Eur. Phys. J. C (2014) 74:3106
DOI 10.1140/epjc/s10052-014-3106-x

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Analysis of the strong $D_2^*(2460)^0 \to D^+\pi^-$ and $D_{s2}^*(2573)^+ \to D^+K^0$ transitions via QCD sum rules

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Received: 8 July 2014 / Accepted: 26 September 2014 / Published online: 21 October 2014

Abstract The strong $D_2^*(2460)^0 \to D^+\pi^-$ and $D_{s2}^*(2573)^+ \to D^+K^0$ transitions are analyzed via three-point QCD sum rules. First we calculate the corresponding strong coupling constants $g_{D_2^*D\pi}$ and $g_{D_{s2}^*DK}$. Then we use them to calculate the corresponding decay widths and branching ratios. Making use of the existing experimental data on the ratio of the decay width in the pseudoscalar D channel to that of the vector D^* channel, finally, we estimate the decay width and branching ratio of the strong $D_2^*(2460)^0 \to D^*(2010)^+\pi^-$ transition.

1 Introduction

Following the first observation, reported in 1986 [1], the past few decades have been a period for the observations of orbitally excited charmed mesons [2–17]. During this period there have also been several theoretical studies on the masses, strong and electromagnetic transitions of these mesons via various methods (for instance see [18–21] and references therein). Among these orbitally excited mesons are the $D_2^*(2460)$ and $D_{s2}^*(2573)$ mesons. The $D_2^*(2460)$ state has the quantum numbers $I(J^P) = \frac{1}{2}(2^+)$. Being not known exactly, $I(J^P) = 0(2^+)$ quantum numbers are favored by the width and decay modes of the $D_{s2}^*(2573)$ state. In this work, it is considered as a charmed strange tensor meson. One may refer to [22–32] and references therein for some experimental and theoretical studies on the properties of the charmed strange mesons.

In the literature, compared to the other types of mesons, there are little theoretical works on the properties of the tensor mesons. Especially, their strong transitions are not studied much. Studying the parameters of these tensor mesons and the comparison of the attained results with the existing experimental results may provide fruitful information about the internal structures and the natures of these mesons. Considering the appearance of these charmed tensor mesons as intermediate states in studying the B meson decays, the results of this work can also be helpful in this respect. Beside all of these, the possibility for searches on the decay properties of D_2^* and D_{s2}^* mesons at LHC is another motivation for theoretical studies on these states.

The present work puts forward the analysis of the strong transitions $D_2^*(2460)^0 \to D^+\pi^-$ and $D_{s2}^*(2573)^+ \to D^+K^0$. For this aim, first we calculate the strong coupling form factors $g_{D_2^*D\pi}$ and $g_{D_{s2}^*DK}$ via QCD sum rules as one of the most powerful and applicable non-perturbative methods to hadron physics [33,34]. These strong coupling form factors are then used to calculate the corresponding decay widths and branching ratios of the transitions under consideration. Making use of the existing experimental data on the ratio of the decay width in the pseudoscalar D channel to that of the vector D^* channel, finally, we evaluate the decay width of the strong $D_2^*(2460)^0 \to D^*(2010)^+\pi^-$ transition.

2 QCD sum rules for the strong coupling form factors $g_{D_2^*D\pi}$ and $g_{D_{s2}^*DK}$

The aim of this section is to present the details of the calculations of the coupling form factors $g_{D_2^*D\pi}$ and $g_{D_{s2}^*DK}$ for which we use the following three-point correlation function:

$$\Pi_{\mu\nu}(p,p',q) = i^2 \int d^4x \int d^4y\, e^{-ip\cdot x}\, e^{ip'\cdot y} \times \langle 0|\mathcal{T}\left(J^D(y)\, J^{\pi[K]}(0)\, J_{\mu\nu}^{D_2^{*\dagger}[D_{s2}^{*\dagger}]}(x)\right)|0\rangle, \quad (1)$$

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where $\mathcal{T}$ is the time ordering operator and $q = p - p'$ is transferred momentum. The interpolating currents appearing in this three-point correlation function can be written in terms of the quark field operators as

$$J^D(y) = i\bar{d}(y)\gamma_5 c(y),$$
$$J^{\pi[K]}(0) = i\bar{u}[\bar{s}](0)\gamma_5 d(0),$$
$$J_{\mu\nu}^{D_2^*[D_{s2}^*]}(x) = \frac{i}{2}\Big[\bar{u}[\bar{s}](x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x)c(x) + \bar{u}[\bar{s}](x)\gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu(x)c(x)\Big], \quad (2)$$

with $\overleftrightarrow{\mathcal{D}}_\mu(x)$ being the two-side covariant derivative that acts on left and right, simultaneously. The covariant derivative $\overleftrightarrow{\mathcal{D}}_\mu(x)$ is defined as

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2}\Big[\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x)\Big], \quad (3)$$

where

$$\overrightarrow{\mathcal{D}}_\mu(x) = \overrightarrow{\partial}_\mu(x) - i\frac{g}{2}\lambda^a A_\mu^a(x),$$
$$\overleftarrow{\mathcal{D}}_\mu(x) = \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a A_\mu^a(x). \quad (4)$$

Here λ^a ($a = 1, 2, \ldots, 8$) are the Gell-Mann matrices and $A_\mu^a(x)$ stand for the external gluon fields. These fields are expressed in terms of the gluon field strength tensor using the Fock–Schwinger gauge ($x^\mu A_\mu^a(x) = 0$), i.e.

$$A_\mu^a(x) = \int_0^1 d\alpha\alpha x_\beta G_{\beta\mu}^a(\alpha x) = \frac{1}{2}x_\beta G_{\beta\mu}^a(0) + \frac{1}{3}x_\eta x_\beta \mathcal{D}_\eta G_{\beta\mu}^a(0) + \cdots, \quad (5)$$

where we keep only the leading term in our calculations and ignore contributions of the derivatives of the gluon field strength tensor.

One follows two different ways to calculate the above mentioned correlation function according to the QCD sum rule approach. It is calculated in terms of hadronic parameters, called the 'hadronic side'. On the other hand, it is calculated in terms of quark and gluon degrees of freedom with the help of the operator product expansion in the deep Euclidean region, called the 'OPE side'. The match of the coefficients of the same structures from both sides provides the QCD sum rules for the intended physical quantities. With the help of a double Borel transformation with respect to the variables p^2 and p'^2 one suppresses the contribution of the higher states and the continuum.

In the hadronic side, the correlation function in Eq. (1) is saturated with complete sets of appropriate $D_2^*[D_{s2}^*]$, $\pi[K]$, and D hadronic states with the same quantum numbers as the ones of the used interpolating currents. Performing the four-integrals over x and y leads to

$$\Pi_{\mu\nu}^{had}(p,p',q) = \frac{\langle 0 \mid J^{\pi[K]} \mid \pi[K](q)\rangle\langle 0 \mid J^D \mid D(p')\rangle\langle D_2^*[D_{s2}^*](p,\epsilon) \mid J_{\mu\nu}^{D_2^*[D_{s2}^*]} \mid 0\rangle}{(p^2 - m^2_{D_2^*[D_{s2}^*]})(p'^2 - m_D^2)(q^2 - m^2_{\pi[K]})} \times \langle \pi[K](q)D(p') \mid D_2^*[D_{s2}^*](p,\epsilon)\rangle + \cdots, \quad (6)$$

where $\cdots$ represents the contributions of the higher states and continuum. The matrix elements appearing in this equation are parameterized as follows:

$$\langle 0 \mid J^{\pi[K]} \mid \pi[K](q)\rangle = i\frac{m^2_{\pi[K]} f_{\pi[K]}}{m_d + m_{u[s]}}, \quad (7)$$

$$\langle 0 \mid J^D \mid D(p')\rangle = i\frac{m_D^2 f_D}{m_d + m_c}, \quad (8)$$

$$\langle D_2^*[D_{s2}^*](p,\epsilon) \mid J_{\mu\nu}^{D_2^*} \mid 0\rangle = m^3_{D_2^*[D_{s2}^*]} f_{D_2^*[D_{s2}^*]} \epsilon_{\mu\nu}^{*(\lambda)}, \quad (9)$$

and

$$\langle \pi[K](q)D(p') \mid D_2^*[D_{s2}^*](p,\epsilon)\rangle = g_{D_2^* D\pi[D_{s2}^* DK]} \epsilon_{\eta\theta}^{(\lambda)} p'_\eta p'_\theta, \quad (10)$$

where $f_{\pi[K]}$, f_D and $f_{D_2^*[D_{s2}^*]}$ are leptonic decay constants of $\pi[K]$, D and $D_2^*[D_{s2}^*]$ mesons, respectively, and $g_{D_2^* D\pi}$ and $g_{D_{s2}^* DK}$ are the strong coupling form factors among the mesons under consideration. In writing Eq. (10) we have used the following relationships of the polarization tensor $\epsilon_{\eta\theta}^{(\lambda)}$ [35]:

$$\epsilon_{\eta\theta}^{(\lambda)} = \epsilon_{\theta\eta}^{(\lambda)}, \quad \epsilon_\eta^{(\lambda)\eta} = 0, \quad p_\eta \epsilon_\lambda^{\eta\theta} = p_\theta \epsilon_\lambda^{\eta\theta} = 0,$$
$$\epsilon_{\eta\theta}^{(\lambda)} \epsilon^{*(\lambda')\eta\theta} = \delta_{\lambda\lambda'}. \quad (11)$$

Using of the matrix elements given in Eqs. (7), (8), (9), and (10) in Eq. (6), the correlation function takes its final form in the hadronic side,

$$\Pi_{\mu\nu}^{had}(p,p',q) = \frac{g_{D_2^* D\pi[D_{s2}^* DK]} m_D^2 m^2_{\pi[K]} f_D f_{\pi[K]} f_{D_2^*[D_{s2}^*]}}{(m_c+m_d)(m_{u[s]}+m_d)(p^2-m^2_{D_2^*[D_{s2}^*]})(p'^2-m_D^2)(q^2-m^2_{\pi[K]})} \times \Big[m_{D_2^*[D_{s2}^*]} p\cdot p' p'_\mu p_\nu - \frac{2(p\cdot p')^2 + m^2_{D_2^*[D_{s2}^*]} p'^2}{3 m_{D_2^*[D_{s2}^*]}} p_\mu p_\nu - m^3_{D_2^*[D_{s2}^*]} p'_\mu p'_\nu + m_{D_2^*[D_{s2}^*]}(p\cdot p') p_\mu p'_\nu + \frac{m_{D_2^*[D_{s2}^*]}(m^2_{D_2^*[D_{s2}^*]} p'^2 - (p\cdot p')^2)}{3} g_{\mu\nu}\Big] + \cdots, \quad (12)$$

where the summation over the polarization tensor has been applied, i.e.

$$\sum_{\lambda}\varepsilon_{\mu\nu}^{(\lambda)}\varepsilon_{\alpha\beta}^{*(\lambda)}=\frac{1}{2}T_{\mu\alpha}T_{\nu\beta}+\frac{1}{2}T_{\mu\beta}T_{\nu\alpha}-\frac{1}{3}T_{\mu\nu}T_{\alpha\beta}, \qquad (13)$$

and

$$T_{\mu\nu}=-g_{\mu\nu}+\frac{p_\mu p_\nu}{m^2_{D_2^*[D_{s2}^*]}}. \qquad (14)$$

Following the application of the double Borel transformation with respect to the initial and final momenta squared, we obtain the hadronic side of the correlation function:

$$\begin{aligned}
&\widehat{\mathbf{B}}\Pi_{\mu\nu}^{had}(q)\\
&=g_{D_2^*D\pi[D_{s2}^*DK]}\frac{f_D f_{D_2^*[D_{s2}^*]}f_{\pi[K]}m_D^2 m_{\pi[K]}^2}{(m_c+m_d)(m_{u[s]}+m_d)(m_{\pi[K]}^2-q^2)}\\
&\times \mathrm{e}^{-\frac{m^2_{D_2^*[D_{s2}^*]}}{M^2}}\mathrm{e}^{-\frac{m_D^2}{M'^2}}\\
&\times\Big\{\frac{1}{12}m_{D_2^*[D_{s2}^*]}\Big(m_D^4+(m^2_{D_2^*[D_{s2}^*]}-q^2)^2\\
&-2m_D^2(m^2_{D_2^*[D_{s2}^*]}+q^2)\Big)g_{\mu\nu}\\
&+\frac{1}{6m_{D_2^*[D_{s2}^*]}}\Big[m_D^4+m_D^2(4m^2_{D_2^*[D_{s2}^*]}-2q^2)\\
&+(m^2_{D_2^*[D_{s2}^*]}-q^2)^2\Big]p_\mu p_\nu\\
&-\frac{1}{2}m_{D_2^*[D_{s2}^*]}(m_D^2+m^2_{D_2^*[D_{s2}^*]}-q^2)p_\nu p'_\mu\\
&+m^3_{D_2^*[D_{s2}^*]}p'_\mu p'_\nu\\
&-\frac{1}{2}m_{D_2^*[D_{s2}^*]}(m_D^2+m^2_{D_2^*[D_{s2}^*]}-q^2)p_\mu p'_\nu\Big\}+\cdots, \qquad (15)
\end{aligned}$$

where M^2 and M'^2 are Borel mass parameters.

In the OPE side, we calculate the aforesaid correlation function in deep Euclidean region, where $p^2\to-\infty$ and $p'^2\to-\infty$. Substituting the explicit forms of the interpolating currents into the correlation function Eq. (1) and after contracting out all quark pairs via Wick's theorem, we get

$$\begin{aligned}
&\Pi_{\mu\nu}^{\mathrm{OPE}}(p,p',q)=\frac{i^5}{2}\int d^4x\int d^4y\,\mathrm{e}^{-ip\cdot x}\mathrm{e}^{ip'\cdot y}\\
&\times\Big\{Tr\Big[\gamma_5 S_d^{ji}(-y)\gamma_5 S_c^{i\ell}(y-x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x)S_{u[s]}^{\ell j}(x)\Big]\\
&+[\mu\leftrightarrow\nu]\Big\}, \qquad (16)
\end{aligned}$$

where $S_c^{i\ell}(x)$ represents the heavy quark propagator which is given by [36]

$$\begin{aligned}
S_c^{i\ell}(x)&=\frac{i}{(2\pi)^4}\int d^4k\mathrm{e}^{-ik\cdot x}\\
&\times\Big\{\frac{\delta_{i\ell}}{\not k-m_c}-\frac{g_sG_{i\ell}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not k+m_c)+(\not k+m_c)\sigma_{\alpha\beta}}{(k^2-m_c^2)^2}\\
&+\frac{\pi^2}{3}\langle\frac{\alpha_sGG}{\pi}\rangle\delta_{i\ell}m_c\frac{k^2+m_c\not k}{(k^2-m_c^2)^4}+\cdots\Big\}, \qquad (17)
\end{aligned}$$

and $S_{u[s]}(x)$ and $S_d(x)$ are the light quark propagators and are given by

$$\begin{aligned}
S_q^{ij}(x)&=i\frac{\not x}{2\pi^2x^4}\delta_{ij}-\frac{m_q}{4\pi^2x^2}\delta_{ij}-\frac{\langle\bar qq\rangle}{12}\Big(1-i\frac{m_q}{4}\not x\Big)\delta_{ij}\\
&-\frac{x^2}{192}m_0^2\langle\bar qq\rangle\Big(1-i\frac{m_q}{6}\not x\Big)\delta_{ij}\\
&-\frac{ig_sG_{\theta\eta}^{ij}}{32\pi^2x^2}\big[\not x\sigma^{\theta\eta}+\sigma^{\theta\eta}\not x\big]+\cdots. \qquad (18)
\end{aligned}$$

After the insertion of the explicit forms of the heavy and light quark propagators into Eq. (16), we use the following transformations in $D=4$ dimensions:

$$\begin{aligned}
\frac{1}{[(y-x)^2]^n}&=\int\frac{d^Dt}{(2\pi)^D}\mathrm{e}^{-it(y-x)}\,i\,(-1)^{n+1}\\
&\times 2^{D-2n}\,\pi^{D/2}\,\frac{\Gamma(D/2-n)}{\Gamma(n)}\Big(-\frac{1}{t^2}\Big)^{D/2-n},\\
\frac{1}{[y^2]^m}&=\int\frac{d^Dt'}{(2\pi)^D}\mathrm{e}^{-it'y}\,i\,(-1)^{m+1}\\
&\times 2^{D-2m}\,\pi^{D/2}\,\frac{\Gamma(D/2-m)}{\Gamma(m)}\Big(-\frac{1}{t'^2}\Big)^{D/2-m}
\end{aligned} \qquad (19)$$

and perform the four-x and four-y integrals after the replacements $x_\mu\to i\frac{\partial}{\partial p_\mu}$ and $y_\mu\to -i\frac{\partial}{\partial p'_\mu}$. The four-integrals over k and t' are performed with the help of the Dirac delta functions which are obtained from the four-integrals over x and y. The remaining four-integral over t is performed via the Feynman parametrization and

$$\int \mathrm{d}^4t\frac{(t^2)^\beta}{(t^2+L)^\alpha}=\frac{i\pi^2(-1)^{\beta-\alpha}\Gamma(\beta+2)\Gamma(\alpha-\beta-2)}{\Gamma(2)\Gamma(\alpha)[-L]^{\alpha-\beta-2}}. \qquad (20)$$

In spite of its smallness we also include the contributions coming from the two-gluon condensate in our calculations.

The correlation function in the OPE side is written in terms of different structures as

$$\begin{aligned}
\Pi_{\mu\nu}^{\mathrm{OPE}}(p,p',q)&=\Pi_1(q^2)p_\mu p_\nu+\Pi_2(q^2)p_\nu p'_\mu\\
&+\Pi_3(q^2)p_\mu p'_\nu+\Pi_4(q^2)p'_\mu p'_\nu+\Pi_5(q^2)g_{\mu\nu}, \qquad (21)
\end{aligned}$$

where each $\Pi_i(q^2)$ function receives contributions from both the perturbative and non-perturbative parts and can be written as

$$\Pi_i(q^2) = \int \mathrm{d}s \int \mathrm{d}s' \frac{\rho_i^{\text{pert}}(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \Pi_i^{\text{non-pert}}(q^2), \tag{22}$$

where the spectral densities $\rho_i(s, s', q^2)$ are given by the imaginary parts of the Π_i functions, i.e., $\rho_i(s, s', q^2) = \frac{1}{\pi} Im[\Pi_i]$. In the present study, we consider the Dirac structure $p_\mu p_\nu$ to obtain the QCD sum rules for the considered strong coupling form factors. The $\rho_1(s, s', q^2)$ and $\Pi_1^{\text{non-pert}}(q^2)$ corresponding to this Dirac structure are obtained as

$$\begin{aligned}\rho_1^{\text{pert}}(s, s', q^2) &= \int_0^1 \mathrm{d}x \int_0^{1-x} \\ &\times \mathrm{d}y \frac{3(1 + 8x^2 - 7y + 8y^2 - 7x + 16xy)}{8\pi^2} \theta[L(s, s', q^2)],\end{aligned} \tag{23}$$

with $\theta[\ldots]$ being the unit-step function and

$$\begin{aligned}
&\Pi_1^{\text{non-pert}}(q^2) \\
&= \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y y \\
&\times \Bigg\{ \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \Bigg[\frac{1}{8L^4} m_c x^3 (1 - 2x - 2y) \\
&\times \Big[m_c m_d m_q (1 - x - y) \\
&+ m_c \Big(p^2 x + q^2 (y - 1) \Big) (x + y - 1)(x + y) \\
&+ m_c p'^2 x (x + y - xy - y^2 - 1) \\
&+ \Big(m_q (x + y - 1) - m_d (x + y) \Big) \\
&\times \Big(p^2 (x - 1)(x + y - 1) + y (p'^2 (1 - x)) \\
&+ q^2 (x + y - 1) \Big) \Big] + \frac{1}{24L^3} \\
&\times \Big[(x - 1)^2 x^2 (2x - 1) \Big(p'^2 - q^2 + p^2 (3x - 2) \Big) \\
&+ xy (x - 1) \Big(q^2 (x - 1)(4 - 13x + 6x^2) \\
&+ p^2 (x - 1)(2 - 17x + 24x^2) \\
&+ p'^2 (3 - 11x + 15x^2 - 6x^3) \Big) \\
&+ q^2 y^2 (3 - 32x + 81x^2 - 75x^3 + 24x^4) \\
&+ xy^2 \Big(p^2 (57x - 90x^2 + 42x^3 - 10) \\
&+ p'^2 (11 - 40x + 50x^2 - 18x^3) \Big) + q^2 y^3 \\
&\times (x - 1)(15 - 62x + 42x^2) \\
&+ xy^3 \Big(p^2 (x - 1)(42x - 19) + 48x p'^2 - 24x^2 p'^2 \\
&- 19 p'^2 \Big) + xy^4 \Big(p'^2 (17 - 18x) - p^2 (17 - 24x) \Big) \\
&+ q^2 y^4 (27 - 73x + 42x^2) \\
&+ 6xy^5 (p^2 - p'^2) + 3y^5 q^2 (8x - 7) \\
&+ 6y^6 q^2 - m_c^2 x^3 (1 + 8x^2 - 7y + 8y^2 - 7x \\
&+ 16xy) - m_c m_q x (x + y - 1)(8x^3 - 3x^2 - 2x \\
&- 5y + 10xy + 8x^2 y + 8y^2) \\
&+ m_c m_q x (8x^4 - 11x^3 + 8x^2 - 3x - 3y + 14xy \\
&- 19x^2 y + 16x^3 y + 7y^2 - 12xy^2 \\
&+ 8x^2 y^2 - 4y^3) \Big] + \frac{1}{48L^2} \\
&\Big[24x^4 + x^3 (72y - 55) + 3x^2 (13 - 48y + 32y^2) \\
&+ (y^2 - y)(8 - 31y + 24y^2) - 8x + 75xy \\
&- 144xy^2 + 72xy^3 \Big] \Bigg] \\
&+ \frac{m_0^2 \langle \bar{d} d \rangle m_q}{24 q^2 (m_c^2 - p'^2)^4} \Big(9m_c^4 - 8m_c^3 m_d - 12 m_c^2 p'^2 \\
&+ 2 m_c m_d p'^2 + 3 p'^4 \Big) \\
&+ \frac{m_0^2 \langle \bar{q} q \rangle m_d}{24 q^2 (m_c^2 - p^2)^4} \Big(9m_c^4 + 8m_c^3 m_q - 12 m_c^2 p'^2 \\
&- 2 m_c m_q p^2 + 3 p^4 \Big) \Bigg\},
\end{aligned} \tag{24}$$

where $\langle \bar{q} q \rangle = \langle \bar{u} u \rangle$, $m_q = m_u$ and $\langle \bar{q} q \rangle = \langle \bar{s} s \rangle$, $m_q = m_s$ for the initial D_2^* and D_{s2}^* states, respectively, and

$$\begin{aligned}L(s, s', q^2) = &-m_c^2 x + sx - sx^2 + q^2 y \\ &- q^2 xy - sxy + s'xy - q^2 y^2.\end{aligned} \tag{25}$$

The final form of the OPE side of the correlation function is obtained after a double Borel transformation as

$$\begin{aligned}\widehat{\mathbf{B}} \Pi_{\mu\nu}^{\text{OPE}}(q^2) = \Bigg\{ &\int \mathrm{d}s \int \mathrm{d}s' \mathrm{e}^{-\frac{s}{M^2}} \mathrm{e}^{-\frac{s'}{M'^2}} \rho_1^{\text{pert}}(s, s', q^2) \\ &+ \widehat{\mathbf{B}} \Pi_1^{nonpert}(q^2) \Bigg\} p_\mu p_\nu + \cdots,\end{aligned} \tag{26}$$

where

$$\begin{aligned}
&\widehat{\mathbf{B}} \Pi_1^{\text{non-pert}}(q^2) \\
&= \int_1^0 \mathrm{d}x \\
&\times \exp\left[\frac{m_c^2 M'^4 x + m_c^2 M^4 x + M^2 M'^2 (-q^2 (x-1)^2 + 2m_c^2 x)}{M^2 M'^2 (M^2 + M'^2) x (x-1)} \right] \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \\
&\times \frac{1}{48} \sqrt{\frac{1}{(x - 1)^2}} \Bigg\{ \frac{M'^{12} (x - 1)^6 (M^2 + M'^2 x)}{x^3 u^6 (M^2 + M'^2)^{10}} \\
&\times \Big[x m_c^2 (M'^4 + M^4) - M'^2 M^2 \\
&\times \Big(q^2 (x - 1)^2 - 2 m_c^2 x \Big) \Big]
\end{aligned}$$

$$+\frac{M'^{12}(x-1)^6(M^2+M'^2x)}{x^3u^5(M^2+M'^2)^9}\Big(M^2q^2(x-1)$$
$$+4M^4x+M'^2(q^2+2M^2x-q^2x)\Big)$$
$$+\frac{M'^8(x-1)^4}{x^2u^4M^2(M^2+M'^2)^7}\Big[m_cm_dM^6$$
$$+M'^6x\Big(M^2(x-1)+m_cm_dx\Big)$$
$$+M^4M'^2\Big(4M^2(1-x)+m_cm_d(1+2x)\Big)$$
$$+M^2M'^4\Big(m_cm_dx(2+x)+M^2(7x-5x^2-2)\Big)\Big]$$
$$-\frac{M'^8(M^2+M'^2x)}{x^2u^3M^2(M^2+M'^2)^5}$$
$$\times(x-1)^4\Big[m_cm_u+M^2\Big]\Big\}\theta\Big[\frac{M^2-M^2x}{M'^2+M^2}\Big] \quad (27)$$

with

$$u=-1+x+\frac{M^2-M^2x}{M^2+M'^2}. \quad (28)$$

Equating the coefficients of the same Dirac structure from both sides of the correlation function, we get the following sum rules for the coupling form factors $g_{D_2^*D\pi}$ and $g_{D_{s2}^*DK}$:

$$g_{D_2^*D\pi[D_{s2}^*DK]}$$
$$=e^{\frac{m^2_{D_2^*[D_{s2}^*]}}{M^2}}e^{\frac{m_D^2}{M'^2}}\frac{6(m_c+m_d)(m_d+m_{u[s]})(m^2_{\pi[K]}-q^2)m_{D_2^*[D_{s2}^*]}}{f_{D_2^*[D_{s2}^*]}f_Df_{\pi[K]}m_D^2m^2_{\pi[K]}}$$
$$\times\frac{1}{\Big[m_D^4+m_D^2(4m^2_{D_2^*[D_{s2}^*]}-2q^2)+(m^2_{D_2^*[D_{s2}^*]}-q^2)^2\Big]}$$
$$\times\Big\{\int_{(m_c+m_{u[s]})^2}^{s_0}ds\int_{(m_c+m_d)^2}^{s'_0}ds'e^{-\frac{s}{M^2}}e^{-\frac{s'}{M'^2}}\rho_1^{\text{pert}}(s,s',q^2)$$
$$+\widehat{\mathbf{B}}\Pi_1^{\text{non-pert}}(q^2)\Big\}, \quad (29)$$

where s_0 and s'_0 are continuum thresholds in $D_2^*[D_{s2}^*]$ and D channels, respectively, and we have used the quark–hadron duality assumption.

3 Numerical results

In this section, we numerically analyze the obtained sum rules for the strong coupling form factors in the previous section and search for the behavior of those couplings with respect to $Q^2=-q^2$. The values of the strong coupling form factors at $Q^2=-m^2_{\pi[K]}$ give the strong coupling constants whose values are then used to find the decay rate and branching ratio of the strong transitions under consideration. To proceed, we use some input parameters, presented in Table 1.

The next task is to find the working regions for the auxiliary parameters M^2, M'^2, s_0, and s'_0. As they are not physical parameters, the strong coupling form factors should roughly be independent of these parameters. In the case of the continuum thresholds, they are not completely arbitrary but are related to the energy of the first excited states with the same quantum numbers as the considered interpolating fields. From a numerical analysis, the working intervals are obtained as $7.6[8.5]\,\text{GeV}^2\le s_0\le 8.8[9.4]\,\text{GeV}^2$ and $4.7\,\text{GeV}^2\le s'_0\le 5.6\,\text{GeV}^2$ for the strong vertex $D_2^*D\pi[D_{s2}^*DK]$. In the case of the Borel mass parameters M^2 and M'^2, we choose their working windows such that they guarantee not only the pole dominance but also the convergence of the OPE. If these parameters are chosen too large, the convergence of the OPE is good but the continuum and higher state contributions exceed the pole contribution. On the other hand if one chooses too small values, although the pole dominates the higher state and continuum contributions, the OPE have a poor convergence. By considering these conditions we choose the windows $3\,\text{GeV}^2\le M^2\le 8\,\text{GeV}^2$ and $2\,\text{GeV}^2\le M'^2\le 5\,\text{GeV}^2$ for the Borel mass parameters. Our analysis shows that, in these intervals, the dependence of the results on the Borel parameters are weak.

Table 1 Input parameters used in calculations

Parameters	Values
m_c	(1.275 ± 0.025) GeV [37]
m_d	$4.8^{+0.5}_{-0.3}$ MeV [37]
m_u	$2.3^{+0.7}_{-0.5}$ MeV [37]
m_s	95 ± 5 MeV [37]
$m_{D_2^*(2460)}$	$(2{,}462.6\pm0.6)$ MeV [37]
$m_{D_{s2}^*(2573)}$	$(2{,}571.9\pm0.8)$ MeV [37]
m_D	$(1{,}869.62\pm0.15)$ MeV [37]
m_π	(139.57018 ± 0.00035) MeV [37]
m_K	(493.677 ± 0.016) MeV [37]
$f_{D_2^*(2460)}$	0.0228 ± 0.0068 [19]
$f_{D_{s2}^*(2573)}$	0.023 ± 0.0011 [20]
f_D	206.7 ± 8.9 MeV [37]
f_π	$130.41\pm0.03\pm0.20$ MeV [37]
f_K	$156.1\pm0.2\pm0.8\pm0.2$ MeV [37]
$\left\langle\frac{\alpha_sG^2}{\pi}\right\rangle$	$(0.012\pm0.004)\,\text{GeV}^4$ [38,39]

Now we proceed to find the variations of the strong coupling form factors with respect to Q^2. Using the working regions for the auxiliary parameters we observe that the following fit function well describes the strong coupling form factors in terms of Q^2:

$$g_{D_2^*D\pi[D_{s2}^*DK]}(Q^2)=c_1\exp\Big[-\frac{Q^2}{c_2}\Big]+c_3, \quad (30)$$

where the values of the parameters c_1, c_2, and c_3 for different structures are presented in Tables 2 and 3 for $D_2^*D\pi$ and D_{s2}^*DK, respectively. From this fit parametrization we obtain the values of the strong coupling constants for each

Table 2 Parameters appearing in the fit function of the coupling form factor for $D_2^* D\pi$ vertex

Structure	c_1 (GeV^{-1})	c_2 (GeV^2)	c_3 (GeV^{-1})
$p_\mu p_\nu$	5.17 ± 1.50	13.21 ± 3.84	$-(0.54 \pm 0.16)$
$p'_\mu p'_\nu$	8.12 ± 2.34	11.14 ± 2.78	12.56 ± 3.77
$p'_\mu p_\nu$	11.57 ± 3.12	12.55 ± 3.51	1.13 ± 0.34
$p_\mu p'_\nu$	11.57 ± 3.12	12.55 ± 3.51	1.13 ± 0.34
$g_{\mu\nu}$	15.24 ± 4.57	10.38 ± 2.91	0.034 ± 0.001

Table 3 Parameters appearing in the fit function of the coupling form factor for $D_{s2}^* DK$ vertex

Structure	c_1 (GeV^{-1})	c_2 (GeV^2)	c_3 (GeV^{-1})
$p_\mu p_\nu$	6.43 ± 1.92	13.31 ± 3.98	$-(0.79 \pm 0.24)$
$p'_\mu p'_\nu$	9.79 ± 2.94	11.85 ± 3.32	10.58 ± 3.17
$p'_\mu p_\nu$	12.03 ± 3.61	12.73 ± 3.18	0.81 ± 0.24
$p_\mu p'_\nu$	12.03 ± 3.61	12.73 ± 3.18	0.81 ± 0.24
$g_{\mu\nu}$	17.75 ± 5.32	10.12 ± 2.84	0.062 ± 0.002

Table 4 Value of the $g_{D_2^* D\pi[D_{s2}^* DK]}$ coupling constant in GeV^{-1} unit for different structures

Structure	$g_{D_2^* D\pi}(Q^2 = -m_\pi^2)$	$g_{D_{s2}^* DK}(Q^2 = -m_K^2)$
$p_\mu p_\nu$	4.63 ± 1.39	5.76 ± 1.84
$p'_\mu p'_\nu$	20.69 ± 6.21	20.59 ± 5.15
$p'_\mu p_\nu$	12.72 ± 3.56	12.85 ± 3.85
$p_\mu p'_\nu$	12.72 ± 3.56	12.85 ± 3.85
$g_{\mu\nu}$	15.30 ± 3.67	18.26 ± 5.48

structure at $Q^2 = -m_{\pi[K]}^2$ as presented in Table 4. The errors appearing in our results belong to the uncertainties in the input parameters as well as errors coming from the determination of the working regions for the auxiliary parameters. From Table 4 we see that the results strongly depend on the selected structure such that the maximum values for the strong couplings in D_2^* and D_{s2}^* channels that belong to the structure $p'_\mu p'_\nu$ are roughly four times greater that those of the minimum values which correspond to the structure $p_\mu p_\nu$. The values obtained using other structures lie between these maximum and minimum values. Note that the coupling constant in the π channel has been estimated in a pioneering study via chiral perturbation theory [40]. By converting the parametrization of the coupling constant used in [40] to our parametrization, Falk [40] finds a value of $g_{D_2^* D\pi} \simeq 16\, GeV^{-1}$ in the π vertex which is close to our prediction obtained via the structure $g_{\mu\nu}$. Our results obtained via the structures $p'_\mu p_\nu$ and $p_\mu p'_\nu$ are comparable with that of [40] within the errors. However, our results obtained via the structure $p'_\mu p'_\nu$ are considerably high and our prediction obtained using the structure $p_\mu p_\nu$ is very low compared to

Table 5 Numerical results for decay width and branching ratio of $D_2^*(2460)^0 \to D^+\pi^-$ transition obtained via different structures

Structure	Γ (GeV)	BR
$p_\mu p_\nu$	$(6.26 \pm 1.87) \times 10^{-4}$	$(1.28 \pm 0.36) \times 10^{-2}$
$p'_\mu p'_\nu$	$(1.25 \pm 0.34) \times 10^{-2}$	$(2.55 \pm 0.74) \times 10^{-1}$
$p'_\mu p_\nu$	$(4.73 \pm 1.42) \times 10^{-3}$	$(9.64 \pm 2.70) \times 10^{-2}$
$p_\mu p'_\nu$	$(4.73 \pm 1.42) \times 10^{-3}$	$(9.64 \pm 2.70) \times 10^{-2}$
$g_{\mu\nu}$	$(5.10 \pm 1.48) \times 10^{-3}$	$(1.04 \pm 0.26) \times 10^{-1}$

Table 6 Numerical results for decay width and branching ratio of $D_{s2}^*(2573)^+ \to D^+K^0$ transition obtained via different structures

Structure	Γ (GeV)	BR
$p_\mu p_\nu$	$(3.70 \pm 1.04) \times 10^{-4}$	$(2.18 \pm 0.59) \times 10^{-2}$
$p'_\mu p'_\nu$	$(4.73 \pm 1.42) \times 10^{-3}$	$(2.78 \pm 0.69) \times 10^{-1}$
$p'_\mu p_\nu$	$(1.84 \pm 0.48) \times 10^{-3}$	$(1.08 \pm 0.27) \times 10^{-1}$
$p_\mu p'_\nu$	$(1.84 \pm 0.48) \times 10^{-3}$	$(1.08 \pm 0.27) \times 10^{-1}$
$g_{\mu\nu}$	$(3.72 \pm 0.97) \times 10^{-3}$	$(2.19 \pm 0.63) \times 10^{-1}$

the result of [40] for the strong coupling constant associated to the $D_2^* D\pi$ vertex.

The final task in the present work is to calculate the decay rates and branching ratios for the strong $D_2^*(2460)^0 \to D^+\pi^-$ and $D_{s2}^*(2573)^+ \to D^+K^0$ transitions. Using the amplitudes of these transitions we find

$$\Gamma = \frac{|M(\mathbf{p}')|^2}{40\pi m^2_{D_2^*[D_{s2}^*]}}|\mathbf{p}'|, \tag{31}$$

where

$$|M(\mathbf{p}')|^2 = g^2_{D_2^* D\pi[D_{s2}^* DK]}\left[\frac{2}{3m^4_{D_2^*[D_{s2}^*]}}\left(m_{D_2^*[D_{s2}^*]}\sqrt{\mathbf{p}'^2 + m_D^2}\right)^4 - \frac{4m_D^2}{3m^2_{D_2^*[D_{s2}^*]}}\left(m_{D_2^*[D_{s2}^*]}\sqrt{\mathbf{p}'^2 + m_D^2}\right)^2 + \frac{2m_D^4}{3}\right], \tag{32}$$

and

$$|\mathbf{p}'| = \frac{1}{2m_{D_2^*[D_{s2}^*]}} \times \sqrt{m^4_{D_2^*[D_{s2}^*]} + m_D^4 + m_\pi^4 - 2m^2_{D_2^*[D_{s2}^*]}m^2_{\pi[K]} - 2m_D^2 m^2_{\pi[K]} - 2m^2_{D_2^*[D_{s2}^*]}m_D^2}. \tag{33}$$

The numerical values of the decay rates for the transitions under consideration are presented in Tables 5 and 6. Using the total widths of the initial particles as $\Gamma_{D_2^*(2460)^0} = (49.0 \pm 1.3)$ MeV, $\Gamma_{D_{s2}^*(2573)^0} = (17 \pm 4)$ MeV [37] we also find the corresponding branching ratios that are also presented in Tables 5 and 6.

Using the following experimental ratio in the π channel [37,41]:

Table 7 Numerical results for decay width and branching ratio of $D_2^*(2460)^0 \rightarrow D^*(2010)^+\pi^-$ transition obtained via different structures

Structure	Γ(GeV)	BR
$p_\mu p_\nu$	$(3.84 \pm 1.15) \times 10^{-4}$	$(7.83 \pm 2.03) \times 10^{-3}$
$p'_\mu p'_\nu$	$(7.67 \pm 2.15) \times 10^{-3}$	$(1.56 \pm 0.44) \times 10^{-1}$
$p'_\mu p_\nu$	$(2.90 \pm 0.87) \times 10^{-3}$	$(5.91 \pm 1.65) \times 10^{-2}$
$p_\mu p'_\nu$	$(2.90 \pm 0.87) \times 10^{-3}$	$(5.91 \pm 1.65) \times 10^{-2}$
$g_{\mu\nu}$	$(3.12 \pm 0.75) \times 10^{-3}$	$(6.38 \pm 1.72) \times 10^{-2}$

$$\frac{\Gamma[D_2^*(2460)^0 \rightarrow D^+\pi^-]}{\Gamma[D_2^*(2460)^0 \rightarrow D^+\pi^-] + \Gamma[D_2^*(2460)^0 \rightarrow D^*(2010)^+\pi^-]} = 0.62 \pm 0.03 \pm 0.02, \quad (34)$$

we also get the values of the decay rate and branching ratio for $D_2^*(2460)^0 \rightarrow D^*(2010)^+\pi^-$ channel for different structures as presented in Table 7.

Considering the fact that the dominant decay modes of $D_2^*(2460)$ are $D_2^*(2460) \rightarrow D\pi$ and $D_2^*(2460) \rightarrow D^*\pi$, from the values presented in Tables 5 and 7, we see that all structures give the results for the total decay width of the $D_2^*(2460)$ tensor meson compatible with the experimental data [37] except for the structure $p_\mu p_\nu$, which gives a result roughly one order of magnitude smaller than the experimental values.

To sum up, we calculated the strong coupling form factors $g_{D_2^* D\pi}(q^2)$ and $g_{D_{s2}^* DK}(q^2)$ in the framework of QCD sum rules. Using the obtained working regions for the auxiliary parameters entering the sum rules of the strong form factors, we found the behavior of those form factors in terms of Q^2. Using $Q^2 = -m^2_{\pi[K]}$, we also found the values of the strong coupling constants $g_{D_2^* D\pi}$ and $g_{D_{s2}^* DK}$, which have then been used to calculate the decay widths and branching ratios of the strong $D_2^*(2460)^0 \rightarrow D^+\pi^-$, $D_2^*(2460)^0 \rightarrow D^*(2010)^+\pi^-$, and $D_{s2}^*(2573)^+ \rightarrow D^+K^0$ transitions. Our results can be used in analyses of the future experimental data, especially at the K channel.

Acknowledgments This work has been supported in part by the Scientific and Technological Research Council of Turkey (TUBITAK) under the research project 114F018.

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