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Letter

Neutron production rates by inverse-beta decay in fully ionized plasmas

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Abstract Recently we showed that the nuclear transmutation rates are largely overestimated in the Widom–Larsen theory of the so-called 'Low Energy Nuclear Reactions'. Here we show that unbound plasma electrons are even less likely to initiate nuclear transmutations.

1 Introduction

Claims of electron–proton conversion into a neutron and a neutrino by inverse beta decay in metallic hydrides have recently been raised [1,2], in the context of the so-called Low Energy Nuclear Reactions (LENR). The condition for the reaction to occur is a considerable mass renormalization of the electrons, to overcome the negative Q-value that, otherwise, would forbid the reaction to occur. Defining a dimensionless parameter, β, in terms of the electron effective mass, $m^\star$,[1] one needs

$$\beta = \frac{m^\star}{m} \geq \frac{m_n - m_p}{m} \approx 2.8, \tag{1}$$

a reference value, $\beta = 20$ was estimated in [1].

It is not clear at all if such spectacularly large values of β can be obtained in metallic hydrides and under which conditions. Nonetheless, *assuming* a given value of β, a calculation of the neutron rate can be obtained in a straightforward fashion from known electroweak physics. A calculation along these lines has been presented in Ref. [3] for the case of an electron bound to a proton, superseding the order-of-magnitude estimate presented in [1].

More recently, the authors of Ref. [2] have argued that nuclear transmutations should most likely be started by unbound plasma electrons. Assuming a fully ionized plasma and completely unscreened electrons, they find a rate which is enhanced, with respect to the value obtained for bound electrons, by the so-called Sommerfeld factor, S_0 ($c = 1$):

$$S_0 = \frac{2\pi\alpha}{v} \tag{2}$$

where α is the fine structure constant and v is the average thermal velocity of the electrons defined by[2]

$$v_{\text{th}} = \sqrt{\frac{3kT}{m^\star}} = \beta^{-1/2}\sqrt{\frac{3kT}{m}} = 3.6 \times 10^{-4} \times \left[\left(\frac{T}{5 \times 10^3\ \text{K}}\right)\left(\frac{20}{\beta}\right)\right]^{1/2}, \tag{3}$$

with the numerical value in correspondence to $\beta = 20$ and to the temperature $T \approx 5 \times 10^3$ K, estimated in [2] as the temperature that can be reached by hydride cathodes. However, the assumption of completely unscreened electrons may be unrealistic. We consider here the situation in the presence of Debye screening, which, in a different context, has been recently analyzed in Refs. [4,5]. We find that at large densities, the plasma enhancement saturates to a value determined by the Debye length, a_D:

$$S_0 \to S = \frac{a_D}{a_B^\star} \tag{4}$$

with

$$a_B^\star = \frac{1}{\alpha m^\star} = \beta^{-1} a_B \tag{5}$$

and a_B the Bohr radius.

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[1] To avoid confusion, we underscore that the mass renormalization in (1) has nothing to do with the velocity dependent relativistic mass. We consider extremely non-relativistic electrons. The situation is closely analogous to muon capture in muonic atoms; in that case $m^\star$ being replaced by the muon mass.

[2] We shall use the numerical values $k = 8.617 \times 10^{-5}$eV/K, $e^2/\hbar c = \alpha = 1/137.043$ and set $c = \hbar c = 1$.

2 Debye length

Static charges are screened in a plasma. The potential of the electric field of a test charge at rest in a plasma is (in Gaussian units)

$$\phi = \frac{e}{r} e^{-r/a_D} \tag{6}$$

where a_D is the Debye length defined by

$$\frac{1}{a_D^2} = \frac{1}{a_e^2} + \frac{1}{a_i^2}. \tag{7}$$

The two lengths $a_{e,i}$ are associated to electrons and ions, respectively, and are given by [6]

$$a_e = \left(\frac{kT_e}{4\pi n_e e^2} \right)^{1/2} \tag{8}$$

and

$$a_i = \left(\frac{kT_i}{4\pi n_i (Ze)^2} \right)^{1/2}. \tag{9}$$

The difference in temperature between electrons and ions is expected to occur naturally because of the large difference of mass which impedes the exchange of energy in electron–ion collisions. Here we will make the approximation $a_D = a_e$, which leads to the numerical value

$$a_D = 4.87\,\text{Å} \times \left[\left(\frac{T}{5000\,\text{K}} \right) \left(\frac{10^{20}\,\text{cm}^{-3}}{n_e} \right) \right]^{1/2} \tag{10}$$

or a Debye mass m_D:

$$m_D = \frac{\hbar}{a_D} = 404\ \text{eV}. \tag{11}$$

We therefore get a Debye length of about nine atoms (compared to $a_B = 0.5$ Å) in correspondence to the reference temperature $T \approx 5 \times 10^3$ K and a reference density $n_e = 10^{20}$ cm^{-3}. When considering the n dependence, we shall restrict ourselves to the range

$$10^{14}\ \text{cm}^{-3} \leq n \leq 6 \times 10^{23}\ \text{cm}^{-3}. \tag{12}$$

Values between 10^8 and 10^{14} cm^{-3} are typical of glow discharges and arcs, whereas a value of about 10^{22} cm^{-3} is the free electron density in copper [7]. Around 2.5×10^{21} cm^{-3} the Debye length equals the Bohr radius.[3]

[3] Electron capture occurs spontaneously during the formation of neutron stars, when the Fermi energy of the electrons increases above the threshold value, due to the gravitational pressure. This occurs at electron densities $\gtrsim 10^{31}$ cm^{-3}.

3 Critical velocity

The Sommerfeld factors in a plasma, Eqs. (40) and (43), can be obtained from an intuitive argument as follows (see the appendix for a derivation from the Schrödinger equation following [4,5]).

We consider the critical value of the velocity as defined by

$$\frac{2\pi\alpha}{v_{\text{crit}}} = \alpha m^\star a_D. \tag{13}$$

Under this condition, the de Broglie wavelength of the particle is equal to the Debye length:[4]

$$\lambda = \frac{2\pi}{m^\star v_{\text{crit}}} = a_D. \tag{14}$$

For larger velocities, the wavelength is smaller and the particle probes a region of space smaller than a_D, where it sees an essentially unscreened Coulomb potential. Under these conditions, we have to use S_0, Eq. (2).

For smaller velocities, as $v \to 0$, the wavelength gets larger than a_D. The Sommerfeld factor saturates to the value on the r.h.s. of (13) since the particle explores increasingly large portions of the neutral plasma, and the screened Sommerfeld factor in Eq. (4) has to be considered.

The critical velocity defined by (13) is

$$v_{\text{crit}} = 2.48 \times 10^{-4} \left(\frac{20}{\beta} \right) \left(\frac{n}{10^{20}\ \text{cm}^{-3}} \right) \left(\frac{5000\ \text{K}}{T} \right). \tag{15}$$

We consider our electrons to be at v_{th}, Eq. (3). At the reference point, this is larger than v_{crit}; hence we should apply the unscreened result, S_0. With increasing density, however, v_{crit} grows above v_{th} (at $n \sim 2 \times 10^{20}$ cm^{-3}) and one should apply the screened result, S.

4 Transmutation rates

To translate the previous discussion into the expected rates for transmutation from electrons in a plasma, we first recall the rate for the transmutation from bound electrons [3]:

$$\begin{aligned} \Gamma(\tilde{e}p \to n\nu_e)_{\text{bound}} &= |\psi(0)|^2 \times \frac{1}{2\pi}(G_F m_e)^2 \\ &\times \left[1 + 3\left(\frac{g_A}{g_V}\right)^2 \right] \times (\beta - \beta_0)^2; \\ |\psi(0)|^2 &= \frac{\beta^3}{\pi a_B^3} \\ \Gamma_{\text{bound}}[\beta = 20] &= 1.8 \times 10^{-3}\ \text{Hz}. \end{aligned} \tag{16}$$

[4] We use $\hbar = 1$, so that $h = 2\pi$.

The total rate is obtained by multiplying the result Γ_{bound} by the volume and by the ion density, which we take equal to the electron density, n, because of global neutrality:

$$\text{Rate}_{\text{bound}} = n \cdot V \cdot \Gamma_{\text{bound}}. \tag{17}$$

In the case of plasma electrons, screened and unscreened rates are obtained by the substitution

$$|\psi(0)|^2 \to n \cdot (S \text{ or } S_0), \tag{18}$$

and the rate is proportional to n^2:

$$\text{Rate}_{\text{plasma}} = n \cdot V \cdot \frac{\Gamma_{\text{bound}}}{|\psi(0)|^2} \cdot n \cdot (S \text{ or } S_0), \tag{19}$$

S and S_0 corresponding, respectively, to the screened Debye plasma and to the unscreened Coulomb case.

For convenience, we normalize the rates in plasma to the rate in Eq. (17), computed for $\beta = 20$, already a considerably large rate, although a factor of ~ 300 smaller than claimed in [1], and we shall see if we can get anywhere close to unity or higher.

The formulas are

$$\eta_{\text{Debye}}(n, \beta) = \frac{\text{Rate}_{\text{Debye}}}{\text{Rate}_{\text{bound}}[\beta = 20]} = n \frac{\pi a_B^3}{\beta^3} \frac{(\beta - \beta_0)^2}{(20 - \beta_0)^2} S$$
$$= \frac{\pi (n a_B^3)}{\beta^2} \frac{a_D}{a_B} \frac{(\beta - \beta_0)^2}{(20 - \beta_0)^2} \tag{20}$$

and

$$\eta_{\text{Coul}}(n, \beta) = \frac{\text{Rate}_{\text{Coul}}}{\text{Rate}_{\text{bound}}[\beta = 20]}$$
$$= n \frac{2\pi\alpha}{v} \frac{\pi a_B^3}{\beta^3} \frac{(\beta - \beta_0)^2}{(20 - \beta_0)^2} \tag{21}$$

for the two cases.

In Fig. 1 we display the ratios corresponding to the screened plasma (Sommerfeld factor S) and to the unscreened one (Sommerfeld factor S_0), for the case $\beta = 20$. The previous discussion indicates that we must use S_0 for $v_{\text{crit}} \leq v_{\text{th}}$ and S for $v_{\text{crit}} \geq v_{\text{th}}$. The result is represented by the thick line.

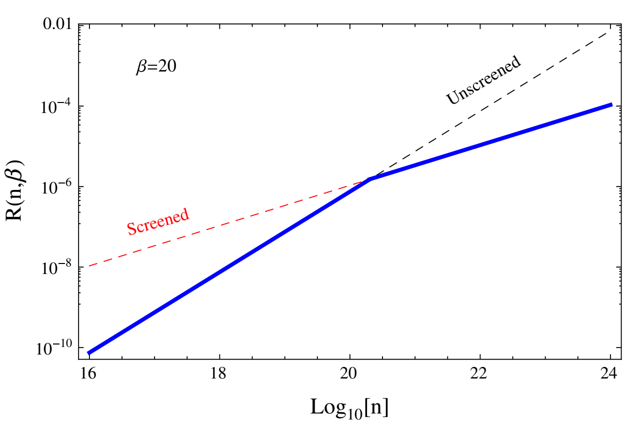

Fig. 1 Ratios corresponding to the screened plasma (Sommerfeld factor S) and to the unscreened one (Sommerfeld factor S_0), for the case $\beta = 20$. The previous discussion indicates that we must use S_0 for $v_{\text{crit}} \leq v_{\text{th}}$ and S for $v_{\text{crit}} \geq v_{\text{th}}$. The result is represented by the *thick line*

The rate for electron capture from plasma never comes anywhere close to the capture rate for bound electrons derived in [3] for the same value of β, let alone to the larger rate quoted in [1]. Our results are in line with the lack of observation of neutrons in plasma discharge experiments recently reported in [9].

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Appendix: Sommerfeld factor for electrons in screened and unscreened plasma

Let us consider an attractive screened potential in the plasma in the form

$$V(r) = -\frac{\alpha}{r} e^{m_D r}. \tag{22}$$

The radial Schrödinger equation for the two body (e^-–ion) wavefunction, $\chi(r)$, reads

$$\frac{d^2\chi(r)}{d^2 r} + 2m^\star \left(m^\star \frac{v^2}{2} - V(r) \right) \chi(r) = 0. \tag{23}$$

Changing r into the nondimensional variable x:

$$r = a_B^\star x = \frac{1}{\alpha m^*} x, \tag{24}$$

we get

$$\chi''(x) + \left(\frac{v^2}{\alpha^2} + \frac{2}{x} e^{-\epsilon x} \right) \chi(x) = 0. \tag{25}$$

In the limit of small or vanishing v we write the equation as

$$\chi''(x) + k^2(x)\chi(x) = 0, \tag{26}$$

and in terms of the effective momentum:

$$k^2(x) = \frac{2}{x} e^{-\epsilon x}. \tag{27}$$

We solve it by the WKB method, which gives

$$\chi(x) = A \frac{1}{\sqrt{k(x)}} e^{\pm i \int^x k(x') dx'}. \tag{28}$$

We can use the WKB approximation as long as

$$\left| \frac{k'(x)}{k^2(x)} \right| \ll 1, \tag{29}$$

that is,

$$\frac{e^{\epsilon x/2}}{2\sqrt{2x}}(1+\epsilon x) \ll 1. \tag{30}$$

At the value where the exponential bends, namely $\epsilon x = 1$, we have

$$\left|\frac{k'(x)}{k^2(x)}\right|_{x=1/\epsilon} = \sqrt{\frac{\epsilon}{2}}e^{1/2} = \frac{\epsilon}{k(x=1/\epsilon)} \equiv \frac{\epsilon}{k_{\mathrm{eff}}}, \tag{31}$$

and the condition that this region is within the range of validity of WKB is then

$$\frac{v_{\mathrm{eff}}}{\alpha} = k_{\mathrm{eff}} \gg \epsilon = \frac{a_B^{\star}}{a_D} = \frac{a_B}{\beta a} \tag{32}$$

with β defined as in Eq. (1).

For $\beta = 20$ and a_D from Eq. (10), we find

$$v_{\mathrm{eff}} > \frac{\alpha a_B}{\beta a_D} \equiv v_{\mathrm{WKB}} \approx 3.9 \times 10^{-5}. \tag{33}$$

On the other hand, the smallest velocity we consider is the thermal velocity, Eq. (3), which is safely within the region of validity of the WKB approximation. Note that v_{WKB} is simply proportional to the critical velocity v_{crit} defined in (13):

$$v_{\mathrm{WKB}} = \frac{v_{\mathrm{crit}}}{2\pi}. \tag{34}$$

We are interested in the square modulus of the wavefunction at the origin relative to its unperturbed value (transmutation is taking place at the origin), the ratio being the Sommerfeld enhancement:

$$S_k \sim |\psi_k(0)|^2 = \left|\frac{R_{k,\ell=0}(x=0)}{Ak}\right|^2 = \left|\frac{\chi_k(0)}{Axk}\right|^2, \tag{35}$$

where we have used the fact that $R_{k\ell}(x) \sim x^{\ell}$ as $x \to 0$. The constant A depends on the normalization of the radial function at large distances.[5] Since $R_{k,\ell=0}$ goes to a constant as $x \to 0$, we need that $\chi_k(x) \to 0$ as $x \to 0$ or

$$\chi_k(x) \to x\chi_k'(0) \quad \text{as } x \to 0, \tag{36}$$

thus giving

$$S_k \sim \left|\frac{\chi_k'(0)}{Ak}\right|^2. \tag{37}$$

Within the region of validity of the WKB approximation, $k \gtrsim \varepsilon$, we have

$$\chi(x) = A\frac{1}{\sqrt{k(x)}}e^{\pm i\int^x \mathrm{d}x' k(x')}, \tag{38}$$

where A is chosen to be the same constant as appears in (35). Therefore

$$S_k \sim \left|\frac{1}{\sqrt{k(x)}}e^{\pm i\int^x \mathrm{d}x' k(x')}\left(\pm i - \frac{1}{2}\frac{k'(x)}{k^2(x)}\right)\right|^2_{x=0}, \tag{39}$$

the last term in parentheses being much smaller than 1. The *maximum* value attainable by S_k is at the border of the WKB approximation limit, *i.e.* for $k \sim \epsilon$, Eq. (32), and we have

$$S \sim \frac{1}{\epsilon} = \frac{a}{a_B^{\star}} = \frac{a}{a_B}\beta. \tag{40}$$

In the limit $\epsilon \to 0$, the Schrödinger equation (23) is solved analytically. The 'in' wavefunction in the continuous spectrum of the attractive Coulomb field is given by

$$\psi_k^{(+)} = e^{\pi k/2}\Gamma(1-i/k)e^{i\mathbf{k}\cdot\mathbf{r}}F(i/k, 1, ikr - i\mathbf{kr}) \tag{41}$$

where $F = {}_1F_1$ is the Kummer function (hypergeometric confluent). Here $\mathbf{k}\cdot\mathbf{r}$ corresponds to $mv \times r$, measured in units $1/m$. Thus it is the nondimensional quantity v/α. The same would hold writing $kr = (k/\alpha m)(\alpha m r)$.

In these respects $k/\alpha m \to k$ is dimensionless, $k = v/\alpha$, and we understand the factor $e^{\pi k/2}$, or the term $\Gamma = (1-i/k)$. The $k = v/\alpha$ appears in the Schrödinger equation (23).

The action of the attractive Coulomb field on the motion of the particle near the origin can be characterized by the ratio of the square modulus of $\psi^{(+)}(0)$ to the square modulus of the wavefunction for free motion, $\psi_k(r) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$. Using $\Gamma^*(z) = \Gamma(z^*)$, $F(i/k, 1, 0) = 1$ and

$$\Gamma(1+i/k)\Gamma(1-i/k) = \frac{\pi}{k\sinh(\pi/k)}, \tag{42}$$

we get the result

$$S = S_0 = |\psi_k^{(+)}(0)|^2 = \frac{2}{k(1-e^{-2\pi/k})} \approx \frac{2\pi}{k} = \frac{2\pi\alpha}{v} \tag{43}$$

for small velocities [2,4,5].

[5] In the conventions of [8], $A = 2$.

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