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Radiating Kerr-like regular black hole

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Abstract We derive a radiating regular rotating black hole solution, radiating Kerr-like regular black hole solution. We achieve this by starting from the Hayward regular black hole solution via a complex transformation suggested by Newman–Janis. The radiating Kerr metric, the Kerr-like regular black hole and the standard Kerr metric are regained in the appropriate limits. The structure of the horizon-like surfaces are also determined.

1 Introduction

The formation of spacetime singularities is a quite common phenomenon in general relativity and, indeed, celebrated theorems, proved by Penrose and Hawking [1], state that under some circumstances singularities are inevitable in general relativity. As these theorems use only the laws of general relativity and some properties of matter, they are valid generally. It is widely accepted that spacetime singularities do not exist in Nature; they are a limitation or creation of the classical theory. The existence of a singularity implies that there exists a point in spacetime where the laws of physics break down or signal a failure of the physical laws. It turns out that what amounts to a singularity in general relativity could be adequately explained by some other theory. If physical laws do exist at those extreme situations, then we should turn ourselves to a theory of quantum gravity. However, we are yet a long distance away from a definite theory of quantum gravity. So a line of action is to understand the inside of a black hole and resolve its singularity by carrying out research of classical or semi-classical black holes, with regular, i.e., nonsingular, properties. This can be motivated by quantum arguments. Sakharov [2] and Gliner [3] proposed that spacetime in the highly dense central region of a black hole should be de Sitter-like for $r \simeq 0$ (see also, [4–7]). This indicates that an unlimited increase of spacetime curvature during a collapse process can stop the collapse if quantum fluctuations dominate the process. This places an upper bound on the value of the curvature and necessitates the formation of a central core.

Bardeen [8] realized concretely the idea of a central matter core, by proposing the first regular black hole solution of the Einstein equations. Bardeen's regular metric is a solution of the Einstein equations in the presence of an electromagnetic field, yielding an alteration of the Reissner–Nordström metric. But near the center the solution tended to a de Sitter core solution. Subsequently, there has been enormous development in investigating the properties of regular black hole solutions [9–17], but most of these regular black hole solutions are more or less based on Bardeen's proposal. In particular, an interesting proposal is made by Hayward [17] for the formation and evaporation of regular black holes, in which the static region is the Bardeen-like black hole. The dynamic regions are Vaidya-like black hole regions, with negative energy flux during evaporation and ingoing radiation of positive energy flux during collapse. The latter is balanced by outgoing radiation of positive energy flux and a surface pressure at a pair creation surface. This is the only non-stationary or dynamical regular black hole. However, these non-rotating metrics cannot be tested by astrophysical observations, as the black hole spin plays an important and fundamental role in any astrophysical process.

The generalization of these stationary regular black holes to the axially symmetric case, the Kerr-like regular black hole, was addressed recently [18–20]. In particular, it was established [18,19] that the rotating regular black hole solutions can be obtained starting from regular black hole solutions by a complex coordinate transformation previously suggested by Newman and Janis [21]. However, this is obviously not the most physical scenario and we would like to consider dynamical black hole solutions, i.e., black holes with

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non-trivial time dependence. Further, the axially symmetric counterpart of the regular Vaidya-like black hole is still unexplored, e.g., the radiating generalization of the regular Kerr-like black hole is still unknown. It is the purpose of this paper to obtain this metric. Thus we extend a recent work [17] on radiating regular black holes to include rotation, and it is also a non-static generalization of the Kerr-like regular black hole solution [18]. We also carry out a detailed analysis of the horizon structure of radiating Kerr-like regular black holes, which also is valid for a static Kerr-like regular black hole and which has not been done earlier. It should be pointed out that the Kerr metric [22] is undoubtedly the single most significant exact solution in the Einstein theory of general relativity, which represents the prototypical black hole that can arise from gravitational collapse. The radiating or non-static counterpart of the Kerr black hole was obtained by Carmeli [23]. We also show that the Kerr-like regular black hole, the Kerr black hole, and the radiating Kerr-like black hole arise as special cases of the radiating Kerr-like regular black hole.

In this paper, we obtain a radiating Kerr-like regular metric in Sect. 2. The Newman–Janis algorithm is applied to spherically symmetric radiating solutions, and radiating rotating solutions are obtained. We investigate the structure and locations of horizons of the radiating Kerr-like regular metric in Sect. 3. The paper ends with concluding remarks in Sect. 4.

We use units which fix the speed of light and the gravitational constant via $G = c = 1$, and use the metric signature $(+, -, -, -)$.

2 Radiating rotating black hole via Newman–Janis

We wish to obtain a radiating rotating regular black hole solution from spherically symmetric black hole solutions via the complex transformation suggested by Newman–Janis [21]. For this purpose, we begin with the "seed metric", expressed in terms of the Eddington (ingoing) coordinate v, as

$$ds^2 = e^{\psi(v,r)} dv \left[f(v,r) e^{\psi(v,r)} dv + 2dr \right] - r^2 d\Omega^2, \quad (1)$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Here $e^{\psi(v,r)}$ is an arbitrary function. It is useful to introduce a local mass function $m(v,r)$ defined by $f(v,r) = 1 - 2m(v,r)/r$. For $m(v,r) = M(v)$ and $\psi(v,r) = 0$, the metric reduces to the standard Vaidya metric. We can always set, without any loss of generality, $\psi(v,r) = 0$. Thus any spherically symmetric radiating black hole is defined by the metric (1). The function $f(v,r)$ is a function of v and r and depends on the matter field.

The Newman–Janis algorithm can be applied to any spherically symmetric static black hole solution of general relativity to generate rotating black hole spacetimes. For example, the Kerr metric can be obtained from the Schwarzschild metric, and the Reissner–Nordström solution leads to the Kerr–Newman solutions, which is based on a complex coordinate transformation. Recently, regular rotating black holes were derived from exact spherically symmetric regular black hole solutions [18]. In what follows, we extend the approach by applying the Newman–Janis algorithm to the general spherically symmetric radiating *seed* metric (1), which can be put in the form

$$ds^2 = f(v,r)dv^2 + 2dvdr - r^2 d\Omega^2, \quad (2)$$

to construct a general radiating rotating black hole solution. The first step of the Newman–Janis algorithm is not required here as the *seed* metric (2) is already in the Eddington–Finkelstein coordinates.

The metric $\tilde{g}_{ab}$ given by Eq. (1) can be written in terms of a null tetrad [21] as

$$\tilde{g}^{ab} = -L^a N^b - L^b N^a + M^a \bar{M}^b + M^b \bar{M}^a, \quad (3)$$

where the null tetrad has the form

$$L^a = \delta^a_r,$$
$$N^a = \delta^a_u - \frac{1}{2} f(v,r) \delta^a_r,$$
$$M^a = \frac{1}{\sqrt{2}r} \left(\delta^a_\theta + \frac{i}{\sin\theta} \delta^a_\phi \right).$$

This tetrad is orthonormal, obeying the conditions

$$L_a M^a = L_a \bar{M}^a = N_a M^a = N_a \bar{M}^a = 0, \quad (4)$$
$$L_a L^a = N_a N^a = M_a M^a = \bar{M}_a \bar{M}^a = 0, \quad (5)$$
$$L_a N^a = -1, \quad M_a \bar{M}^a = 1. \quad (6)$$

Now we allow for some r factors in the null vectors to take on complex values. We rewrite the null vectors in the form [21,24,26]

$$L^a = \delta^a_r,$$
$$N^a = \left[\delta^a_u - \frac{1}{2} f(v,r,\bar{r}) \delta^a_r \right],$$
$$M^a = \frac{1}{\sqrt{2}\bar{r}} \left(\delta^a_\theta + \frac{i}{\sin\theta} \delta^a_\phi \right).$$

Following the Newman–Janis prescription [21], we now write

$$x'^\mu = x^\mu + ia(\Delta^\mu - \delta^\mu_u) \cos\theta \rightarrow \begin{cases} v' = v - ia\cos\theta, \\ r' = r + ia\cos\theta, \\ \theta' = \theta, \phi' = \phi. \end{cases} \quad (7)$$

and we also transform the tetrad $Z^a_s = (L^a, N^a, M^a, \bar{M}^a)$ in the usual way:

$$Z'^a_s = \frac{\partial x'^a}{\partial x^b} Z^b_s, \quad (8)$$

leading to

$$L^a = \delta^a_r, \tag{9}$$

$$N^a = \left[\delta^a_v - \frac{1}{2}\mathcal{F}(v,r,\theta)\delta^a_r\right], \tag{10}$$

$$M^a = \frac{1}{\sqrt{2}(r+ia\cos\theta)}\left[ia\sin\theta\left(\delta^a_v-\delta^a_r\right)+\delta^a_\theta+\frac{i}{\sin\theta}\delta^a_\phi\right], \tag{11}$$

dropping the primes. This transformed tetrad yields a new metric (see Ref. [21,24], for further details) given by the line element

$$\begin{aligned} ds^2 = & \mathcal{F}(v,r,\theta)dv^2+2dvdr-\Sigma(r,\theta)d\theta^2-2a\sin^2\theta dr d\phi \\ & + \left[a^2(\mathcal{F}(v,r,\theta)-2)\sin^2\theta-\Sigma(r,\theta)\right]\sin^2\theta d\phi^2 \\ & +2a\left[1-\mathcal{F}(v,r,\theta)\right]\sin^2\theta dv\, d\phi. \end{aligned} \tag{12}$$

Here $\mathcal{F}(v,r,\theta)$ is a function which depends on $f(r,v)$. It describes the exterior field of the radiating rotating objects. We have applied the aforesaid procedure to radiating models. But the method is general and is applicable to any general radiating spherically symmetric solution to generate a general rotating radiating spacetimes (12). Carmeli [23] was first to obtain the metrics of a rotating radiating spacetime, which in the limit $a = 0$ reduces to the Vaidya spacetime. To further support our analysis, we should be able to rediscover the solution obtained by Carmeli [23], but by using Newman–Janis algorithm. In the Vaidya case

$$f(v,r) = 1 - \frac{2M(v)}{r}. \tag{13}$$

After complex transformations it has the form

$$\mathcal{F}(v,r,\theta) = 1 - \frac{2M(v)r}{\Sigma},$$

and $\Sigma = r^2 + a^2\cos^2\theta$. In the above analysis, all the steps of the Newman–Janis algorithm are applicable to a radiating spacetime to generate the corresponding radiating rotating spacetime. However, to generate the Carmeli radiating rotating spacetime, we must demand that the mass term $M(v)$ remains invariant under the complex transformations. Then we start with the radiating spherically symmetric metric (1), written in Eddington–Finkelstein coordinates; performing the Newman–Janis algorithm with the above $f(v,r)$ given by (13), we derive a radiating rotating solution which takes the form

$$\begin{aligned} ds^2 = & \frac{1}{\Sigma}\left[\Delta - a^2\sin^2\theta\right]dv^2 + 2\left[dv - a\sin^2 d\phi\right]dr \\ & - \Sigma d\theta^2 + \frac{2a}{\Sigma}\left[\Delta(r^2+a^2)-1\right]\sin^2\theta dv d\phi \\ & - \frac{1}{\Sigma}\left[(r^2+a^2)^2 - \Delta a^2\sin^2\theta\right]\sin^2\theta d\phi^2, \end{aligned} \tag{14}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta,$$
$$\Delta = r^2 + a^2 - 2M(v)r.$$

Here $M(v)$ is a function of the retarded time v identified as the mass of the black hole, and a is the angular momentum per unit mass. Thus, the metric (14) bears the same relation to the Kerr case as the Vaidya metric does to the Schwarzschild metric. The metric (14) was originally obtained by Carmeli [23]. Thus we have a kind of radiating rotating metric or radiating Kerr-like solution. Hence for definiteness we shall call the metric (14) the radiating Kerr black hole.

In order to further discuss the physical nature of the radiating Kerr-like black holes, we introduce their kinematical parameters. Following [27–33], the null tetrad of the metric (14) is of the form

$$l_a = \left[1,\ 0,\ 0,\ -a\sin^2\theta\right],$$
$$n_a = \left[1\frac{\Delta}{2\Sigma},\ 1,\ 0,\ \frac{\Delta}{2\Sigma}a\sin^2\theta\right],$$
$$m_a = \frac{1}{\sqrt{2}\rho}\left[ia\sin\theta,\ 0,\ \frac{\Sigma}{\Theta},\ -i(r^2+a^2)\sin\theta\right],$$
$$\bar{m}_a = \frac{1}{\sqrt{2}\bar{\rho}}\left[-ia\sin\theta,\ 0,\ \frac{\Sigma}{\Theta},\ i(r^2+a^2)\sin\theta\right],$$

where $\rho = r + ia\cos\theta$ and $\bar{\rho}$ is its complex conjugate. The null tetrad obeys null, orthogonal, and metric conditions,

$$\begin{aligned} & l_a l^a = n_a n^a = m_a m^a = 0,\ \ l_a n^a = 1, \\ & l_a m^a = n_a m^a = 0,\ m_a\bar{m}^a = -1, \\ & g_{ab} = l_a n_b + l_b n_a - m_a\bar{m}_b - m_b\bar{m}_a, \\ & g^{ab} = l^a n^b + l^b n^a - m^a\bar{m}^b - m^b\bar{m}^a. \end{aligned} \tag{15}$$

It turns out that the metric (14) satisfies the Einstein field equations

$$G_{ab} = T^R_{ab} + T^{NR}_{ab}, \tag{16}$$

where $T^R_{ab} = \chi(v,r,\theta)l_a l_b$ is the energy momentum tensor of the null radiation, $\chi(v,r,\theta)$ the density of the null fluid, and T^{NR}_{ab} represents a non-radiative field [23]. In the stationary case the source, if it exists, is the same for both a black hole and its rotating counterpart, e.g., the vacuum for both Schwarzschild and Kerr black holes, and charge for Reissner–Nordström and Kerr–Newman black holes. But the source for the Vaidya solution is just null radiation, whereas its rotating counterpart (14), in addition to null radiation, has a non-radiation field as source. The radiating Kerr black hole metric (14) is a natural generalization of the stationary Kerr black hole solutions [22], but it is Petrov type II with a twisting, shear free, null congruence—the same as for a Kerr black hole, but the Kerr black hole it is of Petrov type D. Further, all the spin coefficients are identical to those for the Kerr black hole [23]. In addition, replacing $M(v)$ by constant M in metric (14), we get exactly the Kerr metric in the original Kerr coordinates. Further, it may be mentioned that the metric (14) is a radiating rotating metric, which also has the correct static limit and it turns out that the stationary Kerr black hole [22] in Boyer–Lindquist coordinates $(t,\ r,\ \theta,\ \phi)$ can also be obtained by means of local coordinate transformations and replacing $M(v)$ with constant M [26]. Further, in the limit $a = 0$, the metric (14) reduces to the well known Vaidya metric.

2.1 Rotating radiating Hayward black hole

To avoid the black hole singularity problem, Hayward [17] proposed both static and radiating regular black hole models. The radiating Hayward black hole solution is given by the metric (2) with $f(v, r)$ defined by (13) and $M(v)$ replaced by

$$M(v, r) = M(v)\frac{r^3}{r^3 + q^3}. \tag{17}$$

Here $M(v)$ is the radiating black hole mass and q is a constant. Next to get a radiating rotating regular black hole or a rotating radiating Hayward black hole [17], we again have to start with metric (1), and then we apply the Newman–Janis algorithm, as suggested by Newman and Janis [21]. For generating a radiating rotating regular black hole, following Bambi and Modesto [18], we must be able to recover the radiating Kerr black hole or the Carmeli solution (14), in the limit $q = 0$. Thus, following this recipe, we again apply the above complex transformation, and as above we demand that the mass term $M(v, r)$ is invariant under the transformation, and we get

$$\bar{\mathcal{F}}(v, r, \theta) = 1 - \frac{M(v, r)r}{\Sigma}. \tag{18}$$

Hence the metric (14) with the new mass function (18) is the rotating radiating Hayward black hole, and in the limit $q = 0$, it goes over to the radiating Kerr black hole or Carmeli's solution with mass $M(v)$. If the Einstein equations are used for this radiating rotating regular black hole, it is supported by stresses, e.g., the radial pressure T_r^r, transverse pressure T_θ^θ, and another stress such as T_ϕ^θ etc. The supporting stresses (not all mentioned) are given by

$$T_r^r = \frac{6M(v)g^3r^4}{\left(r^3+g^3\right)^3\Sigma^2},$$
$$T_\theta^\theta = \frac{6M(v)g^3r^2\left(a^2\cos^2\theta(2g^3-r^3)+r^2(g^3-2r^3)\right)}{\left(r^3+g^3\right)^3\Sigma^2},$$
$$T_\phi^\theta = \frac{2M(v)a^3r^4\left(dM(v)/dv\right)\cos\theta\sin^3\theta}{\left(r^3+g^3\right)\Sigma^3}.$$

These stresses fall off rapidly at large r for $M(v), dM(v)/dv \neq 0$.

3 Physical parameters and horizons of rotating radiating Hayward black hole

Here we discuss the physical properties of the metric of the radiating rotating regular black hole derived in the previous section. The easiest way to detect a singularity, if it exists, in a spacetime is to observe the divergence of certain invariants of the Riemann tensor. We approach the singularity problem by studying the behavior of the Ricci invariant $\mathrm{R} = R_{ab}R^{ab}$ (R_{ab} is the Ricci tensor) and the Kretschmann invariant $\mathrm{K} = R_{abcd}R^{abcd}$ (R_{abcd} is the Riemann tensor). For the metric (14) they read

$$\mathrm{R} = 288M^2(v)r^4g^6\frac{A\cos^4\theta + B\cos^2\theta + C}{(r+g)^6(r^2-rg+g)^6\Sigma^4},$$
$$\mathrm{K} = 48M^2(v)r^4 \times \frac{D\cos^8\theta+E\cos^6\theta+F\cos^4\theta+G\cos^2\theta+H}{(r+g)^6\Sigma^6}, \tag{19}$$

where $A \ldots H$ are functions of r given by

$$A = \left(g^3-\frac{r^3}{2}\right)^2 r^2,\ \ B = \left(-2\,r^3+g^3\right)\left(g^3-\frac{r^3}{2}\right)r^2a^2,$$
$$C = \frac{r^4}{2}\left(-r^3g^3+g^6+\frac{5r^6}{2}\right),\ \ D = 12\,a^8g^6\left(g^3-\frac{r^3}{2}\right)^2,$$
$$E = -\,4\,a^6r^2\left(g^{12}+\frac{r^{12}}{4}+r^9g^3+\frac{9g^6r^6}{4}+\frac{59}{2}\,r^3g^9\right),$$
$$F = 22\,a^4\left(\frac{21g^6r^6}{2}+\frac{3r^3g^9}{11}+\frac{15}{22}\,r^{12}+\frac{50}{11}\,r^9g^3+g^{12}\right)r^4,$$
$$G = 8\left(\frac{-7r^9g^3}{2}-\frac{15}{8}\,r^{12}+\frac{69}{8}\,g^6r^6+g^{12}-\frac{9r^3g^9}{4}\right)a^2r^6,$$
$$H = 2\left(-r^3g^9+9\,g^6r^6+g^{12}-2\,r^9g^3+\frac{r^{12}}{2}\right)r^8.$$

It is sufficient to study the Kretschmann and Ricci scalars for the investigation of the spacetime curvature singularity(ies). These invariants are regular everywhere including the origin $r = 0$ for $a,\ M(v), \neq 0$. Further, in the limit $\theta = \pi/2$ or $a = 0$, they have the simple form

$$\mathrm{R} = 72\,\frac{g^6\,(M(v))^2\left(-2\,r^3g^3+5\,r^6+2\,g^6\right)}{(r+g)^6\left(r^2-rg+g^2\right)^6}, \tag{20}$$

$$\mathrm{K} = 48\,\frac{\left(r^{12}-4\,r^9g^3+2\,g^{12}-2\,r^3g^9+18\,g^6r^6\right)(M(v))^2}{\left(r^3+g^3\right)^6}. \tag{21}$$

Thus the invariants are everywhere regular for $g \neq 0$. Further, it is easy to obtain these invariants for the radiating Kerr black hole, in the limit $g = 0$ in (19), and they read

$$\mathrm{R} = 0, \tag{22}$$

$$K = \frac{-48M^2(v)}{\Sigma^6} H(r,\theta), \tag{23}$$

with

$$H(r,\theta) = \left(a^6\cos^6\theta - 15r^2a^4\cos^4\theta + 15r^4a^2\cos^2\theta - r^6\right).$$

Thus for the radiating Kerr black hole $\Sigma = 0$ happens to be a scalar polynomial singularity, and such a singularity is given by $r = 0$, $\theta = \pi/2$. The set of points given by $r = 0$ and $\theta = \pi/2$ represent a ring in the equatorial plane of radius a centered on the rotation axis of the black hole, similar to what happens in the stationary Kerr black hole [25].

Inspired by the procedure in Ref. [27,28], a null vector decomposition of the radiating regular Kerr metric (14) is of the form

$$g_{ab} = -n_a l_b - l_a n_b + \gamma_{ab}, \tag{24}$$

where $\gamma_{ab} = m_a\bar{m}_b + m_b\bar{m}_a$. Next we calculate all physical parameters which in turn will help us to study the horizon structure of a radiating Kerr-like regular black hole. The optical behavior of null geodesic congruences is mastered by the Raychaudhuri equation [28–33]

$$\frac{d\Theta}{dv} = \kappa\Theta - R_{ab}l^a l^b - \frac{1}{2}\Theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab}, \tag{25}$$

with expansion Θ, twist ω, shear σ, and surface gravity κ. In our discussion, the surface gravity [28] is

$$\kappa = -n^a l^b \nabla_b l_a. \tag{26}$$

The expansion [28] of the null rays, parameterized by v, is given by

$$\Theta = \nabla_a l^a - \kappa, \tag{27}$$

where ∇ is the covariant derivative. The shear [28] takes the form

$$\sigma_{ab} = \Theta_{ab} - \Theta(\gamma^c_c)\gamma_{ab}. \tag{28}$$

The luminosity due to loss of mass reads $L_M = -dM/dv$, $L_M < 1$, which is measured in the region where d/dv is timelike [28–31].

If we consider radiating regular black holes, it is useful to discuss not only black hole solutions but their horizon structure. In this section, we explore horizons of the radiating regular Hayward black hole, and we discuss the effects which come from the parameter q. In general, a black hole has three important surfaces [28]: the timelike limit surface (TLS), the apparent horizon (AH), and the event horizon (EH). For the non-radiating Schwarzschild black hole, the three surfaces EH, AH, and TLS coincide. For the Vaidya black hole which radiates, we have AH = TLS, but the EH is different from AH. If we break spherical symmetry, preserving stationarity, e.g., in the case of a Kerr black hole, then AH = EH but EH $\neq$ TLS.

Here we shall focus on the investigation of these horizons for the radiating regular Kerr-like black hole. As suggested by York [28], three horizons may be obtained to $O(L)$ by noting that (i) for a radiating black hole, we can define TLS as the locus where $g(\partial_v, \partial_v) = g_{vv} = 0$, AHs are termed surfaces such that $\Theta \simeq 0$, and EHs are surfaces such that $d\Theta/dv \simeq 0$.

The TLS can be null, spacelike, and timelike [28]. First, we find the location of the TLS surface, which for the radiating regular Kerr-like black hole requires that the prefactor of the dv^2 term or g_{vv} in the metric vanishes. It follows from Eq. (14) that the TLS will satisfy $\Delta - a^2\sin^2\theta = 0$ [33], which can be written as

$$g_{vv} = r^2 + a^2\cos^2\theta - 2M(v,r)r = 0. \tag{29}$$

On substituting $M(v,r)$ in Eq. (29), we produce a quintic equation of the form

$$(r^3 + g^3)(r^2 + a^2\cos^2\theta) - 2M(v)r^4 = 0. \tag{30}$$

It is not easy to solve Eq. (29) exactly, and hence we have solved it numerically and plotted the behaviors. In Fig. 1, for a given set of parameters, we show that two positive roots of Eq. (29) are possible, i.e., the solution has two TLS cases, the outer and inner TLSs of a radiating Kerr-like regular black hole. As mentioned above, in the limit $g \to 0$, we get the radiating Kerr black hole solution [24], and Eq. (30) takes the form

$$r^2 + a^2\cos^2\theta - 2M(v)r = 0. \tag{31}$$

This is trivially solved to give

$$r^-_{TLS} = M(v) - \sqrt{M^2(v) - a^2\cos^2\theta},$$
$$r^+_{TLS} = M(v) + \sqrt{M^2(v) - a^2\cos^2\theta}. \tag{32}$$

These are regular outer and inner TLSs for a radiating Kerr black hole [25]. Further in the non-rotating limit $a \to 0$, the solutions (32) reduce to

$$r^{\pm}_{TLS} = 2M(v), \tag{33}$$

which are TLSs of the Bonnor–Vaidya black hole. Thus the radiating regular Kerr-like black hole, in the GR limit and $a \to 0$, reduces to the Vaidya black hole [34]. The TLSs of the radiating regular Kerr-like black hole are shown in Fig. 1 for different values of g and rotation parameter a, and it also shows the TLSs for variable time v. For definiteness we choose $M(v) \sim \lambda v + O(v)$.

The AHs are defined as surfaces such that $\Theta \simeq 0$ [28]. The AH can be either space-like or null, i.e., it can 'move' causally or acausally [28]. The AH is the outermost marginally trapped surface for the outgoing photons. Using Eqs. (15) and (26), we get the surface gravity as

$$\kappa = \frac{1}{2\Sigma}\left[\frac{\partial\Delta}{\partial r} - \frac{2r}{\Sigma}\Delta\right], \tag{34}$$

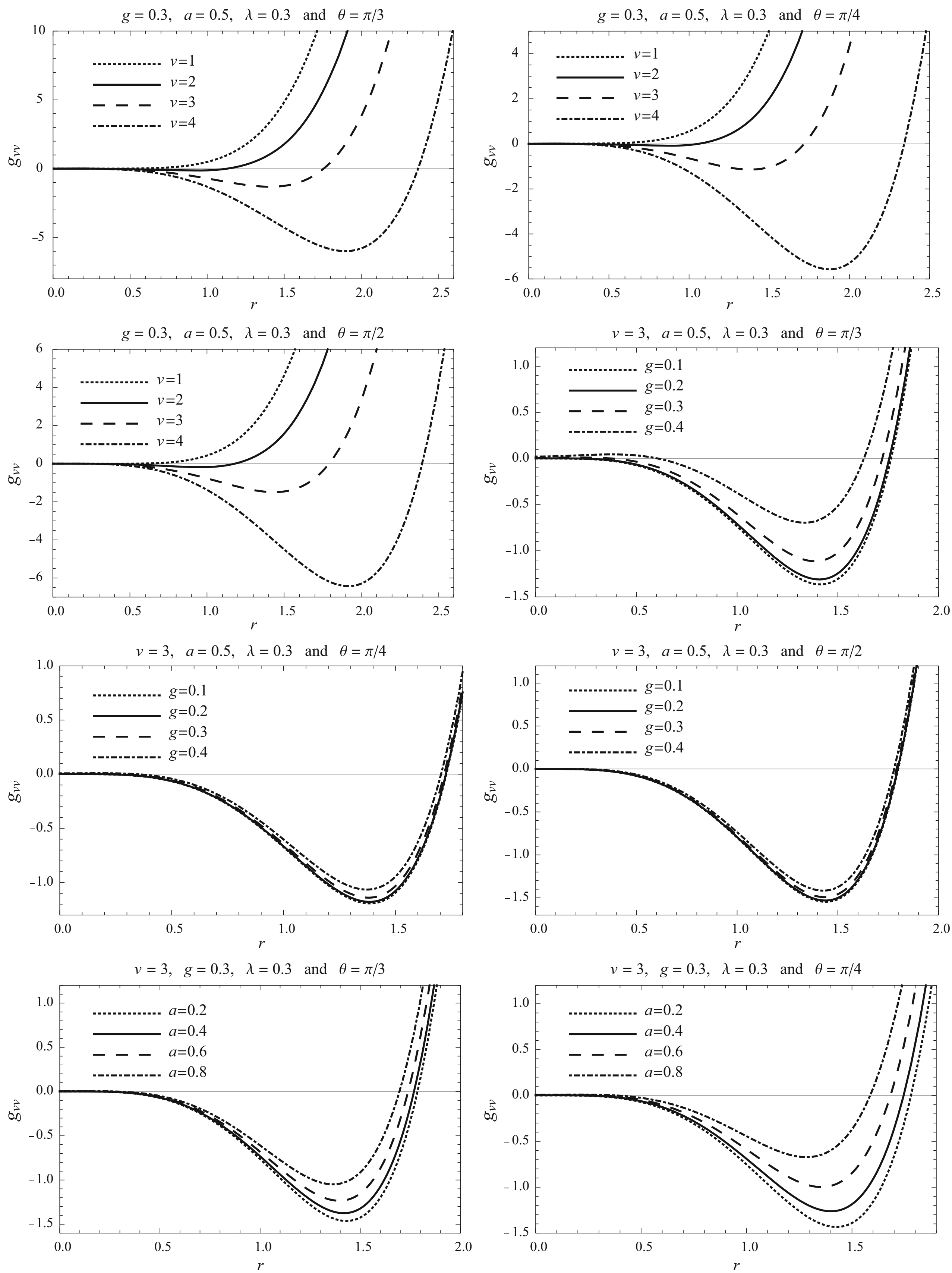

Fig. 1 Plots showing the timelike limit surface (TLS) (g_{vv} vs. r) for radiating rotating regular black hole

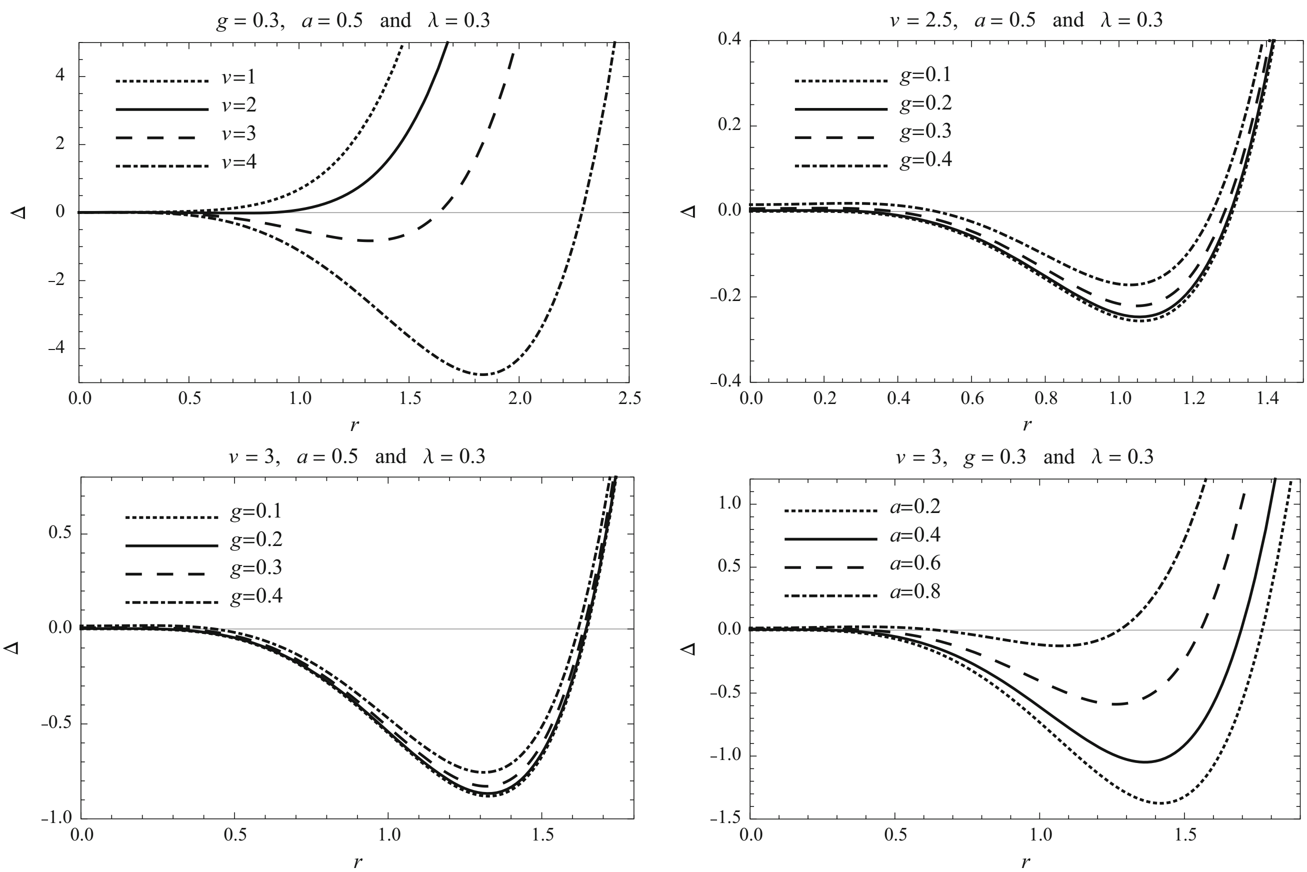

Fig. 2 Plots showing the AH (Δ vs. r) for a radiating rotating regular black hole

which on inserting the Δ expression takes the form

$$\kappa = -\frac{a^2 r}{\Sigma^2} + \frac{-\left(\frac{\partial}{\partial r} M(v,r)\right) r - M(v,r) + r}{\Sigma} + \frac{2M(v,r)r^2 - r^3}{\Sigma^2}. \tag{35}$$

Equations (15), (27), and (34) then yield

$$\Theta = -\frac{r}{\Sigma^2}\Delta = -\frac{r\left(r^2 + a^2 - 2M(v,r)r\right)}{\Sigma^2}. \tag{36}$$

It is obvious that the AHs are zeros of $\Theta = 0$. Thus from Eq. (36), the AHs are zeros of

$$(r^3 + g^3)(r^2 + a^2) - 2M(v)r^4 = 0 = 0. \tag{37}$$

Again in the limit $g \to 0$, we get

$$r^2 - 2M(v)r + a^2 = 0, \tag{38}$$

which admits the solutions

$$r_{AH}^{-} = M(v) - \sqrt{M^2(v) - a^2},\quad r_{AH}^{+} = M(v) + \sqrt{M^2(v) - a^2}. \tag{39}$$

There exist sets of parameters for which two positive roots exist as shown in the Fig. 2. Unlike TLSs, the AHs are not θ dependent. Hence, contrary to non-rotating radiating black holes, TLS, the AH do not coincide in the rotating radiating case. The two roots correspond to inner and outer AHs of black holes. The AH for radiating Kerr-like regular black hole is depicted in Fig. 2 for different values of g and rotation parameter a, and in the same figure we also show the AH for different values of rotation parameter a and time v. The two roots corresponds to, respectively, outer and inner AHs for a radiating Kerr-like regular black hole, and further in the non-rotating limit $a \to 0$, the solutions (39) correspond to AHs of black hole due to Vaidya. Further, Eq. (39) in the limit $a \to 0$ becomes exactly Eq. (32). Thus AHs coincide with TLSs, for the non-rotating but radiating Vaidya case [24]. In the stationary case M is constant whereas in the radiating case $M(v)$ is a function of the retarded time v.

3.1 Event horizon

The above discussion, regarding TLSs and AHs, is also true for the stationary or non-radiating regular Kerr-like black solutions. The AHs and EHs coincide for stationary black holes including the regular Kerr black hole. However, for the non-stationary or radiating Kerr black hole, the three surfaces

AH $\neq$ TLS $\neq$ EH and they are susceptible to any kind of perturbations. Thus, Eqs. (29) and (37) are the same as derived for the corresponding stationary case when $M(v) = M$ with M constant. They are determined via the Raychaudhuri equation (25) to $O(L)$. This definition of the EH requires knowledge of the complete future of the black hole. The EH is a null three-surface which is the locus of outgoing future-directed null geodesic rays that never manage to reach arbitrarily large distances from the black hole and behave such that $d\theta/dv \simeq 0$. Hence, it is difficult to locate the EH exactly in non-stationary spacetime. However, York [28] gave a definition of the EH which is in $O(L)$ equivalent to the photons at EH being un-accelerated in the sense that

$$\frac{d^2r}{dn^2}_{|r=r_{EH}} \approx 0, \tag{40}$$

with $d/dn = n^a \nabla_a$. This criterion enables us to differentiate the AHs and the EHs to the necessary accuracy. It is known that [33]

$$\frac{d^2r}{dn^2} = \frac{1}{\sqrt{A}2\Sigma^2}(r^2+a^2)\frac{\partial \Delta}{\partial v} + \frac{\Delta}{2\Sigma}\kappa. \tag{41}$$

We note that for low luminosity the expression for the EH can be obtained to $O(L)$ [32–34] after evaluating the surface gravity κ at the AH. Then Eqs. (41), (34), and the expression for Δ imply

$$(r^3+g^3)(r^2+a^2) - 2M^*(v)r^4 = 0, \tag{42}$$

where

$$M^*(v) = M(v) + \frac{(r^2+a^2)}{\sqrt{A}\ \kappa\ \Sigma}L.$$

The EHs of the radiating regular Kerr-like black hole are zeros of Eq. (42), which has an interesting mathematical similarity with its counterpart Eq. (37) for AHs. But they are exactly the same for the stationary Kerr BH ($L = 0$), but quite different for radiating Kerr-like black holes. Also, contrary to the AHs, EHs have a θ dependence as the $M^*(v)$ involve Σ or θ. Thus the expression of the EH is exactly the same as its counterpart AH given by Eq. (37) with the mass replaced by the effective mass $M^*(v)$ [29,30,33]. Thus, unlike the stationary case, where AH = EH $\neq$ TLS, we have shown that for a radiating regular Kerr-like black hole, AH $\neq$ EH $\neq$ TLS. The region bounded by the horizon and TLS is called the quantum ergosphere.

4 Conclusion

The rotating Kerr black hole relish many useful properties distinct from the non-rotating counterpart Schwarzschild black hole. However, there is a surprising connection between the two different black holes of general relativity, as analyzed by Newman and Janis [21] in their famous paper. They explicitly demonstrated that, by applying a set of complex transformations, it was possible to construct both the Kerr case starting from the Schwarzschild metric and likewise the Kerr–Newman solutions beginning with the Reissner–Nordström metric [21].

The Newman–Janis algorithm is fruitful in deriving several rotating black hole solutions starting from their non-rotating counterparts [18,19,21,24], which also includes the rotating regular black hole [18,19]. The algorithm is very useful since it directly allows us to generate rotating black holes, which otherwise could be extremely tiresome due to the non-linearity of field equations. For a review of the Newman–Janis algorithm see, e.g., [26]. In this paper, we have generated a radiating (non-static) Kerr-like regular black hole metric, which contains the radiating Kerr metric as the special case when the deviation g vanishes, and also the standard Kerr metric when, in addition to $g = 0$, the mass function $M(v) = M$ is constant. This metric does not arise from any particular set of field equations, but the Newman–Janis algorithm works on the spherical radiating solution to generate radiating rotating solutions. Thus, the derived radiating Kerr-like regular metric (14) bears the same relation with the rotating regular black hole as does the Vaidya metric to the Schwarzschild metric.

The structure of the three surfaces, TLSs, AHs, and EHs, of the derived radiating Kerr-like black hole were investigated by the method developed by York [28] to $O(L)$ by a null vector decomposition of the metric. The analysis presented for determining the structure of the horizons is applicable to the stationary rotating regular case as well, but AHs coincide with EHs because stationary black holes do not accrete, i.e., $L = 0$. However, the three surfaces do not coincide with each other for radiating Kerr-like black holes. For each of TLS, AH, and EH, there exist two surfaces corresponding to the two positive roots r^- and r^+, and they can be viewed, respectively, as inner and outer black hole horizons. Thus it means that as regards the presence of the term q also, we can find values of parameters so that the two inner and outer horizons still exist as in the case of radiating Kerr black hole.

To conclude, the solutions presented here provide necessary grounds to further study geometrical properties, causal structures, and thermodynamics of these black hole solutions, which will be the subject of a future project. Further generalization of such a regular black hole solution is an important direction and will be the subject of our forthcoming papers.

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References

1. S.W. Hawking, G.F.R. Ellis, *The Large Scale Structure of Space-time* (Cambridge University Press, Cambridge, 1973)
2. A.D. Sakharov, JETP **22**, 241 (1966)
3. E.B. Gliner, Sov. Phys. JETP **22**, 378 (1966)
4. M.A. Markov, JETP Lett. **36**, 265 (1982)
5. V.P. Frolov, M.A. Markov, V.F. Mukhanov, Phys. Rev. D **41**, 383 (1990)
6. V.F. Mukhanov, R. Brandenberger, Phys. Rev. Lett. **68**, 1969 (1992)
7. R. Brandenberger, V.F. Mukhanov, A. Sornborger, Phys. Rev. D **48**, 1629 (1993)
8. J.M. Bardeen, in: *Conference Proceedings of GR5* (Tbilisi, USSR, 1968), p. 174
9. E. Ayon-Beato, A. Garcia, Phys. Lett. B **493**, 149 (2000). arXiv:gr-qc/0009077
10. S. Ansoldi, Spherical black holes with regular center: a review of existing models including a recent realization with Gaussian sources. arXiv:0802.0330
11. I. Dymnikova, Gen. Rel. Grav. **24**, 235–242 (1992)
12. I. Dymnikova, Class. Quantum Grav. **21**, 4417–4429 (2004)
13. E. Ayon-Beato, et al., Phys. Rev. Lett. **80**, 5056–5059 (1998). gr-qc/9911046
14. K.A. Bronnikov, Phys. Rev. D **63**, 044005 (2001). gr-qc/0006014
15. K.A. Bronnikov et al., Phys. Rev. Lett. **96**, 251101 (2006). gr-qc/0511109
16. Berej, W., et al., Gen. Rel. Grav. **38**, 885–906 (2006). hep-th/0606185
17. S.A. Hayward, Phys. Rev. Lett. **96**, 031103 (2006)
18. C. Bambi, L. Modesto, Phys. Lett. B **721**, 329 (2013)
19. B. Toshmatov, B. Ahmedov, A. Abdujabbarov, Z. Stuchlik, Phys. Rev. D **89**, 104017 (2014)
20. J.C.S. Neves, A. Saa, Phys. Lett. B **734**, 44 (2014)
21. E.T. Newman, A.I. Janis, J. Math. Phys. **6**, 915 (1965)
22. R.P. Kerr, Phys. Rev. Lett. D **11**, 237 (1963)
23. M. Carmeli, M. Kaye, Ann. Phys. **103**, 97 (1977)
24. S.G. Ghosh, S.D. Maharaj, Phys. Rev. D **85**, 124064 (2012)
25. G.W. Gibbons, S.W. Hawking, Phys. Rev. D **15**, 2738 (1977)
26. R. d'Inverno, *Introducing Einstein's Relativity* (Clarendon, Oxford, 1992)
27. B. Carter, *in General Relativity*, ed. by S.W. Hawking, I. Israel (Cambridge University Press, Cambridge, 1979)
28. J.W. York, Jr., in *Quantum Theory of Gravity: Essays in Honor of Sixtieth Birthday of Bryce S. DeWitt*, ed. by S. Christensen (Hilger, Bristol, 1984), p. 135
29. R.L. Mallett, Phys. Rev. D **33**, 2201 (1986)
30. B.D. Koberlein, R.L. Mallett, Phys. Rev. D **49**, 5111 (1994)
31. B.D. Koberlein, Phys. Rev. D **51**, 6783 (1995)
32. Xu Dian-Yan, Class. Quantum Grav. **15**, 153 (1998)
33. Xu Dian-Yan, Class. Quantum Grav. **16**, 343 (1999)
34. A.K. Dawood, S.G. Ghosh, Phys. Rev. D **70**, 104010 (2003)