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Energy loss of a heavy particle near 3D charged rotating hairy black hole

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Abstract In this paper we consider a charged rotating black hole in three dimensions with a scalar charge and discuss the energy loss of a heavy particle moving near the black-hole horizon. We also study quasi-normal modes and find the dispersion relations. We find that the effect of scalar charge and electric charge increases the energy loss.

1 Introduction

The lower-dimensional theories may be used as toy models to study some fundamental ideas which yield a better understanding of higher-dimensional theories, because they are easier to study [1]. Moreover, these are useful for application of the AdS/CFT correspondence [2–5]. This paper is indeed an application of the AdS/CFT correspondence to probe a moving charged particle near the three-dimensional black holes recently introduced by Refs. [6] and [7], where a charged black hole with a scalar hair in (2+1) dimensions and a rotating hairy black hole in (2+1) dimensions were constructed, respectively. Here, we are interested in the case of a rotating black hole with a scalar hair in (2+1) dimensions. Recently, a charged rotating hairy black hole in three dimensions corresponding to infinitesimal black-hole parameters was constructed [8] and this will be used in this paper. Also, the thermodynamics of such systems was recently studied by Refs. [9] and [10].

In this work we would like to study the motion of a heavy charged particle near the black-hole horizon and calculate the energy loss. The energy loss of a moving heavy charged particle through a thermal medium is known as the drag force. One can consider a moving heavy particle (such as a charm and bottom quark) near the black-hole horizon with a momentum P, mass m, and constant velocity v, which is influenced by an external force F. Thus, one can write the equation of motion as $\dot{P} = F - \zeta P$, where in the non-relativistic motion $P = mv$, and in the relativistic motion $P = mv/\sqrt{1-v^2}$, and ζ is called the friction coefficient. To obtain the drag force, one can consider two special cases. The first case is the constant momentum which yields $F = (\zeta m)v$ for the non-relativistic case. In this case the drag force coefficient (ζm) will be obtained. In the second case, the external force is zero, so one can find $P(t) = P(0)\exp(-\zeta t)$. In other words, by measuring the ratio $\dot{P}/P$ or $\dot{v}/v$ one can determine the friction coefficient ζ without any dependence on the mass m. These methods enable us to obtain the drag force for a moving heavy particle. The moving heavy particle in the context of QCD has a dual picture in string theory, in which an open string is attached to the D-brane and stretched to the horizon of the black hole.

Similar studies have already been performed in several backgrounds [11–22]. Now, we consider the same problem against a charged rotating hairy 3D background. Our motivation for this study is the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence [23–25].

This paper is organized as follows. In the next section, we review a charged rotating hairy black hole in (2+1) dimensions. In Sect. 3, we obtain the equation of motion and in Sect. 4 we try to obtain a solution and discuss the energy loss. In Sect. 5, we give a linear analysis and discuss quasi-normal modes and the dispersion relations. Finally in Sect. 6, we summarize our results and give the conclusion.

2 Charged rotating hairy black hole in (2+1) dimensions

The (2+1)-dimensional gravity with a non-minimally coupled scalar field is described by the following action:

$$S = \frac{1}{2}\int d^3x\sqrt{-g}\left[R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \xi R\phi^2 -2V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \quad (1)$$

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where ξ is a coupling constant between gravity and the scalar field, which will be fixed as $\xi = 1/8$, and $V(\phi)$ is the self-coupling potential. The metric background of this is given by Ref. [7],

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\psi + \omega(r)\,dt)^2, \tag{2}$$

where [8]

$$f(r) = 3\beta - \frac{Q^2}{4} + \left(2\beta - \frac{Q^2}{9}\right)\frac{B}{r} - Q^2\left(\frac{1}{2} + \frac{B}{3r}\right)\ln(r) + \frac{(3r+2B)^2a^2}{r^4} + \frac{r^2}{l^2} + \mathcal{O}(a^2Q^2) \tag{3}$$

where Q is the infinitesimal electric charge, a is the infinitesimal rotational parameter, and l is related to the cosmological constant by $\Lambda = -\frac{1}{l^2}$. Also, β is the integration constant and depends on the black-hole charge and mass as follows:

$$\beta = \frac{1}{3}\left(\frac{Q^2}{4} - M\right), \tag{4}$$

and the scalar charge B is related to the scalar field as follows:

$$\phi(r) = \pm\sqrt{\frac{8B}{r+B}}. \tag{5}$$

The rotational frequency is obtained as follows:

$$\omega(r) = -\frac{(3r+2B)a}{r^3}, \tag{6}$$

and

$$V(\phi) = \frac{2}{l^2} + \frac{1}{512}\left[\frac{1}{l^2} + \frac{\beta}{B^2} + \frac{Q^2}{9B^2}\left(1 - \frac{3}{2}\ln\left(\frac{8B}{\phi^2}\right)\right)\right]\phi^6 + \mathcal{O}(Q^2a^2\phi^8). \tag{7}$$

Also one obtains the following Ricci scalar:

$$R = -\frac{36r^6 - 3l^2Q^2r^4 + 2Bl^2Q^2r^3 + 216Bl^2a^2r + 180l^2a^2B^2}{6l^2r^6}, \tag{8}$$

which is singular at $r = 0$. Finally, in Ref. [8] it is found that

$$r_h^2 = \frac{B}{3Q^2}\left(\frac{7}{6}Q^2 - 2M\right)\left(-1 + \sqrt{1 + \frac{216a^2Q^2}{B(\frac{7}{6}Q^2 - 2M)^2}}\right), \tag{9}$$

where r_h is the radius of the black-hole horizon.

Finally, the black-hole temperature and entropy are obtained by the following relations:

$$T = \frac{f'(r_h)}{4\pi}, \quad s = 4\pi r_h. \tag{10}$$

3 The equations of motion

The moving heavy particle near a black hole may be described by the following Nambu–Goto action:

$$S = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-G}, \tag{11}$$

where $T_0 = \frac{1}{2\pi\alpha'}$ is the string tension. The coordinates τ and σ correspond to the string world-sheet. Also, G_{ab} is the induced metric on the string world-sheet with determinant G obtained as follows:

$$G = -1 - r^2f(r)(x')^2 + \frac{r^2}{f(r)}(\dot{x})^2, \tag{12}$$

where we used the static gauge in which $\tau = t$, $\sigma = r$, and the string only extends in one direction: $x(r,t)$. Then the equation of motion is obtained as follows:

$$\partial_r\left(\frac{r^2f(r)x'}{\sqrt{-G}}\right) - \frac{r^2}{f(r)}\partial_t\left(\frac{\dot{x}}{\sqrt{-G}}\right) = 0. \tag{13}$$

We should obtain the canonical momentum densities associated with the string as follows:

$$\begin{aligned}
\pi_\psi^0 &= \frac{1}{2\pi\alpha'\sqrt{-G}}\frac{r^2}{f(r)}\dot{x},\\
\pi_r^0 &= -\frac{1}{2\pi\alpha'\sqrt{-G}}\frac{r^2}{f(r)}\dot{x}x',\\
\pi_t^0 &= -\frac{1}{2\pi\alpha'\sqrt{-G}}(1 + r^2f(r)(x')^2),\\
\pi_\psi^1 &= \frac{1}{2\pi\alpha'\sqrt{-G}}r^2f(r)x',\\
\pi_r^1 &= -\frac{1}{2\pi\alpha'\sqrt{-G}}\left(1 - \frac{r^2}{f(r)}\dot{x}^2\right),\\
\pi_t^1 &= \frac{1}{2\pi\alpha'\sqrt{-G}}r^2f(r)\dot{x}x'.
\end{aligned} \tag{14}$$

The simplest solution of the equation of motion is a static string, described by $x = const.$, with a total energy of the form

$$E = -\int_{r_h}^{r_m}\pi_t^0\,dr = \frac{1}{2\pi\alpha'}(r_h - r_m) = M_{\text{rest}}, \tag{15}$$

where r_m is an arbitrary location of the D-brane. As we expect, the energy of the static particle can be interpreted as the rest mass.

4 Time-dependent solution

In the general case, we can assume that a particle moves with constant speed $\dot{x} = v$; in that case the equation of motion (13) reduces to

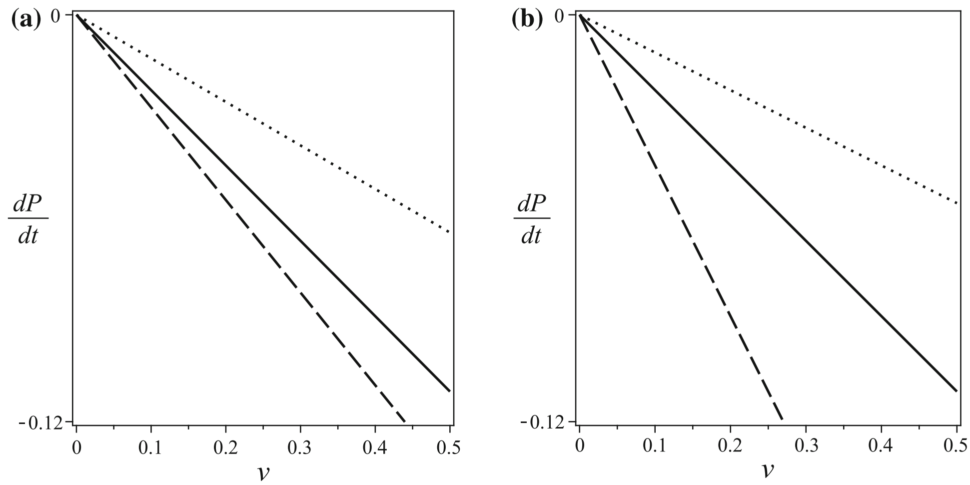

Fig. 1 Drag force in terms of v for $M = 1$. **a** $B = a = 1$, $Q = 1.6$ (*dotted line*), $Q = 2$ (*solid line*), and $Q = 2.4$ (*dashed line*). **b** $a = 1$, $Q = 2$, $B = 0.5$ (*dotted line*), and $B = 1$ (*solid line*), $B = 2$ (*dashed line*)

$$\partial_r\left(\frac{r^2 f(r) x'}{\sqrt{-G}}\right) = 0, \tag{16}$$

where

$$G = -1 - r^2 f(r)(x')^2 + \frac{r^2}{f(r)}v^2. \tag{17}$$

Equation (16) gives the following expression:

$$(x')^2 = \frac{C^2(r^2v^2 - f(r))}{r^2 f(r)^2 (C^2 - r^2 f(r))}, \tag{18}$$

where C is an integration constant which will be determined by using the reality condition of $\sqrt{-G}$. Therefore, we find the following canonical momentum densities:

$$\begin{aligned}\pi_\psi^1 &= -\frac{1}{2\pi\alpha'}C,\\ \pi_t^1 &= \frac{1}{2\pi\alpha'}Cv.\end{aligned} \tag{19}$$

Canonical momentum densities given by the equation (19) gives the following expressions,

$$\begin{aligned}\frac{\mathrm{d}P}{\mathrm{d}t} &= \pi_\psi^1|_{r=r_h} = -\frac{1}{2\pi\alpha'}C,\\ \frac{\mathrm{d}E}{\mathrm{d}t} &= \pi_t^1|_{r=r_h} = \frac{1}{2\pi\alpha'}Cv.\end{aligned} \tag{20}$$

As mentioned before, the reality condition of $\sqrt{-G}$ gives constant C. The expression $\sqrt{-G}$ is real for $r = r_c > r_h$. In the case of small v one obtains

$$r_c = r_h + \left.\frac{r^2v^2}{f(r)'}\right|_{r=r_h} + \mathcal{O}(v^4), \tag{21}$$

which yields

$$C = vr_h^2 + \mathcal{O}(v^3). \tag{22}$$

Therefore we can write the drag force as follows:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{vr_h^2}{2\pi\alpha'} + \mathcal{O}(v^3). \tag{23}$$

In Fig. 1 we can see the behavior of the drag force with the black-hole parameters. We draw the drag force in terms of velocity and, as expected, the value of the drag force is increased by v. Figure 1a, b shows that the black hole electric charge as well as the scalar charge increases the value of the drag force. We find a lower limit for the black-hole charge, which is for example $Q \geq 1.4$ corresponding to $M = a = B = 1$. In this case we find that a slow rotational motion has many infinitesimal effects on the drag force, which may be negligible.

5 Linear analysis

Because of the drag force, the motion of the string yields a small perturbation at a late time. In that case, the speed of the particle is infinitesimal and one can write $G \approx -1$. Also, we assume that $x = \mathrm{e}^{-\mu t}$, where μ is the friction coefficient. Therefore one can rewrite the equation of motion as follows:

$$\frac{f(r)}{r^2}\partial_r(r^2 f(r) x') = \mu^2 x. \tag{24}$$

We assume outgoing boundary conditions near the black-hole horizon and use the following approximation:

$$(4\pi T)^2(r - r_h)\partial_r(r - r_h)x' = \mu^2 x. \tag{25}$$

This suggests the following solutions:

$$x = c(r - r_h)^{-\frac{\mu}{4\pi T}}, \tag{26}$$

where T is the black-hole temperature. In the case of infinitesimal μ, we can use the following expansion:

$$x = x_0 + \mu^2 x_1 + \cdots. \tag{27}$$

Inserting this equation in (26) gives $x_0 = const.$ and

$$x_1' = \frac{A}{r^2 f(r)}\int_{r_h}^{r_m}\frac{r^2}{f(r)}\,\mathrm{d}r, \tag{28}$$

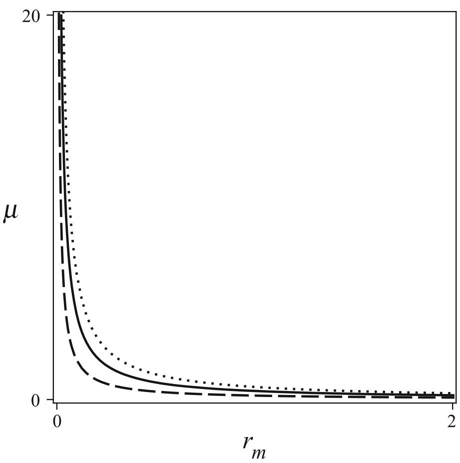

Fig. 2 μ in terms of r_m. $B = Q = a = 1.6$ (*dashed line*), and $B = Q = a = 2$ (*solid line*), $B = Q = a = 2.4$ (*dotted line*)

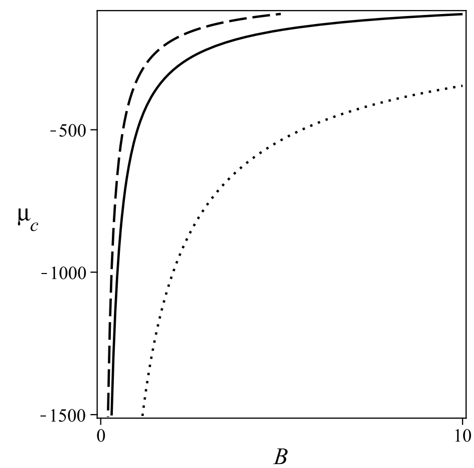

Fig. 3 μ_c in terms of B for $M = 1, l = 1$ and $a = 2$. $Q = 1.6$ (*dotted line*), $Q = 2$ (*solid line*), and $Q = 2.4$ (*dashed line*)

where A is a constant. Assuming the near-horizon limit enables us to obtain the following solution:

$$x_1 \approx \frac{A}{4\pi T r_h^2 (r - r_h)} \left(-r_m + \frac{r_h^2}{4\pi T} \ln (r - r_h) \right). \tag{29}$$

Comparing (26) and (28) gives the following quasi-normal mode condition:

$$\mu = \frac{r_h^2}{r_m}. \tag{30}$$

It is interesting to note that these results recover the drag force (23) for infinitesimal speed. In Fig. 2 we can see the behavior of μ with the black-hole parameters. We find that the black-hole charges increase the value of the friction coefficient.

5.1 Low-mass limit

The low-mass limit means that $r_m \to r_h$, and we use the following assumptions:

$$f(r) \approx 4\pi T (r - r_h) \tag{31}$$

and

$$r^2 = r_h^2 + 2r_h (r - r_h) + \cdots , \tag{32}$$

so by using the relation (24) we can write

$$x(r) = (r - r_h)^{-\frac{\mu}{4\pi T}} (1 + (r - r_h)A + \cdots). \tag{33}$$

We can obtain the constant A as follows:

$$A = \frac{\mu}{2\pi T r_h - \mu r_h}. \tag{34}$$

It tells us that $\mu = 2\pi T$ yields a divergence; therefore we call this the critical behavior of the friction coefficient. We obtain Fig. 3.

5.2 Dispersion relations

Here, we would like to obtain the relation between the total energy E and the momentum P in the low-velocity limit. In that case we obtain

$$\pi_\psi^0 = -\frac{\mu}{2\pi\alpha'} \frac{r^2}{f(r)} = -\frac{1}{2\pi\alpha' \mu} \partial_r (r^2 f(r) x'), \tag{35}$$

which gives the total momentum

$$P = \int \pi_\psi^0 \, \mathrm{d}r = \frac{1}{2\pi\alpha' \mu} \left(r_{\min}^2 f(r_{\min}) x'(r_{\min}) \right), \tag{36}$$

where we use $r_{\min} > r_h$ as an IR cutoff to avoid a divergency. In a similar way we can compute the other momentum density,

$$\pi_t^0 = -\frac{\mu}{2\pi\alpha'} \left[1 + \frac{1}{2} r^2 f(r) (x')^2 + \frac{1}{2} \frac{r^2}{f(r)} v^2 \right], \tag{37}$$

to evaluate the total energy as follows:

$$E = -\int \pi_t^0 \, \mathrm{d}r = \frac{1}{2\pi\alpha'} \left(r_m - r_{\min} - r_{\min}^2 f(r_{\min}) x'(r_{\min}) x(r_{\min}) \right), \tag{38}$$

where we used the equation of motion and the boundary condition $x'(r_m) = 0$. Assuming the following near-horizon solution:

$$x \sim (r - r_h)^{-\frac{\mu}{4\pi T}} \tag{39}$$

and combining Eqs. (36) and (38) give the following relation:

$$E = M_{\text{rest}} + \frac{P^2}{2M_{\text{kin}}}, \tag{40}$$

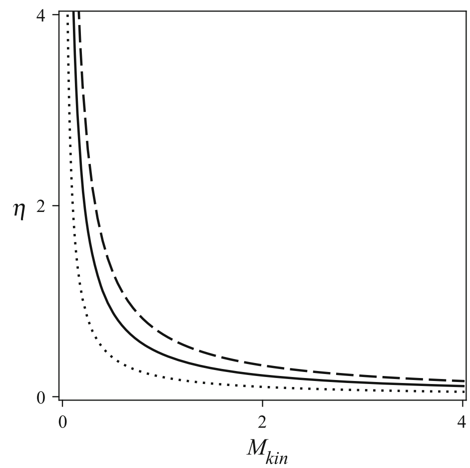

Fig. 4 η in terms of M_{kin}. $Q = B = 1.6$ (*dotted line*), $Q = B = 2$ (*solid line*), and $Q = B = 2.4$ (*dashed line*)

where $M_{\rm rest}$ is given by Eq. (15) with the replacement $r_h \to r_{\rm min}$ and

$$M_{\rm kin} \equiv \frac{r_h^2}{2\pi\alpha'\mu}. \tag{41}$$

It is the usual, non-relativistic dispersion relation for a point particle in which the rest mass is different from the kinetic mass. In Fig. 4, we draw the re-scaled $\eta \equiv 2\pi\alpha'\mu$ in terms of kinetic mass and show that the black-hole charges increase η as expected.

6 Conclusions

In this work we considered the recently constructed charged rotating black hole in three dimensions with a scalar charge and calculated the energy loss of a heavy particle moving near the black-hole horizon. First of all, the important properties of the background were reviewed and then appropriate equations obtained. We have as motivation the AdS/CFT correspondence, and we use a string theory method to study the motion of the particle. This is indeed in the context of $\mathrm{AdS}_3/\mathrm{CFT}_2$, where the drag force on the moving heavy particle is calculated. We found that the black-hole charges, both electric and scalar, increase the value of the drag force, but the infinitesimal value of the rotation parameter has no important effect and may be negligible. We also discussed the quasinormal modes, obtained the friction coefficient and found that black-hole charges increase the value of the friction coefficient, which coincides with increasing the drag force. Finally, we found the dispersion relation between the total energy and the momentum of the particle.

References

1. M. Hortacsu, H.T. Ozcelik, B. Yapiskan, Properties of solutions in 2+1 dimensions. Gen. Relat. Gravit. **35**, 1209 (2003)
2. M. Henneaux, C. Martinez, R. Troncoso, J. Zanelli, Black holes and asymptotics of 2+1 gravity coupled to a scalar field. Phys. Rev. D **65**, 104007 (2002)
3. M. Hasanpour, F. Loran, H. Razaghian, Gravity/CFT correspondence for three dimensional Einstein gravity with a conformal scalar field. Nucl. Phys. B **867**, 483 (2013)
4. D.F. Zeng, An exact hairy black hole solution for AdS/CFT superconductors [arXiv:0903.2620 [hep-th]]
5. B. Chen, Z. Xue, J. Ju Zhang, Note on thermodynamic method of black hole/CFT correspondence. JHEP **1303**, 102 (2013)
6. W. Xu, L. Zhao, Charged black hole with a scalar hair in (2+1) dimensions. Phys. Rev. D **87**, 124008 (2013)
7. L. Zhao, W. Xu, B. Zhu, Novel rotating hairy black hole in (2+1)-dimensions [arXiv:1305.6001 [gr-qc]]
8. J. Sadeghi, B. Pourhassan, H. Farahani, Rotating charged hairy black hole in (2+1) dimensions and particle acceleration, [arXiv:1310.7142 [hep-th]]
9. A. Belhaj, M. Chabab, H. EL Moumni, M.B. Sedra, Critical behaviors of 3D black holes with a scalar hair [arXiv:1306.2518 [hep-th]]
10. J. Sadeghi, H. Farahani, Thermodynamics of a charged hairy black hole in (2+1) dimensions [arXiv:1308.1054 [hep-th]]
11. C. Hoyos-Badajoz, Drag and jet quenching of heavy quarks in a strongly coupled N = 2* plasma. JHEP **0909**, 068 (2009) [arXiv:0907.5036 [hep-th]]
12. J. Sadeghi, B. Pourhassan, Drag force of moving quark at the $\mathcal{N} = 2$ supergravity. JHEP **0812**, 026 (2008). [arXiv:0809.2668 [hep-th]]
13. C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, L.G. Yaffe, Energy loss of a heavy quark moving through $\mathcal{N} = 4$ supersymmetric Yang–Mills plasma. JHEP **0607**, 013 (2006) [arXiv:hep-th/0605158]
14. J. Sadeghi, B. Pourhassan, S. Heshmatian, Application of AdS/CFT in quark-gluon plasma. Adv. High Energy Phys. **2013**, 759804 (2013)
15. C.P. Herzog, Energy loss of heavy quarks from asymptotically AdS geometries. JHEP **0609**, 032 (2006). [arXiv:hep-th/0605191]
16. S.S. Gubser, Drag force in AdS/CFT. Phys. Rev. **D74**, 126005 (2006)
17. E. Nakano, S. Teraguchi, W.Y. Wen, Drag force, jet quenching, and AdS/QCD. Phys. Rev. D **75**, 085016 (2007)
18. E. Caceres, A. Guijosa, Drag force in charged $\mathcal{N} = 4$ SYM plasma. JHEP **0611**, 077 (2006)
19. J.F. Vazquez-Poritz, Drag force at finite 't Hooft coupling from AdS/CFT [arXiv:0803.2890 [hep-th]]
20. A.N. Atmaja, K. Schalm, Anisotropic drag force from 4D Kerr-AdS black holes [arXiv:1012.3800 [hep-th]]
21. B. Pourhassan, J. Sadeghi, STU/QCD correspondence. Can J. Phys. [arXiv:1205.4254 [hep-th]]
22. E. Caceres, A. Guijosa, On drag forces and jet quenching in strongly coupled plasmas. JHEP **0612**, 068 (2006)

23. P. Kraus, Lectures on black holes and the AdS3/CFT2 correspondence. Supersymmetric mechanics, vol. 3. Lecture Notes in Physics, vol. 755, p. 1 (2008)
24. R. Borsato, O.O. Sax, A. Sfondrini, All-loop Bethe ansatz equations for AdS3/CFT2. JHEP **1304**, 116 (2013)
25. D. Momeni, M. Raza, M.R. Setare, R. Myrzakulov, Analytical holographic superconductor with backreaction using AdS3/CFT2. Int. J. Theor. Phys. **52**, 2773 (2013)