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Detecting the long-distance structure of the $X(3872)$

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Abstract We study the $X(3872) \to D^0 \bar{D}^0 \pi^0$ decay within a $D\bar{D}^*$ molecular picture for the $X(3872)$ state. This decay mode is more sensitive to the long-distance structure of the $X(3872)$ resonance than its $J/\psi\pi\pi$ and $J/\psi 3\pi$ decays, which are mainly controlled by the details of the $X(3872)$ wave function at short distances. We show that the $D^0 \bar{D}^0$ final state interaction can be important, and that a precise measurement of this partial decay width can provide valuable information on the interaction strength between the $D^{(*)}\bar{D}^{(*)}$ charm mesons.

1 Introduction

It has been long since mesonic molecules in the charm sector were first theorized [1,2] but there was not any experimental observation until the discovery of the $X(3872)$ in 2003 in the $J/\psi\pi\pi$ channel [3]. However, important details of the inner structure of the resonance are still under debate. Among many different interpretations of the $X(3872)$, the one assuming it to be[1] a $(D\bar{D}^* - D^*\bar{D})/\sqrt{2}$, hadronic molecule (either a bound state [4] or a virtual state [5]) with quantum numbers $J^{PC} = 1^{++}$ (as recently confirmed in Ref. [6]) is the most promising. Quite a lot of work has been done under this assumption; for reviews, see, for instance, Refs. [7,8].

The most discussed decay channels of the $X(3872)$ are those with a charmonium in the final state, which include the $J/\psi\pi\pi$, $J/\psi 3\pi$, $J/\psi\gamma$ and $\psi'\gamma$. In the hadronic molecular picture, these decays occur through the mechanism depicted in Fig. 1. Thus, the charm and anticharm mesons only appear in the intermediate (virtual) state, and the amplitude of such decays is proportional to the appropriate charged or neutral $D\bar{D}^*$ loop integrals [9]. Because the quarks in the two mesons have to recombine to get the charmonium in the final state, the transition from the charm–anticharm meson pair into the J/ψ plus pions (or a photon), occurs at a distance much smaller than both the size of the $X(3872)$ as a hadronic molecule ($\sim$ few fm's)[2] and the range of forces between the D and $\bar{D}^*$ mesons which is of the order of $1/m_\pi \sim 1.5$ fm. In this case, if this transition matrix t in Fig. 1 does not introduce any momentum dependence, the loop integral reduces to the wave function of the $X(3872)$ at the origin, $\Psi(\vec{0})$,[3] (more properly, around the origin, the needed ultraviolet regulator, for which we do not give details here, would smear the wave functions) [11][4],

$$\langle f|X(3872)\rangle = \int d^3q \underbrace{\langle f|D\bar{D}^*(\vec{q})\rangle}_{t} \underbrace{\langle D\bar{D}^*(\vec{q})|X(3872)\rangle}_{\Psi(\vec{q})}$$

$$= t \int d^3q \, \Psi(\vec{q}) = t\,\Psi(\vec{0}) = t\,\hat{g}G \qquad (1)$$

where $\hat{g}$ is the coupling of the $X(3872)$ to the $D\bar{D}^*(\vec{q})$ pair and G is the diagonal loop function for the two intermediate D and $\bar{D}^*$ meson propagators, with the appropriate nor-

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[1] From now on, when we refer to $D^0 \bar{D}^{*0}$, $D^+ \bar{D}^{*-}$, or in general $D\bar{D}^*$ we are actually referring to the combination of these states with their charge conjugate ones in order to form a state with positive C-parity.

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[2] This is approximately given by $1/\sqrt{2\mu\epsilon_X}$ fm where μ is the reduced mass of the D and $\bar{D}^*$ pair and $\epsilon_X = M_{D^0} + M_{D^{*0}} - M_{X(3872)} = 0.16 \pm 0.26$ MeV [10].

[3] The relative distance between the two mesons is zero in the wave function at the origin.

[4] For related discussions in case of the two-photon decay width of a loosely bound hadronic molecule, see Ref. [12].
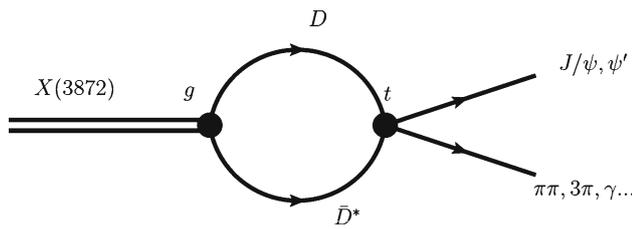
Fig. 1 Mechanism for the decays of the $X(3872)$ into $J/\psi\pi\pi$, $J/\psi 3\pi$, $J/\psi\gamma$, $\psi'\gamma$, ... assuming the $X(3872)$ to be a $D\bar{D}^*$ molecule. The charge conjugated channel is not plotted

malizations that will be discussed below. The last equality follows from the expression of the momentum space wave function [11]

$$\Psi(\vec{q}\,) = \frac{\hat{g}}{M_X - M_D - M_{D^*} - \vec{q}^{\,2}/(2\mu)}, \qquad (2)$$

where μ is the reduced mass of the D and $\bar{D}^*$. This can easily be derived from the Schrödinger equation assuming the coupling of the $X(3872)$ to $D\bar{D}^*$ to be a constant, which is valid since the $X(3872)$ is very close to the threshold. Thus, one can hardly extract information on the long-distance structure of the $X(3872)$ from these decays.

In general, to be sensitive to the long-distance part of the wave function of a hadronic molecule, it is better to investigate the decay processes with one of the constituent hadrons in the final state and the rest of the final particles being products of the decay of the other constituent hadron of the molecule. For instance, for the case of the $X(3872)$ as a $D\bar{D}^*$ molecule, we should use the $X(3872) \to D^0\bar{D}^0\pi^0$ or $X(3872) \to D\bar{D}\gamma$ to study the long-distance structure. In these processes, the relative distance between the $D\bar{D}^*$ pair can be as large as allowed by the size of the $X(3872)$ resonance, since the final state is produced by the decay of the $\bar{D}^*$ meson instead of a rescattering transition. These decay modes have been addressed in some detail in different works, e.g., Refs. [13–19]. The $D^0\bar{D}^0\pi^0$ mode has been already observed by the Belle Collaboration [20,21], which triggered the virtual state interpretation of the $X(3872)$ [5], and it will be studied in detail in this work.

On the other hand, heavy-quark spin symmetry (HQSS) and flavor symmetry [22–25] have been widely used to predict partners of the $X(3872)$ state [26–34]. Moreover, HQSS heavily constrains also the low-energy interactions among heavy hadrons [27,29,31,32,35]. As long as the hadrons are not too tightly bound, they will not probe the specific details of the interaction binding them at short distances. Moreover, each of the constituent heavy hadrons will be unable to see the internal structure of the other heavy hadron. This separation of scales can be used to formulate an effective field theory (EFT) description of hadronic molecules

[29,31,36,37] compatible with the approximate nature of HQSS. At leading order (LO) the EFT is particularly simple and it only involves energy-independent contact-range interactions, since pion exchanges and coupled-channel effects can be considered subleading [31,38]. Moreover, the influence of three-body $D\bar{D}\pi$ interactions on the properties of the $X(3872)$ was found to be moderate in a Faddeev approach [19]. In particular since we will only be interested in the $X(3872)$ mass and its couplings to the neutral and charged $D\bar{D}^*$ pairs, the three-body cut can be safely neglected at LO. As a result of the HQSS, assuming the $X(3872)$ being a $D\bar{D}^*$ molecular state, it is expected to have a spin 2 partner, a $D^*\bar{D}^*$ S-wave hadronic molecule [31–33]. This is because the LO interaction in these two systems are exactly the same due to HQSS. The interaction between a D and a $\bar{D}$, on the contrary, is different. It depends on a different combination of low-energy constants (LECs) [29,31,32]. The $X(3872) \to D^0\bar{D}^0\pi^0$ decay, on one hand, detects the long-distance structure of the $X(3872)$; on the other hand, it provides the possibility to constrain the $D\bar{D}$ S-wave interaction at very low energies. Hence, it is also the purpose of this paper to discuss the effect of the $D\bar{D}$ S-wave final state interaction (FSI) in the $X(3872) \to D^0\bar{D}^0\pi^0$ decay, which can be very large because of the possible existence of a sub-threshold isoscalar state in the vicinity of 3700 MeV [31,32,39].

As mentioned above, this $X(3872)$ decay channel has been previously studied. The first calculation was carried out in Ref. [13] using effective-range theory. In Ref. [16], using an EFT, the results of Ref. [13] was reproduced at LO, and the size of corrections to the LO calculation was estimated. These next-to-leading-order (NLO) corrections to the decay width include effective-range corrections as well as calculable non-analytic corrections from π^0 exchange. It was found that non-analytic calculable corrections from pion exchange are negligible and the NLO correction was dominated by contact interaction contributions. The smallness of these corrections confirms one of the main points raised in [16], namely, that pion exchange can be dealt with using perturbation theory.[5] However, the $D\bar{D}$ FSI effects were not considered in these two works.

The paper is structured as follows: in Sect. 2, we briefly discuss the $X(3872)$ resonance within the hadronic molecular picture and the S-wave low-energy interaction between a charm and an anticharm mesons. The decay $X(3872) \to D^0\bar{D}^0\pi^0$ is discussed in detail in Sect. 3 with the inclusion of the $D\bar{D}$ FSI. Section 4 presents a brief summary.

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[5] This result has been also confirmed in Refs. [19,31,38]. In the latter reference, the range of center-of-mass momenta for which the tensor piece of the one pion exchange potential is perturbative is studied in detail, and it is also argued that the effect of coupled channels is suppressed by at least two orders in the EFT expansion.

2 The $X(3872)$ and the heavy meson S-wave interaction

The basic assumption in this work is that the $X(3872)$ exotic charmonium is a $D\bar{D}^* - D^*\bar{D}$ bound state with quantum numbers $J^{PC} = 1^{++}$. For the sake of completeness we briefly discuss in this section the formalism used in [31,32] to describe this resonance, which is based on solving and finding the poles of the Lippmann–Schwinger equation (LSE). More specific details can be found in these two references.

We use the matrix field $H^{(Q)}$ [$H^{(\bar{Q})}$] to describe the combined isospin doublet of pseudoscalar heavy-meson [antimeson] $P_a^{(Q)} = (P^0, P^+)$ [$P_a^{(\bar{Q})} = (\bar{P}^0, P^-)$] fields and their vector HQSS partners $P_a^{*(Q)}$ [$P_a^{*(\bar{Q})}$] (see for example [40]),

$$H_a^{(Q)} = \frac{1 + \not{v}}{2}(P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5), \quad v \cdot P_a^{*(Q)} = 0,$$

$$H_a^{(\bar{Q})} = (P_{a\mu}^{*(\bar{Q})}\gamma^\mu - P_a^{(\bar{Q})}\gamma_5)\frac{1 - \not{v}}{2}, \quad v \cdot P_a^{*(\bar{Q})} = 0. \quad (3)$$

The matrix field H^c [$H^{\bar{c}}$] annihilates D [$\bar{D}$] and D^* [$\bar{D}^*$] mesons with a definite velocity v. The field $H_a^{(Q)}$ [$H_a^{(\bar{Q})}$] transforms as a $(2, \bar{2})$ [$(\bar{2}, 2)$] under the heavy spin $\otimes$ SU(2)$_V$ isospin symmetry [41]. The definition for $H_a^{(\bar{Q})}$ also specifies our convention for charge conjugation, which is $\mathcal{C}P_a^{(Q)}\mathcal{C}^{-1} = P^{(\bar{Q})a}$ and $\mathcal{C}P_{a\mu}^{*(Q)}\mathcal{C}^{-1} = -P_\mu^{*(\bar{Q})a}$. At very low energies, the interaction between a charm and anticharm meson can be accurately described just in terms of a contact-range potential. The LO Lagrangian respecting HQSS reads [35]

$$\mathcal{L} = \frac{C_A}{4}\text{Tr}[\bar{H}^{(Q)a}H_a^{(Q)}\gamma_\mu]\text{Tr}[H^{(\bar{Q})a}\bar{H}_a^{(\bar{Q})}\gamma^\mu]$$

$$+ \frac{C_A^\tau}{4}\text{Tr}[\bar{H}^{(Q)a}\vec{\tau}_a^{\ b}H_b^{(Q)}\gamma_\mu]\text{Tr}[H^{(\bar{Q})c}\vec{\tau}_c^{\ d}\bar{H}_d^{(\bar{Q})}\gamma^\mu]$$

$$+ \frac{C_B}{4}\text{Tr}[\bar{H}^{(Q)a}H_a^{(Q)}\gamma_\mu\gamma_5]\text{Tr}[H^{(\bar{Q})a}\bar{H}_a^{(\bar{Q})}\gamma^\mu\gamma_5]$$

$$+ \frac{C_B^\tau}{4}\text{Tr}[\bar{H}^{(Q)a}\vec{\tau}_a^{\ b}H_b^{(Q)}\gamma_\mu\gamma_5]\text{Tr}[H^{(\bar{Q})c}\vec{\tau}_c^{\ d}\bar{H}_d^{(\bar{Q})}\gamma^\mu\gamma_5]$$

$$(4)$$

with the hermitian conjugate fields defined as $\bar{H}^{Q(\bar{Q})} = \gamma^0 H^{Q(\bar{Q})\dagger}\gamma^0$, and $\vec{\tau}$ the Pauli matrices in isospin space. Note that in our normalization the heavy-meson or -antimeson fields, $H^{(Q)}$ or $H^{(\bar{Q})}$, have dimensions of $E^{3/2}$ (see [25] for details). This is because we use a non-relativistic normalization for the heavy mesons, which differs from the traditional relativistic one by a factor $\sqrt{M_H}$. For later use, the four LECs that appear above are rewritten into C_{0A}, C_{0B} and C_{1A}, C_{1B} which stand for the counter-terms in the isospin $I = 0$ and $I = 1$ channels, respectively. The relations read

$$C_{0\phi} = C_\phi + 3C_\phi^\tau, \quad C_{1\phi} = C_\phi - C_\phi^\tau, \quad \text{for } \phi = A, B \quad (5)$$

The LO Lagrangian determines the contact interaction potential $V = i\mathcal{L}$, which is then used as kernel of the two-body elastic LSE[6]

$$T(E; \vec{p}\,', \vec{p}\,) = V(\vec{p}\,', \vec{p}\,) + \int \frac{d^3\vec{q}}{(2\pi)^3}V(\vec{p}\,', \vec{q}\,)$$

$$\times \frac{1}{E - \vec{q}^{\,2}/2\mu_{12} - M_1 - M_2 + i\epsilon}T(E; \vec{q}\,, \vec{p}\,). \quad (6)$$

with M_1 and M_2 the masses of the involved mesons, $\mu_{12}^{-1} = M_1^{-1} + M_2^{-1}$, E the center-of-mass (c.m.) energy of the system and $\vec{p}$ ($\vec{p}\,'$) the initial (final) c.m. momentum. Above threshold, we have $E > (M_1 + M_2)$, and the unitarity relation $\text{Im } T^{-1}(E) = \mu_{12}k/(2\pi)$ with $k = \sqrt{2\mu_{12}(E - M_1 - M_2)}$.

When contact interactions are used, the LSE shows an ill-defined ultraviolet (UV) behavior, and requires a regularization and renormalization procedure. We employ a standard Gaussian regulator

$$\langle \vec{p}\,|V|\vec{p}\,'\rangle = C_{I\phi}e^{-\vec{p}^{\,2}/\Lambda^2}e^{-\vec{p}^{\,\prime 2}/\Lambda^2}, \quad (7)$$

with $C_{I\phi}$ the corresponding counter-term deduced from the Lagrangian of Eq. (4). We will take cutoff values $\Lambda = 0.5$–1 GeV [31,32], where the range is chosen such that Λ will be bigger than the wave number of the states but at the same time will be small enough to preserve HQSS and prevent that the theory might become sensitive to the specific details of short-distance dynamics. The dependence of results on the cutoff, when it varies within this window, provides an estimate of the expected size of subleading corrections. On the other hand, in the scheme of Ref. [32] pion exchange and coupled-channel effects are not considered at LO. This is justified since both effects were shown to be small by the explicit calculation carried out in [31] and the power counting arguments established in [38]. Moreover, in what pion exchange respects, this is in accordance with the findings of Refs. [16,19], as mentioned in the Introduction.

Bound states correspond to poles of the T-matrix below threshold on the real axis in the first Riemann sheet of the complex energy. If we assume that the $X(3872)$ state and the isovector $Z_b(10610)$ states[7] are $(D\bar{D}^* - D^*\bar{D})/\sqrt{2}$ and $(B\bar{B}^* + B^*\bar{B})/\sqrt{2}$ bound states, respectively, and use the isospin breaking information of the decays of the $X(3872)$ into the $J/\psi\pi\pi$ and $J/\psi\pi\pi\pi$, we can determine three linear combinations among the four LECs C_{0A}, C_{0B}, C_{1A} and C_{1B} with the help of heavy-quark spin and flavor symmetries [32, 33]. We consider both the neutral $(D^0\bar{D}^{*0} - D^{*0}\bar{D}^0)$ and the

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[6] The extension to the general case of coupled channels is straightforward, as long as only two-body channels are considered: T, V and the two-particle propagator will become matrices in the coupled-channel space, being the latter one diagonal.

[7] The $Z_b(10610)$ observed in Ref. [43] carries electric charge, and its neutral partner was also reported by the Belle Collaboration [44]. We thus assume that its isospin is 1.

charged $(D^+D^{*-} - D^{*+}D^-)$ components in the $X(3872)$. The coupled-channel potential is given by

$$V_{X(3872)} = \frac{1}{2}\begin{pmatrix} C_{0X} + C_{1X} & C_{0X} - C_{1X} \\ C_{0X} - C_{1X} & C_{0X} + C_{1X} \end{pmatrix}, \qquad (8)$$

where $C_{0X} \equiv C_{0A} + C_{0B}$ and $C_{1X} \equiv C_{1A} + C_{1B}$. Using $M_{X(3872)} = (3871.68 \pm 0.17)$ MeV, the isospin violating ratio of the decay amplitudes for the $X(3872) \to J/\psi\pi\pi$ and $X(3872) \to J/\psi\pi\pi\pi$, $R_{X(3872)} = 0.26 \pm 0.07$ [42],[8] and the mass of the $Z_b(10610)$ (we assume that its binding energy is (2.0 ± 2.0) MeV [45]) as three independent inputs, we find

$$C_{0X} = -1.71^{+0.06}_{-0.04}(-0.73^{+0.02}_{-0.01})\ \text{fm}^2,$$
$$C_{1X} = -0.13^{+0.53}_{-0.41}(-0.39 \pm 0.09)\ \text{fm}^2,$$
$$C_{1Z} \equiv C_{1A} - C_{1B} = -0.75^{+0.24}_{-0.14}(-0.30^{+0.03}_{-0.03})\ \text{fm}^2 \qquad (9)$$

for $\Lambda = 0.5(1.0)$ GeV. Errors were obtained from a Monte Carlo (MC) simulation assuming uncorrelated Gaussian errors for the three inputs and using 1,000 samples. Note that the values of the different LEC's are natural, $\sim \mathcal{O}(1\ \text{fm}^2)$, as one would expect. For details of the parameter determination, we refer to Refs. [32–34].

The $X(3872)$ coupling constants to the neutral and charged channels, g_0^X and g_c^X, respectively, are determined by the residues of the T-matrix elements at the $X(3872)$ pole

$$(g_0^X)^2 = \lim_{E \to M_X(3872)} [E - M_X(3872)] \times T_{11}(E),$$
$$g_0^X g_c^X = \lim_{E \to M_X(3872)} [E - M_X(3872)] \times T_{12}(E), \qquad (10)$$

where T_{ij} are the matrix elements of the T-matrix solution of the UV regularized LSE. Their values are slightly different. Using the central values of C_{0X} and C_{1X}, we get

$$g_0^X = 0.35^{+0.08}_{-0.29}(0.34^{+0.07}_{-0.29})\text{GeV}^{-1/2},$$
$$g_c^X = 0.32^{+0.07}_{-0.26}(0.26^{+0.05}_{-0.22})\text{GeV}^{-1/2}, \qquad (11)$$

where, again, the values outside and inside the parentheses are obtained with $\Lambda = 0.5$ and 1 GeV, respectively. Note that when the position of the $X(3872)$ resonance approaches the $D^0\bar{D}^0$ threshold, both couplings g_0^X and g_c^X vanish proportionally to the square root of the binding energy [11,46], which explains the asymmetric errors. Notice that the values of the coupling constants carry important information on the structure of the $X(3872)$. In general, the wave function of the $X(3872)$ is a composition of various Fock states, including the $c\bar{c}$, $D\bar{D}^* - D^*\bar{D}$, $c\bar{c}q\bar{q}(q = u, d)$ and so on. The coupling constants are a measure of the probability of the $X(3872)$ to be a hadronic molecule [11] (for discussions on the relation of the coupling constant with the composite nature of a physical state, we refer to Refs. [47–49]).

Within this model, we will account for the $D\bar{D}$ FSI effects to the $X(3872) \to D^0\bar{D}^0\pi^0$ decay width. The S-wave interaction in the $D\bar{D}$ system with $J^{PC} = 0^{++}$ is not entirely determined by C_{0X}, C_{1X} and C_{1Z}. Indeed, considering again both the neutral and the charged channels $D^0\bar{D}^0$ and D^+D^-, the potential is given by[9]

$$V_{D\bar{D}} = \frac{1}{2}\begin{pmatrix} C_{0A} + C_{1A} & C_{0A} - C_{1A} \\ C_{0A} - C_{1A} & C_{0A} + C_{1A} \end{pmatrix}. \qquad (12)$$

Thus, this interaction is not completely determined from what we have learned from the $X(3872)$ and $Z_b(10610)$ states even if we use heavy-quark spin and flavor symmetries—the value of C_{0A} is still unknown. Depending on the value of C_{0A}, there can be a $D\bar{D}$ S-wave bound state or not. For instance, considering the case for $\Lambda = 0.5$ GeV and taking the central value for $C_{1A} = -0.44$ fm^2, if $C_{0A} = -3.53$ fm^2, then one finds a bound state pole in the $D\bar{D}$ system with a mass 3706 MeV (bound by around 25 MeV); if $C_{0A} = -1.65$ fm^2, there will be a $D^0\bar{D}^0$ bound state at threshold; if the value of C_{0A} is larger, there will be no bound state pole any more. Therefore, the information of C_{0A} will be crucial in understanding the $D\bar{D}$ system and other systems related to it through heavy-quark symmetries [33,34]. Conversely, as we will see, the $X(3872) \to D^0\bar{D}^0\pi^0$ decay width could be used to extract information on the fourth LEC, C_{0A}, thanks to the FSI effects.

3 $X(3872) \to D^0\bar{D}^0\pi^0$ decay

Here, we discuss the decay of the $X(3872)$ into the $D^0\bar{D}^0\pi^0$ mode. This decay can take place directly through the decay of the constituent D^{*0} or $\bar{D}^{*0}$ as shown in Fig. 2a. After emitting a pion, the vector charm meson transit into a pseudoscalar one, and it can interact with the other constituent in the $X(3872)$ as shown in Fig. 2b. Figure 2c presents another possibility, namely the decay can also occur through the decay of the charged vector charm meson, and the virtual charged D^+D^- pair then rescatter into $D^0\bar{D}^0$.

We will use the relevant term in the LO Lagrangian of heavy-meson chiral perturbation theory [41,50–52] to describe the $D^*D\pi$ coupling

$$\mathcal{L}_{\pi HH} = -\frac{g}{2f_\pi}(\text{Tr}[\bar{H}^{(Q)b} H_a^{(Q)}\gamma_\mu\gamma_5]$$
$$+ \text{Tr}[H^{(\bar{Q})b}\bar{H}_a^{(\bar{Q})}\gamma^\mu\gamma_5])(\vec{\tau}\partial_\mu\vec{\phi})_b^a + \cdots \qquad (13)$$

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[8] We have symmetrized the errors provided in [42] to use Gaussian distributions to estimate errors.

[9] The reason for using particle basis, where the interaction is not diagonal, instead of isospin basis is because for some values of the LEC's, a $D^0\bar{D}^0$ bound state close to threshold might be generated. If its binding energy is smaller or comparable to the $D^0\bar{D}^0 - D^+D^-$ threshold difference, as happens in the case of the $X(3872)$ resonance, then it will become necessary to account for the mass difference among the neutral and charged channels.

Fig. 2 Feynman diagrams for the decay $X(3872) \to D^0 \bar{D}^0 \pi^0$. The charge conjugate channel is not shown but included in the calculations

(a)   **(b)**   **(c)**
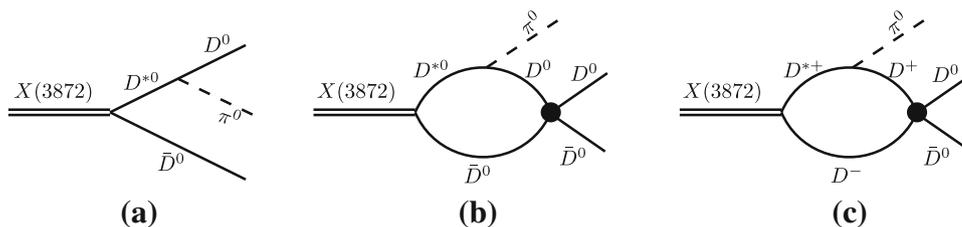
with $\vec{\phi}$ a relativistic field that describes the pion[10], $g \simeq 0.6$ is the $PP^*\pi$ coupling and $f_\pi = 92.2$ MeV the pion decay constant. Note that in our normalization, the pion field has a dimension of energy, while the heavy-meson or -antimeson fields $H^{(Q)}$ or $H^{(\bar{Q})}$ have dimensions of $E^{3/2}$, as we already mentioned.

3.1 Tree-Level Approximation

For the process in question, the charm mesons are highly non-relativistic, thus we can safely neglect higher order terms in $\vec{p}_{\bar{D}^{*0},D^{*0}}/M_{D^*}$. Taking into account the contributions from both the $D^0 \bar{D}^{*0}$ and the $D^{*0} \bar{D}^0$ components of the $X(3872)$, the tree-level amplitude is given by

$$T_{\text{tree}} = -2i \frac{g g_0^X}{f_\pi} \sqrt{M_X} M_{D^{*0}} M_{D^0} \vec{\epsilon}_X \cdot \vec{p}_\pi$$
$$\times \left(\frac{1}{p_{12}^2 - M_{D^{*0}}^2} + \frac{1}{p_{13}^2 - M_{D^{*0}}^2} \right), \quad (14)$$

where $\vec{\epsilon}_X$ is the polarization vector of the $X(3872)$, $\vec{p}_\pi$ is the three-momentum of the pion, p_{12} and p_{13} are the four momenta of the $\pi^0 D^0$ and $\pi^0 \bar{D}^0$ systems, respectively.[11] We have neglected the D^{*0} and $\bar{D}^{*0}$ widths in the above propagators because their inclusion only leads to small numerical variations in the $X(3872) \to D^0 \bar{D}^0 \pi^0$ decay rate of the order of 0.1 keV. As we will see below in Eq. (19), uncertainties on the predicted width induced by the errors in the coupling g_0^X and the mass of the $X(3872)$ resonance, turn out to be much larger (of the order of few keV).

Note that we have approximated the $X(3872)D^0\bar{D}^{*0}$ vertex by g_0^X. It could have some dependence on the momentum of the mesons, which can be expanded in powers of momentum in the spirit of EFT. For the process in question, the momenta of the charm mesons are much smaller than the hard energy scale of the order of the cutoff, we can safely keep only the leading constant term.

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[10] We use a convention such that $\phi = \frac{\phi_x - i\phi_y}{\sqrt{2}}$ creates a π^- from the vacuum or annihilates a π^+, and the ϕ_z field creates or annihilates a π^0.

[11] To obtain the amplitude, we have multiplied by factors $\sqrt{M_{D^{*0}} M_{D^0}}$ and $\sqrt{8 M_X M_{D^{*0}} M_{D^0}}$ to account for the normalization of the heavy-meson fields and to use the coupling constant g_0^X, as defined in Eq. (10) and given in Eq. (11), for the $X(3872)D^0\bar{D}^{*0}$ and $X(3872)D^{*0}\bar{D}^0$ vertices.

Since the amplitude of Eq. (14) depends only on the invariant masses $m_{12}^2 = p_{12}^2$ and $m_{23}^2 = (M_X^2 + m_{\pi^0}^2 + 2M_{D^0}^2 - m_{12}^2 - p_{13}^2)$ of the final $\pi^0 D^0$ and $D^0 \bar{D}^0$ pairs, respectively, we can use the standard form for the Dalitz plot [10]

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M_X^3} \overline{|T|}^2 dm_{12}^2 dm_{23}^2 \quad (15)$$

and thus, we readily obtain

$$\Gamma_{\text{tree}} = \frac{g^2}{192\pi^3 f_\pi^2} \left(g_0^X \frac{M_{D^0} M_{D^{*0}}}{M_X} \right)^2$$
$$\times \int_{(M_{D^0}+m_\pi^0)^2}^{(M_X - M_{D^0})^2} dm_{12}^2 \int_{(m_{23}^2)_{(\text{min})}}^{(m_{23}^2)_{(\text{max})}} dm_{23}^2$$
$$\times \left(\frac{1}{p_{12}^2 - M_{D^{*0}}^2} + \frac{1}{p_{13}^2 - M_{D^{*0}}^2} \right)^2 |\vec{p}_\pi|^2 \quad (16)$$

with

$$|\vec{p}_\pi| = \frac{\lambda^{1/2}(M_X^2, m_{23}^2, m_{\pi^0}^2)}{2M_X} \quad (17)$$

the pion momentum in the $X(3872)$ center-of-mass frame [$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + xz)$ is the Källén function]. In addition, for a given value of m_{12}^2, the range of m_{23}^2 is determined by its values when $\vec{p}_D$ is parallel or antiparallel to $\vec{p}_{\bar{D}}$ [10]:

$$(m_{23}^2)_{(\text{max})} = (E_D^* + E_{\bar{D}}^*)^2 - (p_D^* - p_{\bar{D}}^*)^2$$
$$(m_{23}^2)_{(\text{min})} = (E_D^* + E_{\bar{D}}^*)^2 - (p_D^* + p_{\bar{D}}^*)^2 \quad (18)$$

with $E_D^* = (m_{12}^2 - m_{\pi^0}^2 + M_{D^0}^2)/2m_{12}$ and $E_{\bar{D}}^* = (M_X^2 - m_{12}^2 - M_{D^0}^2)/2m_{12}$ the energies of the D^0 and $\bar{D}^0$ in the m_{12} rest frame, respectively, and $p_{D,\bar{D}}^*$ the moduli of their corresponding three momenta.

Using the couplings given in Eq. (11), the partial decay width for the three-body decay $X(3872) \to D^0 \bar{D}^0 \pi^0$ at tree level is predicted to be

$$\Gamma(X(3872) \to D^0 \bar{D}^0 \pi^0)_{\text{tree}} = 44.0^{+2.4}_{-7.2}(42.0^{+3.6}_{-7.3})\text{keV}, \quad (19)$$

where the values outside and inside the parentheses are obtained with $\Lambda = 0.5$ and 1 GeV, respectively, and the uncertainty reflects the uncertainty in the inputs ($M_{X(3872)}$ and the ratio $R_{X(3872)}$ of decay amplitudes for the $X(3872) \to$

$J/\psi\rho$ and $X(3872) \to J/\psi\omega$ decays). We have performed a Monte Carlo simulation to propagate the errors.

Before studying the effects of the $D\bar{D}$ FSI, we would like to make two remarks:

1. Within the molecular wave-function description of the $X(3872) \to D^0\bar{D}^0\pi^0$ decay, the amplitude of Fig. 2a will read[12]

$$
T_{\text{tree}} \sim \int d^3q \underbrace{\langle D^0\bar{D}^{*0}(\vec{p}_{D^0})|D^0\bar{D}^{*0}(\vec{q}\,)\rangle}_{\propto \delta^3(\vec{q}-\vec{p}_{D^0})}
$$
$$
\times \langle D^0\bar{D}^{*0}(\vec{q}\,)|X(3872)\rangle T_{\bar{D}^{0*}(\vec{p}_{\bar{D}^{0*}})\to\bar{D}^0\pi^0}
$$
$$
= \Psi(\vec{p}_{D^0})T_{\bar{D}^{0*}(\vec{p}_{\bar{D}^{0*}})\to\bar{D}^0\pi^0} \tag{20}
$$

with $\vec{p}_{\bar{D}^{0*}} = -\vec{p}_{D^0}$ in the laboratory frame. Note that this description is totally equivalent to that of Eq. (14) because the $D^0\bar{D}^{*0}$ component of the non-relativistic $X(3872)$ wave function is given by [11]

$$
\Psi(\vec{p}_{D^0}) = \frac{g_0^X}{E_{D^0} + E_{\bar{D}^{*0}} - M_{D^0} - M_{D^{*0}} - \vec{p}_{D^0}^2/2\mu_{D^0D^{*0}}}
$$
$$
= \frac{g_0^X}{E_{\bar{D}^{*0}} - M_{D^{*0}} - \vec{p}_{\bar{D}^{*0}}^2/2M_{D^{*0}}} \tag{21}
$$

with $\mu_{D^0D^{*0}}^{-1} = M_{D^0}^{-1} + M_{D^{*0}}^{-1}$. In the last step, we have used the fact that the D^0 meson is on shell and therefore $(E_{D^0} - M_{D^0} - \vec{p}_{D^0}^2/2M_{D^0}) = 0$. Thus, the wave function in momentum space turns out to be proportional to the coupling g_0^X times the non-relativistic reduction, up to a factor $2M_{D^{*0}}$, of the $\bar{D}^{*0}$ propagator that appears in Eq. (14).

The amplitude in Eq. (20) involves the $X(3872)$ wave function at a given momentum, $\vec{p}_{D^0}$, and the total decay width depends on the wave function in momentum space evaluated only for a limited range of values of $\vec{p}_{D^0}$ determined by energy-momentum conservation. This is in sharp contrast to the decay amplitude into charmonium states, as shown in Fig. 1, in Eq. (1), where there is an integral over all possible momenta included in the wave function. Such an integral can be thought of as a Fourier transform at $\vec{x} = 0$, and thus gives rise to the $X(3872)$ wave function in coordinate space at the origin. This is to say, the width is proportional to the probability of finding the $D\bar{D}^*$ pair at zero (small in general) relative distance within the molecular $X(3872)$ state. This result is intu-

---

[12] For simplicity, we omit the contribution to the amplitude driven by the $D^{*0}\bar{D}^0$ component of the $X(3872)$ resonance, for which the discussion will run in parallel.

itive, since the $D\bar{D}^*$ transitions to final states involving charmonium mesons should involve the exchange of a virtual charm quark, which is only effective at short distances. However, in the $X(3872) \to D^0\bar{D}^0\pi^0$ process, the relative distance of the $D\bar{D}^*$ pair can be as large as allowed by the size of the $X(3872)$ resonance, since the final state is produced by the one-body decay of the $\bar{D}^*$ meson instead of by a strong two-body transition. Thus, this decay channel might provide details on the long-distance part of the $X(3872)$ wave function. Indeed, from Eq. (20) it follows that a future measurement of the $d\Gamma/d|\vec{p}_{D^0}|$ distribution might provide valuable information on the $X(3872)$ wave function $\Psi(\vec{p}_{D^0})$.

2. So far, we have not made any reference to the isospin nature of the $X(3872)$ resonance. We have just used the coupling, g_0^X, of the resonance to the $D^0\bar{D}^{0*}$ pair. In addition to the $J/\Psi\pi^+\pi^-\pi^0$ final state, the $X(3872)$ decay into $J/\Psi\pi^+\pi^-$ was also observed [53,54], pointing out to an isospin violation, at least, in its decays [4]. In the $D\bar{D}^*$ molecular picture, the isospin breaking effects arise due to the mass difference between the $D^0\bar{D}^{*0}$ pair and its charged counterpart, the $D^+\bar{D}^{*-}$ pair, which turns out to be relevant because of the closeness of the $X(3872)$ mass to the D^0D^{*0} threshold [4,9,11,32]. The observed isospin violation in the decays $X(3872) \to \rho J/\psi$, and $X(3872) \to \omega J/\psi$ depends on the probability amplitudes of both the neutral and the charged meson channels near the origin which are very similar [11]. This suggests that, when dealing with these strong processes, the isospin $I = 0$ component will be the most relevant, though the experimental value of the isospin violating ratio, $R_{X(3872)}$, of decay amplitudes could be used to learn details on the weak $D\bar{D}^*$ interaction in the isovector channel [32] (C_{1X} in Eq. (8)). The $X(3872) \to D^0\bar{D}^0\pi^0$ decay mode can shed more light into the isospin dynamics of the $X(3872)$ resonance, since it can be used to further constrain the isovector sector of the $D\bar{D}^*$ interaction. This is the case already at tree level because the numerical value of the coupling g_0^X is affected by the interaction in the isospin one channel, C_{1X}.

We should also stress that in absence of FSI effects that will be discussed below, if C_{1X} is neglected, as in Ref. [11], the $X(3872) \to D^0\bar{D}^0\pi^0$ width will be practically the same independent of whether the $X(3872)$ is considered as an isoscalar molecule or a $D^0\bar{D}^{*0}$ state. In the latter case, the width would be proportional to $\tilde{g}^2$ [11],

$$
\tilde{g}^2 = -\Big(\frac{dG_0}{dE}\Big)^{-1}\Big|_{E=M_X},
$$
$$
G_0(E) = \int_\Lambda \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{E - M_{D^0} - M_{D^{*0}} - \vec{q}^2/2\mu_{D^0D^{*0}}} \tag{22}
$$

where $G_0(E)$ is the UV regularized $D^0 \bar{D}^{*0}$ loop function.[13] However, if the $X(3872)$ were an isoscalar state,

$$|X(3872)\rangle = \frac{1}{\sqrt{2}}(|D^0 \bar{D}^{*0}\rangle + |D^+ D^{*-}\rangle) \qquad (23)$$

one, naively, would expect to obtain a width around a factor two smaller, because now the coupling of the $X(3872)$ state to the $D^0 \bar{D}^{*0}$ pair would be around a factor $\sqrt{2}$ smaller as well [11]

$$(g_0^X)^2 \simeq (g_c^X)^2 \simeq -\left(\frac{dG_0}{dE} + \frac{dG_c}{dE}\right)^{-1}\Bigg|_{E=M_X}, \qquad (24)$$

where G_c is the loop function in the charged charm-meson channel. The approximations would become equalities if the isovector interaction is neglected (it is much smaller than the isoscalar one as can be seen in Eq. (9)). Were $\frac{dG_0}{dE} \simeq \frac{dG_c}{dE}$, the above values would be equal to $\tilde{g}^2/2$ approximately. However, after considering the mass differences between the neutral and charged channels and, since $\frac{dG_i}{dE} \propto 1/\sqrt{B_i}$ [$B_i > 0$ is the binding energy of either the neutral (~ 0.2 MeV) or charged (~ 8 MeV) channels], at the mass of the $X(3872)$ one actually finds

$$\left(\frac{dG_0}{dE}\right)\Bigg|_{E=M_X} \gg \left(\frac{dG_c}{dE}\right)\Bigg|_{E=M_X} \qquad (25)$$

so that $(g_0^X)^2 \simeq \tilde{g}^2$. Therefore, the prediction for the decay width would hardly change.

All these considerations are affected by the $D\bar{D}$ FSI effects which will be discussed next.

3.2 $D\bar{D}$ FSI effects

To account for the FSI effects, we include in the analysis the $D\bar{D} \to D\bar{D}$ T-matrix, which is obtained by solving the LSE (Eq. (6)) in coupled channels with the $V_{D\bar{D}}$ potential given in Eq. (12). We use in Eq. (6) the physical masses of the neutral ($D^0 \bar{D}^0$) and charged ($D^+ D^-$) channels. Thus, considering both the $D^0 \bar{D}^{*0}$ and the $D^{*0} \bar{D}^0$ meson pairs as intermediate states, the decay amplitude for the mechanism depicted in Fig. 2b reads

$$T_{\text{loop}}^{(0)} = -16i \frac{g g_0^X}{f_\pi} \sqrt{M_X} M_{D^{*0}} M_{D^0}^3 \, \vec{\epsilon}_X \cdot \vec{p}_\pi \, T_{00\to00}(m_{23})$$
$$\times I(M_{D^{*0}}, M_{D^0}, M_{D^0}, \vec{p}_\pi), \qquad (26)$$

where $T_{00\to00}$ is the T-matrix element for the $D^0 \bar{D}^0 \to D^0 \bar{D}^0$ process, and the three-point loop function is defined as

$$I(M_1, M_2, M_3, \vec{p}_\pi) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - M_1^2 + i\varepsilon}$$
$$\times \frac{1}{(P-q)^2 - M_2^2 + i\varepsilon} \frac{1}{(q - p_\pi)^2 - M_3^2 + i\varepsilon}, \qquad (27)$$

with $P^\mu = (M_X, \vec{0})$ in the rest frame of the $X(3872)$. This loop integral is convergent. Since all the intermediate mesons in the present case are highly non-relativistic, the three-point loop can be treated non-relativistically. The analytic expression for this loop function at the leading order of the non-relativistic expansion can be found in Eq. (A2) of Ref. [55] (see also Ref. [45]). For the specific kinematics of this decay, the loop function in the neutral channel has an imaginary part, which turns out to be much larger than the real one, except in a narrow region involving the highest pion momenta.

Similarly, the amplitude for the mechanism with charged intermediate charm mesons is given by

$$T_{\text{loop}}^{(c)} = 16i \frac{g g_c^X}{f_\pi} \sqrt{M_X} M_{D^{*0}} M_{D^0} M_{D^\pm}^2 \, \vec{\epsilon}_X \cdot \vec{p}_\pi$$
$$\times T_{+-\to00}(m_{23}) I(M_{D^{*\pm}}, M_{D^\pm}, M_{D^\pm}, \vec{p}_\pi), \qquad (28)$$

where $T_{+-\to00}$ is the T-matrix element for the $D^+ D^- \to D^0 \bar{D}^0$ process. The loop function is now purely real because the $D^+ D^- \pi^0$ channel is closed, and its size is significantly smaller than in the case of the neutral channel. The sign difference between the amplitudes of Eqs. (26) and (28) is due to the sign difference between the $D^{*-} \to D^- \pi^0$ and $\bar{D}^{*0} \to D^0 \pi^0$ transition amplitudes.

For consistency, despite the three-point loop functions in Eqs. (26)–(28) being finite, they should, however, be evaluated using the same UV renormalization scheme as that employed in the $D^{(*)} \bar{D}^{(*)}$ EFT. The applicability of the EFT relies on the fact that long-range physics should not depend on the short-range details. Hence, if the bulk of contributions of the loop integrals came mostly from large momenta (above 1 GeV for instance), the calculation would not be significant. Fortunately, this is not the case, and the momenta involved in the integrals are rather low. Indeed, the biggest FSI contribution comes from the imaginary part of the loop function in the neutral channel, which is hardly sensitive to the UV cutoff. Thus and for the sake of simplicity, FSI effects have been calculated using the analytical expressions for the three-point loop integral mentioned above, valid in the $\Lambda \to \infty$ limit. Nevertheless, we have numerically computed these loop functions with 0.5 and 1 GeV UV Gaussian cutoffs and found small differences[14] in the final results

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[13] Notice that although the loop function is linearly divergent, its derivative with respect E is convergent, and thus it only shows a residual (smooth) dependence on γ/Λ if a gaussian cutoff is used, with $\gamma^2 = 2\mu_{D^0 D^{*0}}(M_{D^0} + M_{D^{*0}} - M_X)$. Were a sharp cutoff used, there would not be any dependence on the cutoff because of the derivative.

[14] The largest changes affect to the charged channel (Fig. 2c). This is because there, the three-meson loop integral is purely real. However,
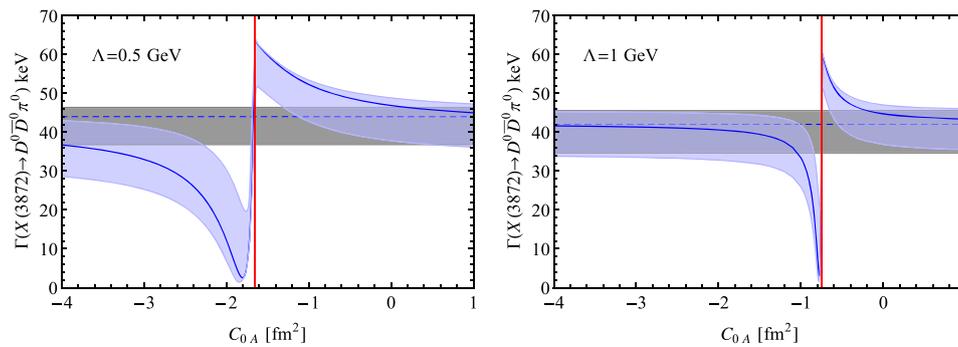
Fig. 3 Dependence of the $X(3872) \to D^0 \bar{D}^0 \pi^0$ partial decay width on the low-energy constant C_{0A}. The UV cutoff is set to $\Lambda = 0.5$ GeV (1 GeV) in the *left* (*right*) *panel*. The *blue error bands* contain $D\bar{D}$ FSI effects, while the *gray bands* stand for the tree-level predictions of Eq. (19). The *solid* (full calculation) and *dashed* (tree level) *lines* stand for the results obtained with the central values of the parameters. The *vertical lines* denote the values of C_{0A} for which a $D\bar{D}$ bound state is generate at the $D^0 \bar{D}^0$ threshold

$[\Gamma(X(3872) \to D^0 \bar{D}^0 \pi^0)$ versus $C_{0A}]$ discussed in Fig. 3. Indeed, the changes turn out to be almost inappreciable for $\Lambda = 1$ GeV, and they are at most of the order of few percent in the $\Lambda = 0.5$ GeV case. Moreover, even then these differences are well accounted for by the error bands shown in the figure.

To compute $T_{00 \to 00}$ and $T_{+- \to 00}$ we need the $D\bar{D}$ potential given in Eq. (12). With the inputs (masses of the $X(3872)$ and $Z_b(10610)$ resonances and the ratio $R_{X(3872)}$) discussed in Sect. 2, three of the four couplings that describe the heavy-meson–antimeson S-wave interaction at LO in the heavy-quark expansion can be fixed. The value of the contact term parameter C_{0A} is undetermined, and thus we could not predict the $D\bar{D}$ FSI effects parameter-free in this $X(3872)$ decay. These effects might be quite large, because for a certain range of C_{0A} values, a near-threshold isoscalar pole could be dynamically generated in the $D\bar{D}$ system [31,32].

To investigate the impact of the FSI, in Fig. 3 we show the dependence of the partial decay width on C_{0A}. For comparison, the tree-level results are also shown in the same plots. The vertical lines denote the values of C_{0A} when there is a $D\bar{D}$ bound state at threshold. When C_{0A} takes smaller values, the binding energy becomes larger; when C_{0A} takes larger values, the pole moves to the second Riemann sheet and becomes a virtual state. Around the values denoted by the vertical lines, the pole is close to threshold no matter on which Riemann sheet it is. One can see an apparent deviation from the tree-level results in this region. The wavy behavior is due to the interference between the FSI and the tree-level terms. The existence of a low-lying $D\bar{D}$ bound state has as a consequence a decrease of the partial decay width to $D^0 \bar{D}^0 \pi^0$, the reason being that there's a substantial probability of a direct

decay to the $D\bar{D}$ bound state and a neutral pion. On the other hand if there is a virtual state near the threshold, the decay width will increase owing to rescattering effects[15].

When the partial decay width will be measured in future experiments, a significant deviation from the values in Eq. (19) will indicate a FSI effect, which could eventually be used to extract the value of C_{0A}. Outside the wavy region, the FSI contribution is small, and it will be unlikely to obtain any conclusive information on C_{0A} from the experimental $\Gamma(X(3872) \to D^0 \bar{D}^0 \pi^0)$ width. However, there exist theoretical hints pointing out the existence of a $D\bar{D}$ bound state close to threshold. In the scheme of Ref. [32], the $Z_b(10610)$ mass input was not used, but, however, there it was assumed that the $X(3915)$ and $Y(4140)$ were $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ molecular states. These two new inputs were used to fix completely the heavy-meson–antimeson interaction, and a $D\bar{D}$ molecular isoscalar state was predicted at around 3,710 MeV. A state in the vicinity of 3,700 MeV was also predicted in Ref. [39], within the hidden gauge formalism, using an extension of the SU(3) chiral Lagrangians to SU(4) that implements a particular pattern of SU(4) flavor symmetry breaking. Experimentally, there is support for that resonance around 3,720 MeV from the analysis of the $e^+e^- \to J/\psi D\bar{D}$ Belle data [56] carried out in [57]. However, the broad bump observed above the $D\bar{D}$ threshold by the Belle Collaboration in the previous reaction could instead be produced by the $\chi_0(2P)$ state [58,59].

In Ref. [60], the authors show that the charged component D^+D^{*-} in the $X(3872)$ is essential to obtain a width for the $X(3872) \to J/\psi\gamma$ compatible with the data. In the process studied in this work, at tree level, the charged component does not directly contribute, though it could indirectly mod-

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Footnote 14 continued
this FSI mechanism, as we will discuss below, provides a very small contribution to the total decay width.

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[15] The mechanism is analogous for instance to the large capture cross-section of thermal neutrons by protons or the near-threshold enhancement of deuteron photo-disintegration, both of which are triggered by the existence of a virtual state in the singlet neutron–proton channel.

ify the $X(3872)D^0\bar{D}^{(*0)}$ coupling g_0^X. However, because the $X(3872)$ resonance is placed so close to the $D^0\bar{D}^{(*0)}$ threshold, we argued that this is not really the case and such a component hardly changes the prediction for the decay width. When the FSI is taken into account, one may ask whether the charged component is important or not since it can now contribute as the intermediate state which radiates the pion. We find, however, this contribution plays a small role here, leading to changes of about ten percent at most for the $\Lambda = 0.5$ GeV case, and much smaller when the UV cutoff is set to 1 GeV. These variations are significantly smaller that the uncertainty bands displayed in Fig. 3. Therefore, we conclude that the relative importance of the charge component in the $X(3872)$ depends on the process in question. When the observable is governed by the wave function of the $X(3872)$ at the origin, it can be important as the case studied in Ref. [60]. For our case, the decay is more sensitive to the long-distance structure of the $X(3872)$, then the charged component is not as important as the neutral one. At this point, we can also comment on the processes $X(3872) \rightarrow D^0\bar{D}^0\gamma$ and $X(3872) \rightarrow D^+D^-\gamma$, where the $D\bar{D}$ system has now a negative C parity in contrast to the pionic decay. The decay amplitudes, when neglecting possible contributions from the $\psi(3770)$, are similar to the one in Eq. (14). Near the $D\bar{D}$ threshold, the intermediate D^{*0} is almost on shell, and the virtuality of the charged D^* is much larger. Thus, the partial decay width into the $D^+D^-\gamma$ should be much smaller than the one into the $D^0\bar{D}^0\gamma$, as discussed in Ref. [15].

4 Summary

In this work, we explored the decay of the $X(3872)$ into the $D^0\bar{D}^0\pi^0$ using an effective field theory based on the hadronic molecule assumption for the $X(3872)$. This decay is unique in the sense that it is sensitive to the long-distance structure of the $X(3872)$ as well as the strength of the S-wave interaction between the D and $\bar{D}$. We show that if there was a near-threshold pole in the $D\bar{D}$ system, the partial decay width can be very different from the result neglecting the FSI effects. Thus, this decay may be used to measure the so far unknown parameter C_{0A} in this situation. Such information is valuable to better understand the interaction between a heavy and an antiheavy meson. In view that some of the XYZ states which are attracting intensive interests are good candidates for the heavy-meson hadronic molecules, it is desirable to carry out a precise measurement of that width.

It is also worth mentioning that since this decay is sensitive to the long-distance structure, the contribution of the $X(3872)$ charged component $(D^+D^{*-} - D^{*+}D^-)$ is not important even when the $D\bar{D}$ FSI is taken into account. We have also discussed how a future measurement of the $d\Gamma/d|\vec{p}_{D^0}|$ distribution might provide valuable information

on the $X(3872)$ wave function at the fixed momentum $\Psi(\vec{p}_{D^0})$.

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