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Letter

Could dynamical Lorentz symmetry breaking induce the superluminal neutrinos?

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Abstract A toy fermion model coupled to the Lagrange multiplier constraint field is proposed. The possibility of superluminal neutrino propagation as a result of dynamical Lorentz symmetry breaking is studied.

The OPERA experiment results indicate toward the possibility that the neutrino speed might exceed the speed of light [1]. Although the experimental results could be in principle fully trustable, they have not been yet confirmed by other experiments. Hence, the superluminal neutrino occurrence remains at present as a quite hypothetical case, especially in relation with the emerging relativistic causality violation. However, it might be interesting to consider an eventual realization of such a theoretical effect in quantum field theory. Several attempts, mainly related to the Lorentz symmetry breaking to give some theoretical explanation for the superluminal neutrino, have just appeared [2–24]. For earlier proposals, see [25].

In the present note we propose a toy model of fermion theory coupled to the Lagrange multiplier constrained field, which induces the dynamical Lorentz symmetry breaking. As a result, the superluminal neutrino propagation appears to be possible. The description of the model is motivated by the analogy with the power-counting renormalizable gravity proposed in [26]. In this model, the Lorentz symmetry or full diffeomorphism invariance is explicitly broken and the dispersion relation of the graviton is modified so that the UV behavior of the quantum field could be improved. Due to the modified dispersion relation, the speed of the graviton can be faster than the light speed.

However, the lack of full diffeomorphism invariance leads to the extra scalar mode in the model [26]. Due to the presence of scalar mode, the general relativity and/or the Newton law cannot be reproduced even in the IR region.

In order to solve this problem, the models with the full diffeomorphism invariance have been investigated [27–33]. The diffeomorphism invariance and/or Lorentz symmetry can be spontaneously broken since the derivative of the scalar field with respect to the coordinates does not vanish. The non-vanishing value is generated by the Lagrange multiplier field, which gives a constraint on the derivative of the scalar field not to vanish. The mechanism is very similar to the Stückelberg formulation of the massive U(1) gauge theory. For the model [33], it has been explicitly shown that the UV behavior of graviton is improved as in the original theory [26] but due to the full diffeomorphism invariance, the extra (scalar/vector) mode does not appear. In such covariant gravity [33], the other fields besides the gravity sector are assumed to be standard ones, which guarantees the experimentally observed Lorentz invariance. Moreover, in such a theory [33], the Lorentz symmetry in the matter sector could be broken if we consider the intermediate states including graviton, though the breakdown could be very small.

Let us show, however, that we can construct a model of spinor which might be identified with the neutrino, where the speed of the spinor can exceed the light speed. In such a construction, the Lorentz symmetry is broken spontaneously as in the models [27–33]. We will estimate the parameters to be roughly consistent with the OPERA experimental data [1].

We now start with the action including the Lagrange multiplier field λ [34–38] and the scalar field ϕ:

$$S_{\rm Lag} = -\int d^4x\, \lambda \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + U_0 \right), \qquad (1)$$

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which gives a constraint,

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0 = 0, \tag{2}$$

that is, the vector $(\partial_\mu\phi)$ is time-like. Therefore, the Lorentz symmetry is broken spontaneously. This mechanism was used to construct a power-counting renormalizable and covariant model of gravity [27–33]. One may choose the direction of time to be parallel to the vector $(\partial_\mu\phi)$ and in the following, we assume

$$\phi = \sqrt{2U_0}t. \tag{3}$$

Here the gravity sector is not included. We would like to construct a model of spinor, whose speed can exceed the light speed, although the action has the full Lorentz symmetry. The action we consider is

$$S = \int d^4x\Big[\bar\psi\big\{\gamma^\mu\partial_\mu + \alpha\big(P_\mu{}^\nu\gamma^\mu\partial_\nu\big)^{2n+1}\big\}\psi - \lambda\Big(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\Big)\Big]. \tag{4}$$

Here α is a constant, n is an integer equal to or greater than 1, and $P_\mu{}^\nu$ is a projection operator defined by [33]

$$P_\mu{}^\nu \equiv \delta_\mu{}^\nu + \frac{\partial_\mu\phi\partial^\nu\phi}{2U_0}. \tag{5}$$

The equation corresponding to the Dirac equation has the following form:

$$0 = \big\{\gamma^\mu\partial_\mu + \alpha\big(P_\mu{}^\nu\gamma^\mu\partial_\nu\big)^{2n+1}\big\}\psi. \tag{6}$$

By using (3), equation (6) looks like

$$0 = \big\{\gamma^0\partial_0 + \gamma^i\partial_i + \alpha\big(\gamma^i\partial_i\big)^{2n+1}\big\}\psi. \tag{7}$$

The dispersion relation for the spinor is then given by

$$\omega = k\sqrt{1+\alpha^2k^{4n}}. \tag{8}$$

Here ω is the angular frequency corresponding to the energy and k is the wave number corresponding to the momentum. In the high energy region, the dispersion relation becomes

$$\omega \sim |\alpha|k^{2n+1}, \tag{9}$$

and therefore the phase velocity v_p and the group velocity v_g are given, respectively, by

$$v_p \equiv \frac{\omega}{k} = |\alpha|k^{2n}, \qquad v_g \equiv \frac{d\omega}{dk} = (2n+1)|\alpha|k^{2n}. \tag{10}$$

When k becomes larger, both v_p and v_g become also larger in an unbounded way and exceed the light speed.

When the breakdown of the Lorentz symmetry is small, we may expand (8) as follows:

$$\omega \sim k\Big(1 + \frac{\alpha^2k^{4n}}{2}\Big). \tag{11}$$

Since we choose the light speed c to be unity, $c = 1$, the OPERA experiment [1] shows

$$\frac{v-c}{c} = \frac{\alpha^2k^{4n}}{2} = (2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.})) \times 10^{-5}, \tag{12}$$

for

$$k \sim E_\nu = 17\ \text{GeV}. \tag{13}$$

Here v and E_ν are the speed and the energy of the neutrino, respectively. Then one finds

$$\alpha^{-\frac{1}{2n}} \sim 10^{1+\frac{5}{4n}}\ \text{GeV}. \tag{14}$$

This suggests that the scale of the Lorentz symmetry breaking could be 10–100 GeV. By using (12) and (13), we may rewrite the deviation from the light speed as

$$\frac{v-c}{c} = \frac{\alpha^2k^{4n}}{2} = 2.48 \times 10^{-5}\Big(\frac{k}{17\ \text{GeV}}\Big)^{4n}. \tag{15}$$

When $k \sim 10$ MeV, a stringent limit was given by the observation of (anti-)neutrinos emitted by the SN1987A supernova [39]:

$$\frac{|v-c|}{c} < 2 \times 10^{-9}, \tag{16}$$

which could be consistent due to the k-dependence in (15). For $k \sim 10$ MeV, (15) gives

$$\frac{v-c}{c} \sim 10^{-5-16n}. \tag{17}$$

Then the constraint (16) can be easily satisfied even for $n = 1$.

If the spinor (4) is identified with neutrino, the usual derivatives should be replaced by the covariant derivatives which include gauge bosons, and therefore new kinds of interactions would emerge. Such an interaction could be suppressed by the k-dependence for the low energy region. In the high energy region, however, the corrections could be large. This completes the construction of our toy model for superluminal neutrino. Of course, a number of questions remain to be understood if such a model has any relation with reality. For instance, the interpretation of Lagrange multiplier scalars introduced originally as the dust of dark energy [28–31, 34] should be developed. Also why does only the neutrino seem to show a superluminal behavior? Moreover, effects related to the manifestation of superluminal propagation at very high energies should be searched for.

Recently there was a claim that a superluminal neutrino would lose energy rapidly due to the bremsstrahlung of electron–positron pairs [40]. For the model without Lorentz invariance, the high energy neutrino can decay into the low energy neutrino itself and other particles like electron–positron pairs, which is not yet strictly proven. Such a process is prohibited for the Lorentz invariant models since we can always choose the coordinate frame such that the (massive) neutrino is not moving, where the decay to the neutrino itself and other particle violates energy conservation. In our model, since there is a Lorentz invariance in the Lagrangian although the invariance is spontaneously broken, there exists a local conserved energy momentum tensor $T_{\mu\nu}$. The conservation law has the standard Lorentz invariant form $\partial^\mu T_{\mu\nu} = 0$. Then the existence of the Lorentz invariantly conserved energy momentum tensor might prohibit the process that the high energy neutrino could decay into the low energy neutrino itself and other particles although we need a more detailed analysis.

Moreover, in [41], it a different scenario was proposed of a background dependent violation of the Lorentz invariance. The proposed scenario is consistent with the OPERA experiment and it passes by the bremsstrahlung problem [40]. The model under discussion in this work may be further generalized to the one with spontaneous background dependent Lorentz symmetry violation. This will be considered elsewhere.

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References

1. T. Adam et al. (OPERA Collaboration), arXiv:1109.4897 [hep-ex]
2. A. Kehagias, arXiv:1109.6312 [hep-ph]
3. A. Drago, I. Masina, G. Pagliara, R. Tripiccione, arXiv:1109.5917 [hep-ph]
4. M. Li, T. Wang, arXiv:1109.5924 [hep-ph]
5. C. Pfeifer, M.N.R. Wohlfarth, arXiv:1109.6005 [gr-qc]
6. S.S. Gubser, arXiv:1109.5687 [hep-th]
7. G.F. Giudice, S. Sibiryakov, A. Strumia, arXiv:1109.5682 [hep-ph]
8. J. Alexandre, arXiv:1109.5629 [hep-ph]
9. D. Autiero, P. Migliozzi, A. Russo, arXiv:1109.5378 [hep-ph]
10. F.R. Klinkhamer, arXiv:1109.5671 [hep-ph]
11. J. Ciborowski, J. Rembielinski, arXiv:1109.5599 [hep-ex]
12. G. Dvali, A. Vikman, arXiv:1109.5685 [hep-ph]
13. G. Amelino-Camelia, G. Gubitosi, N. Loret, F. Mercati, G. Rosati, P. Lipari, arXiv:1109.5172 [hep-ph]
14. G. Cacciapaglia, A. Deandrea, L. Panizzi, arXiv:1109.4980 [hep-ph]
15. Z. Lingli, B.-Q. Ma, arXiv:1109.6097 [hep-ph]
16. R.A. Konoplya, Universe. arXiv:1109.6215 [hep-th]
17. R.B. Mann, U. Sarkar, arXiv:1109.5749 [hep-ph]
18. M.A. Anacleto, F.A. Brito, E. Passos, arXiv:1109.6298 [hep-th]
19. L. Iorio, arXiv:1109.6249 [gr-qc]
20. P. Wang, H. Wu, H. Yang, arXiv:1109.6930 [hep-ph]
21. F.R. Klinkhamer, G.E. Volovik, arXiv:1109.6624 [hep-ph]
22. S. Hannestad, M.S. Sloth, arXiv:1109.6282 [hep-ph]
23. X.-J. Bi, P.-F. Yin, Z.-H. Yu, Q. Yuan, arXiv:1109.6667 [hep-ph]
24. P. Wang, H. Wu, H. Yang, arXiv:1110.0449 [hep-ph]
25. H. Pas, S. Pakvasa, T.J. Weiler, Phys. Rev. D **72**, 095017 (2005). hep-ph/0504096
26. P. Horava, Phys. Rev. D **79**, 084008 (2009). arXiv:0901.3775 [hep-th]
27. S. Nojiri, S.D. Odintsov, Phys. Rev. D **81**, 043001 (2010). arXiv:0905.4213 [hep-th]
28. S. Nojiri, S.D. Odintsov, Phys. Lett. B **691**, 60–64 (2010). arXiv:1004.3613 [hep-th]
29. S. Nojiri, S.D. Odintsov, Phys. Rev. D **83**, 023001 (2011). arXiv:1007.4856 [hep-th]
30. M. Chaichian, M. Oksanen, A. Tureanu, Eur. Phys. J. C **71**, 1657 (2011). arXiv:1101.2843 [gr-qc]
31. G. Cognola, E. Elizalde, L. Sebastiani, S. Zerbini, Phys. Rev. D **83**, 063003 (2011). arXiv:1007.4676 [hep-th]
32. S. Nojiri, S.D. Odintsov, Phys. Rep. **505**, 59 (2011). arXiv:1011.0544 [gr-qc]
33. J. Kluson, S. Nojiri, S.D. Odintsov, Phys. Lett. B **701**, 117–126 (2011). arXiv:1104.4286 [hep-th]
34. E.A. Lim, I. Sawicki, A. Vikman, J. Cosmol. Astropart. Phys. **1005**, 012 (2010). arXiv:1003.5751 [astro-ph.CO]
35. C. Gao, Y. Gong, X. Wang, X. Chen, arXiv:1003.6056 [astro-ph.CO]
36. S. Capozziello, J. Matsumoto, S. Nojiri, S.D. Odintsov, Phys. Lett. B **693**, 198 (2010). arXiv:1004.3691 [hep-th]
37. Y.F. Cai, E.N. Saridakis, Class. Quantum Gravity **28**, 035010 (2011). arXiv:1007.3204 [astro-ph.CO]
38. J. Kluson, arXiv:1101.5880 [hep-th]
39. K. Hirata et al. (KAMIOKANDE-II Collaboration), Phys. Rev. Lett. **58**, 1490–1493 (1987)
40. A.G. Cohen, S.L. Glashow, arXiv:1109.6562 [hep-ph]
41. T. Li, D.V. Nanopoulos, arXiv:1110.3451 [hep-ph]