + \sum_{k=1}^6 \left(\frac{B_{\tau,k}^{\text{ex}} - B_\tau^{\text{th}}}{\sigma_k}\right)^2, \tag{A2}$$

where $B_{\mu,i}^{\text{ex}}$ is the i th experimentally measured branching fraction of $D_s^+ \rightarrow \mu^+ \nu_\mu$ decay, $R_{\mu,j}^{\text{ex}}$ is the j th experimentally measured partial width of $D_s^+ \rightarrow \mu^+ \nu_\mu$ decay relative to the partial width of $D_s^+ \rightarrow \phi\pi^+$ decay, $B_{\tau,k}^{\text{ex}}$ is the k th

Table 6 Measurements of $B(D_s^+ \rightarrow \mu^+ \nu_\mu)$ and $B(D_s^+ \rightarrow \tau^+ \nu_\tau)$

Experiment	$B(D_s^+ \rightarrow \mu^+ \nu_\mu)$ (%)
BES-I [31]	$1.5^{+1.3+0.3}_{-0.6-0.2}$
ALEPH [32]	$0.68 \pm 0.11 \pm 0.18$
CLEO-c [33]	$0.565 \pm 0.045 \pm 0.017$
BaBar [34]	$0.602 \pm 0.038 \pm 0.034$
Belle [35]	$0.531 \pm 0.028 \pm 0.020$
Experiment	$\Gamma(D_s^+ \rightarrow \mu^+ \nu_\mu) / \Gamma(D_s^+ \rightarrow \phi\pi^+)$
BEATRICE [36]	$0.23 \pm 0.06 \pm 0.04$
CLEO-II [37]	$0.173 \pm 0.023 \pm 0.035$
BaBar [38]	$0.143 \pm 0.018 \pm 0.006$
Experiment	$B(D_s^+ \rightarrow \tau^+ \nu_\tau)$ (%)
L3 [39]	$7.4 \pm 2.8 \pm 2.4$
OPAL [40]	$7.0 \pm 2.1 \pm 2.0$
ALEPH [32]	$5.79 \pm 0.77 \pm 1.84$
CLEO-c [41]	$5.58 \pm 0.33 \pm 0.13$
BaBar [34]	$5.00 \pm 0.35 \pm 0.49$
Belle [35]	$5.70 \pm 0.21^{+0.31}_{-0.30}$

experimentally measured branching fraction of $D_s^+ \rightarrow \tau^+ \nu_\tau$ decay, and σ denotes the combined statistical and systematic error of the measured (relative) branching fraction.

The theoretically predicted branching fraction of $D_s^+ \rightarrow \mu^+ \nu_\mu$ is given by

$$B_\mu^{\text{th}} = \Gamma(D_s^+ \rightarrow \mu^+ \nu_\mu) \times \tau_{D_s^+}, \tag{A3}$$

where $\Gamma(D_s^+ \rightarrow \mu^+ \nu_\mu)$ is calculated with Eq. (A1), and $\tau_{D_s^+}$ is the lifetime of D_s^+ meson.

The theoretically expected ratio of $\Gamma(D_s^+ \rightarrow \mu^+ \nu_\mu)$ over $\Gamma(D_s^+ \rightarrow \phi\pi^+)$ is given by

$$R_\mu^{\text{th}} = B_\mu^{\text{th}} / B(D_s^+ \rightarrow \phi\pi^+), \tag{A4}$$

where $B(D_s^+ \rightarrow \phi\pi^+)$ is the branching fraction of $D_s^+ \rightarrow \phi\pi^+$ decay.

The theoretically predicted branching fraction of $D_s^+ \rightarrow \tau^+ \nu_\tau$ is given by

$$B_\tau^{\text{th}} = \Gamma(D_s^+ \rightarrow \tau^+ \nu_\tau) \times \tau_{D_s^+}, \tag{A5}$$

where $\Gamma(D_s^+ \rightarrow \tau^+ \nu_\tau)$ is calculated with Eq. (A1).

To obtain the experimentally measured product $f_{D_s^+} |V_{cs}|$, we perform a χ^2 fit to these measured branching fractions for $D_s^+ \rightarrow \ell^+ \nu_\ell$ decays shown in Table 6. In the fit, we use $m_\mu = (105.6583715 \pm 0.0000035) \text{ MeV}/c^2$, $m_\tau = (1776.82 \pm 0.16) \text{ MeV}/c^2$, $m_{D_s^+} = (1968.30 \pm 0.11) \text{ MeV}/c^2$, $\tau_{D_s^+} = (500 \pm 7) \times 10^{-15} \text{ s}$, and $B(D_s^+ \rightarrow \phi\pi^+) = (4.5 \pm 0.4) \%$ which are all quoted from PDG [13]. The product of the decay

constant and the magnitude of CKM matrix element V_{cs} is the only free parameter in the fit. The fit returns

$$f_{D_s^+}|V_{cs}| = (252.0 \pm 3.7 \pm 1.8) \text{ MeV}, \quad (\text{A6})$$

where the first error is from the statistical and systematic uncertainties in the measured (relative) branching fractions, and the second error is due to the uncertainties in the masses of lepton and D_s^+ meson, the lifetime of D_s^+ meson, and the branching fraction of $D_s^+ \rightarrow \phi\pi^+$ decay.

Dividing the product $f_{D_s^+}|V_{cs}|$ by the value $f_{D_s^+} = (249.0 \pm 0.3_{-1.5}^{+1.1})$ MeV which is the newest and most precise value of decay constant calculated in LQCD with $N_f = 2 + 1 + 1$ quark flavors [42], we obtain

$$|V_{cs}|^{D_s^+ \rightarrow \ell^+ \nu_\ell} = 1.012 \pm 0.015 \pm 0.009, \quad (\text{A7})$$

where the first error is from the statistical and systematic uncertainties in the measured (relative) branching fractions, and the second error is mainly due to the uncertainties in the lifetime of D_s^+ meson, and the $f_{D_s^+}$ calculated in LQCD.

Alternatively, by inserting $|V_{cs}| = 0.97343 \pm 0.00015$ from the SM global fit [13] into Eq. (A6), we determine

$$f_{D_s^+} = (258.9 \pm 3.8 \pm 1.8) \text{ MeV}, \quad (\text{A8})$$

which is the most precisely determined D_s^+ leptonic decay constant.

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