

# QCD analysis of forward neutron production in DIS

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Received: 9 June 2014 / Accepted: 10 August 2014 / Published online: 28 August 2014  
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**Abstract** We consider forward neutron production in DIS within fracture functions formalism. By performing a QCD analysis of available data we extract proton-to-neutron fracture functions exploiting a method which is in close relation with the factorisation theorem for this class of processes.

## 1 Introduction

In hadronic collisions a portion of the produced particle spectrum is characterised by hadrons carrying a sizeable fraction of the available energy and produced at small polar angle with respect to the collision axis. It is phenomenologically observed that for such hadrons their valence-parton composition is almost or totally conserved with respect to the one of initial-state hadrons [1]. Such semi-inclusive processes allow one to test the scaling hypothesis for forward hadron production cross sections [2,3] and give insight on non-perturbative aspects of QCD dynamics in high energy collisions. These hadrons, in fact, are produced at very small transverse momenta with respect to the collision axis, a regime where perturbative techniques cannot be applied. Quite interestingly, forward particle production has also been observed in processes which involve point-like probes in lepton-hadron interactions, such as Semi-Inclusive Deep Inelastic Scattering (SIDIS). At variance with the hadronic collisions mentioned above, such process involves a large momentum transfer at the lepton vertex. The presence of a hard scale is a basic requirement in the derivation of a dedicated factorisation theorem [4–7] which ensures that collinear QCD factorisation holds in the leading-twist approximation for forward particle production in DIS. The relevant cross sections can then be factorised into perturbatively calculable short-distance cross sections and new distributions, fracture functions, which simultaneously encode information both on the interacting parton and on the non-perturbative QCD

dynamics of the spectator fragmentation into the observed forward hadron. In spite of being non-perturbative in nature, their scale dependence can be calculated within perturbative QCD [8]. Fracture functions obey in fact DGLAP [9–11] inhomogeneous evolution equations which result from the structure of collinear singularities in the target-fragmentation region [8,12]. The dedicated factorisation theorem [4–7] guarantees that fracture functions are universal distributions, at least in the context of SIDIS. On this theoretical basis, an impressive experimental program has been pursued at HERA in diffractive DIS which led to accurate determination of the so-called diffractive parton distributions, i.e. proton-to-proton fracture functions in the very forward limit, allowing for the first time a quite accurate investigation of the parton content of the pomeron. The whole formalism has been later used in Ref. [13] to extract proton-to-Lambda fracture functions within a combined perturbative QCD fit to available SIDIS data.

In this paper we will focus on forward neutron production in DIS which provides, with respect to the aforementioned processes, complementary informations on soft QCD dynamics. An intensive physics program with forward neutron tagging has been pursued at HERA as well, where recent results [14,15] show that around 8 % of the DIS events contain a forward neutron. These data are crucial in testing the limiting fragmentation hypothesis [16] and have been used as benchmark in a number of Regge-based models [17–23] which mainly concentrate on the modelisation of forward neutron production mechanisms. In the present paper we adopt instead a complementary approach and no modelisation of neutron production mechanisms is attempted. This strategy is consistent with the factorisation theorem. The resulting set of proton-to-neutron fracture functions (nFFs) could then be used in hard-scattering factorisation tests [24] in forward neutron tagged dijet photoproduction in  $ep$  collisions, as already measured at HERA [25,26], where factorisation is expected to hold only for the so-called direct component of the cross section. Even more intriguing appears to

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be the possibility of using nFFs for predicting the cross section for the associated production of a forward neutron and a Drell–Yan pair or dijet system in hadronic collisions. For this processes the factorisation theorem is not expected to hold and therefore nFFs determined by DIS data alone offer the opportunity to gauge factorisation breaking effects.

The paper is organised as follows. In Sect. 2 we describe the process under study, the kinematic variables relevant to the analysis and the observable used in the fit. In Sect. 3 we discuss the evolution of fracture functions and the general method with which we build initial conditions for the QCD evolution. In Sect. 4 we describe the details of the QCD fit and in Sect. 5 we assess the impact of experimental and theoretical errors on the obtained neutron FF set. In Sect. 6 we summarise our results.

### 2 Data set and observable

In this analysis we consider semi-inclusive DIS events of the type

$$e^+(k) + p(P) \rightarrow e^+(k') + n(P_n) + X(p_X), \quad (1)$$

where, beside the outgoing lepton, an additional neutron  $n$  is detected in the final state. In Eq. (1)  $X$  stands for the unobserved part of the hadronic system and particles four-momenta are indicated in parenthesis. The kinematic variables  $Q^2$ ,  $x_B$  and  $y$  are used to describe the inclusive DIS scattering process. They are defined as

$$q = k - k', \quad Q^2 = -q^2, \quad x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot k}. \quad (2)$$

The kinematic variables used to describe the final state neutron are the neutron transverse momentum  $p_T$  evaluated with respect to the beam axis and the longitudinal momentum fraction  $x_L$  defined by

$$x_L = 1 - \frac{q \cdot (P - P_n)}{P \cdot q} \simeq E_n/E_p, \quad (3)$$

where  $E_n$  and  $E_p$  are the neutron and proton energy in the laboratory frame, respectively. In the following we use the scaled fractional momentum variable  $\beta$  defined by

$$\beta = \frac{x_B}{1 - x_L}, \quad (4)$$

where  $1 - x_L$  is the maximum available fractional momentum of the parton participating in the hard scattering. The analysis is performed on the H1  $ep$  data of Ref. [27] with positrons and protons energies, respectively, of  $E_e = 27.6$  GeV and  $E_p = 920$  GeV, corresponding to a centre-of-mass energy of  $\sqrt{s} = 319$  GeV. The kinematic range of the selected DIS events is  $6 < Q^2 < 100$  GeV<sup>2</sup>,  $0.02 < y < 0.6$  and  $1.5 \cdot 10^{-4} < x_B < 3 \cdot 10^{-2}$ . The values of  $x_L$  range from 0.365

to 0.905. The kinematic  $\beta$ -coverage is  $x_L$ -dependent, in particular  $\beta_{\min} = 3.52 \cdot 10^{-4}$  at  $x_L = 0.365$  and  $\beta_{\max} = 0.22$  at  $x_L = 0.905$ . Forward neutron production is characterised by small values of  $p_T$ . In Ref. [27] an upper limit on  $p_T$  is used to define the semi-inclusive forward neutron cross section which is correspondingly integrated up to  $p_{T,\max} = 0.2$  GeV. Data are presented as a threefold reduced  $e^+p$  cross section,  $\sigma_r^{LN(3)}$ , which depends on the leading neutron transverse and longitudinal structure functions  $F_2^{LN(3)}$  and  $F_L^{LN(3)}$ , respectively. In the one-photon exchange approximation, it reads

$$\sigma_r^{LN(3)}(\beta, Q^2, x_L) = F_2^{LN(3)}(\beta, Q^2, x_L) - \frac{y^2}{1 + (1 - y)^2} F_L^{LN(3)}(\beta, Q^2, x_L). \quad (5)$$

### 3 Theory setup

Hard-scattering factorisation for this class of processes states that the structure functions in Eq. (5) are of the form

$$F_k^{LN(3)}(\beta, Q^2, x_L, p_T^2) = \sum_i \int_{\beta}^1 \frac{d\xi}{\xi} M_{i/P}^N(\beta, \mu_F^2; x_L, p_T^2) \times C_{ki}\left(\frac{\beta}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s(\mu_R^2)\right) + \mathcal{O}\left(\frac{1}{Q^2}\right). \quad (6)$$

The index  $i$  runs on the flavour of the interacting parton. The hard-scattering coefficients  $C_{ki}$  ( $k = 2, L$ ) are perturbatively calculable as a power expansion in the strong coupling  $\alpha_s$  and depend upon  $\mu_F^2$  and  $\mu_R^2$ , the factorisation and renormalisation scales, respectively. The  $C_{ki}$  coefficient functions are the same as in fully inclusive DIS. The proton-to-neutron fracture functions  $M_{i/P}^N(\beta, \mu_F^2, x_L, p_T^2)$  can be interpreted as the number density of interacting partons at a scale  $\mu_F^2$  and fractional momentum  $\beta$  conditional to the observation of a forward neutron in the final state specified by a fractional momentum  $x_L$  and transverse momentum squared  $p_T^2$ . They contain non-perturbative information on the fragmentation of the spectator system which results from the hard interactions. The  $p_T$ -unintegrated nFFs appearing in Eq. (6) obey standard DGLAP [9–11] evolution equations [28]. In the case  $p_T$  is integrated over up to values of order  $Q^2$ , neutron fracture functions obey an inhomogeneous DGLAP-type evolution equations [8]. The additional inhomogeneous term takes into account neutron production from the fragmentation of initial state parton radiation. In the present case, where the  $p_T$  of the neutron is integrated up to some  $p_{T,\max}$  which lies in the non-perturbative region, neutron fracture functions are defined as

$$M_{i/P}^N(\beta, Q^2, x_L) = \int^{p_{T,\max}^2} dp_T^2 M_{i/P}^N(\beta, Q^2, x_L, p_T^2) \quad (7)$$

and again obey familiar DGLAP evolution equations [29]

$$Q^2 \frac{\partial M_{i/P}^N(\beta, Q^2, x_L)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{\beta}^1 \frac{du}{u} P_i^j(u) M_{j/P}^N\left(\frac{\beta}{u}, Q^2, x_L\right), \tag{8}$$

valid at fixed values of  $x_L$ . The central problem of this type of analyses is to find sensible initial conditions for the relevant distributions prior to evolution. It is indeed possible to use phenomenological models to describe forward neutron production. In general at low and intermediate  $x_L$  the dominant mechanism is expected to be proton-remnant fragmentation into neutrons, while at high  $x_L$  the exchange of virtual particles is expected to dominate. On the other hand, in the present analysis, we work at fixed  $x_L$  and no attempts are made to model this non-perturbative dynamics at the proton vertex. Since hard-scattering factorisation in the form Eq. (6) holds at fixed values of  $x_L$  and  $p_T^2$  and this dependence is fully contained in nFFs, these conditional parton distributions are exclusively fixed by the kinematics of the outgoing neutron and they are, at least in principle, different for different values of  $x_L$  and  $p_T^2$ . The approach we describe in this paper fully takes into account these important recipes in the construction of sensible input for nFFs distributions focusing on parton dynamics as explored by the virtual photon once an additional forward neutron is detected in the final state. This new approach has already been used in the extraction of diffractive parton densities from diffractive DIS data in Ref. [30]. This idea is realised in practice performing a series of QCD fits at fixed values of  $x_L$  with a common initial condition controlled by a set of parameters  $\{p_i\}$ . This procedure guide us to deduce the approximate dependence of parameters  $\{p_i\}$  on  $x_L$  allowing the construction of a generalised initial condition in the  $(\beta, x_L)$ -space to be used in a  $x_L$ -combined QCD fit. This procedure can be generalised to take into account the  $p_T$  dependence. In particular, if four-differential cross sections were available, we could test whether, at fixed  $x_L$ , the parton content probed by the virtual photon is the same (apart from normalisation) in different neutron  $p_T$  ranges.

#### 4 Fitting procedure

In this section we describe QCD fits at fixed values of  $x_L$ . The distributions of neutron FFs in the quark sector at large  $\beta$  could show valence-like structures for some quark-flavour combinations. However, the accessible values of  $\beta$  in the experimental data are quite low. In view of this fact, and in order to reduce the number of free parameters, we assume that all light quark distributions are equal to each other, so that only the singlet and gluon distributions are required. We assume for the latter, at the arbitrary scale  $Q_0^2$  and for any given value of  $x_L$ , a momentum distributions of the type:

$$\beta M_{\Sigma/P}^N(\beta, Q_0^2) = A_q \beta^{B_q} (1 - \beta)^{C_q}, \tag{9}$$

$$\beta M_{g/P}^N(\beta, Q_0^2) = A_g \beta^{B_g} (1 - \beta)^{C_g}. \tag{10}$$

These distributions are then evolved with the QCDNUM1.7 [31] package within a zero mass (ZM) variable flavour number scheme (VFNS) at next-to-leading order. Within this scheme heavy flavours distributions are generated radiatively above their respective mass thresholds which are set to  $m_c = 1.4$  GeV and  $m_b = 4.5$  GeV for charm and bottom quark, respectively. In the VFNS the initial conditions must be imposed at  $Q_0^2 < m_c^2$ . The factorisation and renormalisation scale are both set equal to  $Q^2$ . The coupling constant is set to  $\alpha_s(M_Z^2) = 0.118$ . The convolution engine of QCDNUM1.7 is used to obtain  $F_2^{LN(3)}$  and  $F_L^{LN(3)}$  structure functions at next-to-leading order which are then used to calculate the reduced cross section in Eq. (5) and minimised against data by using the MINUIT program [32]. We adopt the generalised  $\chi^2$  definition proposed in Ref. [33]

$$\chi^2 = \sum_i \left( \frac{m_i - f_i(\mathbf{p}, \mathbf{s})}{\sigma_i} \right)^2 + \sum_k s_k^2, \tag{11}$$

where systematics effects are incorporated in theory model predictions

$$f_i(\mathbf{p}, \mathbf{s}) = t_i(\mathbf{p}) + \sum_k s_k \Delta_{ik}. \tag{12}$$

In the above formulae,  $m_i$  is the measurement of data point  $i$ ,  $t_i$  is the model prediction depending on a set of parameters  $\mathbf{p}$ ,  $\sigma_i$  are the uncorrelated and statistical errors on data point  $i$  added in quadrature and  $\Delta_{ik}$  is the correlated systematic error from source  $k$  on the data  $i$ . The variables  $s_k$  denote Gaussian random variables with zero mean and unit variance. In the present Section we use the above definitions with  $s_k = 0$ .

No cut on the invariant mass of the hadronic system  $X$  nor on the minimum  $Q^2$  of data to be included in the fit is applied. As already discussed above, given the kinematic coverage of the data, we found that the singlet large- $\beta$  coefficient  $C_q$  is loosely constrained by data. For the gluon distribution, which is only indirectly constrained by scaling violations, we found that  $B_g$  strongly correlates with  $A_g$ , when the former is left free to vary in the fit. Therefore we set tentatively  $C_q = 0.5$ ,  $C_g = 1$  and  $B_g = 0$  so that the initial condition contains three free parameters in each  $x_L$ -bin. We performed a combined scan on the value of initial scale  $Q_0^2$ . Given the quite stiff functional form of the initial conditions, there is a mild dependence of the  $\chi^2$  on  $Q_0^2$  which is then fixed at  $Q_0^2 = 1$  GeV<sup>2</sup>. An essential condition for the  $x_L$ -combination procedure to work is that good quality fits must be obtained in each  $x_L$ -bin with the common initial conditions, Eqs. (9, 10). The  $\chi^2$  values of the fixed- $x_L$  fits, obtained with statistical and uncorrelated uncertainties added in quadrature, are

**Table 1**  $\chi^2$  values for fits at fixed values of  $x_L$  with initial condition in Eqs. (9, 10). The total  $\chi^2$ , calculated as the sum of partial  $\chi^2$  at fixed  $x_L$ , and the total number of fitted points are also indicated

$x_L$	$\chi^2$	Fitted points
0.365	12.0	29
0.455	25.5	29
0.545	19.9	29
0.635	21.0	29
0.725	23.6	29
0.815	17.1	29
0.905	15.7	29
Sum	134.8	203

presented in Table 1. From these values we may conclude that initial conditions provided by Eqs. (9, 10), supplemented by the constraints on  $C_q$ ,  $C_g$  and  $B_g$ , are general enough to describe the data in all  $x_L$ -bins. With these results at hand we may now proceed and discuss the  $x_L$ -combined fit. The generalised initial conditions have now the form

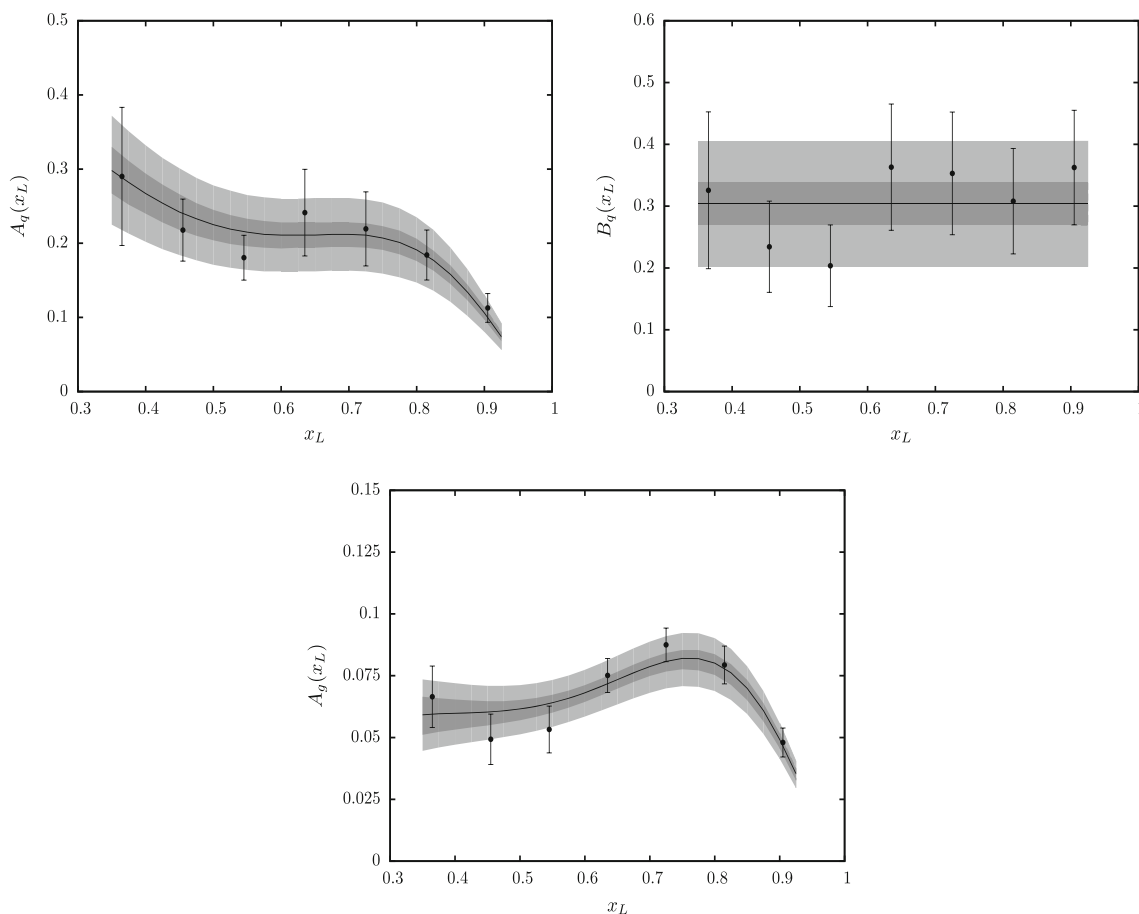
**Table 2**  $\chi^2$  values for the combined- $x_L$  fit. In the third column is indicated the  $\chi^2$  difference between combined- $x_L$  and fixed- $x_L$  fits in each  $x_L$ -bin. The total  $\chi^2$  of the combined- $x_L$  fit and the combination penalty are also indicated

$x_L$	$\chi^2$	$\Delta\chi^2$
0.3650	12.7	+0.7
0.4550	27.5	+2.0
0.5450	22.0	+2.1
0.6350	22.3	+1.3
0.7250	25.5	+1.9
0.8150	17.3	+0.2
0.9050	16.3	+0.6
Tot	143.6	+8.8

$$\beta M_{\Sigma/P}^N(\beta, Q_0^2, x_L) = A_q(x_L) \beta^{B_q(x_L)} (1 - \beta)^{C_q(x_L)},$$

$$\beta M_{g/P}^N(\beta, Q_0^2, x_L) = A_g(x_L) \beta^{B_g(x_L)} (1 - \beta)^{C_g(x_L)}, \quad (13)$$

where the  $\beta$  dependence is the same as in Eqs. (9, 10) and the various coefficients are now  $x_L$ -dependent. The dependences of the  $A_q$ ,  $B_q$  and  $A_g$  free parameters on  $x_L$  may be inferred



**Fig. 1** The black points represent parameters  $A_q$ ,  $B_q$  and  $A_g$  and their errors as a function of  $x_L$ , as obtained from fixed- $x_L$  fits. The best fit parametrisations according to Eqs. (18–20) is displayed as a black solid

line. The dark and light grey bands are obtained by propagating statistical and uncorrelated errors in the  $x_L$ -combined fit, as described in the text, with the condition  $\Delta\chi^2 = 1$  and  $\Delta\chi^2 = 9$ , respectively

**Table 3** Best fit parameter values and their errors

Parameters	$p_i \pm \delta p_i$
$a_1$	$0.62 \pm 0.07$
$c_1$	$17.2 \pm 1.8$
$d_1$	$6.25 \pm 0.17$
$e_1$	$1.77 \pm 0.05$
$a_2$	$0.30 \pm 0.03$
$a_3$	$0.32 \pm 0.06$
$b_3$	$0.90 \pm 0.27$

inspecting Fig. 1. We adopt a redundant parametrisation of the coefficients of the type

$$A_q(x_L) = a_1 x_L^{b_1} (1 + c_1 x_L^{d_1})(1 - x_L)^{e_1}, \tag{14}$$

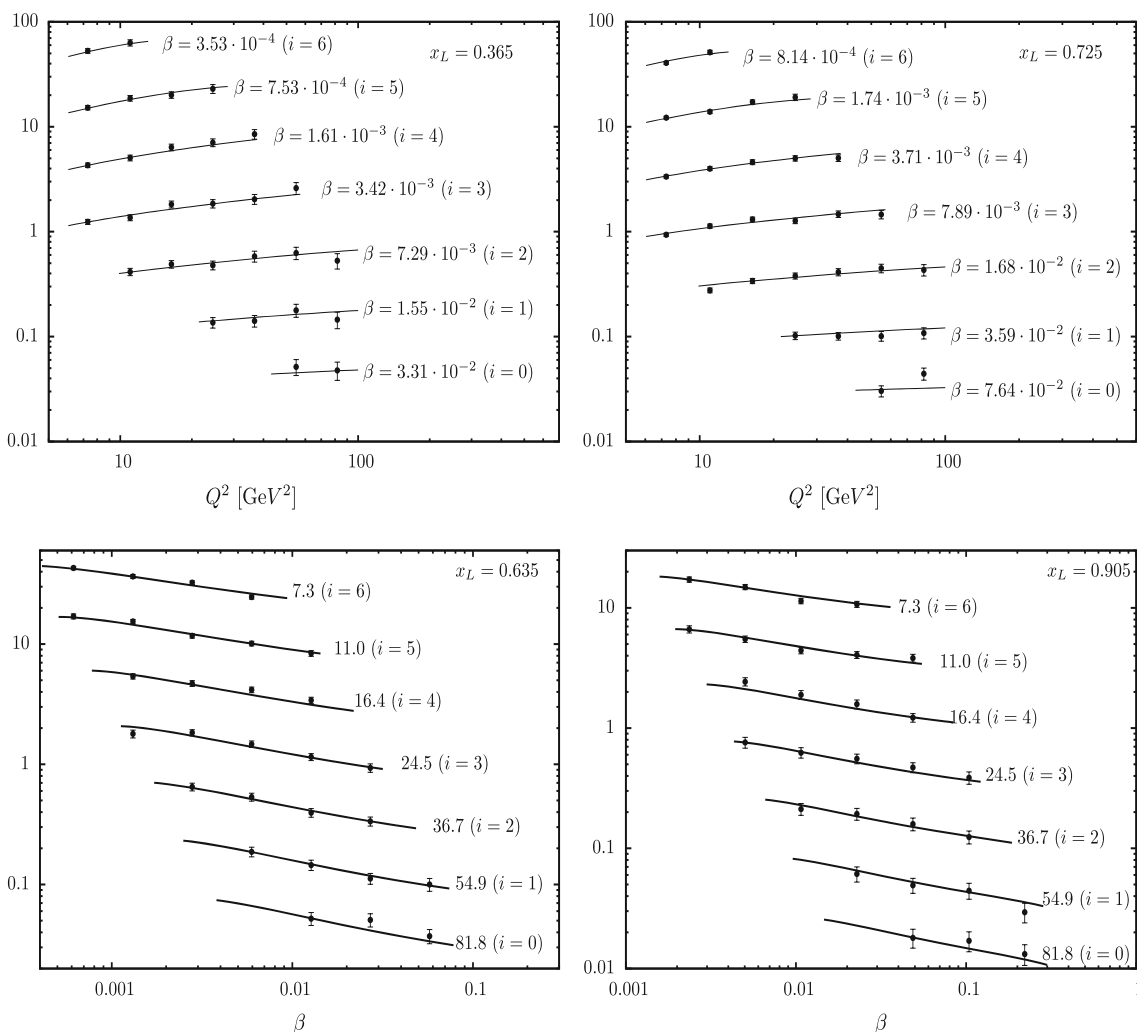
$$B_q(x_L) = a_2 + b_2 x_L + c_2 x_L^2, \tag{15}$$

$$A_g(x_L) = a_3 x_L^{b_3} (1 + c_3 x_L^{d_3})(1 - x_L)^{e_3}, \tag{16}$$

where the term  $1 + c x_L^d$  is included to describe the relative maximum of the normalisation  $A_q$  and  $A_g$  at intermediate values of  $x_L$ . For  $B_q$  we assumed a second order polynomial in  $x_L$ . The parameters  $C_q = 0.5$ ,  $C_g = 1$  and  $B_g = 0$  are still kept fixed to these values. All the QCD settings are the same as in fixed- $x_L$  fits. With the help of Eq. (13) and Eqs. (14–16) we perform a series of  $x_L$ -combined fits. At each iteration we study the eigenvalues of the covariance matrix of the fit parameters. Small eigenvalues, in fact, are associated to (combination of) parameters which are loosely determined by data. We found that, at large  $x_L$ , both the singlet and the gluon normalisations can be described by a common parametrisation  $g(x_L)$

$$g(x_L) = (1 + c_1 x_L^{d_1})(1 - x_L)^{e_1}. \tag{17}$$

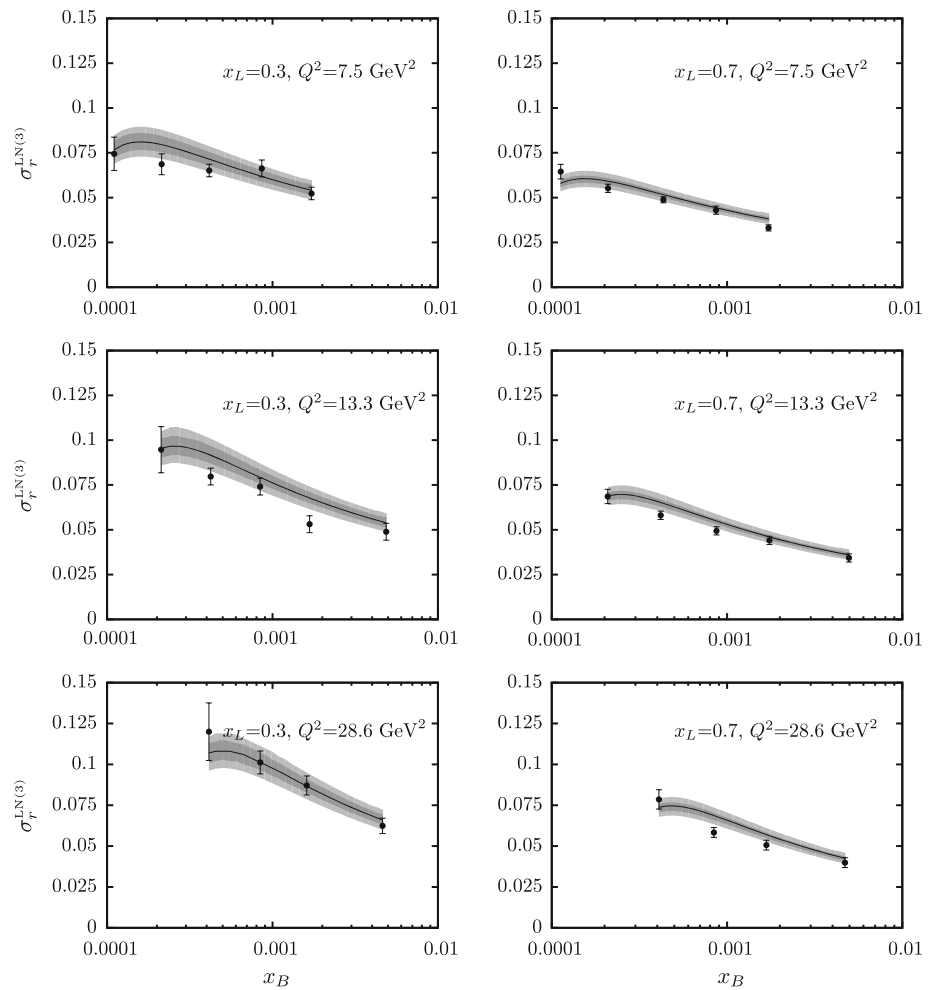
The  $B_q$  coefficient is found to be compatible with a constant so that only the parameter  $a_2$  is left free to vary in the fit.



**Fig. 2** *Top* reduced cross section as a function of  $Q^2$  at different  $\beta$  in two bins of  $x_L$ . *Bottom* reduced cross section as a function of  $\beta$  at different  $Q^2$  (in  $\text{GeV}^2$  units) in two bins of  $x_L$ . The reduced cross section is

scaled by a factor  $3^i$  for better visibility. H1 points from Ref. [27]. The error bars associated with the data points show the sum in quadrature of the statistical and total systematic uncertainty

**Fig. 3** Best fit predictions compared to ZEUS data [37]. The error bars associated with the data points show the sum in quadrature of the statistical and total systematic uncertainty. The dark grey error band corresponds to  $\Delta\chi^2 = 9$  and it is constructed with  $S_{k=1,\dots,14}$ . The light grey error band is constructed as the quadratic sum of statistical ( $S_{k=1,\dots,14}$ ) and systematic ( $S_{k=15,\dots,34}$ ) errors



Finally we found that  $b_1$  is determined with rather large error and compatible with zero, so that we fix it to this value. The final form of the parametrisations of the coefficients is then

$$A_q(x_L) = a_1 g(x_L), \tag{18}$$

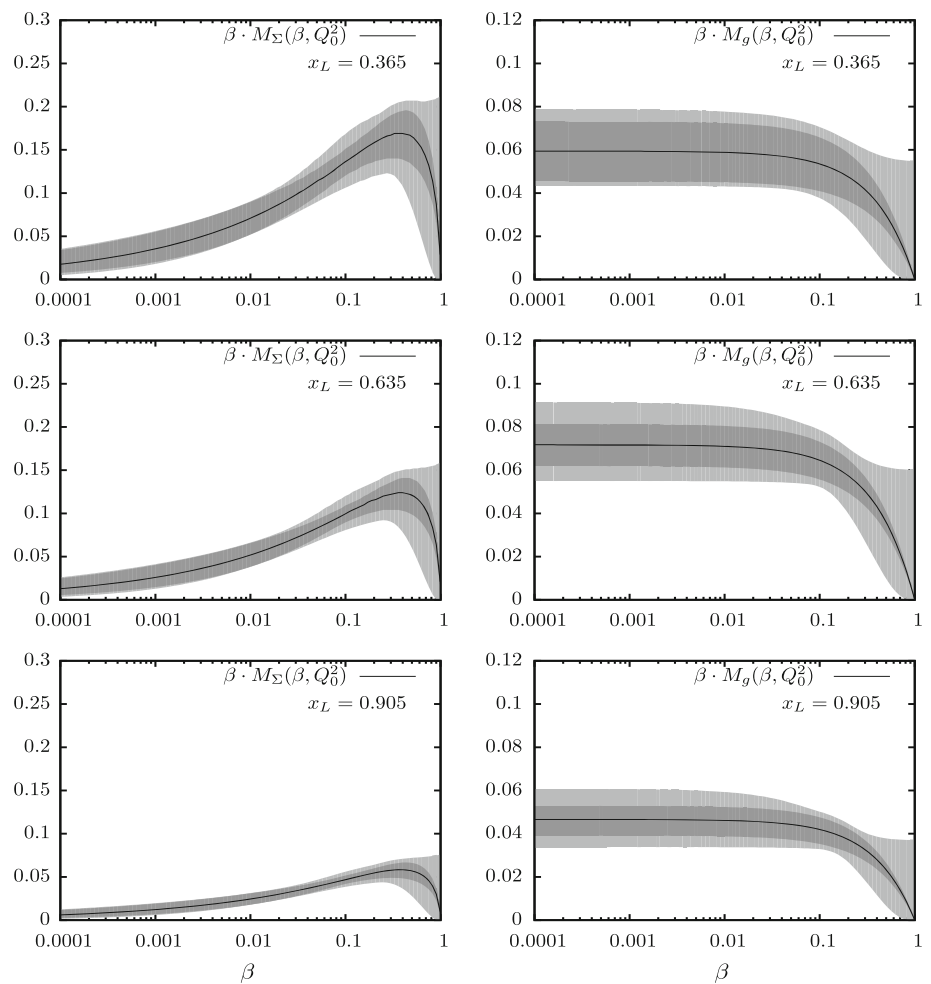
$$B_q(x_L) = a_2, \tag{19}$$

$$A_g(x_L) = a_3 x_L^{b_3} g(x_L), \tag{20}$$

for a total of seven free parameters. The best fit, with statistical and uncorrelated errors added in quadrature, returns a value of  $\chi^2 = 143.6$  for 196 degrees of freedom. The resulting  $\chi^2$  in each  $x_L$ -bin is presented in Table 2. In the third column of the same table is indicated the increase of the  $\chi^2$  generated by the combination procedure in each  $x_L$ -bin with respect to the result presented for fixed- $x_L$  fit in Table 1. Comparing the partial  $\chi^2$  for each value of  $x_L$  in Table 2 and Table 1 we conclude that the combined- $x_L$  fit does not introduce any misrepresentation in the description of any given  $x_L$  bin. We further quantify the quality of the chosen coefficient parametrisations comparing the sum of the  $\chi^2$  obtained by fixed- $x_L$  (134.8) with the  $\chi^2$  from the combined fit (143.6). The combination procedure induces an increase in the over-

all  $\chi^2$  of 8.8 units and, on average, around one unit across the  $x_L$  bins. The best fit parameters and relative errors are reported in Table 3. The best fit predictions are compared to H1 data in Fig. 2 in two representative bins,  $x_L = 0.365$  and  $x_L = 0.725$ , respectively. From the plot it appears that the hard-scattering formula in Eq. (6) together with nFF initial conditions describe data down to the lowest accessible value of  $Q^2$ . This in turn implies also that, in the kinematical range covered by the experiment, no additional power-suppressed terms are required to describe the data. The presence of large and positive scale violations up to the largest parton fractional momentum,  $\beta$ , reveals the substantial contribution to  $\sigma_r^{LN(3)}$  of the gluon nFF distribution induced by QCD evolution. As a final remark we note that the singlet and gluon normalisation coefficients,  $A_q$  and  $A_g$ , in Eq. (13), have a different behaviour at small  $x_L$ . This in turn implies a violation of the so-called proton-vertex factorisation. If this hypothesis is enforced, that is, if we set  $b_1 = b_3$  and let this parameter free to vary in the fit, we obtain a  $\chi^2 = 150$ . Therefore, in the explored kinematical range and given the accuracy of the present data, proton-vertex factorisation holds to a good approximation.

**Fig. 4** Singlet (*left*) and gluon (*right*) momentum distributions at the initial scale  $Q_0^2 = 1 \text{ GeV}^2$  in three representative bins of  $x_L$ . The dark grey error band is obtained with the  $\Delta\chi^2 = 9$  criterion, while the light grey one with the additional  $S_{k=35,\dots,38}$  parametrisations added in quadrature



### 5 Error estimation and propagation

In order to judge the agreement with other data sets and observables or to assess effects beyond the ones taken into account by the theoretical model, the obtained nFF parametrisation must be supplemented, fully exploiting the potential of the data, by a careful error analysis. The general method with which experimental and theoretical uncertainties are propagated to a generic observable  $F$  is based on the construction of alternative nFFs parametrisation sets  $S_k$ . By defining the difference  $r_k = F(S_k) - F(S_0)$  and indicating with  $S_0$  the best fit parametrisation, the uncertainties on a  $F$  are given by

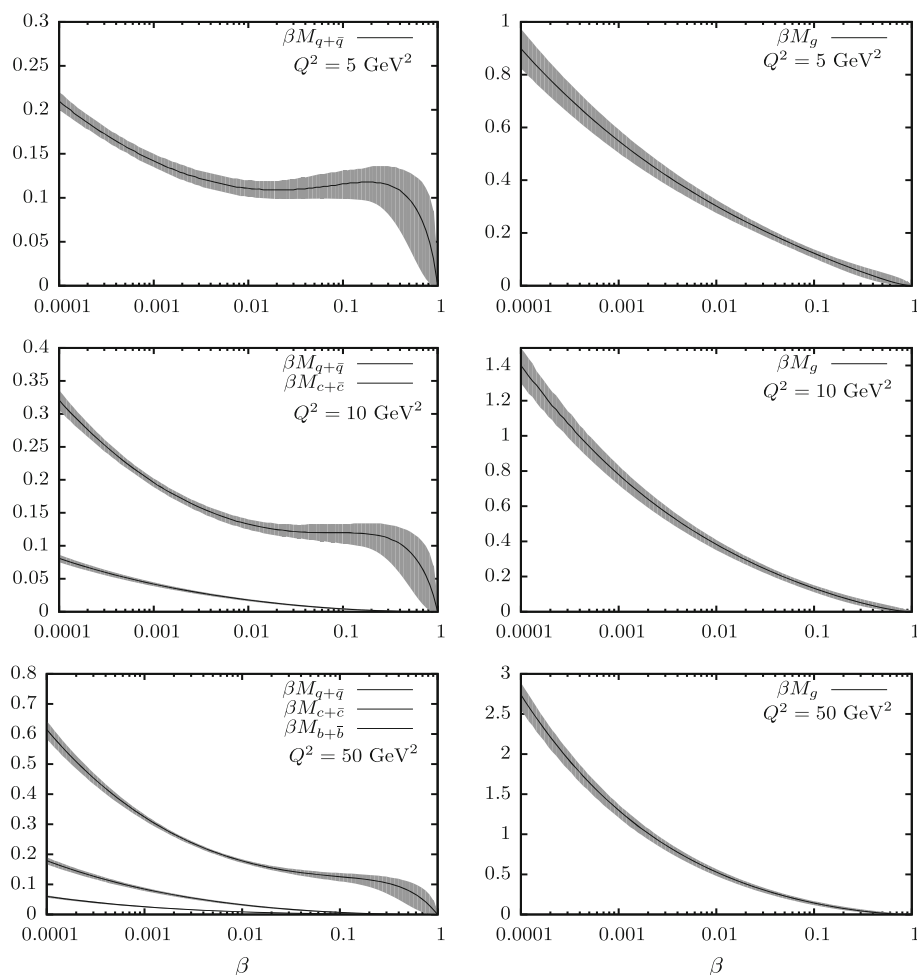
$$\Delta F^+ = \left[ \sum_{k=1}^n r_k^2 \theta(r_k) \right]^{1/2}, \quad \Delta F^- = \left[ \sum_{k=1}^n r_k^2 \theta(-r_k) \right]^{1/2}, \tag{21}$$

where  $\theta$  is the Heaviside step function. In order to propagate statistical and uncorrelated uncertainties, following Refs. [34,35], we have diagonalised the covariance matrix of the best fit parameters and constructed a set of alternative parametrisations  $S_{k=1,\dots,14}$  according to the standard

$\Delta\chi^2 = 1$  criterion. The error band constructed with the help of Eq. (21) and the  $S_{k=1,\dots,14}$  parametrisation set is shown in Fig. 1. The latter is narrower than individual errors on parameters obtained from fixed- $x_L$  fits. This error reduction is in fact due to the  $x_L$ -combination, and can be understood considering for example  $B_q$ , which is just a constant as a function of  $x_L$ . In the combined fit, this parameter is constrained by 203 points rather than 29 of a single  $x_L$  bin and so it is determined far more precisely. Since, however, fixed- $x_L$  fits, by construction, represent the best parametrisations of the data and Eqs. (18–20) are interpreted as a mere interpolating tool, we require that the accuracy of the  $x_L$ -combined fit does not exceed the one of the fixed- $x_L$  fits. We found that a conservative  $\Delta\chi^2 = 9$  criterion matches these requirements, as shown in Fig. 1. We also note that, incidentally, this number is close to the combination penalty reported in Table 2.

We now turn to the inclusion of systematics in the error analysis. In data from Ref. [27] nine systematics sources are identified plus the luminosity uncertainty [36]. For each of them we performed, according to the so-called offset method, alternative fits in which each  $s_k$  is held fixed in turn either to  $-1$  or  $+1$  and produced the parametrisation set  $S_{k=15,\dots,34}$ .

**Fig. 5** Momentum distributions at various  $Q^2$  at  $x_L = 0.635$ . The error band, corresponding to the  $\Delta\chi^2 = 9$  criterion, is obtained with the  $S_{k=1,\dots,14}$  and  $S_{k=35,\dots,38}$  parametrisations added in quadrature



For some sources, for example the 5% luminosity uncertainty common to all data points, the shifts induce steady variation of the  $\chi^2$  and mostly correlates with the central values of the normalisations coefficients  $a_1$  and  $a_3$ . The impact of the propagation of systematic errors are presented in Fig. 3 where the best fit predictions are compared to ZEUS data [37]. The latter are presented in terms of reduced cross sections as a function of  $x_B$  in different bins of  $x_L$  and  $Q^2$  and integrated up to the same  $p_{T,\max} = 0.2$  GeV, as in the H1 analysis from which nFFs are obtained. The effects induced by systematic errors are significant, as shown by the light grey band in Fig. 3. After taking into account all error sources we find that predictions based on nFFs describe ZEUS data both in shape and normalisation.

We conclude this section attempting an estimate of theoretical errors. Among them we mention the ones related to pQCD settings and the ones related to the choice of the parametrisation of initial conditions. The latter are by far dominant in the present analysis so we focus on them in the following. The impact of the functional form used for the  $x_L$ -combination is estimated in  $8.8 \chi^2$  units, which corresponds to the combination penalty term. The uncertainties associated

to the functional forms chosen for the  $\beta$  dependence are not so easily quantified. In the following we restrict ourselves to study the impact of the main assumptions in the parametrisation of the initial conditions at large  $\beta$ . The parameters controlling the large- $\beta$  behaviour, being almost unconstrained by the data, were held fixed in the fit, i.e.  $C_q = 0.5$  and  $C_g = 1$ . This implies that the error propagation produces, as shown in Fig. 4, an artificial shrinkage of the (dark grey) error band at large  $\beta$  and by no means represents a correct error estimate in this limit. In order to quantify the errors introduced by these assumptions, we performed four additional fits in which the  $C_q$  and  $C_g$  parameters are kept fixed in the minimisation but chosen in the following combinations:  $(C_q, C_g) = (0, 1), (2, 1), (0.5, 0), (0.5, 3)$ . The latter values are chosen so that fits return a difference of around  $\Delta\chi^2 = 9$  with respect to best fit. The latter four alternative parametrisations,  $S_{k=35,\dots,38}$ , are used to produce the light grey error band shown in Fig. 4 and shows the degree of underdetermination of the distributions in this region. These additional parametrisations can be especially useful to propagate uncertainties to observables which require nFFs large- $\beta$  extrapolation, for example jet cross section. Less problem-



atic appear the  $Q^2$  extrapolation, since the latter is fully predicted by the theory. In Fig. 5 we present the initial condition at  $x_L = 0.635$  for three different values of  $Q^2$ . It is interesting to note that the uncertainties on the nFFs increase as  $Q^2$  decrease. This effect can be partly ascribed to the fact that no data point with  $Q^2$  below  $7.3 \text{ GeV}^2$  is included in the fit. But, more importantly, it has to be ascribed to QCD evolution: small displacements of the parametrisations at high  $Q^2$ , where they are actually constrained by the data, turn into large fluctuations of the initial conditions at  $Q_0^2$ , due to the logarithmic nature of QCD evolution equations.

## 6 Conclusions

In this paper we have presented a perturbative QCD analysis and extraction of neutron fracture functions from forward neutron production in DIS in HERA kinematics. Data can be described by the leading-twist approximation implied by the hard-scattering factorisation formula and perturbative QCD evolution down to the lowest values of  $Q^2$  accessed by the experiment. The results of the fit and within the precision of the present data, indicate that the proton-vertex factorisation hypothesis is supported to good accuracy, a fact which is likely to be related to the relative low  $\beta$  regime accessed by the measurements. The nFFs low- $Q^2$  extrapolation, although with large uncertainties, can be used to address the impact of absorptive effects going from the DIS to the photoproduction regime. The predictions based on the obtained nFFs have been successfully compared to ZEUS data and an error estimation on nFFs has been provided. The obtained NLO QCD nFFs parametrisation is a quantitative tools which can be used in factorisation tests in processes with a tagged forward neutron. In this context we mention dijet photoproduction in  $ep$  collisions and dijet or Drell–Yan pair production in hadronic collisions. For the latter process, next-to-leading order corrections have been estimated in Refs. [38,39]. The nFFs parametrisation and error set are available upon request to the author and are provided as Fortran steering file for the QCDNUM1.7 package.

**Acknowledgments** We warmly thank Armen Bunyatyan and Vitaly Dodonov for numerous discussions on the measurements and for providing us the breakdown of the systematics errors.

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