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Photon and photino as Nambu–Goldstone zero modes in an emergent SUSY QED

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Abstract We argue that supersymmetry with its well-known advantages, such as naturalness, grand unification, and dark matter candidate seems to possess one more attractive feature: it may trigger, through its own spontaneous violation in the visible sector, a dynamical generation of gauge fields as massless Nambu–Goldstone modes during which physical Lorentz invariance itself is ultimately preserved. We consider the supersymmetric QED model extended by an arbitrary polynomial potential of a massive vector superfield that breaks gauge invariance in the SUSY invariant phase. However, the requirement of vacuum stability in such a class of models renders both supersymmetry and Lorentz invariance spontaneously broken. As a consequence, the massless photino and photon appear as the corresponding Nambu–Goldstone zero modes in the emergent SUSY QED, and also a special gauge invariance is simultaneously generated. Due to this invariance all observable relativistically non-invariant effects appear to be completely canceled out among themselves and physical Lorentz invariance is recovered. Nevertheless, such theories may have an inevitable observational evidence in terms of the goldstino–photino like state present in the low-energy particle spectrum. Its study is of special interest for this class of SUSY models, which, apart from some indications of the emergence nature of QED and the Standard Model, may appreciably extend the scope of SUSY breaking physics being actively studied in recent years.

1 Introduction and overview

It has long been believed that spontaneous Lorentz invariance violation (SLIV) may lead to the emergence of massless Nambu–Goldstone (NG) zero modes [1,2], which are identified with photons and other gauge fields appearing in the Standard Model. This old idea [3–5], supported by a close analogy with the dynamical origin of massless particle excitations for spontaneously broken internal symmetries, has gained new impetus in recent years. On the other hand, besides its generic implication for the possible origin of physical gauge fields [6–12] in a conventional quantum field theory (QFT) framework, there are many different contexts in the literature where Lorentz violation may stem itself from string theory [13,14], quantum gravity [15] or any unspecified dynamics at an ultraviolet scale perhaps related to the Planck scale [16–22]. Though we are mainly focused on the spontaneous Lorentz violation in QFT, particularly in QED and the Standard Model, we give below some brief comments on other approaches and we make clearer the aims and results of the present work.

1.1 Vector NG bosons in gauge theories. Inactive SLIV

When speaking about SLIV, one important thing to notice is that, in contrast to the spontaneous violation of internal symmetries, it seems not to necessarily imply a physical breakdown of Lorentz invariance. Rather, when appearing in a gauge theory framework, this may ultimately result in a non-covariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory. In substance, the SLIV ansatz, due to which the vector field develops a vacuum expectation value (VEV)

$$\langle A_\mu(x)\rangle = n_\mu M \tag{1}$$

(where n_μ is a properly oriented unit Lorentz vector, $n^2 = n_\mu n^\mu = \pm 1$, while M is the proposed SLIV scale) may itself be treated as a pure gauge transformation with a gauge function linear in the coordinates, $\omega(x) = n_\mu x^\mu M$. From this viewpoint gauge invariance in QED leads to the conversion of SLIV into gauge degrees of freedom of the massless photon emerged.

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A good example for such a kind of SLIV, which we call the "inactive" SLIV hereafter, is provided by the nonlinearly realized Lorentz symmetry for the underlying vector field $A_\mu(x)$ through the length-fixing constraint

$$A_\mu A^\mu = n^2 M^2. \tag{2}$$

This constraint in the gauge invariant QED framework was first studied by Nambu a long ago [23], and in more detail in recent years [24–28]. The constraint (2) is in fact very similar to the constraint appearing in the nonlinear σ-model for pions [29], $\sigma^2 + \pi^2 = f_\pi^2$, where f_π is the pion decay constant. Rather than imposing it by postulate, the constraint (2) may be implemented into the standard QED Lagrangian extended by the invariant Lagrange multiplier term

$$\mathcal{L} = L_{\mathrm{QED}} - \frac{\lambda}{2}\left(A_\mu A^\mu - n^2 M^2\right) \tag{3}$$

provided that the initial values for all fields (and their momenta) involved are chosen so as to restrict the phase space to values with a vanishing multiplier function $\lambda(x)$, $\lambda = 0$. Otherwise, as was shown in [30] (see also [27]), it might be problematic to have a ghost-free QED model with a positive Hamiltonian.[1]

One way or the other, the constraint (2) means in essence that the vector field A_μ develops the VEV (1) and Lorentz symmetry $SO(1,3)$ breaks down to $SO(3)$ or $SO(1,2)$, depending on whether the unit vector n_μ is time-like ($n^2 > 0$) or space-like ($n^2 < 0$). The point, however, is that, in sharp contrast to the nonlinear σ-model for pions, the nonlinear QED theory, due to gauge invariance in the starting Lagrangian L_{QED}, leaves physical Lorentz invariance intact. Indeed, the nonlinear QED contains a plethora of Lorentz and CPT violating couplings when it is expressed in terms of the pure vector NG boson modes (a_μ) associated with a physical photon,

$$A_\mu = a_\mu + n_\mu (M^2 - n^2 a^2)^{\frac{1}{2}}, \quad n_\mu a_\mu = 0 \quad (a^2 \equiv a_\mu a^\mu), \tag{4}$$

including the effective Higgs mode given by the second term in (4) properly expanded in a power series of a^2. However, the contributions of all these couplings to physical processes completely cancel out among themselves, as was shown in the tree [23] and one-loop approximations [24]. Actually, the nonlinear constraint (2) implemented as a supplementary condition can be interpreted in essence as a possible gauge choice for the starting vector field A_μ. Meanwhile the S-matrix remains unaltered under such a gauge convention unless gauge invariance in the theory turns out to be really broken (see the next subsection) rather than merely being restricted by gauge condition (2). A later similar result concerning the inactive SLIV in gauge theories was also confirmed for spontaneously broken massive QED [25], non-Abelian theories [26], and tensor field gravity [28].

Remarkably enough, the nonlinear QED model (3) may be considered in some sense as originating from a conventional QED Lagrangian extended by vector field potential energy terms,

$$\mathcal{L}' = L_{\mathrm{QED}} - \frac{\boldsymbol{\lambda}}{4}\left(A_\mu A^\mu - n^2 M^2\right)^2 \tag{5}$$

(where $\boldsymbol{\lambda}$ is a coupling constant) rather than by the Lagrange multiplier term. This is the simplest example of a theory sometimes referred to as the "bumblebee" model (see [11,12] and references therein) where physical Lorentz symmetry could in principle be spontaneously broken due to the presence of an active Higgs mode in the model. On the other hand, the Lagrangian (5) taken in the limit $\boldsymbol{\lambda} \to \infty$ can formally be regarded as the nonlinear QED. Actually, both models are physically equivalent in the infrared energy domain, where the Higgs mode is considered infinitely massive. However, as was argued in [30], a bumblebee-like model appears generally unstable; its Hamiltonian is not bounded from below unless the phase space sector is not limited by the nonlinear vector field constraint $A_\mu A^\mu = n^2 M^2$ (2). With this condition imposed, the massive Higgs mode never appears, the Hamiltonian is positive, and the model is physically equivalent to the constraint-based nonlinear QED (3) with the inactive SLIV, which does not lead to physical Lorentz violation.[2]

To summarize, we have considered above the standard QED with vector field constraint (2) being implemented into the Lagrangian through the Lagrange multiplier term (3). In crucial contrast to internal symmetry breaking (say, the breaking of a chiral $SU(2) \times SU(2)$ symmetry in the nonlinear σ-model for pions), SLIV caused by a similar σ-model type vector field constraint, (2), does not lead to physical Lorentz violation. Indeed, though SLIV induces the vector Goldstone-like states (4), all observable SLIV effects appear to be completely canceled out among themselves due to the generic gauge invariance of QED. We call it the inactive SLIV in the sense that one may have Goldstone-like states in a theory but may have not a non-zero symmetry breaking effect. This is a somewhat new and unusual situation that just happens with SLIV in gauge invariant theories (and never in an internal symmetry breaking case). More precisely there

[1] Note that this solution with the basic Lagrangian multiplier field $\lambda(x)$ vanishing can technically be realized by introducing some additional Lagrange multiplier term of the type $\xi\lambda^2$, where $\xi(x)$ is a new multiplier field. One can now easily confirm that a variation of the modified Lagrangian $\mathcal{L} + \xi\lambda^2$ with respect to the ξ field leads to the condition $\lambda = 0$, whereas a variation with respect to the basic multiplier field λ preserves the vector field constraint (2).

[2] Apart from its generic instability, the "bumblebee" model, as we will see shortly, cannot be technically realized in a SUSY context, whereas the nonlinear QED model successfully matches supersymmetry.

are, in essence, two different aspects regarding the inactive SLIV (different, though related to each other). The first is the generation of Goldstone modes which inevitably happens once the nonlinear σ-model type constraint (2) is put on the vector field. The second is that gauge invariance, even being restricted by this constraint (interpreted as a gauge condition), provides a cancelation mechanism for physical Lorentz violation. As a consequence, emergent gauge theories induced by the inactive SLIV mechanism are in fact indistinguishable from conventional gauge theories. Their emergent nature can only be seen when a gauge condition is taken to be the vector field length-fixing constraint (2). Any other gauge, e.g. the Coulomb gauge, is not in line with the emergent picture, since it explicitly breaks Lorentz invariance. As to the observational evidence in favor of emergent theories, the only way for SLIV to be activated may appear if gauge invariance in these theories turns out to be broken in an explicit rather than spontaneous way. As a result, the SLIV cancelation mechanism does not work any longer and one inevitably ends up with physical Lorentz violation.

1.2 Activating SLIV by gauge symmetry breaking

Looking for some appropriate examples of physical Lorentz violation in a QFT framework one necessarily comes across the problem of proper suppression of gauge non-invariant high-dimension couplings where such a violation can in principle occur. Remarkably enough, for QED type theories with the supplementary vector field constraint (2) gauge symmetry breaking naturally appears only for five- and higher-dimensional couplings. Indeed, all dimension-four couplings are generically gauge invariant, if the vector field kinetic term has a standard $F_{\mu\nu}F^{\mu\nu}$ and, apart from relativistic invariance, the restrictions related to the conservation of parity, charge-conjugation symmetry, and fermion number conservation are generally imposed on the theory [31,32]. With these restrictions taken, one can easily confirm that all possible dimension-five couplings are also combined by themselves in some would-be gauge invariant form provided that the vector field is constrained by the SLIV condition (2). Indeed, for charged matter fermions interacting with vector field such couplings generally amount to

$$L_{\dim 5}=\frac{1}{\mathcal{M}}\check{D}^{*}_{\mu}\overline{\psi}\cdot\check{D}^{\mu}\psi+\frac{G}{\mathcal{M}}A_{\mu}A^{\mu}\overline{\psi}\psi,\quad A_{\mu}A^{\mu}=n^{2}M^{2}. \tag{6}$$

Such couplings could presumably become significant at an ultraviolet scale $\mathcal{M}$, probably close to the Planck scale M_P. They, besides covariant derivative terms, also include an independent "sea-gull" fermion–vector field term with the coupling constant G being in general of the order 1. The main point regarding the Lagrangian (6) is that, while it is gauge invariant in itself, the coupling constant $\check{e}$ in the covariant derivative $\check{D}^{\mu}=\partial^{\mu}+i\check{e}A^{\mu}$ differs in general from the coupling e in the covariant derivative $D^{\mu}=\partial^{\mu}+ieA^{\mu}$ in the standard Dirac Lagrangian (3)

$$L_{\rm QED}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\overline{\psi}(i\gamma_{\mu}D^{\mu}-m)\psi\ . \tag{7}$$

Therefore, gauge invariance is no longer preserved in the total Lagrangian $L_{\rm QED}+L_{\dim 5}$. It is worth noting that, though the high-dimension Lagrangian part $L_{\dim 5}$ (6) usually only gives some small corrections to a conventional QED Lagrangian (7), the situation may drastically change when the vector field A_{μ} develops a VEV and SLIV occurs.

Actually, putting the SLIV parameterization (4) into the basic QED Lagrangian (7) one obtains the truly emergent model for QED being essentially nonlinear in the vector Goldstone modes a_{μ} associated with photons. This model contains, among other terms, the inappropriately large (while false; see below) Lorentz-violating fermion bilinear $-eM\overline{\psi}(n_{\mu}\gamma^{\mu})\psi$. This term appears when the effective Higgs mode expansion in Goldstone modes a_{μ} [as is given in the parametrization (4)] is applied to the fermion current interaction term $-e\overline{\psi}\gamma_{\mu}A^{\mu}\psi$ in the QED Lagrangian (7). However, due to local invariance this bilinear term can be gauged away by making an appropriate redefinition of the fermion field $\psi\rightarrow e^{-ie\omega(x)}\psi$ with a gauge function $\omega(x)$ linear in coordinates, $\omega(x)=(n_{\mu}x^{\mu})M$. Meanwhile, the dimension-five Lagrangian $L_{\dim 5}$ (6) is substantially changed under this redefinition, which significantly modifies the fermion bilinear terms

$$L_{\overline{\psi}\psi}=\overline{\psi}(i\gamma_{\mu}\partial^{\mu}-m)\psi-i\Delta e\frac{M}{\mathcal{M}}n_{\mu}\overline{\psi}\overleftrightarrow{\partial^{\mu}}\psi$$
$$+[G+(\Delta e)^{2}n^{2}]\frac{M^{2}}{\mathcal{M}}\overline{\psi}\psi\ , \tag{8}$$

where we retained the notation ψ for the redefined fermion field and denoted, as usually, $\overline{\psi}\overleftrightarrow{\partial^{\mu}}\psi=\overline{\psi}(\partial^{\mu}\psi)-(\partial^{\mu}\overline{\psi})\psi$. Note that the extra fermion derivative terms given in (8) are produced just due to the gauge invariance breaking that is determined by the electromagnetic charge difference $\Delta e=\check{e}-e$ in the total Lagrangian $L_{\rm QED}+L_{\dim 5}$. As a result, there appears the entirely new, SLIV inspired, dispersion relation for a charged fermion (taken with 4-momentum p_{μ}) of the type

$$p_{\mu}^{2}\cong[m_{f}+2\delta(p_{\mu}n^{\mu}/n^{2})]^{2},\quad m_{f}=\left(m-G\frac{M^{2}}{\mathcal{M}}\right)-\delta^{2}n^{2}\mathcal{M}, \tag{9}$$

given to an accuracy of $O(m_{\rm f}^{2}/\mathcal{M}^{2})$ with a properly modified total fermion mass $m_{\rm f}$. Here δ stands for the small characteristic, positive or negative, parameter $\delta=(\Delta e)M/\mathcal{M}$ of physical Lorentz violation that reflects the joint effect as is given, on the one hand, by the SLIV scale M and, on the other hand, by the charge difference Δe being a measure of an internal gauge non-invariance. Notably, the spacetime in itself still possesses Lorentz invariance, however, fermions with SLIV contributing into their total mass m_f (9) propagate and interact in it in the Lorentz non-covariant way. At the same time, the photon dispersion relation is still retained in the order $1/\mathcal{M}$ considered.

So, we have shown in the above that SLIV caused by the vector field VEV (1), while being superficial in gauge invariant theory, becomes physically significant for some high value of the SLIV scale M close to the scale $\mathcal{M}$, which is proposed to be located near the Planck scale M_P. This may happen even at relatively low energies provided the gauge non-invariance caused by high-dimension couplings of the matter and vector fields is not vanishingly small. This leads, as was demonstrated in [31,32], through special dispersion relations appearing for matter, charged fermions, to a new class of phenomena, which could be of distinctive observational interest in particle physics and astrophysics. They include a significant change in the GZK cutoff for UHE cosmic-ray nucleons, stability of high-energy pions and W bosons, modification of nucleon beta decays, and some others just in the presently accessible energy area in cosmic-ray physics.

However, though one could speculate about some generically broken or partial gauge symmetry in a QFT framework [31,32], this seems to be too high a price for an actual Lorentz violation which may stem from SLIV. What is more: is there really any strong theoretical reason left for Lorentz invariance to be physically broken, if emergent gauge fields are anyway generated through the "safe" inactive SLIV models which recover conventional Lorentz invariance?

1.3 Direct Lorentz non-invariant extensions of SM and gravity

Nevertheless, it must not be ruled out that physical Lorentz invariance might be explicitly, rather than spontaneously, broken at high energies. This has attracted considerable attention in recent years as an interesting phenomenological possibility appearing in direct Lorentz non-invariant extensions of SM [16–22]. They are generically regarded as originating in a more fundamental theory at some large scale, probably related to the Planck scale M_P. These extensions are in a certain measure motivated [13,14] by string theory, according to which an explicit (from a QFT point of view) Lorentz violation might be in essence a spontaneous Lorentz violation related to hypothetical tensor-valued fields acquiring non-zero VEVs in some non-perturbative vacuum. These VEVs appear effectively as a set of external background constants so that interactions with these coefficients have preferred spacetime directions in an effective QFT framework. The full SM extension (SME) [18–20] is then defined as the effective gauge invariant field theory obtained when all such Lorentz-violating vector and tensor field backgrounds are contracted term by term with SM (and gravitational) fields. However, without a completely viable string theory, it is not possible to assign definite numerical values to these coefficients. Moreover, not to have disastrous consequences (especially when these coefficients are contracted with non-conserved currents) one also has to additionally propose that observable violating effects in the low-energy theory with a laboratory scale m should be suppressed by some power of the ratio m/M_P, dependent on the dimension of the Lorentz breaking couplings. Therefore, one has in this sense a pure phenomenological approach treating the above arbitrary coefficients as quantities to be bounded in experiments as if they would simply appear due to explicit Lorentz violation. Actually, in sharp contrast to the above formulated SLIV in a pure QFT framework, there is nothing in the SME itself that requires that these Lorentz-violation coefficients emerge due to a process of a spontaneous Lorentz violation. Indeed, neither the corresponding massless vector (tensor) NG bosons are required to be generated, nor do these bosons have to be associated with photons or any other gauge fields of SM.

Apart from Lorentz violation in the Standard Model, one can generally think that the vacuum in quantum gravity may also determine a preferred rest frame at the microscopic level. If such a frame exists, it must be very much hidden in low-energy physics since, as was mentioned above, numerous observations severely limit the possibility of Lorentz-violating effects for the SM fields [16–22]. However, the constraints on Lorentz violation in the gravitational sector are generally far weaker. This allows one to introduce a pure gravitational Lorentz violation having no significant impact on the SM physics. An elegant way, close in spirit to our SLIV model (3, 4), seems to appear in the so-called Einstein-aether theory [15]. This is in essence a general covariant theory in which local Lorentz invariance is broken by some vector "aether" field u_μ defining the preferred frame. This field is similar to our constrained vector field A_μ, apart from that this field is taken to be unit, $u_\mu u^\mu = 1$. It spontaneously breaks Lorentz symmetry down to a rotation subgroup, just like as our constrained vector field A_μ does it for a time-like Lorentz violation. So, they both give a nonlinear realization of Lorentz symmetry thus leading to its spontaneous violation and inducing the corresponding Goldstone-like modes. The crucial difference is that, while modes related to the vector field A_μ are collected into the physical photon, modes associated with the unit vector field u_μ (one helicity-0 and two helicity-1 modes) exist by them own appearing in some effective SM and gravitational couplings. Some of them might disappear being absorbed by the corresponding spin-connection fields related to local Lorentz symmetry in the Einstein-aether theory. In any case, while the aether field u_μ can significantly change the dispersion relations of fields involved, thus leading to many gravitational and cosmological consequences of preferred frame effects, it certainly cannot be a physical gauge field candidate (say, the photon in QED).

1.4 Lorentz violation and supersymmetry. The present paper

There have been a few active attempts [33,34] over the last decade to construct Lorentz-violating operators for matter and gauge fields in the supersymmetric standard model through their interactions with external vector and tensor field backgrounds. These backgrounds, according to the SME approach [18–20] discussed above, are generated by some Lorentz-violating dynamics at an ultraviolet scale of order the Planck scale. As some advantages over the ordinary SME, it was shown that in the supersymmetric standard model the lowest possible dimension for such operators is five, just as we had above in the high-dimensional SLIV case (6). Therefore, they are suppressed by at least one power of an ultraviolet energy scale, providing a possible explanation for the smallness of Lorentz violation and its stability against radiative corrections. All possible dimension-five and -six Lorentz-violating operators in the SUSY QED were classified [34], their properties at the quantum level analyzed, and their observational consequences in this theory described. These operators, as was confirmed, do not induce destabilizing D-terms, gauge anomaly, and the Chern–Simons term for the photons. Dimension-five Lorentz-violating operators were shown to be constrained by low-energy precision measurements at 10^{-10}–10^{-5} level in units of the inverse Planck scale, while the Planck-scale suppressed dimension-six operators are allowed by the observational data.

Also, the supersymmetric extension has been constructed of the Einstein-aether theory [35] discussed above. It has been found that the dynamics of the super-aether is somewhat richer than of its non-SUSY counterpart. In particular, the model possesses a family of inequivalent vacua exhibiting different symmetry breaking patterns while remaining stable and ghost free. Interestingly enough, as long as the aether VEV preserves spatial supersymmetry (SUSY algebra without boosts), the Lorentz breaking does not propagate into the SM sector at the renormalizable level. The eventual breaking of SUSY, which must be incorporated in any realistic model, is unrelated to the dynamics of the aether. It is assumed to come from a different source, characterized by a lower energy scale. However, in spite of its own merits, a significant final step which would lead to natural accommodation of this super-aether model into the supergravity framework has not yet been done.

In contrast, we strictly focus here on a spontaneous Lorentz violation in an actual gauge QFT framework related to the Standard Model rather than in an effective low-energy theory with some hypothetical remnants in terms of external tensor-valued backgrounds originating somewhere around the Planck scale. In essence, we try to extend to their supersymmetric analogs the emergent gauge theories with SLIV and the associated emergence of gauge bosons as massless vector Nambu–Goldstone modes studied earlier [6–12] (see also [24–28]). Generally speaking, it may turn out that SLIV is not the only reason why massless photons could dynamically appear, if spacetime symmetry is further enlarged. In this connection, special interest may be in supersymmetry, as was recently argued in [36]. Actually, the situation is changed remarkably in the SUSY inspired emergent models which, in contrast to non-SUSY theories, could naturally have some clear observational evidence. Indeed, as we discussed above (Sect. 1.2), ordinary emergent theories admit some experimental verification only if gauge invariance is properly broken being caused by some high-dimension couplings. Their SUSY counterparts, and primarily emergent SUSY QED, generically appear with supersymmetry being spontaneously broken in a visible sector to ensure stability of the theory. Therefore, the verification is now related to the inevitable emergence of a goldstino-like photino state in the SUSY particle spectrum at low energies, while physical Lorentz invariance is still left intact.[3] In this sense, a generic source for the massless photon to appear may be spontaneously broken supersymmetry rather than physically manifest spontaneous Lorentz violation.

To see how such a scenario may work, we consider the supersymmetric QED model extended by an arbitrary polynomial potential of a massive vector superfield that induces the spontaneous SUSY violation in the visible sector. As a consequence, a massless photino emerges as the fermion NG mode in the broken SUSY phase, and a photon as a photino companion, also massless in the tree approximation (Sect. 2). However, the requirement of vacuum stability in such a class of models renders Lorentz invariance spontaneously broken as well. As a consequence, the massless photon has now appeared as the vector NG mode, and also a special gauge invariance is simultaneously generated in an emergent SUSY QED. This invariance is only restricted by the supplemented vector field constraint being invariant under supergauge transformations (Sect. 3). Due to this invariance all observable SLIV effects appear to be completely canceled out among themselves, and physical Lorentz invariance is restored. Meanwhile, the photino being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector largely appears in the form of a light pseudo-goldstino, whose physics seems to be of

[3] Of course, physical Lorentz violation will also appear if one admits some gauge non-invariance in the emergent SUSY theory as well. This may happen, for example, through high-dimension couplings being supersymmetric analogs of the couplings (6).

special observational interest (Sect. 4). Finally, we conclude (Sect. 5).

2 Extended supersymmetric QED

We start by considering a conventional SUSY QED extended by an arbitrary polynomial potential of a general vector superfield $V(x,\theta,\overline{\theta})$, which in the standard parametrization [37–39] has the form

$$V(x,\theta,\overline{\theta}) = C(x) + i\theta\chi - i\overline{\theta}\overline{\chi} + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta}\overline{\theta}S^{*} - \theta\sigma^{\mu}\overline{\theta}A_{\mu} + i\theta\theta\overline{\theta}\overline{\lambda'} - i\overline{\theta}\overline{\theta}\theta\lambda' + \frac{1}{2}\theta\theta\overline{\theta}\overline{\theta}D', \tag{10}$$

where its vector field component A_{μ} is usually associated with a photon. Note that, apart from the conventional photino field λ and the auxiliary D field, the superfield (10) contains in general the additional degrees of freedom in terms of the dynamical C and χ fields and non-dynamical complex scalar field S (we have used the brief notations, $\lambda' = \lambda + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\overline{\chi}$ and $D' = D + \frac{1}{2}\Box C$ with $\sigma^{\mu} = (1, \overrightarrow{\sigma})$ and $\overline{\sigma}^{\mu} = (1, -\overrightarrow{\sigma})$). The corresponding SUSY invariant Lagrangian may be written as

$$\mathcal{L} = L_{\mathrm{SQED}} + \sum_{n=1} b_n V^n|_D \tag{11}$$

where the terms in this sum (b_n are some constants) for the vector superfield (10) are given through the polynomial D-term $V^n|_D$ expansion into the component fields. It can readily be checked that the first term in this expansion appears to be the known Fayet–Iliopoulos D-term, while the other terms only contain bilinear, trilinear, and quadrilinear combination of the superfield components A_{μ}, S, λ, and χ, respectively.[4] Actually, there appear higher-degree terms only for the scalar field component $C(x)$. Expressing them all in terms of the C field polynomial

$$P(C) = \sum_{n=1} \frac{n}{2} b_n C^{n-1}(x) \tag{12}$$

and its first three derivatives with respect to the C field

$$P' \equiv \frac{\partial P}{\partial C}, \quad P'' \equiv \frac{\partial^2 P}{\partial C^2}, \quad P''' \equiv \frac{\partial^3 P}{\partial C^3}, \tag{13}$$

[4] Note that all terms in the sum in (11) except the Fayet–Iliopoulos D-term explicitly break gauge invariance which is then recovered in the SUSY broken phase (see below). For simplicity, we could restrict ourselves to the third degree superfield polynomial potential in the Lagrangian $\mathcal{L}$ (11) to eventually have a theory with dimensionless coupling constants in the interactions of the component fields. However, for the sake of completeness, we will proceed with a general superfield potential.

one has for the whole Lagrangian $\mathcal{L}$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \frac{1}{2}D^2 + P\left(D + \frac{1}{2}\Box C\right) + P'\left(\frac{1}{2}SS^{*} - \chi\lambda' - \overline{\chi}\overline{\lambda'} - \frac{1}{2}A_{\mu}A^{\mu}\right) + \frac{1}{2}P'' \times \left(\frac{i}{2}\overline{\chi}\overline{\chi}S - \frac{i}{2}\chi\chi S^{*} - \chi\sigma^{\mu}\overline{\chi}A_{\mu}\right) + \frac{1}{8}P'''(\chi\chi\overline{\chi}\overline{\chi}), \tag{14}$$

where, for more clarity, we still omitted matter superfields in the model reserving them for Sect. 4. One can see that the superfield component fields C and χ become dynamical due to the potential terms in (14) rather than from the properly constructed supersymmetric field strengths, as appear for the vector field A_{μ} and its gaugino companion λ. A very remarkable point is that the vector field A_{μ} may only appear with bilinear mass terms in the polynomially extended Lagrangian (14). Hence it follows that the "bumblebee" type model mentioned above (5) with nontrivial vector field potential containing both a bilinear mass term and a quadrilinear stabilizing term can in no way be realized in a SUSY context. Meanwhile, the nonlinear QED model, as will become clear below, successfully matches supersymmetry.

Varying the Lagrangian $\mathcal{L}$ with respect to the D field we obtain

$$D = -P(C), \tag{15}$$

which finally gives the standard potential energy for the field system considered

$$U(C) = \frac{1}{2}P^2 \tag{16}$$

provided that the other superfield components do not develop VEVs. The potential (16) may lead to the spontaneous SUSY breaking in the visible sector if the polynomial P (12) has no real roots, while its first derivative has,

$$P \neq 0, \quad P' = 0. \tag{17}$$

This requires $P(C)$ to be an even degree polynomial with properly chosen coefficients b_n in (12), which will force its derivative P' to have at least one root, $C = C_0$, in which the potential (16) is minimized and supersymmetry is spontaneously broken. As an immediate consequence, one can readily see from the Lagrangian $\mathcal{L}$ (14) that a massless photino λ being a Goldstone fermion in the broken SUSY phase make all the other component fields in the superfield $V(x,\theta,\overline{\theta})$ including the photon also massless. However, the question then arises whether this masslessness of the photon will be stable against radiative corrections since gauge invariance is explicitly broken in the Lagrangian (14). We show below that it could be the case if the vector superfield $V(x, \theta, \overline{\theta})$ would appear to be properly constrained.

3 Constrained vector superfield

3.1 Instability of superfield polynomial potential

Let us first analyze the possible vacuum configurations for the superfield components in the polynomially extended QED case taken above. In general, besides the "standard" potential energy expression (16) determined solely by the scalar field component $C(x)$ of the vector superfield (10), one also has to consider other field component contributions into the potential energy. A possible extension of the potential energy (16) seems to appear only due to the pure bosonic field contributions, namely due to the couplings of the vector and auxiliary scalar fields, A_μ and S, in (14)

$$\mathcal{U} = \frac{1}{2}P^2 + \frac{1}{2}P'(A_\mu A^\mu - SS^*), \tag{18}$$

rather than due to the potential terms containing the superfield fermionic components.[5] It can immediately be seen that these new couplings in (18) can make the potential unstable since the vector and scalar fields mentioned may in general develop any arbitrary VEVs. This happens, as emphasized above, due the fact that their bilinear term contributions are not properly compensated by appropriate four-linear field terms, which are generically absent in a SUSY theory context.

For more details we consider the extremum conditions for the entire potential (18) with respect to all fields involved: C, A_μ, and S. They are given by the appropriate first partial derivative equations

$$\begin{aligned} &\mathcal{U}'_C = PP' + \tfrac{1}{2}P''(A_\mu A^\mu - SS^*) = 0, \\ &\mathcal{U}'_{A_\mu} = P'A^\mu = 0, \ \ \mathcal{U}'_S = -P'S^* = 0, \end{aligned} \tag{19}$$

where all the VEVs are denoted by the corresponding field symbols (supplied below with the lower index 0). One can see that there can occur a local minimum for the potential (18) with the unbroken SUSY solution.[6]

$$C = C_0, \ \ P(C_0) = 0, \ P'(C_0) \neq 0\,; \ \ A_{\mu 0} = 0, \ \ S_0 = 0 \tag{20}$$

with the vanishing potential energy

$$\mathcal{U}^s_{\min} = 0, \tag{21}$$

provided that the polynomial P (12) has some real root $C = C_0$. Otherwise, a local minimum with the broken SUSY solution can occur for some other C field value (though denoted by the same letter C_0)

$$\begin{aligned} &C = C_0, \quad P(C_0) \neq 0, \quad P'(C_0) = 0; \\ &A_{\mu 0} \neq 0, \quad S_0 \neq 0, \quad A_{\mu 0}A^\mu_0 - S_0 S^*_0 = 0. \end{aligned} \tag{22}$$

In this case one has the non-zero potential energy

$$\mathcal{U}^{as}_{\min} = \frac{1}{2}[P(C_0)]^2, \tag{23}$$

as directly follows from the extremum equations (19) and potential energy expression (18).

However, as shown by the standard second partial derivative test, the fact is that the local minima mentioned above are minima with respect to the C field VEV (C_0) only. Actually, for all three fields VEVs, the potential (18) has indeed saddle points with "coordinates" indicated in (20) and (22), respectively. For testing convenience this potential can be rewritten in the form

$$\begin{aligned} \mathcal{U} \quad &= \tfrac{1}{2}P^2 + \tfrac{1}{2}P' g^{\Theta\Theta'} B_\Theta B_{\Theta'}, \\ g^{\Theta\Theta'} &= diag\ (1, -1, -1, -1, -1, -1) \end{aligned} \tag{24}$$

with only two variable fields C and B_Θ, where the new field B_Θ unifies the A_μ and S field components, $B_\Theta = (A_\mu, S_a)$ ($\Theta = \mu, a;\ \mu = 0, 1, 2, 3;\ a = 1, 2$).[7] The complex S field is now taken in a real basis, $S_1 = (S + S^*)/\sqrt{2}$ and $S_2 = (S - S^*)/i\sqrt{2}$, so that the "vector" B_Θ field has one time and five space components. As a result, one finally comes to the following Hessian 7×7 matrix [being in fact the second-order partial derivatives matrix taken in the extremum point $(C_0, A_{\mu 0}, S_0)$ (20)]:

$$\begin{aligned} &H(\mathcal{U}^s) = \begin{bmatrix} [P'(C_0)]^2 & 0 \\ 0 & P'(C_0)g^{\Theta\Theta'} \end{bmatrix}, \\ &|H(\mathcal{U}^s)| = -[P'(C_0)]^8\,. \end{aligned} \tag{25}$$

This matrix clearly has the negative determinant $|H(\mathcal{U}^s)|$, as is indicated above, which confirms that the potential definitely has a saddle point for the solution (20). This means the VEVs of the A_μ and S fields can take in fact any arbitrary value making the potential (18, 24) to be unbounded from below in the unbroken SUSY case that is certainly inaccessible.

One might think that in the broken SUSY case the situation would be better since due to the conditions (22) the B_Θ term completely disappears from the potential $\mathcal{U}$ (18, 24) in the ground state. Unfortunately, the direct second partial

[5] Actually, this restriction is not essential for what follows and is taken just for simplicity. Generally, the fermion bilinears involved could also develop VEVs.

[6] Hereafter by $P(C_0)$ and $P'(C_0)$ are meant the C field polynomial P (12) and its functional derivative P' (13) taken in the potential extremum point C_0.

[7] Interestingly, the B_Θ term in the potential (24) possesses the accidental $SO(1, 5)$ symmetry. This symmetry, though it is not shared by kinetic terms, appears in fact to be stable under radiative corrections since the S field is non-dynamical and, therefore, can always be properly arranged.

derivative test in this case is inconclusive, since the determinant of the corresponding Hessian 7 × 7 matrix appears to vanish,

$$H(\mathcal{U}^{as}) = \begin{bmatrix} P(C_0)P''(C_0) & P''(C_0)g^{\Theta\Theta'}B_{\Theta'} \\ & P''(C_0)g^{\Theta\Theta'}B_{\Theta'}0 \end{bmatrix}, \\ |H(\mathcal{U}^{as})| = 0\,. \tag{26}$$

Nevertheless, since in general the B_Θ term can take both positive and negative values in small neighborhoods around the vacuum point $(C_0, A_{\mu 0}, S_0)$ where the conditions (22) are satisfied, this point also turns out to be a saddle point. Thus, the potential $\mathcal{U}$ (18, 24) appears generically unstable both in the SUSY invariant and the SUSY broken phase.

3.2 Stabilization of vacuum by constraining vector superfield

The only possible way to stabilize the ground states (20) and (22) seems to seek the proper constraints on the superfield component fields (C, A_μ, S) themselves rather than on their expectation values. Indeed, if such (potential bounding) constraints are physically realizable, the vacua (20) and (22) will be automatically stabilized.

In a SUSY context a constraint can only be put on the entire vector superfield $V(x, \theta, \overline{\theta})$ (10), rather than individually on its field components. Actually, we can constrain our vector superfield $V(x, \theta, \overline{\theta})$ by analogy with the constrained vector field in the nonlinear QED model [see (3)]. This will be done again through some invariant Lagrange multiplier coupling simply adding its D term to the above Lagrangian (11, 14)

$$\mathcal{L}_{\text{tot}} = \mathcal{L} + \frac{1}{2}\Lambda(V - C_0)^2|_D, \tag{27}$$

where $\Lambda(x, \theta, \overline{\theta})$ is some auxiliary vector superfield, while C_0 is the constant background value of the C field for which the potential U (16) vanishes as is required for the supersymmetric minimum or has some non-zero value corresponding to the SUSY breaking minimum (17) in the visible sector. We will consider both cases simultaneously using the same notation C_0 for either of the potential minimizing the values of the C field.

Note first of all, that the Lagrange multiplier term in (27) has in fact the simplest possible form that leads to some nontrivial constrained superfield $V(x, \theta, \overline{\theta})$. The alternative minimal forms, such as the bilinear form $\Lambda(V - C_0)$ or trilinear one $\Lambda(V^2 - C_0^2)$, appear to be too restrictive. One can easily confirm that they eliminate most component fields in the superfield $V(x, \theta, \overline{\theta})$ including the physical photon and photino fields, which is definitely inadmissible. As to the appropriate non-minimal high linear multiplier forms, they basically lead to the same consequences as follow from the minimal multiplier term taken in the total Lagrangian (27).

Writing down its invariant D term through the component fields, one finds

$$\begin{aligned} &\Lambda(V - C_0)^2|_D \\ &= C_\Lambda \left[\widetilde{C}D' + \left(\frac{1}{2}SS^* - \chi\lambda' - \overline{\chi}\overline{\lambda'} - \frac{1}{2}A_\mu A^\mu\right)\right] \\ &\quad + \chi_\Lambda\left[2\widetilde{C}\lambda' + i(\chi S^* + i\sigma^\mu\overline{\chi}A_\mu)\right] \\ &\quad + \overline{\chi}_\Lambda[2\widetilde{C}\overline{\lambda'} - i(\overline{\chi}S - i\chi\sigma^\mu A_\mu)] \\ &\quad + \frac{1}{2}S_\Lambda\left(\widetilde{C}S^* + \frac{i}{2}\overline{\chi\chi}\right) + \frac{1}{2}S_\Lambda^*\left(\widetilde{C}S - \frac{i}{2}\chi\chi\right) \\ &\quad + 2A_\Lambda^\mu(\widetilde{C}A_\mu - \chi\sigma_\mu\overline{\chi}) + 2\lambda'_\Lambda(\widetilde{C}\chi) + 2\overline{\lambda}'_\Lambda(\widetilde{C}\overline{\chi}) \\ &\quad + \frac{1}{2}D'_\Lambda\widetilde{C}^2 \end{aligned} \tag{28}$$

where

$$C_\Lambda,\ \chi_\Lambda,\ S_\Lambda,\ A_\Lambda^\mu,\ \lambda'_\Lambda = \lambda_\Lambda + \frac{i}{2}\sigma^\mu\partial_\mu\overline{\chi}_\Lambda, \\ D'_\Lambda = D_\Lambda + \frac{1}{2}\Box C_\Lambda \tag{29}$$

are the component fields of the Lagrange multiplier superfield $\Lambda(x, \theta, \overline{\theta})$ in the standard parametrization (10) and $\widetilde{C}$ stands for the difference $C(x) - C_0$. Varying the Lagrangian (27) with respect to these fields and properly combining their equations of motion

$$\frac{\partial\mathcal{L}_{\text{tot}}}{\partial\left(C_\Lambda, \chi_\Lambda, S_\Lambda, A_\Lambda^\mu, \lambda_\Lambda, D_\Lambda\right)} = 0 \tag{30}$$

we find the constraints which appear to be put on the V superfield components

$$C = C_0, \quad \chi = 0, \quad A_\mu A^\mu = SS^*. \tag{31}$$

Again, as before in non-SUSY case (3), we only take a solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (29), which will provide a ghost-free theory with a positive Hamiltonian.[8]

Remarkably, the constraints (31) do not touch on the physical degrees of freedom of the superfield $V(x, \theta, \overline{\theta})$ related to the photon and photino fields. The point is, however, that apart from the constraints (31), one has the equations of motion for all fields involved in the basic superfield $V(x, \theta, \overline{\theta})$. With vanishing multiplier component fields (29), as was proposed above, these equations appear in fact as extra constraints on components of the superfield

[8] As in the non-supersymmetric case discussed above (see footnote[1]), this solution with all vanishing components of the basic Lagrangian multiplier superfield $\Lambda(x, \theta, \overline{\theta})$ can be reached by introducing some extra Lagrange multiplier term.

$V(x, \theta, \overline{\theta})$. Indeed, the equations of motion for the fields C, S and χ obtained by the corresponding variations of the total Lagrangian $\mathcal{L}$ (14) turned out to be, respectively,

$$P(C_0)P'(C_0) = 0, \quad S(x)P'(C_0) = 0, \quad \lambda(x)P'(C_0) = 0 \tag{32}$$

where the basic constraints (31) emerging at the potential extremum point $C = C_0$ have also been used. One can immediately see now that these equations turn out to become trivial identities in the broken SUSY case, in which the factor $P'(C_0)$ in each of them appears to be identically vanished, $P'(C_0) = 0$ (22). In the unbroken SUSY case, in which the potential (16) vanishes instead, i.e. $P(C_0) = 0$ (20), the situation is drastically changed. Indeed, though the first equation in (32) still automatically turns into an identity at the extremum point $C(x) = C_0$, the other two equations require that the auxiliary field S and photino field λ have to be identically vanishing as well. This causes in turn the photon field to vanish, according to the basic constraints (31). Besides, the D field component in the vector superfield is also vanished in the unbroken SUSY case according to (15), $D = -P(C_0) = 0$. Thus, one is ultimately left with a trivial superfield $V(x, \theta, \overline{\theta})$, which only contains the constant C field component C_0, and that is unacceptable. So, we have to conclude that the unbroken SUSY fails to provide stability of the potential (18) even by constraining the superfield $V(x, \theta, \overline{\theta})$. In contrast, in the spontaneously broken SUSY case extra constraints do not appear at all, and one has a physically meaningful theory; this is what we basically consider in what follows.

Actually, substituting the constraints (31) into the total Lagrangian $\mathcal{L}_{\text{tot}}$ (27, 14) we eventually come to the emergent SUSY QED appearing in the broken SUSY phase

$$\mathcal{L}^{\mathbf{em}}_{\text{tot}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \frac{1}{2}D^2 + P(C_0)D,$$
$$A_{\mu}A^{\mu} = SS^* \tag{33}$$

supplemented by the vector field constraint as its vacuum stability condition. Remarkably, for the constrained vector superfield involved, we have

$$\widehat{V}(x, \theta, \overline{\theta}) = C_0 + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta\theta}S^*$$
$$-\theta\sigma^{\mu}\overline{\theta}A_{\mu} + i\theta\theta\overline{\theta\lambda} - i\overline{\theta\theta}\theta\lambda + \frac{1}{2}\theta\theta\overline{\theta\theta}D, \tag{34}$$

we have the almost standard SUSY QED Lagrangian with the same states—photon, photino and an auxiliary scalar D field—in its gauge supermultiplet, while another auxiliary complex scalar field S gets only involved in the vector field constraint. The linear (Fayet–Iliopoulos) D-term with the effective coupling constant $P(C_0)$ in (33) shows that the supersymmetry in the theory is spontaneously broken, due to which the D field acquires the VEV, $D = -P(C_0)$. Taking the non-dynamical S field in the constraint (31) to be some constant background field (for a more formal discussion, see below) we come to the SLIV constraint (2) which we discussed above regarding an ordinary non-supersymmetric QED theory (Sect. 1). As is seen from this constraint in (33), one may only have the time-like SLIV in a SUSY framework but never the space-like one. There also may be a light-like SLIV, if the S field vanishes.[9] So, any possible choice for the S field corresponds to the particular gauge choice for the vector field A_{μ} in an otherwise gauge invariant theory.

3.3 Constrained superfield: a formal view

We conclude this section by showing that the extended Lagrangian $\mathcal{L}_{\text{tot}}$ (27, 14), underlying the emergent QED model described above, as well as the vacuum stability constraints on the superfield component fields (31) appearing due to the Lagrange multiplier term in (27) are consistent with supersymmetry. The first part of this assertion is somewhat immediate, since the Lagrangian $\mathcal{L}_{\text{tot}}$, aside from the standard supersymmetric QED part L_{SQED} (11), only contains D-terms of various vector superfield products. They are, by definition, invariant under conventional SUSY transformations [37–39], which for the component fields (10) of a general superfield $V(x, \theta, \overline{\theta})$ (10) are written as

$$\delta_{\xi}C = i\xi\chi - i\overline{\xi}\overline{\chi}, \quad \delta_{\xi}\chi = \xi S + \sigma^{\mu}\overline{\xi}(\partial_{\mu}C + iA_{\mu}),$$
$$\frac{1}{2}\delta_{\xi}S = \overline{\xi\lambda} + \overline{\sigma}_{\mu}\partial^{\mu}\chi,$$
$$\delta_{\xi}A_{\mu} = \xi\partial_{\mu}\chi + \overline{\xi}\partial_{\mu}\overline{\chi} + i\xi\sigma_{\mu}\overline{\lambda} - i\lambda\sigma_{\mu}\overline{\xi},$$
$$\delta_{\xi}\lambda = \frac{1}{2}\xi\sigma^{\mu}\overline{\sigma}^{\nu}F_{\mu\nu} + \xi D,$$
$$\delta_{\xi}D = -\xi\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \overline{\xi}\sigma^{\mu}\partial_{\mu}\lambda. \tag{35}$$

However, there may still be left a question as to whether supersymmetry remains in force when the constraints (31) on the field space are "switched on", thus leading to the final Lagrangian $\mathcal{L}^{\mathbf{em}}_{\text{tot}}$ (33) in the broken SUSY phase with both dynamical fields C and χ eliminated. This Lagrangian appears similar to the standard supersymmetric QED taken in the Wess–Zumino gauge, except that the supersymmetry is spontaneously broken in our case. In both cases the photon stress tensor $F_{\mu\nu}$, the photino λ, and the non-dynamical scalar D field form an irreducible representation of the supersymmetry algebra [the last two lines in (35)]. Nevertheless, any reduction of the component fields in the vector superfield is not consistent in general with the linear superspace version

[9] Indeed, this case, first mentioned in [23], may also mean spontaneous Lorentz violation with a non-zero VEV $< A_{\mu} > = (\widetilde{M}, 0, 0, \widetilde{M})$ and Goldstone modes $A_{1,2}$ and $(A_0 + A_3)/2 - \widetilde{M}$. The "effective" Higgs mode $(A_0 - A_3)/2$ can then be expressed through Goldstone modes so as the light-like condition $A^2_{\mu} = 0$ to be satisfied.

of supersymmetry transformations, whether it is the Wess–Zumino gauge case or our constrained superfield $\widehat{V}$ (34). Indeed, a general SUSY transformation does not preserve the Wess–Zumino gauge: a vector superfield in this gauge acquires some extra terms when being SUSY transformed. The same occurs with our constrained superfield $\widehat{V}$ as well. The point, however, is that in both cases a total supergauge transformation,

$$V \to V + i(\Omega - \Omega^*), \tag{36}$$

where Ω is a chiral superfield gauge transformation parameter, can always restore the superfield initial form. Actually, the only difference between these two cases is that whereas the Wess–Zumino supergauge leaves the ordinary gauge freedom untouched, in our case this gauge is unambiguously fixed in terms of the above vector field constraint (31). However, this constraint remains under the supergauge transformation (36) applied to our superfield $\widehat{V}$ (34). Indeed, the essential part of this transformation which directly acts on the constraint (31) has the form

$$\widehat{V} \to \widehat{V} + i\theta\theta F - i\overline{\theta\theta} F^* - 2\theta\sigma^\mu\overline{\theta}\partial_\mu\varphi, \tag{37}$$

where the real and complex scalar field components, φ and F, in the chiral superfield parameter Ω are properly activated. As a result, the vector and scalar fields, A_μ and S, in the supermultiplet $\widehat{V}$ (34) transform as

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu(2\varphi), \quad S \to S' = S + 2F\ . \tag{38}$$

It can be immediately seen that our basic Lagrangian $\mathcal{L}^{\mathbf{em}}_{\mathrm{tot}}$ (33), being gauge invariant and containing no scalar field S, is automatically invariant under either of these two transformations individually. In contrast, the supplementary vector field constraint (31), though it also turned out to be invariant under the supergauge transformations (38), but only if they are made jointly. Indeed, for any choice of the scalar φ in (38) there can always be found such a scalar F (and vice versa) that the constraint remains invariant,

$$A_\mu A^\mu = SS^* \to A'_\mu A'^\mu = S' S'^*. \tag{39}$$

In other words, the vector field constraint is invariant under supergauge transformations (38) but not invariant under an ordinary gauge transformation. As a result, in contrast to the Wess–Zumino case, the supergauge fixing in our case will also lead to the ordinary gauge fixing. We will use this supergauge freedom to reduce the S field to some constant background value and find the final equation for the gauge function $\varphi(x)$. So, for the parameter field F chosen in such a way to have

$$S' = S + 2F = Me^{i\alpha(x)}\ , \tag{40}$$

where M is some constant mass parameter (and $\alpha(x)$ is an arbitrary phase), we end by (39) in

$$(A_\mu - 2\partial_\mu\varphi)(A^\mu - 2\partial^\mu\varphi) = M^2\ , \tag{41}$$

which is precisely our old SLIV constraint (2) being varied by the gauge transformation (38). Recall that this constraint, as was thoroughly discussed in the Introduction (Sect. 1.1), only fixes the gauge (which such a gauge function $\varphi(x)$ has to satisfy), rather than that it physically breaks gauge invariance. Notably, in contrast to the non-SUSY case where this constraint was merely postulated, it now follows from the vacuum stability and supergauge invariance in the emergent SUSY QED. Besides, this constraint, as mentioned above, may only be time-like (and light-like if the mass parameter M is taken to be zero). When such inactive time-like SLIV is properly developed one ends with the essentially nonlinear emergent SUSY QED, in which the physical photon arises as a three-dimensional Lorentzian NG mode (just as is the case in non-SUSY for the time-like SLIV; see Sect. 1.1).

To finalize, it was shown that the vacuum stability constraints (31) on the allowed configurations of the physical fields in a general polynomially extended Lagrangian (27) appear entirely consistent with supersymmetry. In the broken SUSY phase one eventually comes to the standard SUSY QED type Lagrangian (33) being supplemented by the vector field constraint which is invariant under supergauge transformations. One might think that, unlike the gauge invariant linear (Fayet–Iliopoulos) superfield term, the quadratic and higher order superfield terms in the starting Lagrangian (27) would seem to break gauge invariance. However, this fear proves groundless. Actually, as was shown above, this breaking amounts to the gauge fixing determined by the nonlinear vector field constraint (39). It is worth noting that this constraint formally follows from the SUSY invariant Lagrange multiplier term in (27) for which the phase space is required to be restricted to vanishing values of all the multiplier component fields (29). The total vanishing of the multiplier superfield provides the SUSY invariance of such restrictions. Any non-zero multiplier component field left in the Lagrangian would immediately break supersymmetry and, even worse, would eventually lead to ghost modes in the theory and a Hamiltonian unbounded from below.

4 Broken SUSY phase: photino as pseudo-goldstino

Let us now turn to matter superfields, which have not yet been included in the model. In their presence spontaneous SUSY breaking in the visible sector, which fundamentally underlies our approach, might be phenomenologically ruled out by the well-known supertrace sum rule [37–39] for actual masses of quarks and leptons and their superpartners.[10] However, this sum rule is acceptably relaxed when taking into account large radiative corrections to the masses of the supersymmetric particles, which, according to the proposal, stem from the hidden sector. This is just what one may expect in conventional supersymmetric theories with the standard two-sector paradigm, according to which SUSY breaking entirely occurs in a hidden sector; then this spontaneous breaking is mediated to the visible sector by some indirect interactions whose nature depends on a particular mediation scenario [37–39]. The emergent QED approach advocated here requires some modification of this idea in such a way that, while the hidden sector is largely responsible for spontaneous SUSY breaking, supersymmetry can also be spontaneously broken in the visible sector, which ultimately leads to a double spontaneous SUSY breaking pattern.

We may suppose, just for uniformity, only D-term SUSY breaking both in the visible and hidden sectors.[11] Properly stated, our supersymmetric QED model may be further extended by some extra local $U'(1)$ symmetry which is proposed to be broken at the very high-energy scale M' (for some appropriate anomaly mediated scenarios, see [40] and references therein). It is natural to think that due to the decoupling theorem all effects of the $U'(1)$ are suppressed at energies $E << M'$ by powers of $1/M'$ and only the D'-term of the corresponding vector superfield $V'(x,\theta,\overline{\theta})$ remains in essence when going down to low energies. Actually, this term with a proper choice of messenger fields and their couplings naturally provides the M_{SUSY} order contributions to the masses of the scalar superpartners.

As a result, the simplified picture discussed above (in Sects. 2 and 3) is properly changed: a strictly massless fermion eigenstate, the true goldstino ζ_g, should now be some mix of the visible sector photino λ and the hidden sector goldstino λ'

$$\zeta_g = \frac{\langle D\rangle \lambda + \langle D'\rangle \lambda'}{\sqrt{\langle D\rangle^2 + \langle D'\rangle^2}}\,, \tag{42}$$

where $\langle D\rangle$ and $\langle D'\rangle$ are the corresponding D-component VEVs in the visible and hidden sectors, respectively. Another orthogonal combination of them may be referred to as the pseudo-goldstino ζ_{pg},

$$\zeta_{pg} = \frac{\langle D'\rangle \lambda - \langle D\rangle \lambda'}{\sqrt{\langle D\rangle^2 + \langle D'\rangle^2}}\,. \tag{43}$$

In the supergravity context, the true goldstino ζ_g is eaten through the super-Higgs mechanism to form the longitudinal component of the gravitino, while the pseudo-goldstino ζ_{pg} gets some mass proportional to the gravitino mass from supergravity effects. Due to the large soft masses required to be mediated, one may generally expect that SUSY is much stronger broken in the hidden sector than in the visible one, $\langle D'\rangle >> \langle D\rangle$, which means in turn that the pseudo-goldstino ζ_{pg} is largely the photino λ,

$$\zeta_{pg} \simeq \lambda\,. \tag{44}$$

These pseudo-goldstonic photinos seem to be of special observational interest in the model, which, apart from some indication of the QED emergence nature, may shed light on SUSY breaking physics. The possibility that the supersymmetric Standard Model visible sector might also spontaneously break SUSY, thus giving rise to some pseudo-goldstino state, was also considered, though in a different context, in [41,42].

Interestingly enough, our polynomially extended SQED Lagrangian (11) is not only SUSY invariant but also generically possesses a continuous R-symmetry $U(1)_R$ [37–39]. Indeed, vector superfields always have zero R-charge, since they are real. Accordingly, it follows that the physical field components in the constrained vector superfield $\widehat{V}$ (34) transform as

$$A_\mu \to A_\mu, \quad \lambda \to e^{i\alpha}\lambda, \quad D \to D \tag{45}$$

and so have R charges 0, 1, and 0, respectively. Along with that, we assume a suitable R-symmetric matter superfield setup as well making a proper R-charge assignment for basic fermions and scalars (and messenger fields) involved. This will lead to the light pseudo-goldstino in the gauge-mediated scenario. Indeed, if the visible sector possesses an R-symmetry which is preserved in the course of mediation the pseudo-goldstino mass is protected up to the supergravity effects which violate an R-symmetry. As a result, the pseudo-goldstino mass appears proportional to the gravitino mass, and, eventually, the same region of parameter space simultaneously solves both gravitino and pseudo-goldstino overproduction problems in the early universe [42].

Apart from cosmological problems, many other sides of new physics related to pseudo-goldstinos appearing through the multiple SUSY breaking were also studied recently (see [41–46] and references therein). The point, however, is that

[10] Note that an inclusion of direct soft mass terms for scalar superpartners in the model would mean in general that the visible SUSY sector is explicitly, rather than spontaneously, broken, which could immediately invalidate the whole idea of the massless photons as the zero Lorentzian modes triggered by the spontaneously broken supersymmetry.

[11] In general, both D- and F-type terms can be simultaneously used in the visible and hidden sectors (usually just F-term SUSY breaking is used in both sectors [37–39]).

there have been exclusively used non-vanishing F-terms as the only mechanism of the visible SUSY breaking in the models considered. In this connection, our pseudo-goldstonic photinos solely caused by non-vanishing D-terms in the visible SUSY sector may lead to somewhat different observational consequences. One of the most serious differences may be related to the Higgs boson decays when the present SUSY QED is further extended to the supersymmetric Standard Model. For the cosmologically safe masses of pseudo-goldstino and gravitino ($\lesssim 1keV$, as typically follows from the R-symmetric gauge mediation) these decays are appreciably modified. Actually, the dominant channel becomes the conversion of the Higgs boson (say, the lighter CP-even Higgs boson h^0) into a conjugated pair of corresponding pseudo-sgoldstinos ϕ_{pg} and $\overline{\phi}_{pg}$ (being superpartners of pseudo-goldstinos ζ_{pg} and $\overline{\zeta}_{pg}$, respectively),

$$h^0 \to \phi_{pg} + \overline{\phi}_{pg} \,, \tag{46}$$

once it is kinematically allowed. This means that the Higgs boson will dominantly decay invisibly for F-term SUSY breaking in a visible sector [42]. By contrast, for the D-term SUSY breaking case considered here the roles of pseudo-goldstino and pseudo-sgoldstino are just played by photino and photon, respectively, which could enhance the standard two-photon decay channel of the Higgs boson even somewhat. In the light of the recent discovery of the Higgs-like state [47,48] just through its visible decay modes, the F-term SUSY breaking in the visible sector seems to be disfavored by data, while the SUSY breaking D-term is not in trouble with them.

5 Concluding remarks

It is well known that spontaneous Lorentz violation in general vector field theories may lead to an appearance of massless Nambu–Goldstone modes, which are identified with photons and other gauge fields in the Standard Model. Nonetheless, it may turn out that SLIV is not the only reason for emergent massless photons to appear, if the spacetime symmetry is further enlarged. In this connection, special interest may be in supersymmetry and its possible theoretical and observational relation to SLIV.

To see how such a scenario may work we have considered supersymmetric QED model extended by an arbitrary polynomial potential of a general vector superfield $V(x, \theta, \overline{\theta})$ whose pure vector field component $A_\mu(x)$ is associated with a photon in the Lorentz invariant phase. Gauge non-invariant couplings other than potential terms are not included into the theory. For the theory in which gauge invariance is not required from the outset this is in fact the simplest generalization of a conventional SUSY QED. This superfield potential (18) is turned out to be generically unstable unless SUSY is spontaneously broken. However, it appears not to be enough. To provide an overall stability of the potential one additionally needs the special direct constraint being put on the vector superfield itself that is made by an appropriate SUSY invariant Lagrange multiplier term (27). Remarkably enough, when this term is written in field components it leads precisely to the nonlinear σ-model type constraint of type (2) which one has in the non-SUSY case. So, we come again to the picture, which we called the inactive SLIV, with a Goldstone-like photon and special (SLIV restricted) gauge invariance providing the cancelation mechanism for physical Lorentz violation. But now this picture follows from the vacuum stability and supergauge invariance in the extended SUSY QED rather than being postulated as is in the non-SUSY case. This allows one to think that a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz non-invariance.

In more exact terms, in the broken SUSY phase one eventually comes to the almost standard SUSY QED Lagrangian (33) possessing some special gauge invariance emerged. This invariance is only restricted by the gauge condition put on the vector field, $A_\mu A^\mu = |S|^2$, which appears to be invariant under supergauge transformations. One can use this supergauge freedom to reduce the non-dynamical scalar field S to some constant background value so as to eventually come to the nonlinear vector field constraint (2). As a result, the inactive time-like SLIV is properly developed, thus leading to essentially nonlinear emergent SUSY QED in which the physical photon arises as a three-dimensional Lorentzian NG mode. So, figuratively speaking, the photon passes through three evolution stages being initially the massive vector field component of a general vector superfield (14), then the three-level massless companion of an emergent photino in the broken SUSY stage (17) and finally a generically massless state as an emergent Lorentzian mode in the inactive SLIV stage (31).

As to an observational status of emergent SUSY theories, one can see that, as in an ordinary QED, physical Lorentz invariance is still preserved in the SUSY QED model at the renormalizable level and can only be violated if some extra gauge non-invariant couplings [being supersymmetric analogs of the high-dimension couplings (6)] are included into the theory. However, one may have some specific observational evidence in favor of the inactive SLIV even in the minimal (gauge invariant) supersymmetric QED and Standard Model. Indeed, since as mentioned above the vacuum stability is only possible in spontaneously broken SUSY case, this evidence is related to an existence of an emergent goldstino–photino type state in the SUSY visible sector. Being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector this state largely turns into the light pseudo-goldstino. Its study seem to be of special observational interest for this class of models that, apart from some indication of an emergence nature of QED and the Standard Model, may appreciably extend the scope of SUSY breaking physics being actively studied in recent years. We may return to this important issue elsewhere.

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