

Effects of time-varying β in SNLS3 on constraining interacting dark energy models

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Abstract It has been found that, for the Supernova Legacy Survey three-year (SNLS3) data, there is strong evidence for the redshift evolution of the color–luminosity parameter β . In this paper, adopting the w -cold-dark-matter (w CDM) model and considering its interacting extensions (with three kinds of interaction between dark sectors), we explore the evolution of β and its effects on parameter estimation. In addition to the SNLS3 data, we also use the latest Planck distance priors data, the galaxy clustering data extracted from Sloan Digital Sky Survey data release 7 and baryon oscillation spectroscopic survey, as well as the direct measurement of Hubble constant H_0 from the Hubble Space Telescope observation. We find that, for all the interacting dark energy (IDE) models, adding a parameter of β can reduce χ^2 by ~ 34 , indicating that a constant β is ruled out at 5.8σ confidence level. Furthermore, it is found that varying β can significantly change the fitting results of various cosmological parameters: for all the dark energy models considered in this paper, varying β yields a larger fractional CDM densities Ω_{c0} and a larger equation of state w ; on the other side, varying β yields a smaller reduced Hubble constant h for the w CDM model, but it has no impact on h for the three IDE models. This implies that there is a degeneracy between h and coupling parameter γ . Our work shows that the evolution of β is insensitive to the interaction between dark sectors, and then highlights the importance of considering β 's evolution in the cosmology fits.

1 Introduction

Cosmic acceleration is one of the biggest puzzles in modern cosmology [1–17]. There are mainly two approaches to explain this extremely counterintuitive phenomenon: dark energy (DE) [18–88] and modified gravity (MG) [89–98]. For recent reviews, see [99–108].

Cosmological observations are of essential importance to understanding cosmic acceleration, and one of the most important observations is type Ia supernovae (SNe Ia) [109–113]. In 2010, the Supernova Legacy Survey (SNLS) group released their 3 years data, i.e. SNLS3 dataset [114]. Soon after, using this dataset, Conley et al. [115] and Sullivan et al. [116] presented the SN-only cosmological results and the joint cosmological constraints, respectively. Unlike other supernova (SN) group, the SNLS team treated two important quantities, stretch–luminosity parameter α and color–luminosity parameter β of SNe Ia, as free model parameters.

There are many factors that can lead to systematic uncertainties of SNe Ia. One of the most important factors is the potential SN evolution, i.e. the possibility for the redshift evolution of α and β . The current studies show that α is consistent with a constant, but the hints for the evolution of β have been found in [117–121]. For example, in [122], using a linear $\beta(z) = \beta_0 + \beta_1 z$, Mohlabeng and Ralston studied the case of Union2.1 dataset and found that β deviates from a constant at 7σ confidence levels (CL). Wang and Wang [123] found, for the SNLS3 data, β increases significantly with z at the 6σ CL; moreover, they proved that this conclusion is insensitive to the lightcurve fitter models, or the functional form of $\beta(z)$ assumed [123]. Therefore, the evolution of β is a common phenomenon for various SN datasets, and should be taken into account seriously.

It is very interesting to study the effects of a time-varying β on parameter estimation. Wang et al. [124] explored this issue by considering the Λ -cold-dark-matter (Λ CDM) model, the w CDM model, and the Chevallier–Polarski–Linder (CPL) model. Soon after, Wang et al. [125] studied the case of holographic dark energy (HDE) model, which is a physically plausible DE candidate based on the holographic principle. It is found that, for all these DE models, adding a parameter of β can reduce χ_{\min}^2 by ~ 36 ; in addition, considering the evolution of β is helpful in reducing the tension between SN and other cosmological observations. It should be mentioned

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that, in principle, there is always an important possibility that DE directly interacts with CDM. This factor was not considered in [124, 125]. To do a comprehensive analysis on the effects of a time-varying β , it is necessary to extend the corresponding discussions to the case of interacting dark energy (IDE) models.

In this paper, we explore the effects of a time-varying β on the cosmological constraints of the IDE model. Three kinds of interaction terms are taken into account. In addition to the SNLS3 data, we also use the Planck distance prior data [126], the galaxy clustering (GC) data from sloan digital sky survey (SDSS) data release 7 (DR7) [127] and baryon oscillation spectroscopic survey (BOSS) [128], as well as the direct measurement of Hubble constant $H_0 = 73.8 \pm 2.4$ km/s/Mpc from the Hubble Space Telescope (HST) observation [17].

We describe our method in Sect. 2, present our results in Sect. 3, and conclude in Sect. 4. In this paper, we assume today's scale factor $a_0 = 1$, thus the redshift $z = a^{-1} - 1$. The subscript "0" always indicates the present value of the corresponding quantity, and the natural units are used.

2 Methodology

2.1 Theoretical models

In this paper, we consider a non-flat universe. The Friedmann equation can be written as

$$3M_{\text{pl}}^2 H^2 = \rho_c + \rho_{de} + \rho_r + \rho_b + \rho_k, \tag{1}$$

where $M_{\text{pl}} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass, $\rho_c, \rho_{de}, \rho_r, \rho_b$ and ρ_k are the energy densities of CDM, DE, radiation, baryon and curvature, respectively. The reduced Hubble parameter $E(z) \equiv H(z)/H_0$ satisfies

$$E^2 = \Omega_{c0} \frac{\rho_c}{\rho_{c0}} + \Omega_{de0} \frac{\rho_{de}}{\rho_{de0}} + \Omega_{r0} \frac{\rho_r}{\rho_{r0}} + \Omega_{b0} \frac{\rho_b}{\rho_{b0}} + \Omega_{k0} \frac{\rho_k}{\rho_{k0}}, \tag{2}$$

where $\Omega_{c0}, \Omega_{de0}, \Omega_{r0}, \Omega_{b0}$ and Ω_{k0} are the present fractional densities of CDM, DE, radiation, baryon and curvature, respectively. Since $\Omega_{c0} + \Omega_{de0} + \Omega_{r0} + \Omega_{b0} + \Omega_{k0} = 1$, we do not treat Ω_{de0} as an independent parameter in this paper. In addition, $\rho_r = \rho_{r0}(1+z)^4, \rho_b = \rho_{b0}(1+z)^3, \rho_k = \rho_{k0}(1+z)^2, \Omega_{r0} = \Omega_{m0}/(1+z_{\text{eq}})$, where $\Omega_{m0} = \Omega_{c0} + \Omega_{b0}$ and $z_{\text{eq}} = 2.5 \times 10^4 \Omega_{m0} h^2 (T_{\text{cmb}}/2.7 \text{ K})^{-4}$ (here we take $T_{\text{cmb}} = 2.7255 \text{ K}$).

Considering the interaction between dark sectors, the dynamical evolutions of CDM and DE become

$$\dot{\rho}_c + 3H\rho_c = Q, \tag{3}$$

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q, \tag{4}$$

where the over dot denotes the derivative with respect to the cosmic time $t, p_{de} = w\rho_{de}$ is the pressure of DE, w is the equation of state of DE, and Q is the interaction term, which describes the energy transfer rate between CDM and DE. Notice that $a = \frac{1}{1+z}$ and $H = \frac{\dot{a}}{a}$, we have $\frac{d}{dt} = -H(1+z)\frac{d}{dz}$. Then Eqs. (3) and (4) can be rewritten as

$$(1+z) \frac{d\rho_c}{dz} - 3\rho_c = -Q/H, \tag{5}$$

$$(1+z) \frac{d\rho_{de}}{dz} - 3(1+w)\rho_{de} = Q/H. \tag{6}$$

The solutions of these two equations depend on the specific forms of Q .

So far, the microscopic origin of interaction between dark sectors is still a big puzzle to us. To study the issue of interaction, one has to write down the possible forms of Q by hand. In this paper we consider the following four cases:

$$Q_0 = 0, \tag{7}$$

$$Q_1 = 3\gamma H\rho_c, \tag{8}$$

$$Q_2 = 3\gamma H\rho_{de}, \tag{9}$$

$$Q_3 = 3\gamma H \frac{\rho_c \rho_{de}}{\rho_c + \rho_{de}}, \tag{10}$$

where γ is a dimensionless coupling parameter describing the strength of interaction. Notice that the model with Q_0 denotes the case without dark sector interaction; the models with Q_1 and Q_2 are very popular, and both of them have been widely studied in the literature (see, e.g., [129–153]); the model with Q_3 is proposed in [154], and it can solve the early-time superhorizon instability and future unphysical CDM density problems at the same time. For simplicity, hereafter we call them w CDM model, Iw CDM1 model, Iw CDM2 model, and Iw CDM3 model, respectively.

For the w CDM model, the reduced Hubble parameter $E(z) \equiv H(z)/H_0$ can be written as

$$E(z) = (\Omega_{r0}(1+z)^4 + (\Omega_{c0} + \Omega_{b0})(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{de0}(1+z)^{3(1+w)})^{1/2}. \tag{11}$$

For the Iw CDM1 model, Eq. (5) has a general solution

$$\rho_c = \rho_{c0}(1+z)^{3(1-\gamma)}. \tag{12}$$

Substituting Eq. (12) into Eq. (6) and using the initial condition $\rho_{de}(z=0) = \rho_{de0}$, we get

$$\rho_{de} = \frac{\gamma\rho_{c0}}{\gamma+w} \left((1+z)^{3(1+w)} - (1+z)^{3(1-\gamma)} \right) + \rho_{de0}(1+z)^{3(1+w)}. \tag{13}$$

Then substituting Eqs. (12) and (13) into Eq. (2), we obtain

$$E(z) = \left(\Omega_{r0}(1+z)^4 + \Omega_{b0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{de0}(1+z)^{3(1+w)} + \Omega_{c0} \left(\frac{\gamma}{w+\gamma} (1+z)^{3(1+w)} + \frac{w}{w+\gamma} (1+z)^{3(1-\gamma)} \right) \right)^{1/2}. \tag{14}$$

For the $Iw\text{CDM}2$ model, Eq. (6) has a general solution

$$\rho_{de} = \rho_{de0}(1+z)^{3(1+w+\gamma)}. \tag{15}$$

Substituting Eq. (15) into Eq. (5) and using the initial condition $\rho_c(z=0) = \rho_{c0}$, we get

$$\rho_c = \rho_{c0}(1+z)^3 + \frac{\gamma\rho_{de0}}{w+\gamma}(1+z)^3 - \frac{\gamma\rho_{de0}}{w+\gamma}(1+z)^{3(1+w+\gamma)}. \tag{16}$$

Then substituting Eqs. (15) and (16) into Eq. (2), we have

$$E(z) = \left(\Omega_{r0}(1+z)^4 + (\Omega_{c0} + \Omega_{b0})(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{de0} \left(\frac{\gamma}{w+\gamma} (1+z)^3 + \frac{w}{w+\gamma} (1+z)^{3(1+w+\gamma)} \right) \right)^{1/2}. \tag{17}$$

For the $Iw\text{CDM}3$ model, the energy densities of CDM and DE satisfy

$$\rho_c = \rho_{c0}(1+z)^3 \times \left(\frac{\rho_{c0}}{\rho_{c0} + \rho_{de0}} + \frac{\rho_{de0}}{\rho_{c0} + \rho_{de0}} (1+z)^{3(w+\gamma)} \right)^{-\frac{\gamma}{w+\gamma}}, \tag{18}$$

$$\rho_{de} = \rho_{de0}(1+z)^{3(1+w+\gamma)} \times \left(\frac{\rho_{c0}}{\rho_{c0} + \rho_{de0}} + \frac{\rho_{de0}}{\rho_{c0} + \rho_{de0}} (1+z)^{3(w+\gamma)} \right)^{-\frac{\gamma}{w+\gamma}}. \tag{19}$$

Substituting Eqs. (18) and (19) into Eq. (2), we get

$$E(z) = \left(\Omega_{c0}C(z)(1+z)^3 + \Omega_{de0}C(z)(1+z)^{3(1+w+\gamma)} + \Omega_{r0}(1+z)^4 + \Omega_{b0}(1+z)^3 + \Omega_{k0}(1+z)^2 \right)^{1/2}, \tag{20}$$

where

$$C(z) = \left(\frac{\Omega_{c0}}{\Omega_{c0} + \Omega_{de0}} + \frac{\Omega_{de0}}{\Omega_{c0} + \Omega_{de0}} (1+z)^{3(w+\gamma)} \right)^{-\frac{\gamma}{w+\gamma}}. \tag{21}$$

Note that in Eqs. (11), (14), (17), (20), and (21), Ω_{de0} is not an independent parameter, which is given by $\Omega_{de0} = 1 - \Omega_{c0} - \Omega_{b0} - \Omega_{r0} - \Omega_{k0}$.

2.2 Observational data

In this subsection, we introduce how to include the SNLS3 data into the χ^2 analysis.

For the SNLS3 sample, the observable is m_B , which is the rest-frame peak B-band magnitude of the SN. By considering three functional forms (linear case, quadratic case, and step function case), Wang and Wang [123] showed that the evolutions of α and β are insensitive to functional form of α and β assumed. So in this paper, we just adopt a constant α and a linear $\beta(z) = \beta_0 + \beta_1 z$. Then the predicted magnitude of an SN becomes

$$m_{\text{mod}} = 5 \log_{10} \mathcal{D}_L(z) - \alpha(s - 1) + \beta(z)\mathcal{C} + \mathcal{M}, \tag{22}$$

where s and \mathcal{C} are the stretch measure and the color measure for the SN light curve. Here \mathcal{M} is a parameter representing some combination of SN absolute magnitude M and Hubble constant H_0 . It must be emphasized that, to include host-galaxy information in the cosmological fits, Conley et al. [115] split the SNLS3 sample based on host-galaxy stellar mass at $10^{10} M_\odot$, and made \mathcal{M} to be different for the two samples. Therefore, unlike other SN samples, there are two values of \mathcal{M} , \mathcal{M}_1 and \mathcal{M}_2 , for the SNLS3 data. Moreover, Conley et al. removed \mathcal{M}_1 and \mathcal{M}_2 from cosmology fits by analytically marginalizing over them (for more details, see the appendix C of [115], as well as the the public code which is available at <https://tspace.library.utoronto.ca/handle/1807/24512>). In this paper, we just follow the recipe of [115]. The luminosity distance $\mathcal{D}_L(z)$ is defined as

$$\mathcal{D}_L(z) \equiv H_0(1+z_{\text{hel}})r(z), \tag{23}$$

where z and z_{hel} are the CMB rest frame and heliocentric redshifts of SN. In addition, the comoving distance $r(z)$ is given by

$$r(z) = H_0^{-1} |\Omega_{k0}|^{-1/2} \text{sinn}(|\Omega_{k0}|^{1/2} \Gamma(z)), \tag{24}$$

where $\Gamma(z) = \int_0^z \frac{dz'}{E(z')}$, and $\text{sinn}(x) = \sin(x)$, x , $\sinh(x)$ for $\Omega_{k0} < 0$, $\Omega_{k0} = 0$, and $\Omega_{k0} > 0$, respectively.

For a set of N SNe with correlated errors, the χ^2 function is

$$\chi_{SN}^2 = \Delta \mathbf{m}^T \cdot \mathbf{C}^{-1} \cdot \Delta \mathbf{m}, \tag{25}$$

Table 1 Fitting results for various constant β and linear $\beta(z)$ cases, where the SNe+CMB+GC+ H_0 data are used

Parameters	wCDM		lwCDM1		lwCDM2		lwCDM3	
	Const β	Linear $\beta(z)$	Const β	Linear $\beta(z)$	Const β	Linear $\beta(z)$	Const β	Linear $\beta(z)$
α	$1.444^{+0.079}_{-0.115}$	$1.423^{+0.087}_{-0.095}$	$1.424^{+0.104}_{-0.094}$	$1.398^{+0.110}_{-0.066}$	$1.427^{+0.096}_{-0.097}$	$1.421^{+0.084}_{-0.101}$	$1.445^{+0.082}_{-0.115}$	$1.393^{+0.121}_{-0.068}$
β_0	$3.251^{+0.113}_{-0.098}$	$1.518^{+0.326}_{-0.378}$	$3.272^{+0.087}_{-0.116}$	$1.438^{+0.367}_{-0.372}$	$3.275^{+0.084}_{-0.112}$	$1.474^{+0.308}_{-0.369}$	$3.248^{+0.110}_{-0.084}$	$1.505^{+0.292}_{-0.402}$
β_1		$4.926^{+1.011}_{-0.869}$		$5.102^{+0.988}_{-0.924}$		$4.970^{+1.015}_{-0.819}$		$4.886^{+1.191}_{-0.747}$
Ω_{c0}	$0.224^{+0.010}_{-0.010}$	$0.231^{+0.011}_{-0.009}$	$0.232^{+0.012}_{-0.015}$	$0.244^{+0.016}_{-0.013}$	$0.226^{+0.012}_{-0.013}$	$0.238^{+0.020}_{-0.012}$	$0.225^{+0.014}_{-0.011}$	$0.244^{+0.013}_{-0.016}$
Ω_{b0}	$0.042^{+0.002}_{-0.002}$	$0.044^{+0.002}_{-0.002}$	$0.041^{+0.003}_{-0.002}$	$0.040^{+0.003}_{-0.002}$	$0.041^{+0.003}_{-0.002}$	$0.041^{+0.003}_{-0.002}$	$0.042^{+0.002}_{-0.002}$	$0.042^{+0.002}_{-0.003}$
Ω_{k0}	$0.00046^{+0.004}_{-0.003}$	$0.0032^{+0.0038}_{-0.0041}$	$0.0039^{+0.0044}_{-0.0061}$	$0.0095^{+0.0050}_{-0.0059}$	$0.0061^{+0.0142}_{-0.0162}$	$0.0192^{+0.0180}_{-0.0165}$	$0.0046^{+0.0160}_{-0.0131}$	$0.0194^{+0.0194}_{-0.0159}$
γ			$-0.0028^{+0.0043}_{-0.0031}$	$-0.0053^{+0.0035}_{-0.0026}$	$-0.0105^{+0.0310}_{-0.0295}$	$-0.0322^{+0.0300}_{-0.0396}$	$-0.0198^{+0.0613}_{-0.0752}$	$-0.0732^{+0.0684}_{-0.0823}$
w	$-1.118^{+0.065}_{-0.071}$	$-1.042^{+0.068}_{-0.072}$	$-1.105^{+0.075}_{-0.069}$	$-1.016^{+0.075}_{-0.063}$	$-1.124^{+0.070}_{-0.062}$	$-1.052^{+0.070}_{-0.068}$	$-1.116^{+0.059}_{-0.072}$	$-1.038^{+0.068}_{-0.080}$
h	$0.725^{+0.014}_{-0.014}$	$0.716^{+0.014}_{-0.015}$	$0.739^{+0.019}_{-0.023}$	$0.743^{+0.016}_{-0.024}$	$0.734^{+0.018}_{-0.025}$	$0.735^{+0.017}_{-0.024}$	$0.732^{+0.022}_{-0.021}$	$0.729^{+0.027}_{-0.018}$
χ^2_{\min}	422.696	388.508	422.376	387.128	422.674	387.814	422.642	387.716

where $\Delta m \equiv m_B - m_{\text{mod}}$ is a vector with N components, and \mathbf{C} is the $N \times N$ covariance matrix of the SN, given by

$$\mathbf{C} = \mathbf{D}_{\text{stat}} + \mathbf{C}_{\text{stat}} + \mathbf{C}_{\text{sys}}. \tag{26}$$

\mathbf{D}_{stat} is the diagonal part of the statistical uncertainty, given by [115]

$$\begin{aligned} D_{\text{stat},ii} = & \sigma_{m_{B,i}}^2 + \sigma_{\text{int}}^2 + \sigma_{\text{lensing}}^2 + \sigma_{\text{host correction}}^2 \\ & + \left[\frac{5(1+z_i)}{z_i(1+z_i/2)\ln 10} \right]^2 \sigma_{z,i}^2 \\ & + \alpha^2 \sigma_{s,i}^2 + \beta(z_i)^2 \sigma_{C,i}^2 + 2\alpha C_{m_{B},s,i} \\ & - 2\beta(z_i) C_{m_{B},C,i} - 2\alpha\beta(z_i) C_{s,C,i}, \end{aligned} \tag{27}$$

where $C_{m_{B},s,i}$, $C_{m_{B},C,i}$, and $C_{s,C,i}$ are the covariances between m_B , s , and C for the i th SN, $\beta_i = \beta(z_i)$ are the values of β for the i th SN. Notice that $\sigma_{z,i}^2$ includes a peculiar velocity residual of 0.0005 (i.e., 150 km/s) added in quadrature. Following [115], we fix the intrinsic scatter σ_{int} to ensure that $\chi^2/\text{dof} = 1$. Varying σ_{int} could have a significant impact on parameter estimation; see [119, 155] for details.

We define $\mathbf{V} \equiv \mathbf{C}_{\text{stat}} + \mathbf{C}_{\text{sys}}$, where \mathbf{C}_{stat} and \mathbf{C}_{sys} are the statistical and systematic covariance matrices, respectively. After treating β as a function of z , \mathbf{V} is given in the form,

$$\begin{aligned} V_{ij} = & V_{0,ij} + \alpha^2 V_{a,ij} + \beta_i \beta_j V_{b,ij} + \alpha V_{0a,ij} + \alpha V_{0a,ji} \\ & - \beta_j V_{0b,ij} - \beta_i V_{0b,ji} - \alpha \beta_j V_{ab,ij} - \alpha \beta_i V_{ab,ji}. \end{aligned} \tag{28}$$

It must be stressed that, while V_0 , V_a , V_b , and V_{0a} are the same as the “normal” covariance matrices given by the SNLS data archive, V_{0b} , and V_{ab} are *not* the same as the ones given there. This is because the original matrices of SNLS3 are produced by assuming β is constant. We have used the V_{0b} and V_{ab}

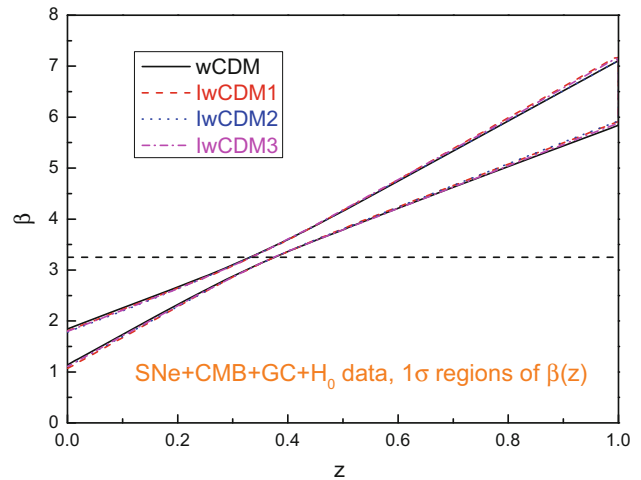


Fig. 1 1σ confidence constraints for the evolution of $\beta(z)$, given by the SNe+CMB+GC+ H_0 data, for the wCDM model, the lwCDM1 model, the lwCDM2 model, and the lwCDM3 model. The solid black lines denote the wCDM model, the dashed red lines denote the lwCDM1 model, the dotted blue lines denote the lwCDM2 model, and the dashed-dotted pink lines denote the lwCDM3 model. To make a comparison, for the wCDM model, the best-fit result of the constant β case is also plotted, shown as the horizontal dashed black line

matrices for the “Combined” set that are applicable when varying $\beta(z)$ (A. Conley, private communication, 2013).

To improve the cosmological constraints, we also use some other cosmological observations, including the Planck distance prior data [126], the GC data extracted from SDSS DR7 [127] and BOSS [128], as well as the direct measurement of Hubble constant $H_0 = 73.8 \pm 2.4$ km/s/Mpc from the HST observations [17]. For the details of including Planck and GC data into the χ^2 analysis, see [124]. Now the total

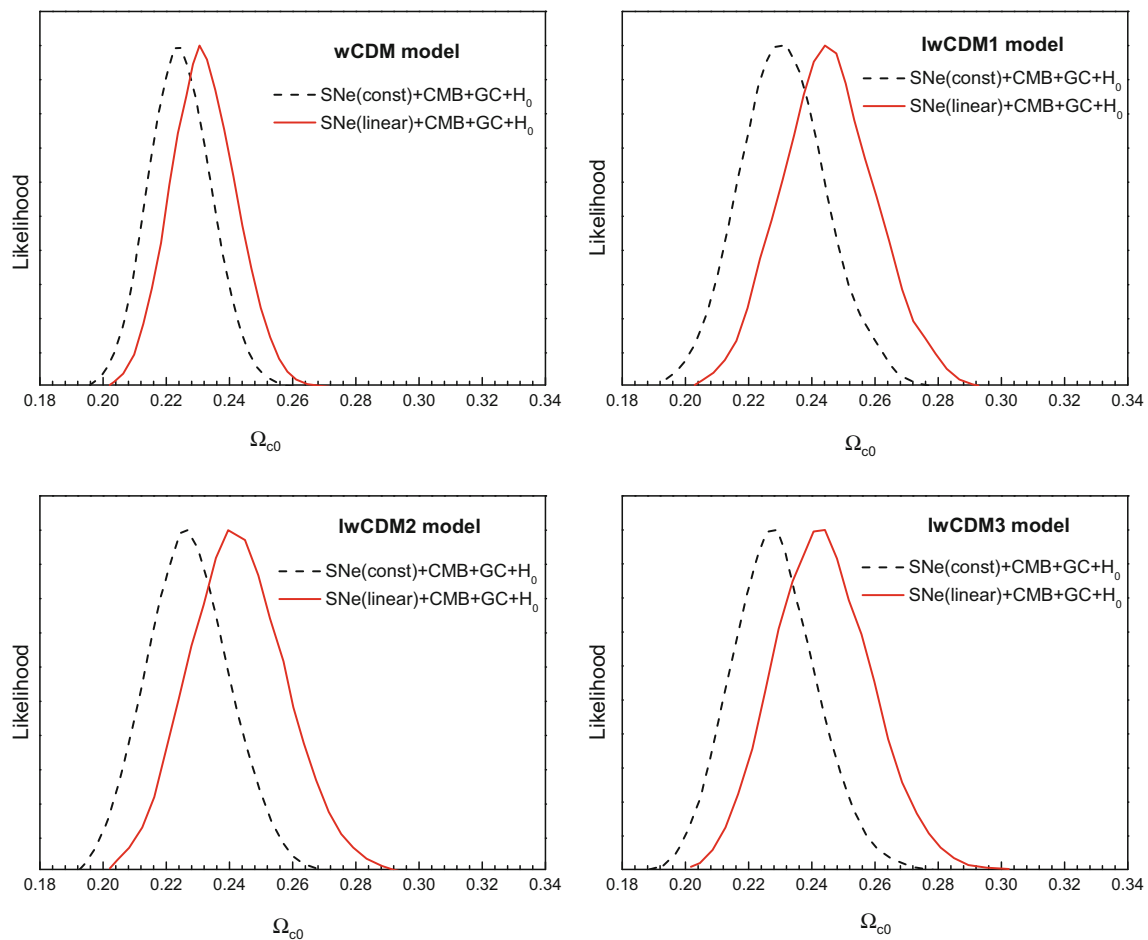


Fig. 2 The 1D marginalized probability distributions of Ω_{c0} , given by the SNe+CMB+GC+ H_0 data, for the w CDM model and the three IDE models. Both the results of the constant β (dashed black lines) and the linear $\beta(z)$ (solid red lines) cases are presented

χ^2 function is

$$\chi^2 = \chi_{SN}^2 + \chi_{CMB}^2 + \chi_{GC}^2 + \chi_{H_0}^2. \tag{29}$$

In addition, assuming the measurement errors are Gaussian, the likelihood function satisfies

$$\mathcal{L} \propto e^{-\chi^2/2}, \quad \text{Likelihood} \equiv \mathcal{L}/\mathcal{L}_{max} = \mathcal{L}/\mathcal{L}(\chi_{min}^2). \tag{30}$$

It should be mentioned that in this paper we just use the purely geometric measurements, and do not consider the cosmological perturbations in the DE models. As analyzed in detail in [156], adopting a new framework for calculating the perturbations, the cosmological perturbations will always be stable in all IDE models (for a related discussion concerning the stability, see [154]). Therefore, the use of the Planck distance prior is sufficient for our purpose.

Finally, we perform an MCMC likelihood analysis [157] to obtain $\mathcal{O}(10^6)$ samples for each model considered in this paper.

3 Results

3.1 Evolution of β

In this subsection, we explore the evolution of β in the frame of IDE.

In Table 1, we list the fitting results for various constant β and linear $\beta(z)$ cases, where the SNe+CMB+GC+ H_0 data are used. An obvious feature of this table is that varying β can significantly improve the fitting results: for all the models, adding a parameter of β can reduce the best-fit values of χ^2 by ~ 34 . Based on the Wilk theorem, 34 units of χ^2 is equivalent to a Gaussian fluctuation of 5.8σ . This means that the result of $\beta_1 = 0$ is ruled out at 5.8σ confidence level (CL). As shown in [124, 125], for the cases of various DE models (such as Λ CDM, w CDM, CPL, and HDE model) without interaction, β deviates from a constant at 6σ CL. Therefore, we further confirm the redshift evolution of β for the SNLS3 data.

In Fig. 1, using the SNe+CMB+GC+ H_0 data, we plot the 1σ confidence constraints of $\beta(z)$, for the w CDM model,

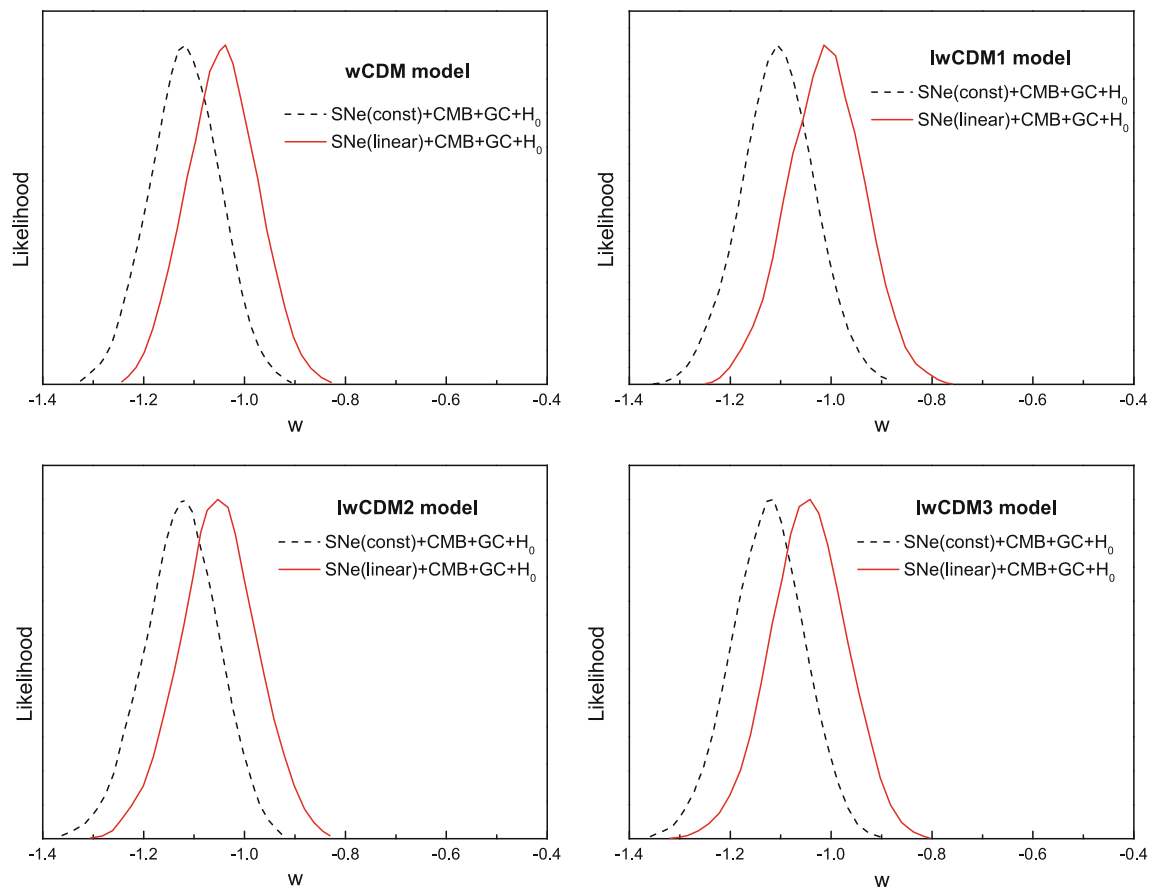


Fig. 3 The 1D marginalized probability distributions of w , given by the SNe+CMB+GC+ H_0 data, for the w CDM model and the three IDE models. Both the results of the constant β (dashed black lines) and the linear $\beta(z)$ (solid red lines) cases are presented

the Iw CDM1 model, the Iw CDM2 model, and the Iw CDM3 model. For comparison, we also plot the best-fit result of the constant β case for the w CDM model. From this figure one can see that the 1σ regions of $\beta(z)$ of all these models are almost overlapping. This shows that the evolution of β is independent of the IDE models. In addition, for all the models, $\beta(z)$ rapidly increases with z . This result is consistent with the results of [124, 125], showing that the evolution of β is insensitive to dark energy models including those with interaction between dark sectors.

It should be pointed out that the evolutionary behaviors of $\beta(z)$ depends on the SN samples used. Mohlabeng and Ralston [122] found that, for the Union 2.1 SN data, $\beta(z)$ decreases with z . This is similar to the case of the Pan-STARRS1 SN data [121].

It is interesting to study how different segments of the SNLS3 dataset give rise to different behavior of β . To do this, we perform the following test: (1) Per [115], we evenly divide the redshift range $[0, 1]$ into 9 bins and assume that both α and β are constant within each bin. (2) For each redshift bin, we make a small covariance matrix corresponding to only SNe in that bin. (3) Since we have already proved that the evolution of β is insensitive to dark energy models, per

[115], we just adopt a fixed cosmological background (a flat Λ CDM model with $\Omega_{m0} = 0.26$) to do this test. (4) We fit α and β separately for the 9 redshift bins. Based on the best-fit analysis, it is found that β is relatively flat till the seventh bin, and then it rapidly increases along with redshift z . In other words, the rapid increase of $\beta(z)$ is mainly due to the contributions from high-redshift ($z > 0.7$) SN samples of the SNLS3 dataset. It should be mentioned that, to keep this paper focused on its main purpose, here we just briefly present the conclusion, instead of describing all the detailed results of the test. To understand why high-redshift SNLS3 samples will yield this kind of evolutionary behavior of β , some numerical simulation studies may be needed. We will study this issue in future works.

3.2 Effects of time-varying β

In this subsection, we discuss the effects of varying β on parameter estimation of IDE models.

In Fig. 2, using SNe+CMB+GC+ H_0 data, we plot the 1D marginalized probability distributions of Ω_{c0} , for all the models considered in this paper. We find that varying β yields a larger Ω_{c0} : for the constant β case, the best-fit results are

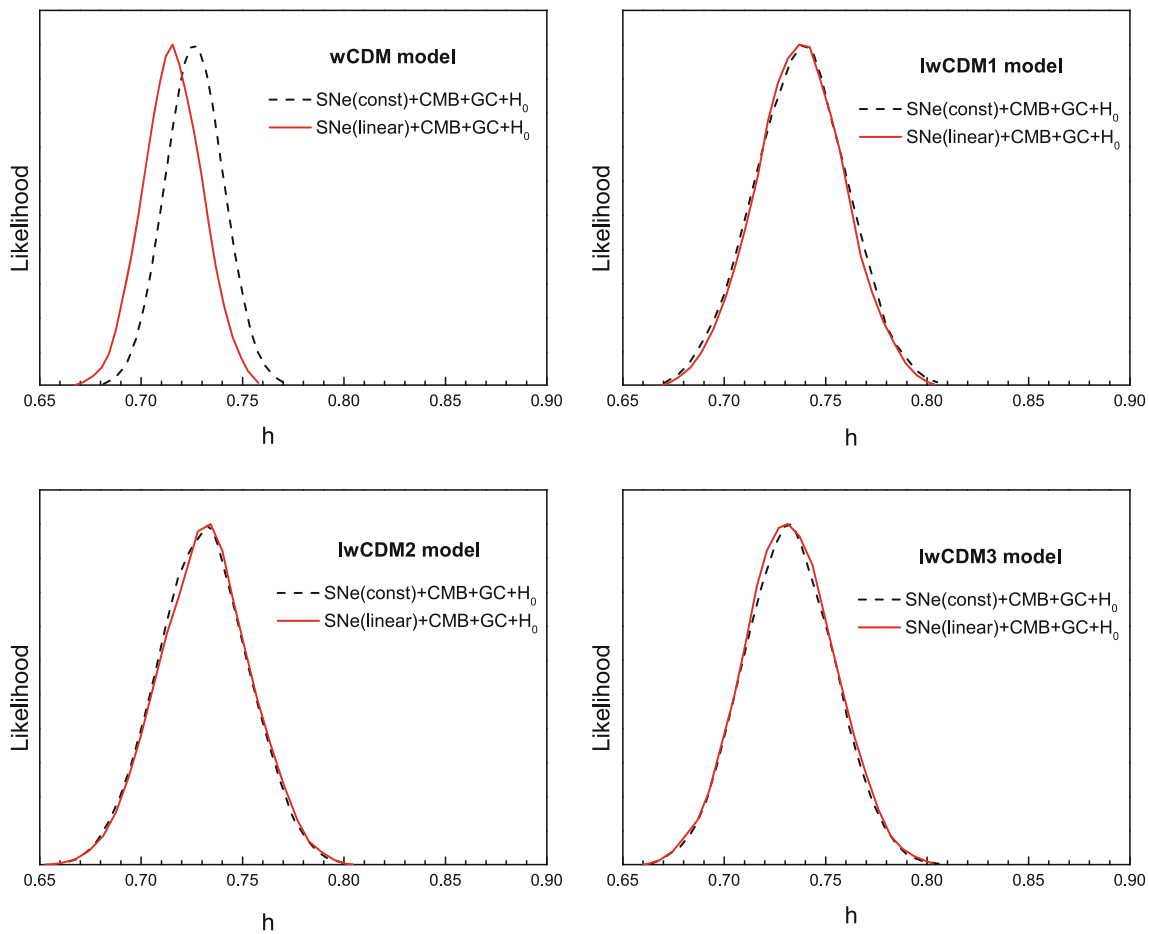


Fig. 4 The 1D marginalized probability distributions of h , given by the SNe+CMB+GC+ H_0 data, for the w CDM model and the three IDE models. Both the results of the constant β (dashed black lines) and the linear $\beta(z)$ (solid red lines) cases are presented

$\Omega_{c0} = 0.224, 0.232, 0.226,$ and 0.225 , for the w CDM, the Iw CDM1, the Iw CDM2, and the Iw CDM3 model, respectively; while for the linear $\beta(z)$ case, the best-fit results are $\Omega_{c0} = 0.231, 0.244, 0.238,$ and 0.244 , for the w CDM, the Iw CDM1, the Iw CDM2, and the Iw CDM3 model, respectively. In addition, as shown in [124,125], for various DE models without interaction term, a time-varying β also yields a larger fractional matter density $\Omega_{m0} \equiv \Omega_{c0} + \Omega_{b0}$. Therefore, we can conclude that the effects of varying β on the present fractional matter density are insensitive to the interaction between dark sectors.

For all the models considered in this paper, the 1D marginalized probability distributions of w are plotted in Fig. 3. It is found that varying β yields a larger w : for the constant β case, $w = -1.118^{+0.065}_{-0.071}, -1.105^{+0.075}_{-0.069}, -1.124^{+0.070}_{-0.062},$ and $-1.116^{+0.059}_{-0.072}$, for the w CDM model, the Iw CDM1 model, the Iw CDM2 model, and the Iw CDM3 model, respectively; while for the linear $\beta(z)$ case, $w = -1.042^{+0.068}_{-0.072}, -1.016^{+0.075}_{-0.063}, -1.052^{+0.070}_{-0.068},$ and $-1.038^{+0.068}_{-0.080}$, for the w CDM model, the Iw CDM1 model, the Iw CDM2 model,

and the Iw CDM3 model, respectively. In other words, $w < -1$ is preferred at more than 1σ CL for the constant β case, while w is consistent with -1 at 1σ CL for the linear $\beta(z)$ case. This means that, compared to the constant β case, the results from the varying β case are in better agreement with a cosmological constant. This conclusion is consistent with the noninteracting cases [124,125], showing that the effects of varying β on w are insensitive to the interaction between dark sectors.

In Fig. 4, we plot the 1D marginalized probability distributions of h , for all the models considered in this paper. It can be seen that, for the w CDM model, varying β yields a smaller h ; this result is consistent with the noninteracting cases [124,125]. However, for all the IDE models, the 1D distribution results of h of the linear β case are almost same with those of the constant β case. In other words, once considering the interaction between dark sectors, varying β will not change the fitting results of h . This result is quite different from the results of Figs. 2 and 3, showing that there is a degeneracy between h and γ .

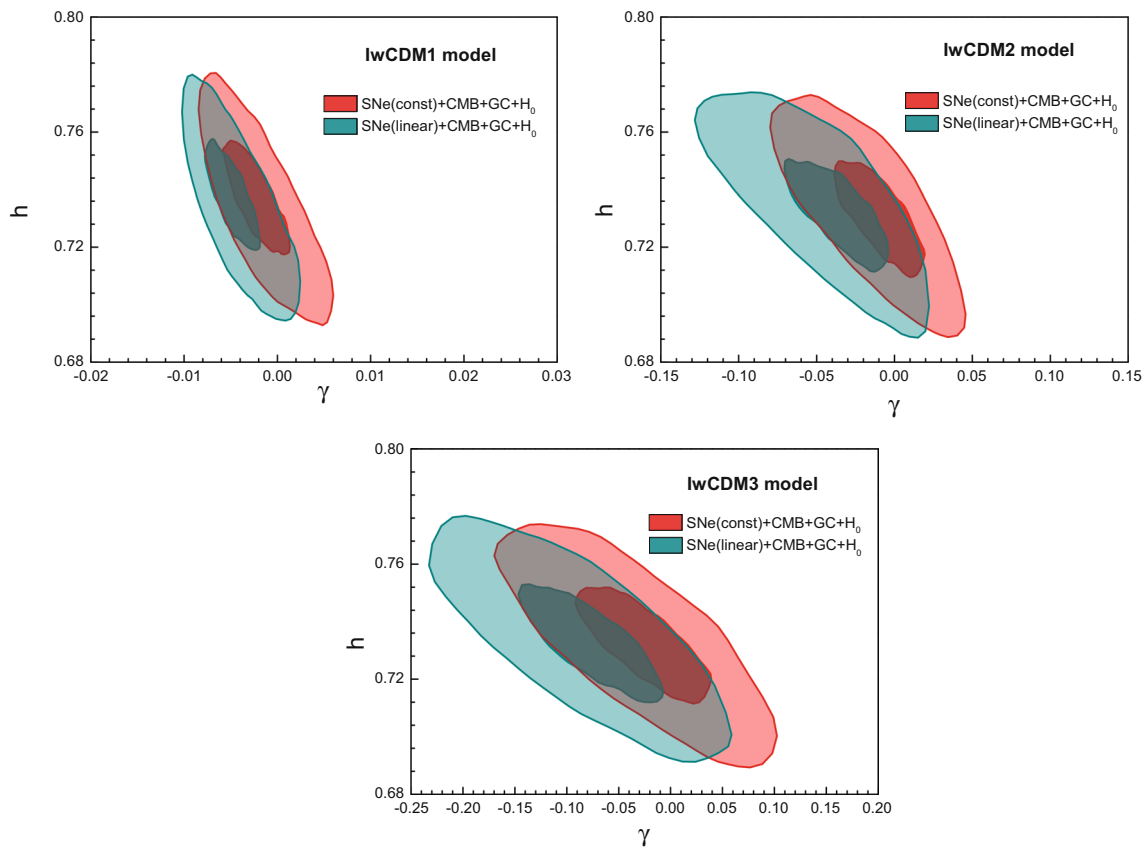


Fig. 5 The 1σ and 2σ confidence contours for $\{\gamma, h\}$, for the three IDE models. Both the results of the constant β (red regions) and the linear $\beta(z)$ (dark cyan regions) cases are presented

Next, we turn to the constraints on interaction parameter γ . In Fig. 5, we plot 1σ and 2σ confidence contours for $\{\gamma, h\}$, for all the IDE models. Again, one can see that varying β has no impact on h . To the contrary, varying β yields a smaller γ : for the constant β case, the best-fit results are $\gamma = -0.0028, -0.0105,$ and -0.0198 , for the $IwCDM1$ model, the $IwCDM2$ model, and the $IwCDM3$ model, respectively; while for the linear $\beta(z)$ case, the best-fit results are $\gamma = -0.0053, -0.0322,$ and -0.0732 , for the $IwCDM1$ model, the $IwCDM2$ model, and the $IwCDM3$ model, respectively. In other words, $\gamma < 0$ is slightly more favored in the linear $\beta(z)$ case. This means that energy will transfer from dark matter to dark energy. In addition, we find that γ and h are anti-correlated, showing that there is a degeneracy between h and γ .

In Fig. 6, to make a visual comparison among three interaction forms, we plot the 2σ confidence contours for $\{\Omega_{c0}, \gamma\}$, based on the linear $\beta(z)$ case, for all the IDE models. From this figure one can see that γ is tightly constrained in the $IwCDM1$ model; to the contrary, γ cannot be well constrained in the $IwCDM2$ and $IwCDM3$ models. This result is consistent with the result of [158], in which only the constant β case was considered.

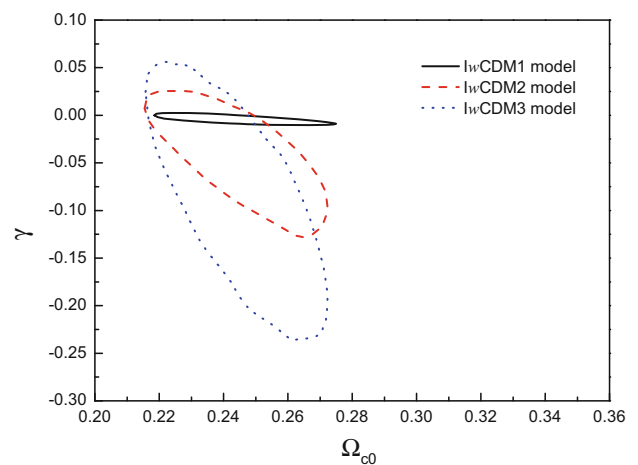


Fig. 6 The 2σ confidence contours for $\{\Omega_{c0}, \gamma\}$, based on the linear $\beta(z)$ case, for the $IwCDM1$ model (solid black line), the $IwCDM2$ model (dashed red line), and the $IwCDM3$ model (dotted blue line)

4 Discussion and summary

In recent years, more and more SNe Ia have been discovered, and the systematic errors of SNe Ia have drawn more

and more attentions. One of the most important systematic uncertainties for SNe Ia is the potential SN evolution. The hints for the evolution of β have been found [117–121]. For example, Mohlabeng and Ralston [122] studied the case of Union2.1 and found that β deviates from a constant at 7σ CL. Wang and Wang [123] found that, for the SNLS3 data, β increases significantly with z at the 6σ CL; moreover, they proved that this conclusion is insensitive to the lightcurve fitter models, or the functional form of $\beta(z)$ assumed [123].

It is clear that a time-varying β will have significant impact on parameter estimation. Adopting a constant α and a linear $\beta(z) = \beta_0 + \beta_1 z$, Wang et al. [124] explored this issue by considering the Λ CDM model, the w CDM model, and the CPL model. Soon after, Wang et al. [125] studied this issue in the frame of HDE model, which is a physically plausible DE candidate based on the holographic principle. It is found that, for all these DE models, β deviates from a constant at 6σ CL; in addition, considering the evolution of β is helpful in reducing the tension between SN and other cosmological observations. It should be pointed out that, in principle, there is always an important possibility that DE directly interacts with CDM. This factor was not considered in [124, 125].

In this paper, we extend the corresponding discussions to the case of IDE model. To perform the cosmology fits, the w CDM model is adopted. Moreover, three kinds of interaction forms are considered: $Q_1 = 3\gamma H\rho_c$, $Q_2 = 3\gamma H\rho_{de}$, and $Q_3 = 3\gamma H \frac{\rho_c \rho_{de}}{\rho_c + \rho_{de}}$. In addition to the SNLS3 SN data, we also use the Planck distance priors data, the GC data extracted from SDSS DR7 and BOSS, as well as the direct measurement of Hubble constant from the HST observation.

We further confirm the redshift evolution of β for the SNLS3 data: for all the IDE models, adding a parameter of β can reduce χ^2 by ~ 34 , indicating that $\beta_1 = 0$ is ruled out at 5.8σ CL. In addition, we find that the 1σ regions of $\beta(z)$ of all these models are almost overlapping, showing that the evolution of β is insensitive to the interaction between dark sectors. These results further verify the importance of considering the evolution of β in the cosmology fits.

Furthermore, we find that a time-varying β has significant effects on the results of parameter estimation: for all the models considered in this paper, varying β yields a larger Ω_{c0} and a larger w ; on the other side, varying β yields a smaller h for the w CDM model, while varying β has no influence on h for the three IDE models. Moreover, we find that γ and h are anti-correlated, showing that there is a degeneracy between h and γ . In addition, we find that γ is tightly constrained in the Iw CDM1 model, but it cannot be well constrained in the Iw CDM2 and Iw CDM3 models.

In all, these results show that the evolution of β is insensitive to the interaction between dark sectors, and they highlight the importance of considering β 's evolution in the cosmology fits.

So far, only the effects of varying β on DE models are considered. It is of great interest to study the effects of varying β on parameter estimation in MG models. In addition, some other factors, such as the evolution of σ_{int} [155], may also cause the systematic uncertainties of SNe Ia. These issues will be studied in future works.

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References

1. A.G. Riess et al., *Astron. J.* **116**, 1009 (1998)
2. S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999)
3. D.N. Spergel et al., *Astrophys. J. Suppl.* **148**, 175 (2003)
4. C.L. Bennet et al., *Astrophys. J. Suppl.* **148**, 1 (2003)
5. D.N. Spergel et al., *Astrophys. J. Suppl.* **170**, 377 (2007)
6. L. Page et al., *Astrophys. J. Suppl.* **170**, 335 (2007)
7. G. Hinshaw et al., *Astrophys. J. Suppl.* **170**, 263 (2007)
8. M. Tegmark et al., *Phys. Rev. D* **69**, 103501 (2004)
9. M. Tegmark et al., *Astrophys. J.* **606**, 702 (2004)
10. M. Tegmark et al., *Phys. Rev. D* **74**, 123507 (2006)
11. E. Komatsu et al., *Astrophys. J. Suppl.* **180**, 330 (2009)
12. E. Komatsu et al., *Astrophys. J. Suppl.* **192**, 18 (2011)
13. W.J. Percival et al., *Mon. Not. R. Astron. Soc.* **401**, 2148 (2010)
14. A.G. Sanchez et al., [arXiv:1203.6616](https://arxiv.org/abs/1203.6616) (*Mon. Not. R. Astron. Soc.* accepted)
15. M. Drinkwater et al., *Mon. Not. R. Astron. Soc.* **401**, 1429 (2010)
16. C. Blake et al., [arXiv:1108.2635](https://arxiv.org/abs/1108.2635) (*Mon. Not. R. Astron. Soc.* accepted)
17. A.G. Riess et al., *Astrophys. J.* **730**, 119 (2011)
18. P.J.E. Peebles, B. Ratra, *Astrophys. J.* **325**, L17 (1988)
19. C. Wetterich, *Nucl. Phys. B* **302**, 668 (1988)
20. R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998)
21. I. Zlatev, L. Wang, P.J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999)
22. R.R. Caldwell, *Phys. Lett. B* **545**, 23 (2002)
23. S.M. Carroll, M. Hoffman, M. Trodden, *Phys. Rev. D* **68**, 023509 (2003)
24. R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, *Phys. Rev. Lett.* **91**, 071301 (2003)
25. X. Zhang, *Eur. Phys. J. C* **59**, 755 (2009)
26. X. Zhang, *Eur. Phys. J. C* **60**, 661 (2009)
27. X.D. Li et al., *Sci. China Phys. Mech. Astron.* **55**, 1330 (2012)
28. C. Armendariz-Picon, T. Damour, V. Mukhanov, *Phys. Lett. B* **458**, 209 (1999)
29. C. Armendariz-Picon, V. Mukhanov, P.J. Steinhardt, *Phys. Rev. D* **63**, 103510 (2001)

30. T. Chiba, T. Okabe, M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000)
31. A.Y. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B **511**, 265 (2001)
32. M.C. Bento, O. Bertolami, A.A. Sen, Phys. Rev. D **66**, 043507 (2002)
33. X. Zhang, F.Q. Wu, J. Zhang, JCAP **01**, 003 (2006)
34. K. Liao, Y. Pan, Z.H. Zhu, Res. Astron. Astrophys. **13**, 159 (2013)
35. T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002)
36. J.S. Bagla, H.K. Jassal, T. Padmanabhan, Phys. Rev. D **67**, 063504 (2003)
37. M. Li, Phys. Lett. B **603**, 1 (2004)
38. Q.G. Huang, M. Li, JCAP **08**, 013 (2004)
39. X. Zhang, F.Q. Wu, Phys. Rev. D **72**, 043524 (2005)
40. Z. Chang, F.Q. Wu, X. Zhang, Phys. Lett. B **633**, 14 (2006)
41. X. Zhang, F.Q. Wu, Phys. Rev. D **76**, 023502 (2007)
42. J.-F. Zhang, X. Zhang, H.-Y. Liu, Eur. Phys. J. C **52**, 693 (2007)
43. M. Li, C.S. Lin, Y. Wang, JCAP **05**, 023 (2008)
44. M. Li, X.D. Li, S. Wang, X. Zhang, JCAP **06**, 036 (2009)
45. X. Zhang, Phys. Lett. B **683**, 81 (2010)
46. Y.H. Li, S. Wang, X.D. Li, X. Zhang, JCAP **02**, 033 (2013)
47. H. Wei, R.G. Cai, D.F. Zeng, Class. Quantum Gravity **22**, 3189 (2005)
48. H. Wei, R.G. Cai, Phys. Rev. D **72**, 123507 (2005)
49. H. Wei, N. Tang, S.N. Zhang, Phys. Rev. D **75**, 043009 (2007)
50. W. Zhao, Y. Zhang, Class. Quantum Gravity **23**, 3405 (2006)
51. T.Y. Xia, Y. Zhang, Phys. Lett. B **656**, 19 (2007)
52. S. Wang, Y. Zhang, T.Y. Xia, JCAP **10**, 037 (2008)
53. S. Wang, Y. Zhang, Phys. Lett. B **669**, 201 (2008)
54. X. Zhang, Phys. Lett. B **648**, 1 (2007)
55. X. Zhang, Phys. Rev. D **74**, 103505 (2006)
56. J. Zhang, X. Zhang, H. Liu, Phys. Lett. B **651**, 84 (2007)
57. J. Zhang, X. Zhang, H. Liu, Eur. Phys. J. C **54**, 303 (2008)
58. X. Zhang, Phys. Rev. D **79**, 103509 (2009)
59. D. Comelli, M. Pietroni, A. Riotto, Phys. Lett. B **571**, 115 (2003)
60. X. Zhang, Mod. Phys. Lett. A **20**, 2575 (2005)
61. X. Zhang, Phys. Lett. B **611**, 1 (2005)
62. J.A. Frieman, C.T. Hill, A. Stebbins, I. Waga, Phys. Rev. Lett. **75**, 2077 (1995)
63. M. Chevallier, D. Polarski, Int. J. Mod. Phys. D **10**, 213 (2001)
64. E.V. Linder, Phys. Rev. Lett. **90**, 091301 (2003)
65. D. Huterer, G. Starkman, Phys. Rev. Lett. **90**, 031301 (2003)
66. D. Huterer, A. Cooray, Phys. Rev. D **71**, 023506 (2005)
67. Y. Wang, M. Tegmark, Phys. Rev. Lett. **92**, 241302 (2004)
68. Y. Wang, M. Tegmark, Phys. Rev. D **71**, 103513 (2005)
69. Y. Wang, K. Freese, Phys. Lett. B **632**, 449 (2006)
70. Y. Wang, P. Mukherjee, Astrophys. J. **650**, 1 (2006)
71. Y. Wang, P. Mukherjee, Phys. Rev. D **76**, 103533 (2007)
72. Y. Wang, Phys. Rev. D **78**, 123532 (2008)
73. U. Alam, V. Sahni, T.D. Saini, A.A. Starobinsky, Mon. Not. R. Astron. Soc. **344**, 1057 (2003)
74. U. Alam, V. Sahni, T.D. Saini, A.A. Starobinsky, Mon. Not. R. Astron. Soc. **354**, 275 (2004)
75. A. Shafieloo, U. Alam, V. Sahni, A.A. Starobinsky, Mon. Not. R. Astron. Soc. **366**, 1081 (2006)
76. U. Alam, V. Sahni, A.A. Starobinsky, JCAP **02**, 011 (2007)
77. V. Sahni, A. Shafieloo, A.A. Starobinsky, Phys. Rev. D **78**, 103502 (2008)
78. A. Shafieloo, V. Sahni, A.A. Starobinsky, Phys. Rev. D **80**, 101301(R) (2009)
79. A. Shafieloo, V. Sahni, A.A. Starobinsky, Phys. Rev. D **86**, 103527 (2012)
80. J.F. Zhang, X. Zhang, H.Y. Liu, Mod. Phys. Lett. A **23**, 139 (2008)
81. Q.G. Huang, M. Li, X.D. Li, S. Wang, Phys. Rev. D **80**, 083515 (2009)
82. S. Wang, X.D. Li, M. Li, Phys. Rev. D **82**, 103006 (2010)
83. M. Li, X.D. Li, X. Zhang, Sci. China Phys. Mech. Astron. **53**, 1631 (2010)
84. S. Wang, X.D. Li, M. Li, Phys. Rev. D **83**, 023010 (2011)
85. Y.H. Li, X. Zhang, Eur. Phys. J. C **71**, 1700 (2011)
86. X.D. Li et al., JCAP **07**, 011 (2011)
87. J.Z. Ma, X. Zhang, Phys. Lett. B **699**, 233 (2011)
88. H. Li, X. Zhang, Phys. Lett. B **713**, 160 (2012)
89. V. Sahni, S. Habib, Phys. Rev. Lett. **81**, 1766 (1998)
90. L. Parker, A. Raval, Phys. Rev. D **60**, 063512 (1999)
91. G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B **485**, 208 (2000)
92. S. Nojiri, S.D. Odintsov, M. Sasaki, Phys. Rev. D **71**, 123509 (2005)
93. A. Nicolis, R. Rattazzi, E. Trincherini, Phys. Rev. D **79**, 064036 (2009)
94. W. Hu, I. Sawicki, Phys. Rev. D **76**, 064004 (2007)
95. A.A. Starobinsky, J. Exp. Theor. Phys. Lett. **86**, 157 (2007)
96. G.R. Bengochea, R. Ferraro, Phys. Rev. D **79**, 124019 (2009)
97. E.V. Linder, Phys. Rev. D **81**, 127301 (2010)
98. T. Harko, F.S.N. Lobo, S. Nojiri, S.D. Odintsov, Phys. Rev. D **84**, 024020 (2011)
99. E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006)
100. J. Frieman, M. Turner, D. Huterer, Ann. Rev. Astron. Astrophys. **46**, 385 (2008)
101. E.V. Linder, Rept. Prog. Phys. **71**, 056901 (2008)
102. R.R. Caldwell, M. Kamionkowski, Ann. Rev. Nucl. Part. Sci. **59**, 397 (2009)
103. J.-P. Uzan, [arXiv:0908.2243](https://arxiv.org/abs/0908.2243)
104. S. Tsujikawa, [arXiv:1004.1493](https://arxiv.org/abs/1004.1493)
105. S. Nojiri, S.D. Odintsov, Phys. Rep. **505**, 59 (2011)
106. M. Li, X.D. Li, S. Wang, Y. Wang, Commun. Theor. Phys. **56**, 525 (2011)
107. T. Clifton, P.G. Ferreira, A. Padilla, C. Skordis, Phys. Rep. **513**, 1 (2012)
108. Y. Wang, *Dark Energy* (Wiley-VCH, New York, 2010)
109. M. Kowalski et al., Astrophys. J. **686**, 749 (2008)
110. M. Hicken et al., Astrophys. J. **700**, 1097 (2009)
111. M. Hicken et al., Astrophys. J. **700**, 331 (2009)
112. R. Amanullah et al., Astrophys. J. **716**, 712 (2010)
113. N. Suzuki et al., Astrophys. J. **746**, 85 (2012)
114. J. Guy et al., A&A **523**, 7 (2010)
115. A. Conley et al., Astrophys. J. Suppl. **192**, 1 (2011)
116. M. Sullivan et al. [arXiv:1104.1444](https://arxiv.org/abs/1104.1444)
117. P. Astier et al., Astron. Astrophys. **447**, 31 (2006)
118. R. Kessler et al., Astrophys. J. Suppl. **185**, 32 (2009)
119. J. Marriner et al. [arXiv:1107.4631](https://arxiv.org/abs/1107.4631)
120. D. Scolnic et al. [arXiv:1306.4050](https://arxiv.org/abs/1306.4050) (ApJ in press)
121. D. Scolnic et al. [arXiv:1310.3824](https://arxiv.org/abs/1310.3824)
122. G. Mohlabeng, J. Ralston. [arXiv:1303.0580](https://arxiv.org/abs/1303.0580)
123. S. Wang, Y. Wang, Phys. Rev. D **88**, 043511 (2013)
124. S. Wang, Y.H. Li, X. Zhang, Phys. Rev. D **89**, 063524 (2014)
125. S. Wang, J.J. Geng, Y.L. Hu, X. Zhang. [arXiv:1312.0184](https://arxiv.org/abs/1312.0184)
126. Y. Wang, S. Wang, Phys. Rev. D **88**, 043522 (2013)
127. C.H. Chuang, Y. Wang, Mon. Not. R. Astron. Soc. **426**, 226 (2012)
128. C.H. Chuang et al. [arXiv:1303.4486](https://arxiv.org/abs/1303.4486)
129. G.R. Farrar, P.J.E. Peebles, Astrophys. J. **604**, 1 (2004)
130. G. Olivares, F. Atrio-Barandela, D. Pavon, Phys. Rev. D **71**, 063523 (2005)
131. T. Koivisto, Phys. Rev. D **72**, 043516 (2005)
132. H.M. Sadjadi, M. Alimohammadi, Phys. Rev. D **74**, 103007 (2006)
133. Z.K. Guo, N. Ohta, S. Tsujikawa, Phys. Rev. D **76**, 023508 (2007)
134. C.G. Boehmer, G. Caldeira-Cabral, R. Lazkoz, R. Maartens, Phys. Rev. D **78**, 023505 (2008)
135. M. Quartin, M.O. Calvao, S.E. Joras, R.R.R. Reis, I. Waga, JCAP **05**, 007 (2008)

136. J. Valiviita, E. Majerotto, R. Maartens, J. Cosmol. Astropart. Phys. **07**, 020 (2008)
137. R. Bean, E.E. Flanagan, I. Laszlo, M. Trodden, Phys. Rev. D **78**, 123514 (2008)
138. S. Chongchitnan, Phys. Rev. D **79**, 043522 (2009)
139. B.M. Jackson, A. Taylor, A. Berera, Phys. Rev. D **79**, 043526 (2009)
140. J. Zhang, H. Liu, X. Zhang, Phys. Lett. B **659**, 26 (2008)
141. M. Li, X.-D. Li, S. Wang, Y. Wang, X. Zhang, JCAP **0912**, 014 (2009)
142. L. Zhang, J. Cui, J. Zhang, X. Zhang, Int. J. Mod. Phys. D **19**, 21 (2010)
143. J. Cui, X. Zhang, Phys. Lett. B **690**, 233 (2010)
144. Y. Li, J. Ma, J. Cui, Z. Wang, X. Zhang, Sci. China Phys. Mech. Astron. **54**, 1367 (2011)
145. T.-F. Fu, J.-F. Zhang, J.-Q. Chen, X. Zhang, Eur. Phys. J. C **72**, 1932 (2012)
146. Z. Zhang, S. Li, X.-D. Li, X. Zhang, M. Li, JCAP **1206**, 009 (2012)
147. T. Clemson, K. Koyama, G.B. Zhao, R. Maartens, J. Valiviita, Phys. Rev. D **85**, 043007 (2012)
148. J. Zhang, L. Zhao, X. Zhang, Sci. China Phys. Mech. Astron. **57**, 387 (2014)
149. J.H. He, B. Wang, JCAP **06**, 010 (2008)
150. J.H. He, B. Wang, P. Zhang, Phys. Rev. D **80**, 063530 (2009)
151. J.H. He, B. Wang, E. Abdalla, D. Pavon, JCAP **12**, 022 (2010)
152. X.D. Xu, B. Wang, E. Abdalla, Phys. Rev. D **85**, 083513 (2012)
153. X.D. Xu, B. Wang, P. Zhang, F. Atrio-Barandela, JCAP **12**, 001 (2013)
154. Y.H. Li, X. Zhang, Phys. Rev. D **89**, 083009 (2014)
155. A. Kim. [arXiv:1101.3513](https://arxiv.org/abs/1101.3513)
156. Y.H. Li, J.F. Zhang, X. Zhang, Phys. Rev. D **90**, 063005 (2014)
157. A. Lewis, S. Bridle, Phys. Rev. D **66**, 103511 (2002)
158. J.J. Geng, J.F. Zhang, X. Zhang, JCAP **07**, 006 (2014)