Eur. Phys. J. C (2014) 74:2680
DOI 10.1140/epjc/s10052-013-2680-7

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Thermodynamics of string black hole with hyperscaling violation

J. Sadeghi[1,2,3,a], **B. Pourhassan**[1,b], **A. Asadi**[2,c]

[1] Young Researchers Club, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran
[2] Department of Physics, Mazandaran University, Babolsar, Iran
[3] Department of Physics, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran

Received: 8 September 2012 / Accepted: 29 November 2013 / Published online: 23 January 2014

Abstract In this paper, we start with a black brane and construct a specific space-time which violates hyperscaling. To obtain the string solution, we apply the Null-Melvin Twist and *KK* reduction. Using the difference action method, we study the thermodynamics of the system to obtain a Hawking–Page phase transition. To have hyperscaling violation, we need to consider $\theta = \frac{d}{2}$. In this case, the free energy F is always negative and our solution is thermal radiation without a black hole. Therefore, we find that there is no Hawking–Page transition. Also, we discuss the stability of the system and all thermodynamical quantities.

1 Introduction

As is well known, the AdS/CFT correspondence provides an analytic approach to the study of strongly coupled field theory [1–4]. Recently, there have appeared several papers on the development of AdS gravity theories and their conformal field theory dual. In that case the metric background is generalized and the result is dual to scale-invariant field theories instead of conformally invariant. The scale invariance is provided by the dynamical critical exponent $z \neq 1$ ($z = 1$ corresponds to the case of the AdS metric) on the following metric [8]:

$$\mathrm{d}s^2 = -\frac{1}{r^{2z}}\mathrm{d}t^2 + \frac{1}{r^2}\left(\mathrm{d}r^2 + \mathrm{d}x_i^2\right). \tag{1}$$

The corresponding metric will be invariant under the following scale transformation:

$$t \to \lambda^z t, \qquad x_i \to \lambda x_i, \qquad r \to \lambda r. \tag{2}$$

The resulting metric may be a solution of field equations with theories with coupling to matter with negative cosmological constant which also include an abelian field in the bulk. Space-time metrics that transform covariantly under dilatation have recently been reinterpreted as a holography dual to a stress tensor of quantum field theories which violates hyperscaling [5–7]. Recently, the large class of scaling metrics containing an abelian gauge field and scalar dilaton has been considered [8–22], which is presented by the following equation [13]:

$$\mathrm{d}s^2 = r^{-2(d-\theta)/d}\left(r^{-2(z-1)}\mathrm{d}t^2 + \mathrm{d}r^2 + \mathrm{d}x_i^2\right), \tag{3}$$

where θ is the hyperscaling violation exponent. Note that this metric is not invariant under a scale transformation (2), but transforms covariantly as

$$\mathrm{d}s = \lambda^{\theta/d}\mathrm{d}s, \tag{4}$$

which defines the property of hyperscaling violation in the language of holography. The corrections of the conformal hyperscaling relation from the conformal point of view in the case of large N_f QCD as a concrete dynamical model are given in Ref. [23]. Such examples show that QCD can be a candidate for the use of hyperscaling. On the other hand, we have a strong motivation to study a metric with hyperscaling violation. As we know, scale invariance is broken under quantum effects in some theories, e.g. QCD. Especially on a large scale or at low energy, scale invariance is broken in massive theories. In that case, using an AdS metric which has scale invariance is not appropriate. For example, we have some problems in calculating the form factor or quantum mass spectrum in QCD. Therefore, it is necessary to modify the original metric [24]. Instead of a modification of the AdS metric, it is appropriate to choose a suitable metric, such as a hyperscale metric which has scale violation. In case of a large scale (or $r \to \infty$), there are good applications of a metric with hyperscaling violation in QCD or string theory. In this paper we use the metric of Ref. [8] to obtain a string solution. Next we discuss physical properties (especially thermodynamics) of the metric mentioned to verify the validity of working on the basis of our motivation. If our solution will

[a] e-mail: pouriya@ipm.ir
[b] e-mail: b.pourhassan@umz.ac.ir
[c] e-mail: ali.asadi89@stu.umz.ac.ir

coincide with known physical rules, then one can use the hyperscale metric instead of AdS metric for future work.

An important concept in our study is the Galilean holography, which is developed in Refs. [25,26], where the non-relativistic generalizations of the AdS/CFT correspondence are extracted. An expansion of the Galilean algebra can be obtained by adding a dilation operator and a special conformal transformation identically to the time and space scale. A discussion of the non-relativistic conformal symmetry generalization which is known as the Schrödinger generalization has been explained in Ref. [3]. In this discussion, the time and space geometry of d dimensions with isometry group has Schrödinger symmetry and has been established over the AdS/CFT correspondence. One has suggested that gravity is the holographic dual of the non-relativistic conformal field theories at strong couplings. The next development of Galilean holography is the finite-temperature generalization [27–29]. In the AdS/CFT correspondence of finite-temperature, planar black-brane solutions were suggested in the Schrödinger space as the holographic dual of the non-relativistic conformal field theory at finite temperature. The investigation of AdS_5 geometry near the horizon of a D3-brane in flat space is investigated in [27,28]. Then, the known Null-Melvin-Twist (NMT) [30–32] is applied to this system. Ref. [29] started with a solution of asymptotical black-hole metrics which leads to the string solution. This characterizes the specific non-relativistic conformal field theories to which they are dual. An analysis of these black-hole space-time thermodynamics shows that they describe the dual conformal field theory at finite temperature and finite density. It has been shown that, after doing NMT by applying a KK reduction over the S^5 geometry, the result is an extremal black brane; also the asymptotic limit is reduced to Schrödinger geometry. The thermodynamic solutions of such a black hole are discussed in Refs. [27,28].

The new regularization method has been suggested by Ref. [33]; it is the oldest regularization method [34,35] with some modification which involves subtraction with an unusual boundary matching.

In the recent work [36], the thermodynamics of the Schrödinger black holes with hyperscaling violation has been studied. Some overlaps may be found between our work and the paper mentioned; however, we should note that our system is completely different, and application of hyperscaling violation in both systems yields independent results, which are interesting in themselves. While the primary metrics of the two papers are similar, we use the NMT method to obtain a string solution and also discuss the phase transition and the thermodynamical stability.

This paper is organized as follows. In the next section, we begin with a black-brane metric and present the corresponding metric which violates hyperscaling. In that case, we apply NMT and a KK reduction to obtain the string solution of this geometry. In Sect. 3, we use the difference action method and extract the thermodynamics of the system in Sect. 4 and discuss the Hawking–Page phase transition and the thermodynamical stability. In Sect. 5, we summarize our results.

2 String black brane

Now, we consider the non-extremal D3-brane geometry [33] near horizon, which is obtained by the following action [25, 28]:

$$\begin{aligned} \mathrm{d}s^2 &= \left(\frac{r}{R}\right)^2\left(-f\mathrm{d}t^2+\mathrm{d}y^2+\mathrm{d}x_i^2\right) \\ &\quad + \left(\frac{R}{r}\right)^2 f^{-1}\mathrm{d}r^2 + R^2\mathrm{d}\Omega_5^2, \\ &\quad \phi=0, \qquad B=0, \qquad f(r)=1-\left(\frac{r_{\mathrm{H}}}{r}\right)^4, \end{aligned} \tag{5}$$

where R is the AdS scale, $x_i=(x_1,x_2)$, and $r=r_{\mathrm{H}}$ is the location of the horizon, so the metric at $r_{\mathrm{H}}=0$ reduces to the extremal case. ϕ is the dilaton and B is an $NS-NS$ two-form. A particularly convenient choice for $\mathrm{d}\phi$ is given by a Hopf fibration, $s^1 \rightarrow s^5 \rightarrow p^2$, with the following metric:

$$\mathrm{d}\Omega_5^2 = \mathrm{d}s_{p^2}^2 + (\mathrm{d}\chi + \mathcal{A}), \tag{6}$$

where χ is the local coordinate on the Hopf fiber and $\mathcal{A}$ is the one-form on P^2; $\mathrm{d}s_{p^2}^2$ is the metric on P^2 [29]. We need to consider two isometry directions, $\mathrm{d}y$ and $\mathrm{d}\phi$, for the Melvinization process, where $\mathrm{d}y$ is along the world-volume, $\mathrm{d}\phi$ is along the S^5, and y is one of the three spatial coordinates. Now, we begin with the metric (5) including hyperscaling violation in the black-hole solution according to Ref. [8]:

$$\begin{aligned} &\mathrm{d}s_{d+2}^2 \\ &= \left(\frac{r}{R}\right)^2\left(\frac{r_F}{r}\right)^{2\theta/d}\left(-\left(\frac{R^2}{r}\right)^{-2(z-1)} f\mathrm{d}t^2+\mathrm{d}y^2+\mathrm{d}x_i^2\right) \\ &\quad + \left(\frac{R}{r}\right)^2\left(\frac{r_F}{r}\right)^{2\theta/d}\left(\frac{\mathrm{d}r^2}{f}+r^2\mathrm{d}\Omega_5^2\right), \\ &\quad f=1-\left(\frac{r_{\mathrm{H}}}{r}\right)^{d+z-\theta}, \end{aligned} \tag{7}$$

where $d=3$, and r_F is the scale which is obtained from dimensional analysis [8]. Finite temperature effects in theories with hyperscaling violation are studied; in that case as regards gravity, we have $r_F < r_h$. From the null energy condition (NEC), $T_{\mu\nu}n^\mu n^\nu \geq 0$ [8,37], and the null vectors satisfy the $n^\mu n^\nu = 0$ condition. The above conditions lead to the following relations:

$$\begin{aligned} &(d-\theta)(d(z-1)-\theta) \geq 0, \\ &(z-1)(d+z-\theta) \geq 0. \end{aligned} \tag{8}$$

To match the following results with Eq. (4), we need to consider $z = 1$ (because $z = 1$ in the $\theta \to 0$ limit gives the AdS metric). From the first relation of (8) one obtains

$$(\theta \le 0,\ d \ge \theta), \quad \text{or} \quad (\theta \ge 0,\ d \le \theta). \tag{9}$$

Now, we apply NMT to the metric (7) with $z = 1$ and obtain

$$\begin{aligned}
&\mathrm{d}s^2_{d+2} \\
&= K^{-1}\left(\frac{r}{R}\right)^2 M\Big[-(1+b^2r^2M^2)f\mathrm{d}t^2 - 2b^2r^2fM^2\mathrm{d}t\,\mathrm{d}y \\
&\quad + (1-b^2r^2fM^2)\mathrm{d}y^2 + K\mathrm{d}x_i^2\Big], \\
&\quad + M\left(\frac{R}{r}\right)^2 f^{-1}\mathrm{d}r^2 + MK^{-1}R^2\eta^2 + MR^2\mathrm{d}s^2_{p^2},
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
&\phi = -\frac{1}{2}\ln K, \\
&B = \frac{M^2}{K}\left(\frac{r}{R}\right)^2 b(f\mathrm{d}t + \mathrm{d}y)\wedge\eta, \\
&K = 1-(f-1)br^2M^2,
\end{aligned} \tag{11}$$

where $\eta = (\mathrm{d}\chi + \mathcal{A})$, $M = \left(\frac{r_F}{r}\right)^{(2\theta/d)}$, and b has dimension $[L^{-1}]$. If we perform the KK reduction on S^5 for the non-extremal solution (10), we obtain

$$\begin{aligned}
&\mathrm{d}s^2_{d+2} \\
&= K^{-2/3}\left(\frac{r}{R}\right)^2 M\Big[-(1+b^2r^2M^2)f\mathrm{d}t^2 - 2b^2r^2fM^2\mathrm{d}t\,\mathrm{d}y \\
&\quad + (1-b^2r^2fM^2)\mathrm{d}y^2 + K\mathrm{d}x_i^2\Big] \\
&\quad + K^{1/3}M\left(\frac{R}{r}\right)^2 f^{-1}\mathrm{d}r^2,
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
&\phi = -\frac{1}{2}\ln K, \\
&A = \frac{M^2}{K}\left(\frac{r}{R}\right)^2 b(f\mathrm{d}t + \mathrm{d}y),
\end{aligned} \tag{13}$$

where A is a one-form field in an Einstein frame. It is useful to work in the following light-cone coordinates:

$$x^+ = bR(t+y), \quad \text{and} \quad x^- = \frac{1}{2bR}(t-y). \tag{14}$$

Therefore, the solution is

$$\begin{aligned}
\mathrm{d}s^2_{d+2} = {} & K^{-2/3}\left(\frac{r}{R}\right)^2 M\Big[-\left(\frac{f-1}{(2bR)^2} - \left(\frac{r}{R}\right)^2 fM^2\right)\mathrm{d}x^{+2} \\
& -(1+f)\mathrm{d}x^+\mathrm{d}x^- + (bR)^2(1-f)\mathrm{d}x^{-2} \\
& + K\mathrm{d}x_i^2\Big] + K^{1/3}M\left(\frac{R}{r}\right)^2 f^{-1}\mathrm{d}r^2,
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
&\phi = -\frac{1}{2}\ln K, \\
&A = \frac{M^2}{K}\left(\frac{r}{R}\right)^2 b\left[\frac{f+1}{2bR}\mathrm{d}x^+ + bR(1-f)\mathrm{d}x^-\right].
\end{aligned} \tag{16}$$

Equation (15) is the same as Eq. (5) in Ref. [33] with additionally $M = \left(\frac{r_F}{r}\right)^{2\theta/d}$. By considering the x^+ coordinate as the time, the recently found metric under the scale transformation $x^+ \to \lambda^z x^+$, $x_i \to \lambda x_i$, $r \to \lambda^{-1} r$, $x_- \to \lambda^{2-z} x_-$ and with $d = 2\theta$ transforms covariantly as in Eq. (4), and it violates hyperscaling.

The extremal case comes from $f = 1$, and the non-extremal case approaches this at asymptotically large r. The last metric on the light-cone coordinates in Eq. (14) gives an extremal case which is independent of the parameter b. Therefore, b is unphysical and thus cannot lead to any physical quantity. One can interpret this result in the zero-temperature limit [29]. The metric background (15) is a solution of the effective action. In the non-extremal case, if $\theta = 0$, we have the following action [33]:

$$\begin{aligned}
S_5 = {} & \frac{1}{16\pi G_5}\int \mathrm{d}x^5\sqrt{-g}\Big[\mathcal{R} - \frac{4}{3}(\partial_\mu\phi)(\partial^\mu\phi) \\
& -\frac{1}{4}R^2 e^{-8\phi/3}F_{\mu\nu}F^{\mu\nu} - 4A_\mu A^\mu - \frac{V}{R^2}\Big],
\end{aligned} \tag{17}$$

where G_5, g, and R are the five-dimensional Newton constant, the determinant of the five-dimensional metric, and the scalar curvature, respectively. $F = \mathrm{d}A$ is a two-form field and the potential V is defined by the following expression:

$$V = 4e^{2\phi/3}(e^{2\phi} - 4). \tag{18}$$

By setting $\phi = 0$, the above action reduces to the extremal action [25]. As is well known, in case of $\theta \neq 0$ the shape of the action (17) will be conserved, but in this process the potentials V and the corresponding field ϕ will be changed. This is because K will be changed by the parameter θ.

The ADM form of metric is

$$\begin{aligned}
\mathrm{d}s^2_{d+2} = {} & K^{1/3}\left(\frac{r_F}{r}\right)^{(2\theta/d)}\left(\frac{R}{r}\right)^2 f^{-1}\mathrm{d}r^2 \\
& + K^{-2/3}\left(\frac{r}{R}\right)^2\left(\frac{r_F}{r}\right)^{(2\theta/d)} \\
& \times\left[K\mathrm{d}x_i^2 - \left(\frac{1}{(bR)^2(1-f)} + \left(\frac{r}{R}\right)^2\left(\frac{r_F}{r}\right)^{(4\theta/d)}\right)f\mathrm{d}x^{+2}\right] \\
& + K^{-2/3}\left(\frac{r}{R}\right)^2\left(\frac{r_F}{r}\right)^{(2\theta/d)} \\
& \times\left[(bR)^2(1-f)\left(\mathrm{d}x^- - \frac{(1+f)}{2(bR)^2(1-f)}\mathrm{d}x^+\right)^2\right].
\end{aligned} \tag{19}$$

Using the corresponding metric, we obtain the angular velocity of the horizon Ω_{H}, which is interpreted as the chemical potential associated with the conserved quantities along the

x^- direction,

$$\Omega_{\rm H} = \frac{1}{2(bR)^2}. \tag{20}$$

Note that we have mentioned two kinds of hypersurfaces; the time-like boundary at large fixed r and the space-like surface at fixed time x^+ whose time is described by the ADM form. In the extremal case, there is a problem with the g_{--} component in the calculation of the difference action; ($g_{--} = 0$).

3 The difference action

The metric (15) gives the extremal solution near the boundary (that is, large r) and this is interpreted as the finite-temperature generalization of the Galilean holography [27–29]. We want to consider the thermodynamics of this system in the case of a finite temperature. To calculate the thermodynamics, we use the difference action method [33–35].

In accordance with Ref. [33], first we analytically continue x^+ to ix^+ and put the system inside a box by the cutoff $r = r_{\rm B}$. The cutoff $r_{\rm B}$ is larger than the scale R, but it is finite. We subtract the action of the extremal solution from the non-extremal one. We note here that each action includes two terms, such as the bulk and the Gibbons–Hawking surface term. To proceed with such a process, we have to match the geometries of the metrics in the $r = r_{\rm B}$ wall. As mentioned earlier, the g_{--} component of the extremal case is degenerate in the metric (15), so we cannot match the metrics in the wall. To remove this problem, we match the boundary metric of the extremal geometry to the non-extremal one only for constant x^-. Thus, we appropriately rescale three-dimensional slices (x^+, x^i). We obtain the scaled extremal metric:

$$\begin{aligned} {\rm d}s^2_{d+2} = &\left(\frac{r}{R}\right)^2\left(\frac{r_F}{r}\right)^{(2\theta/d)}\left[\left(\frac{r}{R}\right)^2\left(\frac{r_F}{r}\right)^{(4\theta/d)} H_{\rm B}^2{\rm d}x^{+2}\right.\\ &\left. - 2iH_{\rm B}{\rm d}x^+{\rm d}x^- + G_{\rm B}^2{\rm d}x_i{}^2\right]\\ &+\left(\frac{R}{r}\right)^2\left(\frac{r_F}{r}\right)^{(2\theta/d)}{\rm d}r^2, \end{aligned} \tag{21}$$

$$\phi = 0,$$

$$A = i\left(\frac{r}{R}\right)^2\left(\frac{r_F}{r}\right)^{\frac{4\theta}{d}}\frac{H_{\rm B}}{R}{\rm d}x^+,$$

where

$$\begin{aligned} H_{\rm B} = &\left[K(r_{\rm B})^{-2/3}\left(\frac{f(r_{\rm B})-1}{(2bR)^2}\right.\right.\\ &\left.\left.+\left(\frac{r_{\rm B}}{R}\right)^2\left(\frac{r_F}{r_{\rm B}}\right)^{(4\theta/d)}f(r_{\rm B})\right)\right]^{1/2}\\ &\times\left(\frac{r_{\rm B}}{R}\right)^{-1}\left(\frac{r_F}{r_{\rm B}}\right)^{(-2\theta/d)},\\ G_{\rm B} = &\,K(r_{\rm B})^{1/6}. \end{aligned} \tag{22}$$

The difference action $(S - S_0)$ will be

$$S_0 = S_{0\rm bulk} + S_{0\rm GH}, \quad \text{and} \quad S = S_{\rm bulk} + S_{\rm GH}, \tag{23}$$

where both $S_{0\rm bulk}$ and $S_{\rm bulk}$ are the actions (17), but $S_{0\rm bulk}$ evaluates the extremal solution (21) and the $S_{\rm bulk}$ calculates the non-extremal solution (15). Also $S_{0\rm GH}$ and $S_{\rm GH}$ both are the Gibbons–Hawking surface term

$$S_{0\rm GH} = -\frac{1}{8\pi G_5}\int {\rm d}x^4\sqrt{g_{\rm B}}\,(TrK_0), \tag{24}$$

where $g_{\rm B}$ is the determinant of the boundary's first fundamental form, and (TrK_0) is the trace of the boundary's second fundamental form. We calculate the difference action in the limit of $r_{\rm B} \to \infty$, which is not divergent

$$\lim_{r_{\rm B}\to\infty}(S - S_0) = \frac{V_4}{16\pi G_5}\frac{r_{\rm H}^4}{R^5}\left(1-\frac{\theta}{d}\right)\left(\frac{r_F}{r_{\rm H}}\right)^{3\theta/d}, \tag{25}$$

where V_4 is the volume of four-dimensional space-time. It has been shown that this result agrees with Ref. [33] without hyperscaling violation.

4 Thermodynamics

Now we use the results of the previous section to study the thermodynamics of the system. In that case the Hawking temperature can be obtained from the surface gravity through $\beta = \frac{2\pi}{\kappa}$ where κ is the surface gravity,

$$\kappa^2 = -\frac{1}{2}\left(\nabla^a\xi^b\right)(\nabla_a\xi_b), \tag{26}$$

where ξ is the Killing vector field which is obtained by the following expression:

$$\xi = \frac{1}{bR}\frac{\partial}{\partial t} = \partial_+ + \Omega_{\rm H}\partial_-, \tag{27}$$

and the corresponding β is obtained by

$$\beta = \frac{4}{d+1-\theta}\frac{\pi bR^3}{r_{\rm H}}. \tag{28}$$

The Killing generator of the event horizon (27) not only has components along the boundary time translation direction x^+, but also along the light-like direction x^-. From the gravitational point of view, it is, therefore, a system with chemical potential for the x^- directions,

$$\mu = \frac{1}{2(bR)^2}. \tag{29}$$

To study the thermodynamics of the system, we use the following free energy [33,38,39]:

$$
\begin{aligned}
F &= -(16\pi G_5)V_3^{-1} \lim_{r_{\rm B}\to\infty}(S - S_0)\\
&= -\beta\left(\frac{r_{\rm H}^4}{R^5}\right)\left(1-\frac{\theta}{d}\right)\left(\frac{r_F}{r_{\rm H}}\right)^{3\theta/d}\\
&= -\frac{\pi^4 R^3}{4\mu^2\beta^3}\left(1-\frac{\theta}{d}\right)\left(\frac{\beta r_F}{\pi R^2}\right)^{3\theta/d}\left(\frac{4}{d+1-\theta}\right)^{4-3\theta/d}\\
&\quad\times(2\mu)^{3\theta/2d},
\end{aligned}
\tag{30}
$$

where V_3 is the integration over $x^{-,i}$, equal to $V_4\beta^{-1}$. So we obtain the entropy as

$$
\begin{aligned}
S &= \beta\left(\frac{\partial F}{\partial\beta}\right)_\mu - F\\
&= \frac{4\pi b r_{\rm H}^3}{R^2}\frac{(1-\frac{\theta}{d})}{(d+1-\theta)}\left(4-\frac{3\theta}{d}\right)\left(\frac{r_F}{r_{\rm H}}\right)^{3\theta/d}.
\end{aligned}
\tag{31}
$$

These equations, in the case of $\theta = 0$, agree with Ref. [33]. Also we obtain

$$
\begin{aligned}
E &= \left(\frac{\partial F}{\partial\beta}\right)_\mu - \mu\beta^{-1}\left(\frac{\partial F}{\partial\mu}\right)_\beta\\
&= \frac{r_{\rm H}^4}{R^5}\left(1-\frac{\theta}{d}\right)\left(1-\frac{3\theta}{2d}\right)\left(\frac{r_F}{r_{\rm H}}\right)^{3\theta/d},\\
Q &= -\beta^{-1}\left(\frac{\partial F}{\partial\mu}\right)_\beta\\
&= -\frac{4b^2 r_{\rm H}^4}{R^3}\left(1-\frac{\theta}{d}\right)\left(1-\frac{3\theta}{4d}\right)\left(\frac{r_F}{r_{\rm H}}\right)^{3\theta/d}.
\end{aligned}
\tag{32}
$$

In Eq. (30), we have two conditions for F, $F > 0$ and $F < 0$. In the case of $F < 0$, we have the two conditions $\theta < d$ and $\theta > d+1$, and in the case of $F > 0$ we have $d < \theta < d+1$. So, in the cases of $\theta = d$ and $\theta = d+1$ we have a Hawking–Page phase transition. As mentioned before, we take $\theta = d/2$, so we always have negative F. Thus, our solution is thermal radiation without a black hole and we do not have a Hawking–Page phase transition. As always, in order to calculate the stability of system, we need to obtain the Hessian of $\beta(E-\mu Q) - S$ with respect to the thermodynamic variables $(r_{\rm H}, b)$ and evaluate it at the on-shell values of (β, μ). In the case of $\theta = 0$, it recovers the results of Ref. [33]. In the case of a hyperscaling violation with the condition $\theta > \frac{1}{2}$, the results will be positive and the system is thermodynamically stable. We can check the first law of thermodynamics: $dE = T dS + \Omega_{\rm H} dQ$, which is satisfied by the above quantities.

5 Conclusion

In this paper, we considered the black-brane metric and presented the corresponding metric which violates hyperscaling. Using the difference action method, we obtained the thermodynamical quantities such as β, Q, S, E, and F. In the case of $F > 0$, we achieved two conditions: $\theta < d$ or $\theta > d+1$. Also for $F > 0$ we arrived at $d < \theta < d+1$. The above two conditions lead to a Hawking–Page phase transition ($\theta = d, \theta = d+1$). But in this paper, we always have negative F, because our condition was $\theta = \frac{d}{2}$; thus we have no such phase transition. Also, we discussed the stability of the system which agrees with Ref. [33] in the case $\theta = 0$. We have shown that in the case of hyperscaling violation we must have $\theta > \frac{1}{2}$, which is covered by our condition. In general, we can say that the system has thermodynamical stability. Therefore, one can use the hyperscale metric instead of the AdS metric to avoid technical problems in the boundary because of scale-symmetry breaking. In future work, we shall focus on this subject and use a hyperscale metric instead of an AdS metric to calculate the form factor in QCD. Finally, we verified that the first law of thermodynamics is valid.

References

1. J.M. Maldacena, The large N limit of superconformal field theories and supergravity. Adv. Theor. Math. Phys. **2**, 231 (1998). [Int. J. Theor. Phys. 1113 (1999)] [arXiv:hep-th/9711200]
2. S.S. Gubster, I.R. Klebanov, A.M. Polyakov, Gauge theory correlators from non-critical string theory. Phys. Lett. B **428**, 105 (1998). [arXiv:hep-th/9802109]
3. E. Witten, Anti-de Sitter space and holography. Adv. Theor. Math. Phys. **2**, 253 (1998). [arXiv:hep-th/9802150]
4. O. Aharony, S.S. Gubster, J.M. Maldacena, H. Ooguri, Y. Oz, Large N field theories, string theory and gravity. Phys. Rept. **323**, 183 (2000). [arXiv:hep-th/9905111]
5. S. Kachru, X. Liu, M. Mulligan, Gravity dual of Lifshitz-like fixed points. Phys. Rev. D **78**, 106005 (2008). [arXiv:0808.1725]
6. M. Tylor, Non-relativitic holography. [arXiv:0812.0530]
7. M. Cadoni, S. Mignemi, Phase transition and hyperscaling violationfor scalar black branes. JHEP **1206**, 056 (2012). [arXiv:1205.0412 [hep-th]]
8. X. Dong, S. Harrison, S. Kachru, G. Torroba, H. Wang, Aspects of holography for theories with hyperscaling violation. JHEP **1206**, 041 (2012). [arXiv:1201.1905v4 [hep-th]]
9. S.S. Gubser, F.D. Rocho, Peculiar properties of a charged dilatonic black hole in Ads5. Phys. Rev. D **81**, 046001 (2010). [arXiv:0911.2898 [hep-th]]
10. K. Goldstein, S. Kachru, S. Prakash, S.P. Trivedi, Holography of charged dilaton black hole. JHEP **1008**, 078 (2010). [arXiv:0911.3586 [hep-th]]
11. M. Cadoni, G. D'Appollonio, P. Pani, Phase transitions between Ressner–Nordstrom and dilatonic black hole in 4D Ads spacetime. JHEP **1003**, 100 (2010). [arXiv:0912.3520 [hep-th]]
12. M. Cadoni, P. Pani, Holography of charged dilatonic black branes at finite temperature. JHEP **1104**, 049 (2011). [arXiv:1102.3820 [hep-th]]
13. C. Charmousis, B. Gouteraux, B.S. Kim, E. Kiritsis, R. Meyer, Effective Holographic Theories for low-temperature condensed

matter system. JHEP **1011**, 151 (2010). [arXiv:1005.4690 [hep-th]].
14. E. Perlmutter, Domain wall holography for finite temperature scaling solutions. JHEP **1102**, 013 (2011). [arXiv:1006.2124 [hep-th]].
15. G. Bertoldi, B.A. Burrington, A.W. Peet, Thermal behavior of charged dilatonic black branes in Ads and UV completions of Lifshitz-like geometries. Phys. Rev. D **82**, 106013 (2010). [arXiv:1007.1464 [hep-th]]
16. G. Bertoldi, B.A. Burrington, A.W. Peet, I.G. Zadeh, Lifshitz-like black brane thermodynamics in higher dimensions. Phys. Rev. D **83**, 126006 (2011). [arXiv:1101.1980 [hep-th]]
17. K. Goldstein, N. Iizuka, S. Kachru, S. Prakash, S.P. Trivedi, A. Westphal, Holography of dyonic dilaton black branes. JHEP **1010**, 027 (2010). [arXiv:1007.2490 [hep-th]]
18. N. Izuka, N. Kundu, P. Narayan, S.P. Trivedi, Holographic Fermi and non-Fermi liquids with transitions in dilaton gravity, [arXiv:1105.1162 [hep-th]]
19. P. Berglund, J. Bhattacharyya, D. Mattingly, Charged dilatonic ads black brane in arbitrary dimensions. [arXiv:1107.3096 [hep-th]]
20. N. Ogawa, T. Takayanagi, T. Ugajin, Holographic Fermi surface and entanglement. Entropy. [arXiv:1111.1023 [hep-th]]
21. L. Huijse, S. Sachdev, B. Swingle, Hidden Fermi surface in compressible state of gauge-gravity duality. Phys. Rev B **85**, 035121 (2012). [arXiv:1112.0573 [cond-mat.str-el]]
22. E. Shaghoulian, Holograghic entanglement entropy and Fermi surfaces. [arXiv:1112.2702 [hep-th]]
23. Y. Aoki, T. Aoyama, M. Kurachi, T. Mashawa, K. Nagai, H. Ohki, A. Shibata, K. Yamawaki, T. Yamazaki, Study of the conformal hyperscaling relation through the Schwinger–Dyson equation, [arXiv:1201.4157 [hep-lat]]
24. S.J. Brodsky, G.F. de Teramond, AdS/CFT and Light-Front QCD. [arXiv:0802.0514 [hep-ph]]
25. D.T. Son, Toward an Ads/cold atoms correspondence: a geometric realization of the Schrodinger symmetry. Phys. Rev. D **78**, 046003 (2003). [arXiv:0804.3972 [hep-lat]]
26. K. Balasubramanian, J. McGreevy, Gravity duals for non-relativitic CFTs. Phys. Rev. Lett. **101**, 061601 (2008). [arXiv:0804.4053 [hep-lat]]
27. J. Maldacena, D. Martelli, Y. Tachikawa, Comments on String theory backgrounds with non-relativitic conformal symmetry. JHEP **0810**, 072 (2008). [arXiv:0807.1100 [hep-th]]
28. C.P. Herzog, M. Rangamani, S.F. Ross, Heating up Galilean holography. JHEP **0811**, 080 (2008). [arXiv:0807.1099 [hep-th]]
29. A. Adams, K. Balasubramanian, J. McGreevy, Hot spacetime for cold atoms. JHEP **0811**, 059 (2008). [arXiv:0807.1111 [hep-th]]
30. M. Alishahiha, O.J. Ganor, Twisted backgrounds, pp-waves and nonlocal field theories. JHEP **0303**, 006 (2003). [arXiv:hep-th/0301080]
31. E.G. Gimon, A. Hashimoto, V.E. Hubeny, O. Lunin, M. Rangamani, Black string in asymptotically plane wave geometries. JHEP **0308**, 035 (2003). [arXiv:hep-th/0306131]
32. J. Sadeghi, B. Pourhassan, Energy loss and jet quenching parameter in a thermal non-relativistic, non-commutative Yang–Mills plasma. Acta Phys. Polonica B **43**, 1825 (2012). [arXiv:1002.1596 [hep-th]]
33. D. Yamada, Thermodynamics of black holes in Schrödinger spacs. Class. Quant. Grav. **26**, 075006 (2009). [arXiv:0809.4928 [hep-th]]
34. S.W. Hawking, D.N. Page, Thermodynamics of black hole in Anti-De Sitter space. Commun. Math. Phys. **87**, 577 (1983)
35. G.W. Gibbons, S.W. Hawking, Action integrals and partition function in quantum gravity. Phys. Rev. D **15**, 2752 (1977)
36. J. Sadeghi, B. Pourhassan, F. Pourasadollah, Thermodynamics of Schrodinger black holes with hyperscaling violation. Phys. Lett. B **720**, 244–249 (2013). [arXiv:1209.1874 [hep-th]]
37. K. Narayan, On Lifshitz scaling and hyperscaling violation in string theory. [arXiv:1202.5935 [hep-th]]
38. J. Sadeghi, B. Pourhassan, M. Rostami, Z. Sadeghi, Thermodynamics of near-extremal solutions of Einstein–Maxwell–Scalar theory. Int. J. Theor. Phys. **52**, 2564 (2013)
39. A. Pourdarvish, J. Sadeghi, H. Farahani, B. Pourhassan, Thermodynamics and statistics of Godel black hole with logarithmic correction. Int. J. Theor. Phys. **52**, 3560 (2013)