

Super-Yang–Mills, Chern–Simons couplings and their all order α' corrections in IIB superstring theory

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Abstract We explore the closed form of the correlation function of four spin operators (including one closed string Ramond–Ramond (RR) and two open string fermions) and one current in ten dimensions, to be able to find the complete and the closed form of the amplitude of one closed string Ramond–Ramond, one gauge field and two fermionic strings (with the same chirality) to all orders in α' in IIB superstring theory. In particular we use a special gauge fixing to the amplitude and apply fermions' equations of motion to $\langle V_C V_A V_{\bar{\psi}} V_{\psi} \rangle$ correlator. The string amplitude implicated that neither there should be any u -channel gauge poles for $p = n + 2$ case nor there are couplings between two fermions and two gauge fields for $p = n$ case in the field theory of type IIB. All infinite u -channel scalar poles and t, s -channel fermion poles of the string amplitude are useful in discovering new couplings of type IIB. More specifically, by making use of the SYM couplings of one scalar, one gauge and two fermions and their all order α' higher derivative corrections, we are able to exactly produce all infinite $(s + t + u)$ -channel scalar poles of $\langle V_C V_A V_{\bar{\psi}} V_{\psi} \rangle$.

1 Introduction

D-branes [1–4] are fundamental non-perturbative objects in string theory. They play a key role in diverse subjects as well as in superstring theories. To be able to talk about the derivation of AdS/CFT one has to deal with these fundamental objects. Let us point out one of their dynamical aspects. In order to be able to describe the different transitions of open/closed strings, we consider [5] an interesting paper.

For completeness, to get familiar with string dualities and to observe various dual descriptions, we introduce [6] to the interested reader. As an example one may talk about a particu-

lar configuration such as the $D0/D4$ system where its importance as well as its applications/explanations are addressed in [7].

To be more specific, one can see the presence of world volume theory from a supergravity point of view [8]. Basically, in [8], we have just employed a new version of ADM reduction and it is applied to type IIB superstring theory. Indeed this kind of ADM reduction has to be deduced to five dimensional hyperboloidal space. In this particular formalism we could understand the appearance of either an AdS or dS space.

One has to apply suitable boundary conditions [9, 10], to be able to observe that D_p -branes must be seen as some hypersurfaces in decompactification space (ten dimensions of flat space time) where p is interpreted as spatial dimension of a D_p -brane. IIA (IIB) does include BPS D_p -branes with even (odd) p and indeed supersymmetry is guaranteed. BPS branes also carry RR(C-field) charge. What can we say about the dynamical aspects of branes? In order to deal with dynamical aspects of D_p -branes we need to discover the general form of the effective actions. Basically we might work with either bosonic effective actions or their supersymmetric versions and these bosonic effective actions for diverse brane configurations have already been found in [11].

One should argue that the supersymmetrized versions of the bosonic actions [11] have not been entirely explored yet, nevertheless we point out to an interesting and pioneering work [12]. Let us just highlight the most important references. In order to talk about the effective action of just a single bosonic D_p -brane [13] must be taken into account. To work with the supersymmetric action of a D_p -brane, we refer to [14–18].

It is worth mentioning that Myers terms, and the Chern–Simons, Wess–Zumino (WZ), and Born–Infeld actions are completely derived in [19–22]. More importantly, we have explained how to look for all the standard effective field theory methods of Myers terms, Taylor, and pull-back techniques where their all order α' corrections are found in [23].

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However, we discussed in [22] that certainly the presence of some other methods apart from those three standard ways is needed. In fact in order to obtain all the infinite higher derivative corrections in string theory one should go further and construct new methods for both BPS [22, 24] and non-BPS branes [25–29].

Open strings have provided various features and their importance can be extracted by dealing with some formal aspects of scattering amplitude arguments. For instance we point out to two different conjectures on taking quantum effects of the BPS strings [30] and a given prescription of all order α' higher derivative corrections to BPS and non-BPS branes [31] for which the first conjecture shows the effects on the curvature of the host branes.

We have been following several motivations for our continuous works [22], so let us emphasize some of them once more. Several important results appeared in [32–34] and we want to explore the entire form of the effective actions to describe dynamical aspects of branes. One of the goals of this paper is to actually prepare more information on the general structure of all order Chern–Simons and Born–Infeld effective actions in superstring theory. For completeness we highlight [35–43] and consider some of the papers which are related to either scattering of BPS branes or related to the formal applications of the BPS and non-BPS branes [44–48].

It is important to have some tools to be able to work with higher point functions of the superstring theory, given the fact that the derivation of AdS/CFT correspondence is unknown. Because we know that by going through them and inside the AdS/CFT there exists a very straightforward relation of a closed and an open string. So by working with mixture open-closed amplitudes one might hope to shed light in understandings all order α' corrections and to the other future works. For instance, we have recently derived various recent WZ effective couplings with their all order α' higher derivative corrections and just showed that those corrections must be taken into account to clarify the N^3 entropy of M5 brane, where for further explanations we just refer the reader to [7].

In fact α' corrections come from all the infinite couplings of the branes with lower dimensions with RR field. Thus one understands the dissolution of soliton objects or lower dimensional branes [7] within branes with higher dimensions. One might talk about a particular application to Myers terms as follows. In the system of $D(-1)/D3$, one employs higher order α' Myers terms to be able to explore this configuration carries N^2 entropy relation. To observe several applications to some of the recent α' corrections, to WZ couplings in flux vacua and M-theory, Refs. [49–51] are worth considering.

If we try to work out some of the recent works on Myers terms and new kind of WZ effective actions [21, 27–29, 31, 52, 53] then we will be able to find some of the corrections in string theory [54]. Basically in order to produce all infinite scalar poles of the amplitude of $\langle V_C V_A V_{\bar{\psi}} V_{\psi} \rangle$, an

infinite number of α' corrections to two fermion–one scalar–one gauge field couplings are needed. The other important point is as follows. Unlike $\langle V_C V_{\phi} V_{\bar{\psi}} V_{\psi} \rangle$ correlators, the closed form of the $\langle V_C V_A V_{\bar{\psi}} V_{\psi} \rangle$ amplitude includes just infinite u -channel scalar poles in its final form and the direct computations of this paper show that there is not even one single u -channel gauge field pole left over.

Direct computations of this paper show that there are no corrections to two fermions and two gauge field couplings of type IIB for $p = n$ case where n is the rank of RR field strength (H). This fact is in favor of carrying out direct conformal field theory techniques, instead of doing a T-duality transformation to $\langle V_C V_{\phi} V_{\bar{\psi}} V_{\psi} \rangle$ amplitude, see [22] for more promising reasons.¹

If we find the infinite corrections to one scalar, one gauge, and two fermion fields, then we are able to find all infinite scalar ($t+s+u$)-channel poles of the string theory for $p+2 = n$ case. Hence by comparing all infinite scalar poles of the string theory amplitude in ($t+s+u$)-channel with field theory vertices we obtain an infinite corrections of two fermions, one on-shell gauge and one off-shell scalar field. It is important to highlight the fact that these all order corrections of type IIB cannot be used in type IIA and may have diverse application to either M-theory [7, 50, 51] or F-theory [55].

Therefore this paper among other things clearly indicates that SYM vertex operators, including one on-shell gauge, one off-shell scalar field, and two fermion fields will give rise exactly to all the same infinite scalar poles that appeared in the string amplitude of $\langle V_C V_A V_{\bar{\psi}} V_{\psi} \rangle$ as well.

Here is the outline of this paper.

In the second section we carry out direct conformal field theory techniques to be able to find the entire form of the amplitude of a closed string RR, one gauge field, and two fermion fields $\langle V_C V_A V_{\bar{\psi}} V_{\psi} \rangle$. Note that in this paper both fermions carry the same chirality, hence the calculations of this paper work just for IIB and the corrections that we are getting to make derivations in IIB theory cannot be applied to IIA.

We also expand our S-matrix to be able to derive all infinite extensions to the unknown vertices. In particular we explicitly show that there are just infinite scalar u -channel massless poles for the $p+2 = n$ case. Indeed all infinite gauge poles that appeared in $\langle V_C V_{\phi} V_{\bar{\psi}} V_{\psi} \rangle$, for the $p = n$ case, have disappeared in the closed form of $\langle V_C V_A V_{\bar{\psi}} V_{\psi} \rangle$ and this is an interesting fact in favor of carrying out direct computations. Then we try to produce all infinite scalar poles of the amplitude in ($t+s+u$)-channel poles. We also explicitly write down all the desired couplings of two fermions and two gauge fields and show that they do not match with string theory amplitude. This clearly confirms that there are no cor-

¹ As is seen, the entire result of $\langle V_C V_{\phi} V_A V_A \rangle$ cannot be derived by applying a T-duality transformation to $\langle V_C V_A V_A V_A \rangle$ of [21].

rections to two fermion–two gauge field couplings of type IIB. We also produce all infinite t, s -channel fermion poles involving their extensions to new couplings.

Eventually we conclude and point out a fascinating relation between open and closed string amplitudes. One might look at Appendices A and B of [23, 53] to get used to standard notations and for some other details.

It is worth mentioning that this paper may shed light in understanding the universal behavior of all order α' higher derivative corrections of the string theory [31]. Having used the results of this paper, we are able to judge that the universal conjecture given in [31] does apply even to fermionic S-matrices. This conjecture may also be applied to obtain all the infinite singularities of five and six point BPS functions without any knowledge of world sheet integrals.

2 Complete form of $RR A \bar{\psi} \psi$ amplitude in type IIB

To obtain the entire and closed form of the S-matrix elements of one closed string RR (C-field), one gauge field and two fermions with the same chirality (which makes sense in the world volume of type IIB), one has to deal with the direct CFT techniques. It is worth addressing several important references on superstring theory [56–58], higher point BPS functions [21, 31, 53, 59–65] and non-BPS [23, 26] tree level calculations.

For completeness we point out the relevant vertex operators for our computations as follows:

$$\begin{aligned}
 V_A^{(0)}(x) &= \xi_a \left(\partial X^a(x) + \alpha' ik \cdot \psi \psi^a(x) \right) e^{\alpha' ik \cdot X(x)}, \\
 V_A^{(-2)}(y) &= e^{-2\phi(y)} V_A^{(0)}(y), \\
 V_{\bar{\psi}}^{(-1/2)}(x) &= \bar{u}^A e^{-\phi(x)/2} S_A(x) e^{\alpha' iq \cdot X(x)} \\
 V_{\psi}^{(-1/2)}(x) &= u^B e^{-\phi(x)/2} S_B(x) e^{\alpha' iq \cdot X(x)} \\
 V_C^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \not{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_{\alpha}(z) e^{i \frac{\alpha'}{2} p \cdot X(z)} \\
 &\quad \times e^{-\phi(\bar{z})/2} S_{\beta}(\bar{z}) e^{i \frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})},
 \end{aligned}
 \tag{1}$$

the Majorana–Weyl wave function u^A is also introduced in ten dimensions. The on-shell condition for RR, fermion, and scalar fields is $p^2 = q^2 = k^2 = 0$. For the other notations on charge conjugation, the definition of the traces and field strength of RR in IIB, reference [24] should be considered. Note also that in order to work with holomorphic propagators, doubling tricks were used [23], however, let us point out the standard correlators

$$\begin{aligned}
 \langle X^{\mu}(z) X^{\nu}(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z-w), \\
 \langle \psi^{\mu}(z) \psi^{\nu}(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} (z-w)^{-1}, \\
 \langle \phi(z) \phi(w) \rangle &= -\log(z-w).
 \end{aligned}
 \tag{2}$$

The one gauge field and two fermions amplitude $\langle V_A V_{\bar{\psi}} V_{\psi} \rangle$ is computed in [66]. If we normalize its S-matrix with the $(iT_p 2^{1/2} \pi \alpha')$ coefficient, then one can show that the S-matrix should be produced by extracting the kinetic term of the fermion fields and taking into account the commutator in $D^a \psi$ as follows:

$$(2\pi \alpha' T_p) \text{Tr} (\bar{\psi} \gamma^a D_a \psi), \quad D^a \psi = \partial^a \psi - i[A^a, \psi].$$

The closed form of our amplitude is given by taking the closed form of the following correlator:

$$\langle V_A^{(0)}(x_1) V_{\bar{\psi}}^{(-1/2)}(x_2) V_{\psi}^{(-1/2)}(x_3) V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle. \tag{3}$$

Notice that the complete result of our calculations should not be used for IIA as here we are considering both fermions with the same chirality, thus all order corrections in this paper cannot be used for IIA.

The amplitude has two different parts, for the first part we need to have the correlation function of four spin operators in ten dimensions [67, 68]:

$$\begin{aligned}
 I_{1\gamma\delta\alpha\beta} &= \langle S_{\gamma}(x_2) S_{\delta}(x_3) S_{\alpha}(x_4) S_{\beta}(x_5) \rangle \\
 &= \left[(\gamma^{\mu} C)_{\alpha\beta} (\gamma_{\mu} C)_{\gamma\delta} x_{25} x_{34} \right. \\
 &\quad \left. - (\gamma^{\mu} C)_{\gamma\beta} (\gamma_{\mu} C)_{\alpha\delta} x_{23} x_{45} \right] \\
 &\quad \times \frac{1}{2(x_{23} x_{24} x_{25} x_{34} x_{35} x_{45})^{3/4}}
 \end{aligned}$$

with $x_{ij} = x_i - x_j$, $x_4 = z = x + iy$, $x_5 = \bar{z} = x - iy$. Having replaced $I_{1\gamma\delta\alpha\beta}$ in the first part of the amplitude, we obtain

$$\begin{aligned}
 \mathcal{A}_1^{CA\bar{\psi}\psi} &\sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \not{H}_{(n)} M_p)^{\alpha\beta} \xi_{1a} \bar{u}_1^{\gamma} u_2^{\delta} \\
 &\quad \times (x_{23} x_{24} x_{25} x_{34} x_{35} x_{45})^{-1/4} I_{1\gamma\delta\alpha\beta} I_2^a \text{Tr} (\lambda_1 \lambda_2 \lambda_3),
 \end{aligned}
 \tag{4}$$

with

$$\begin{aligned}
 I_2 &= |x_{12}|^{\alpha^2 k_1 \cdot k_2} |x_{13}|^{\alpha^2 k_1 \cdot k_3} |x_{14} x_{15}|^{\frac{\alpha^2}{2} k_1 \cdot p} |x_{23}|^{\alpha^2 k_2 \cdot k_3} \\
 &\quad \times x_{24} x_{25} |x_{34} x_{35}|^{\frac{\alpha^2}{2} k_2 \cdot p} |x_{45}|^{\frac{\alpha^2}{4} p \cdot D \cdot p}, \\
 I_3^a &= ik_2^a \left(\frac{x_{42}}{x_{14} x_{12}} + \frac{x_{52}}{x_{15} x_{12}} \right) + ik_3^a \left(\frac{x_{43}}{x_{14} x_{13}} + \frac{x_{53}}{x_{15} x_{13}} \right).
 \end{aligned}$$

Now we can obviously see that the amplitude has the $SL(2, R)$ invariance property. We shall use the very specific gauge fixing ($x_1 = 0, x_2 = 1, x_3 = \infty$) and in particular we are going to employ the following definitions for the Mandelstam variables:

$$\begin{aligned}
 s &= -\frac{\alpha'}{2} (k_1 + k_3)^2, \quad t = -\frac{\alpha'}{2} (k_1 + k_2)^2, \\
 u &= -\frac{\alpha'}{2} (k_3 + k_2)^2.
 \end{aligned}$$

Thus having gauge fixed it, one can find the complete form of the first part of the amplitude as follows:

$$\begin{aligned} \mathcal{A}_1^{CA\bar{\psi}\psi} &\sim (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \xi_{1a} \bar{u}_1^\gamma u_2^\delta \left(\frac{-1}{2}\right) \\ &\times \int \int dz d\bar{z} |z|^{2t+2s} |1-z|^{2t+2u-2} (z-\bar{z})^{-2(t+s+u)-1}, \\ &\times \left[(\gamma^\mu C)_{\gamma\delta} (\gamma_\mu C)_{\alpha\beta} (1-\bar{z}) + (z-\bar{z}) (\gamma^\mu C)_{\gamma\beta} (\gamma_\mu C)_{\alpha\delta} \right] \\ &\times \left(2ik_2^a - \frac{(z+\bar{z})(ik_2^a + ik_3^a)}{|z|^2} \right) \text{Tr}(\lambda_1 \lambda_2 \lambda_3). \end{aligned} \tag{5}$$

In order to find the complete result of the amplitude to all orders in α' one has to take integrations on the location of closed string RR where all details of these integrals can be partially found in [69] and they can be completely found in Appendix B of [23]. Therefore the complete form of the first part of the amplitude appears as

$$\begin{aligned} \mathcal{A}_1^{CA\bar{\psi}\psi} &\sim (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \xi_{1a} \bar{u}_1^\gamma u_2^\delta \left(\frac{-1}{2}\right) \left\{ (\gamma^\mu C)_{\gamma\delta} (\gamma_\mu C)_{\alpha\beta} \right. \\ &\times \left[ik_2^a (usL_1 - 2sL_2) - ik_3^a (utL_1 - 2tL_2) \right] \\ &\left. + (\gamma^\mu C)_{\gamma\beta} (\gamma_\mu C)_{\alpha\delta} (2ik_2^a us - 2ik_3^a ut) L_1 \right\} \text{Tr}(\lambda_1 \lambda_2 \lambda_3), \end{aligned} \tag{6}$$

with

$$\begin{aligned} L_1 &= (2)^{-2(t+s+u)} \\ &\times \pi \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}, \\ L_2 &= (2)^{-2(t+s+u)-1} \\ &\times \pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}. \end{aligned} \tag{7}$$

Unlike the first part of $C\phi\bar{\psi}\psi$ amplitude, this part does not seem to have any u -channel gauge nor scalar poles. Let us talk about the second part of the $CA\bar{\psi}\psi$ amplitude. For this part one needs to have the closed form of the correlation function of four spin operators (with the same chirality) and one current. The details for deriving this correlator have been given in section 2 of [24], however, for completeness here we are going to write down the complete form of that correlator as follows:

$$\langle : \psi^b \psi^a(x_1) : S_\alpha(x_2) : S_\beta(x_3) : S_\gamma(x_4) : S_\delta(x_5) : \rangle = I_{\alpha\beta\gamma\delta}^{ba},$$

with

$$\begin{aligned} I_{\alpha\beta\gamma\delta}^{ba} &= \frac{(x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{-3/4}}{4(x_{12}x_{13}x_{14}x_{15})} \\ &\times \left[\left((\gamma^b C)_{\gamma\beta} (\gamma^a C)_{\alpha\delta} - (\gamma^b C)_{\alpha\delta} (\gamma^a C)_{\gamma\beta} \right) \right. \\ &\left. \times (x_{12}x_{14}x_{35} - x_{15}x_{13}x_{24}) \right. \end{aligned}$$

$$\begin{aligned} &\times x_{23}x_{45} - \left((\gamma^b C)_{\alpha\beta} (\gamma^a C)_{\gamma\delta} + (\gamma^b C)_{\gamma\delta} (\gamma^a C)_{\alpha\beta} \right) \\ &\times x_{25}x_{34} (x_{15}x_{13}x_{24} + x_{14}x_{12}x_{35}) \\ &+ (\Gamma^{ba\lambda} C)_{\alpha\beta} (\gamma_\lambda C)_{\gamma\delta} x_{23}x_{25}x_{34} (x_{14}x_{15}) \\ &+ (\Gamma^{ba\lambda} C)_{\gamma\delta} (\gamma_\lambda C)_{\alpha\beta} x_{25}x_{34}x_{45} (x_{12}x_{13}) \\ &- (\Gamma^{ba\lambda} C)_{\alpha\delta} (\gamma_\lambda C)_{\gamma\beta} x_{23}x_{25}x_{45} (x_{14}x_{13}) \\ &\left. + (\Gamma^{ba\lambda} C)_{\gamma\beta} (\gamma_\lambda C)_{\alpha\delta} x_{23}x_{34}x_{45} (x_{12}x_{15}) \right]. \end{aligned} \tag{8}$$

By substituting the second part of the gauge field's vertex operator (in zero picture) and taking the above correlator into the amplitude we find

$$\begin{aligned} \mathcal{A}_2^{CA\bar{\psi}\psi} &\sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathbb{H}_{(n)} M_p)^{\gamma\delta} \xi_{1a} (2ik_{1b}) \\ &\times \bar{u}_1^\alpha u_2^\beta (x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{-1/4} I_{\alpha\beta\gamma\delta}^{ba}. \end{aligned} \tag{9}$$

Concerning the above gauge fixing and evaluating the integrals on the closed string position, the closed form of the second part of our S-matrix is given by

$$\begin{aligned} \mathcal{A}_2^{CA\bar{\psi}\psi} &\sim \left(\mathcal{A}_{21} + \mathcal{A}_{22} + \mathcal{A}_{23} + \mathcal{A}_{24} + \mathcal{A}_{25} + \mathcal{A}_{26} \right) \text{Tr}(\lambda_1 \lambda_2 \lambda_3), \end{aligned}$$

meanwhile

$$\begin{aligned} \mathcal{A}_{21} &= \frac{1}{4} (P_- \mathbb{H}_{(n)} M_p)^{\gamma\delta} \xi_{1a} (2ik_{1b}) \bar{u}_1^\alpha u_2^\beta (\Gamma^{ba\lambda} C)_{\alpha\beta} (\gamma_\lambda C)_{\gamma\delta} \\ &\times \left[L_2 + \frac{u}{2} (t+s) L_1 \right] \\ \mathcal{A}_{22} &= \frac{-1}{4} (P_- \mathbb{H}_{(n)} M_p)^{\gamma\delta} \xi_{1a} (2ik_{1b}) \bar{u}_1^\alpha u_2^\beta (\Gamma^{ba\lambda} C)_{\gamma\delta} (\gamma_\lambda C)_{\alpha\beta} \\ &\times \left[L_1(st) + \frac{1}{2} L_3 \right] \\ \mathcal{A}_{23} &= \frac{-1}{4} (P_- \mathbb{H}_{(n)} M_p)^{\gamma\delta} \xi_{1a} (2ik_{1b}) \bar{u}_1^\alpha u_2^\beta (\Gamma^{ba\lambda} C)_{\alpha\delta} (\gamma_\lambda C)_{\gamma\beta} \\ &\times \left[-L_1(su) + \frac{1}{2} L_3 \right] \\ \mathcal{A}_{24} &= \frac{1}{4} (P_- \mathbb{H}_{(n)} M_p)^{\gamma\delta} \xi_{1a} (2ik_{1b}) \bar{u}_1^\alpha u_2^\beta (\Gamma^{ba\lambda} C)_{\gamma\beta} (\gamma_\lambda C)_{\alpha\delta} \\ &\times \left[L_1(tu) - \frac{1}{2} L_3 \right] \\ \mathcal{A}_{25} &= \frac{1}{4} (P_- \mathbb{H}_{(n)} M_p)^{\gamma\delta} \xi_{1a} (2ik_{1b}) \bar{u}_1^\alpha u_2^\beta \left[u(s+t)L_1 + L_3 \right] \\ &\times \left((\gamma^b C)_{\gamma\beta} (\gamma^a C)_{\alpha\delta} - (\gamma^b C)_{\alpha\delta} (\gamma^a C)_{\gamma\beta} \right) \\ \mathcal{A}_{26} &= \frac{-1}{4} (P_- \mathbb{H}_{(n)} M_p)^{\gamma\delta} \xi_{1a} (2ik_{1b}) \bar{u}_1^\alpha u_2^\beta \\ &\times \left[-\frac{1}{2} u(s-t)L_1 + (-t+s)L_2 \right] \\ &\times \left(-(\gamma^a C)_{\gamma\delta} (\gamma^b C)_{\alpha\beta} + (\gamma^a C)_{\alpha\beta} (\gamma^b C)_{\gamma\delta} \right), \end{aligned} \tag{10}$$

with L_3 as follows:

$$L_3 = (2)^{-2(t+s+u)+1} \pi \times \frac{\Gamma(-u + \frac{1}{2})\Gamma(-s + \frac{1}{2})\Gamma(-t + \frac{1}{2})\Gamma(-t - s - u + 1)}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)}. \tag{11}$$

Note that L_3 does not include any singularities as the expansion is a low energy expansion and all the gamma functions appearing in L_3 have no singularities. It is seen that the amplitude involves infinite s, t, u , and $(t + s + u)$ -channel poles and once we are dealing with all massless strings, the expansion is just a low energy expansion in which by sending α' to zero we should be able to get to the desired poles in field theory side ($t + s + u = -p^a p_a$).

To be more clearly one of the ideas for deriving the exact form of the amplitude is to be able to discover all the infinite higher derivative corrections to the SYM couplings and also to fix the coefficients of these couplings by producing the S-matrix element. Notice that our amplitude is antisymmetric under interchanging the fermions. Now we try to produce all the infinite poles by exploring new SYM couplings as well as their all order α' higher derivative corrections.

3 Infinite u -channel singularities and their contact interactions for $p = n$ case

Unlike the $\langle V_C V_\phi V_{\bar{\psi}} V_\psi \rangle$, the closed form of the $\langle V_C V_A V_{\bar{\psi}} V_\psi \rangle$ correlators just gives us infinite u -channel massless scalar poles. The stL_1 expansion is

$$stL_1 = -\pi^{3/2} \left[\sum_{n=-1}^{\infty} b_n \left(\frac{1}{u} (t+s)^{n+1} \right) + \sum_{p,n,m=0}^{\infty} e_{p,n,m} u^p (st)^n (s+t)^m \right], \tag{12}$$

with

$$b_{-1} = 1, \quad b_0 = 0, \quad b_1 = \frac{1}{6}\pi^2, \quad b_2 = 2\zeta(3),$$

$$e_{0,1,0} = 2\zeta(3), \quad e_{1,0,0} = \frac{1}{6}\pi^2, \quad e_{0,0,1} = \frac{1}{3}\pi^2;$$

the b_n coefficients do have a universal structure [21]. Let us consider the first term of \mathcal{A}_{22} which has infinite u -channel poles. One may think γ^λ in $\Gamma^{ba\lambda}$ can have both world volume ($\lambda = a$) and transverse components ($\lambda = i$) and accordingly there can be both u -channel gauge and scalar pole (sounds ambiguity), however, since the integrations for the Chern–Simons action should be taken on the world volume (the sum of the world volume direction should be $p + 1$) and there is no coupling between one RR, $(p - 3)$ form field (C_{p-3}) and one gauge field and two fermions (even though

$$\frac{(2\pi\alpha')^2 \mu_p}{2!(p-2)!} \int d^{p+1} \sigma \text{Tr} (C_{(p-3)} \wedge F \wedge F)$$

is allowed), we conclude that λ cannot have a component in the world volume direction. Thus all u -channel scalar poles for the $p = n$ case in string theory should be written as

$$\frac{\mu_p}{(p)!} (\varepsilon^v)^{a_0 \dots a_{p-2} ba} H_{a_0 \dots a_{p-2}}^i 2\pi \xi_{1a} (2i k_{1b}) \bar{u}_1^\alpha (\gamma_i)_{\alpha\beta} u_2^\beta \times \sum_{n=-1}^{\infty} b_n \left(\frac{1}{u} (t+s)^{n+1} \right) \text{Tr} (\lambda_1 \lambda_2 \lambda_3),$$

where we have normalized the amplitude by a factor of $\frac{\mu_p}{2\pi^{1/2}}$. In the above the trace is replaced as

$$\text{Tr} (P_- \not{H}_{(n)} M_p \Gamma^{bai}) = \frac{32}{2(p)!} (\varepsilon^v)^{a_0 \dots a_{p-2} ba} H_{a_0 \dots a_{p-2}}^i. \tag{13}$$

The first u -channel pole (for $n = -1$) can be produced by taking its Feynman rule on the field theory side as below:

$$\mathcal{A} = V_\alpha^i (C_{p-1}, A_1, \phi) G_{\alpha\beta}^{ij} (\phi) V_\beta^j (\phi, \bar{\Psi}_1, \Psi_2). \tag{14}$$

Let us just mention that to deal with the field theory of RR and scalar fields, we need to work out either Wess–Zumino (WZ) terms [11] or a pull-back or Taylor expansion (for extensive discussions [23] is suggested).

By applying a Taylor expansion,

$$i \frac{\lambda^2 \mu_p}{(p)!} \int d^{p+1} \sigma \text{Tr} \left(\partial_i C_{(p-1)} \wedge F \phi^i \right), \tag{15}$$

one can obtain the vertex of $V_\alpha^i (C_{p-1}, A_1, \phi)$ where $\lambda = 2\pi\alpha'$ so that

$$V_\alpha^i (C_{p-1}, A_1, \phi) = i \frac{\lambda^2 \mu_p}{(p)!} H_{a_0 \dots a_{p-2}}^i k_{1a_{p-1}} \xi_{1a_p} (\varepsilon^v)^{a_0 \dots a_p} \text{Tr} (\lambda_1 \lambda^\alpha), \tag{16}$$

the scalar propagator has been derived by taking into account the kinetic term of scalar fields in the DBI action $\left(-T_p \frac{(2\pi\alpha')^2}{2} \text{Tr} (D_a \phi^i D^a \phi_i) \right)$ as follows:

$$G_{\alpha\beta}^{ij} (\phi) = \frac{-i \delta_{\alpha\beta} \delta^{ij}}{T_p (2\pi\alpha')^2 k^2} = \frac{-i \delta_{\alpha\beta} \delta^{ij}}{T_p (2\pi\alpha')^2 u}. \tag{17}$$

In particular $V_\beta^j (\phi, \bar{\Psi}_1, \Psi_2)$ should be derived by considering the fixed kinetic term of fermion fields $\left(-T_p (2\pi\alpha') \text{Tr} (-\bar{\Psi} \gamma^a D_a \Psi) \right)$ as well as extracting the covariant derivative of fermion field such that

$$V_j^B (\bar{\Psi}_1, \Psi_2, \phi) = T_p (2\pi\alpha') \bar{u}_1^A \gamma_{AB}^j u_2^B \left(\text{Tr} (\lambda_2 \lambda_3 \lambda^\beta) - \text{Tr} (\lambda_3 \lambda_2 \lambda^\beta) \right). \tag{18}$$

Having substituted (16), (17) and (18) in the field theory amplitude of (14), we are able to precisely produce the

first simple massless scalar u-channel pole, however, as is obvious from the stL_1 expansion, our S-matrix does involve infinite u-channel poles. These infinite singularities can be found by postulating an infinite number of higher derivative corrections to the vertex of $V_\alpha^i(C_{p-1}, A_1, \phi)$ (note that the scalar propagator and all kinetic terms will not receive any corrections [21,23,53] as they have already been fixed in the DBI action; see [24]) as below:

$$i \frac{\lambda^2 \mu_p}{p!} \int d^{p+1} \sigma \sum_{n=-1}^\infty b_n(\alpha')^{n+1} \times \text{Tr} \left(\partial_i C_{(p-1)} \wedge D^{a_0} \dots D^{a_n} F D_{a_0} \dots D_{a_n} \phi^i \right). \tag{19}$$

Notice that all commutator terms in the definitions of covariant derivatives in (19) should be neglected. Now the infinite extension of the corrected vertex operator to all orders in α' is given by

$$V_\alpha^i(C_{p-1}, A_1, \phi) = i \frac{\lambda^2 \mu_p}{p!} H_{a_0 \dots a_{p-2}}^i k_{1a_{p-1}} \xi_{1a_p} (\varepsilon^v)^{a_0 \dots a_p} \times \sum_{n=-1}^\infty b_n(\alpha' k_1 \cdot k)^{n+1} \text{Tr} (\lambda_1 \lambda^\alpha), \tag{20}$$

where

$$\alpha' k_1 \cdot k = t + s, \quad (k_1 + k_2 + k_3 + p)^a = 0, \quad p^a (\varepsilon^v)^{a_0 \dots a_{p-1} a} = 0,$$

and we have employed the following standard kinetic terms in superstring theory:

$$-T_p(2\pi\alpha') \text{Tr} \left(\frac{(2\pi\alpha')}{2} D_a \phi^i D^a \phi_i - \frac{(2\pi\alpha')}{4} F_{ab} F^{ba} - \bar{\Psi} \gamma^a D_a \Psi \right). \tag{21}$$

Momentum conservation in world volume direction as well as the constraint for RR are also used. Now (20) is the so called all order α' extension of (15). By replacing (20) into (14), keeping the fixed scalar propagator and $V_\beta^j(\phi, \bar{\Psi}_1, \Psi_2)$, we can exactly derive all infinite u-channel scalar poles of $CA\bar{\psi}\psi$. Therefore the RR $(p-1)$ -form field has just induced an infinite number of higher derivative corrections to an on-shell gauge and one off-shell scalar field. This is a property of closed string RR which proposes all order extensions to all kinds of BPS [21,24,31,53] and non-BPS open strings [23] where we have called it the universal property of all order higher derivative corrections. Let us end this section by constructing a new coupling and fixing its coefficient by comparing it with all contact interactions in the stL_1 expansion. First consider the following coupling:

$$\frac{(2\pi\alpha')^2 \mu_p}{(p)!} \int d^{p+1} \sigma \text{Tr} (C_{(p-1)} \wedge F \bar{\psi} \gamma^i D_i \psi). \tag{22}$$

In order to be able to produce all contact interactions in the stL_1 expansion (the second terms inside (12)), one can generalize (22) to all orders in α' as follows:

$$\sum_{p,n,m=0}^\infty e_{p,n,m}(\alpha')^{2n+m-2} \left(\frac{\alpha'}{2} \right)^p \frac{(2\pi\alpha')^2 \mu_p}{\pi(p)!} \times \int d^{p+1} \sigma \text{Tr} (C_{(p-1)} \wedge D^{a_1} \dots D^{a_n} D^{a_{n+1}} \dots D^{a_{2n}} \times D^{a_1} \dots D^{a_m} F (D^a D_a)^p D_{a_1} \dots D_{a_m} \times (D_{a_1} \dots D_{a_n} \bar{\psi} \gamma^i D_{a_{n+1}} \dots D_{a_{2n}} D_i \psi)). \tag{23}$$

3.1 An infinite number of scalar poles for $p + 2 = n$ case

In this section, in order to match an infinite number of massless poles of the string theory amplitude of $C_{p+1} A \bar{\Psi} \Psi$ with field theory poles, we need to obtain an infinite number of higher derivative corrections to two fermions (with the same chirality), one scalar, and one gauge field in type IIB.

More importantly we would like to show that the universal conjecture for all order α' corrections [31] holds for the string amplitudes including fermionic strings as well.

The needed terms for these poles are related to the second and the fourth terms of \mathcal{A}_1 ; thus, by extracting the traces, we are able to write down singular terms in the string amplitude as

$$\mathcal{A} = - \frac{2i\alpha' \pi^{-1/2} \mu_p}{(p+1)!} (\varepsilon^v)^{a_0 \dots a_{p-1}} H_{a_0 \dots a_{p-1}}^i \bar{u}_1^\gamma (\gamma^i)_{\gamma\delta} u_2^\delta \times \left[-2k_2 \cdot \xi_1 s + 2k_3 \cdot \xi_1 t \right] L_2 \text{Tr} (\lambda_1 \lambda_2 \lambda_3). \tag{24}$$

Notice that since in $\langle V_C V_A V_{\bar{\psi}} V_\psi \rangle$ we do not have an external scalar field, we reveal that these two fermions (with the same chirality), one gauge, and one scalar couplings must be found just by comparing them with the string theory amplitude.

The L_2 expansion is

$$L_2 = - \frac{\pi^{5/2}}{2} \left(\sum_{n=0}^\infty c_n (s+t+u)^n + \frac{\sum_{n,m=0}^\infty c_{n,m} [s^n t^m + s^m t^n]}{(t+s+u)} + \sum_{p,n,m=0}^\infty f_{p,n,m} (s+t+u)^p [(s+t)^n (st)^m] \right), \tag{25}$$

which clearly shows that it involves an infinite number of $(t+s+u)$ -channel massless scalar poles. In [24] we have derived the vertex of one off-shell scalar and one $RR-(p+1)$ form field as well as a scalar propagator as below:

$$G_{\alpha\beta}^{ij}(\phi) = \frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2 k^2} = \frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2 (t+s+u)}, \quad V_\alpha^i(C_{p+1}, \phi) = i(2\pi\alpha') \mu_p \frac{1}{(p+1)!} (\varepsilon^v)^{a_0 \dots a_p} H_{a_0 \dots a_p}^i \text{Tr} (\lambda_\alpha). \tag{26}$$

The following rule should be considered to be able to produce an infinite number of scalar $(t + s + u)$ -channel poles:

$$\mathcal{A} = V_\alpha^i(C_{p+1}, \phi) G_{\alpha\beta}^{ij}(\phi) V_\beta^j(\phi, \bar{\Psi}, \Psi, A). \tag{27}$$

Moreover, if we take the following couplings:

$$\frac{T_p(2\pi\alpha')^3}{4} \left[\bar{\Psi} \gamma^i D_b \Psi D^a \phi_i F_{ab} + \bar{\Psi} \gamma^i D_b \Psi F_{ab} D^a \phi_i \right], \tag{28}$$

overlook the commutator terms inside all the covariant derivative terms, take into account $\text{Tr}(\lambda_2 \lambda_3 \lambda_\beta \lambda_1)$ ordering to the first coupling in (28), and also apply $\text{Tr}(\lambda_2 \lambda_3 \lambda_1 \lambda_\beta)$ ordering to the second coupling in (28) (λ_β holds for Abelian scalar field), then one can easily find the following vertex operator:

$$V_\beta^j(\phi, \bar{\Psi}, \Psi, A) = -i \frac{T_p(2\pi\alpha')^3}{2} \bar{u}^\gamma(\gamma^j)_{\gamma\delta} u^\delta \times \left(-\frac{t}{2} k_3 \cdot \xi_1 + \frac{s}{2} k_2 \cdot \xi_1 \right) \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta), \tag{29}$$

where momentum conservation is also used.

By replacing (29) into (27), keeping the second term of the expansion of L_2 for $n = m = 0$ (appearing in (25)) inside (27), and comparing it with the string amplitude, we are able to clarify that just the first simple $(t + s + u)$ -channel scalar pole in (24) can be precisely produced.

But our string amplitude has infinite $(t + s + u)$ -channel scalar poles, so keeping fixed the simple propagator and $V_\alpha^i(C_{p+1}, \phi)$ (there are no corrections to them), one immediately explores that all infinite massless scalar poles should be derived by making use of all order corrections to two on-shell fermions, one on-shell gauge, and one off-shell scalar field of type IIB as follows:

$$\begin{aligned} \mathcal{L}^{n,m} = \pi^3 \alpha'^{n+m+3} T_p \left(a_{n,m} \text{Tr} \left[\mathcal{D}_{nm} \left(\bar{\Psi} \gamma^i D_b \Psi D^a \phi^i F_{ab} \right) \right. \right. \\ \left. \left. + \mathcal{D}_{nm} \left(\bar{\Psi} \gamma^i D_b \Psi F_{ab} D^a \phi^i \right) + h.c. \right] \right. \\ \left. + i b_{n,m} \text{Tr} \left[\mathcal{D}'_{nm} \left(\bar{\Psi} \gamma^i D_b \Psi D^a \phi^i F_{ab} \right) \right. \right. \\ \left. \left. + \mathcal{D}'_{nm} \left(\bar{\Psi} \gamma^i D_b \Psi F_{ab} D^a \phi^i \right) + h.c. \right] \right), \tag{30} \end{aligned}$$

where the following definitions for the higher derivative operators of $\mathcal{D}_{nm}, \mathcal{D}'_{nm}$ should be taken as well:

$$\begin{aligned} \mathcal{D}_{nm}(EFGH) &\equiv D_{b_1} \cdots D_{b_m} D_{a_1} \cdots D_{a_n} E F D^{a_1} \\ &\quad \cdots D^{a_n} G D^{b_1} \cdots D^{b_m} H \\ \mathcal{D}'_{nm}(EFGH) &\equiv D_{b_1} \cdots D_{b_m} D_{a_1} \cdots D_{a_n} E D^{a_1} \\ &\quad \cdots D^{a_n} F G D^{b_1} \cdots D^{b_m} H. \end{aligned}$$

First, let us deal with the terms carrying $a_{n,m}$ coefficients. If we consider the first and the second term of (30) as well as their hermitian conjugate (with the suitable ordering, mentioned earlier on), we obtain

$$V_\beta^j(\phi, \bar{\Psi}, \Psi, A) = -i \frac{T_p(2\pi\alpha')^3}{4} \bar{u}^\gamma(\gamma^j)_{\gamma\delta} u^\delta (t^m s^n + t^n s^m) \times \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta) a_{n,m} \left(-\frac{t}{2} k_3 \cdot \xi_1 + \frac{s}{2} k_2 \cdot \xi_1 \right). \tag{31}$$

Now if we would substitute (31) into (27) and would keep the second term of the expansion of L_2 for general n, m inside (27), we would be able to show that all infinite $(t + s + u)$ -channel scalar poles of string amplitude (24) are exactly produced in field theory as well.

If we use the same standard field theory techniques (with the above ordering) for the terms carrying the coefficients of $b_{n,m}$ then we derive the following vertex to all orders of α' :

$$V_\beta^j(\phi, \bar{\Psi}, \Psi, A) = -i \frac{T_p(2\pi\alpha')^3}{4} \bar{u}^\gamma(\gamma^j)_{\gamma\delta} u^\delta (t^m u^n + s^m u^n) \times \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta) b_{n,m} \left(-\frac{t}{2} k_3 \cdot \xi_1 + \frac{s}{2} k_2 \cdot \xi_1 \right). \tag{32}$$

Now by applying the on-shell condition $(t + s + u = 0)$ at each order of α' to the above vertex, one can easily see that the common coefficient in both string and field theory amplitudes is precisely re-constructed. It means that all order α' corrections for the terms including $b_{n,m}$ coefficients are exact.

On the other hand, one can show that all order α' corrections to two fermions and two scalar fields of type IIB are given by

$$\begin{aligned} \mathcal{L}^{n,m} = \pi^3 \alpha'^{n+m+3} T_p \left(a_{n,m} \text{Tr} \left[\mathcal{D}_{nm} \left(\bar{\Psi} \gamma^a D_b \Psi D^a \phi^i D^b \phi_i \right) \right. \right. \\ \left. \left. + \mathcal{D}_{nm} \left(D^a \phi^i D^b \phi_i \bar{\Psi} \gamma^a D_b \Psi \right) + h.c. \right] \right. \\ \left. + i b_{n,m} \text{Tr} \left[\mathcal{D}'_{nm} \left(\bar{\Psi} \gamma^a D_b \Psi D^a \phi^i D^b \phi_i \right) \right. \right. \\ \left. \left. + \mathcal{D}'_{nm} \left(D^a \phi^i D^b \phi_i \bar{\Psi} \gamma^a D_b \Psi \right) + h.c. \right] \right). \tag{33} \end{aligned}$$

It is worth trying to point out some comments on two fermion–two gauge couplings which carry three momenta in world volume directions. Consider the following coupling:

$$\bar{\Psi} \gamma^a D_a \Psi F_{bc} F^{bc}. \tag{34}$$

If we take all possible orderings $\text{Tr}(\lambda_2 \lambda_3 \lambda_1 \lambda_\beta)$ and $\text{Tr}(\lambda_2 \lambda_3 \lambda_\beta \lambda_1)$ where λ_β is related to the Abelian gauge field, then we obtain the vertex of two on-shell fermion fields and one on-shell/one off-shell gauge field on the field theory side as

$$v_b^\beta(\bar{\Psi}_2, \Psi_3, A_1, A) = \bar{u} \gamma^a u (-ik_3 a) \times \left[(t + s) \xi^b + 2k_1^b (-k_2 \cdot \xi_1 - k_3 \cdot \xi_1) \right] \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta). \tag{35}$$

However, if we apply the on-shell equation for the fermion fields to (35) then we understand that the above coupling

(34) does not have any contribution to the field theory amplitude. By the same analysis, one can explicitly show that $\bar{\Psi}\gamma^a D_b \Psi F_{bc} F^{ac}$ produces some extra terms which do not appear in the string theory amplitude of $CA\bar{\Psi}\Psi$ of type IIB. Therefore we conclude that there are no α' corrections to two fermion–two gauge field couplings of type IIB. One may try to find out this S-matrix in type IIA to see whether or not there are α' corrections to IIA theory [70, 71].

4 An infinite number of t, s -channel fermion poles

If we consider the fact that the exchanged strings must have just a non-zero fermion number, then we believe neither gauge/scalar nor any other strings (except fermions) can be propagated. Therefore to be able to find all infinite t, s -channel poles, fermions with the same chirality should be propagated. If we would simplify all the terms appearing in our S-matrix and make use of various identities then we could find that both all infinite t - and s -channel poles make sense just for the $p = n$ case and at the end of the day they do come from A_{26} as follows:

$$A = \frac{\alpha' \mu_p \pi \xi_{1a} (2ik_{1b})}{(p)!} \bar{u}_1^A (\gamma^b)_{AB} u_2^B \sum_{n=-1}^{\infty} b_n \frac{1}{t} (u+s)^{n+1} \times (\varepsilon^v)^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} \text{Tr} (\lambda_1 \lambda_2 \lambda_3). \tag{36}$$

Note that in the above we have also all infinite s -channel poles, however, as we know, the amplitude is antisymmetric with respect to $(s \leftrightarrow t)$. Thus we just produce all t -channel poles.

The Feynman rule for producing all infinite fermionic t -channel poles is

$$A = V_\alpha(C_{p-1}, \Psi_3, \bar{\Psi}) G_{\alpha\beta}(\Psi) V_\beta(\Psi, \bar{\Psi}_2, A_1). \tag{37}$$

To produce the fermionic propagator, one needs to make use of the last term of (21). Note that in order to be able to read off $V_\beta(\Psi, \bar{\Psi}_2, A_1)$, one must extract the covariant derivative of fermion inside its kinetic term ($D^a \psi = \partial^a \psi - i[A^a, \psi]$) and in particular take into account all the desired orderings of the fermions and the scalars, such that

$$V_\beta(\Psi, \bar{\Psi}_2, A_1) = -iT_p (2\pi\alpha') \bar{u}_1^A \gamma_A^\alpha \xi_{1a} (\text{Tr} (\lambda_1 \lambda_2 \lambda^\beta) - \text{Tr} (\lambda_2 \lambda_1 \lambda^\beta))$$

$$G_{\alpha\beta}(\psi) = \frac{-i\delta_{\alpha\beta}}{T_p(2\pi\alpha')k} = \frac{-i\delta_{\alpha\beta} \gamma^a (k_1 + k_2)_a}{T_p(2\pi\alpha')t}. \tag{38}$$

Now if we take the following coupling of one on-shell RR $(p - 1)$ -form field, an on-shell/an off-shell fermion, then $V_\alpha(C_{p-1}, \bar{\Psi}, \Psi)$ can be explored as

$$i \frac{(2\pi\alpha') \mu_p}{(p)!} \text{Tr} \left(C_{a_0 \dots a_{p-2}} \bar{\Psi} \gamma^b \partial_b \Psi \right) (\varepsilon^v)^{a_0 \dots a_{p-2}}, \tag{39}$$

where one has to keep in mind the equations of motion for fermions ($k_{2a} \bar{u} = k_{3a} u = 0$) as well.

One can now apply some field theory methods to the above coupling (39) to be able to discover the vertex of one RR $(p - 1)$ -form field and an on-shell/an off-shell fermion as

$$V_\alpha(C_{p-1}, \Psi_3, \bar{\Psi}) = i \frac{(2\pi\alpha') \mu_p}{(p)!} (\varepsilon^v)^{a_0 \dots a_{p-2}} H_{a_0 \dots a_{p-2}}^b \gamma^b u_2 \text{Tr} (\lambda_3 \lambda^\alpha). \tag{40}$$

If we substitute (38) and (40) to the Feynman rule in (37) then the field theory amplitude will give rise just to the first massless t -channel fermion pole of the string amplitude (for $n = -1$ in (36)).

However, as we can see the string amplitude has infinite t, s -channel fermion poles. To be able to obtain all t -channel fermion poles, we need to propose all the infinite extensions of the higher derivative corrections to (39). Indeed the same idea held here as well, namely the kinetic term of the fermion fields is already fixed in the effective action so it will not receive any correction, likewise the simple fermion pole has no correction. Thus we need to work out an infinite number of higher derivative corrections to the vertex of $V_\alpha(C_{p-1}, \Psi_3, \bar{\Psi})$ as

$$i \frac{(2\pi\alpha') \mu_p}{(p)!} \sum_{n=-1}^{\infty} b_n (\alpha')^{n+1} \text{Tr} \times \left(C_{a_0 \dots a_{p-2}} D^{a_0} \dots D^{a_n} \bar{\Psi} \gamma^b D_{a_0} \dots D_{a_n} \partial_b \Psi \right) \times (\varepsilon^v)^{a_0 \dots a_{p-2}}. \tag{41}$$

By making use of (41), we are able to define all infinite extensions of $V_\alpha(C_{p-1}, \Psi_3, \bar{\Psi})$ to all orders in α' as follows:

$$V_\alpha(C_{p-1}, \Psi_3, \bar{\Psi}) = i \frac{(2\pi\alpha') \mu_p}{(p)!} (\varepsilon^v)^{a_0 \dots a_{p-2}} H_{a_0 \dots a_{p-2}}^b \gamma^b u_2 \times \text{Tr} (\lambda_3 \lambda^\alpha) \sum_{n=-1}^{\infty} b_n (\alpha' k_3 \cdot k)^{n+1}. \tag{42}$$

Two remarks are in order. Basically one has to overlook all the connections inside the definitions of the covariant derivatives and the equations of motion should have been applied to the field theory amplitude to be able to obtain the infinite t -channel fermion poles of the string amplitude.

If we substitute (42) into (37) then we observe that all the infinite fermion poles, either t - or s -channel, of (36) are precisely reproduced.

Hence we have seen that not only the RR $(p - 1)$ -form field proposed all infinite α' corrections to two fermions but also it imposed all order α' corrections to one on-shell gauge and an off-shell scalar field in the world volume of BPS branes in type IIB.

Therefore we conclude that this phenomenon seems to be universal and indeed is useful to determine all the massless

poles of the higher point functions in type IIB superstring theory.

Finally it would be nice to observe whether there are α' higher derivative corrections to four fermions or to two fermion–two gauge field couplings of type IIA [70]; more significantly, to see whether or not this universal conjecture on all order α' corrections of type IIB holds for type IIA.

5 Conclusions

In this paper we have carried out the conformal field theory calculations and obtained the entire $\langle V_C V_{\bar{\psi}} V_{\psi} V_A \rangle$ amplitude in IIB superstring theory. Unlike the $\langle V_C V_{\bar{\psi}} V_{\psi} V_{\phi} \rangle$ correlator, here we just found an infinite number of scalar poles for the $p = n$ case. All infinite t, s -channel fermion poles are also discovered.

We have seen that $V_i^{\alpha}(C_{p+1}, \phi)$ and simple poles do not receive any corrections. Thus infinite $(t + s + u)$ -channel massless scalar poles help us in exploring all order α' corrections to one off-shell scalar, one on-shell gauge field, and two on-shell fermion fields of type IIB. In particular the computations of this paper showed that there are no α' corrections to two fermion–two gauge field couplings of type IIB superstring theory.

As we have clarified, inside the RR vertex operator winding modes are not included in a non-compact space. Hence we come to the fact that one should not apply T-duality to the previous results. For instance in the $\langle V_C V_{\bar{\psi}} V_{\psi} V_{\phi} \rangle$ amplitude in type IIB, we derived all infinite corrections to two fermion–two scalar couplings while in this paper using direct computations we have shown that there are no corrections to two gauge–two fermion fields and more importantly there is not even one single u -channel gauge pole either. Therefore one must apply direct calculations and should follow some prescriptions to the S-matrices in superstring theory.

Our computations are done in such a way that all the propagators have been found by the conformal field theory formalism and we used the doubling trick, however, RR has two sectors with $(\alpha_n, \tilde{\alpha}_n)$ oscillators. It is not clear to us how to deal with $\tilde{\alpha}_n$. Just for completeness, we refer to [72] for further information. Basically one has to use an analytic continuation, which means that the closed string must be regarded just as a composite state of the open strings.

Therefore background fields in the DBI effective action must be some functions of SYM fields. One has to consider all background fields as composite states and eventually all background fields should include Taylor expansions as has been discussed in [11].

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