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Erratum

Erratum to: Arnowitt–Deser–Misner representation and Hamiltonian analysis of covariant renormalizable gravity

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The source of error

The idea of the notation convention was simple: denote tensors on the spacetime with the ${}^{(4)}$ prefix and tensors that are tangent to the spatial hypersurface Σ_t without the prefix. According to this convention it makes sense to denote the component of the Ricci tensor ${}^{(4)}R_{\mu\nu}$ that is tangent to Σ_t by $R_{\mu\nu}$. However, R is already used to denote the intrinsic curvature of Σ_t. Since these two things are not the same, there is a misleading conflict in the notation.

Erratum

On page 4, the paragraph that contains (2.30)–(2.36) should be corrected as follows.

The Ricci tensor ${}^{(4)}R_{\mu\nu}$ of spacetime can be decomposed as

$$ {}^{(4)}R_{\mu\nu} = {}^{(4)}_{\perp}R_{\mu\nu} - {}^{(4)}_{\perp}R_{\mu n}n_\nu - {}^{(4)}_{\perp}R_{\nu n}n_\mu + {}^{(4)}R_{nn}n_\mu n_\nu, \tag{2.30} $$

where we have defined

$$ \begin{aligned} {}^{(4)}_{\perp}R_{\mu\nu} &\equiv g^\rho{}_\mu g^\sigma{}_\nu {}^{(4)}R_{\rho\sigma} \\ &= R_{\mu\nu} + KK_{\mu\nu} - 2K_{\mu\rho}K^\rho{}_\nu - \frac{1}{N}D_\mu D_\nu N \\ &\quad + \frac{1}{N}\mathcal{L}_{Nn}K_{\mu\nu}, \end{aligned} \tag{2.31} $$

$$ {}^{(4)}_{\perp}R_{\mu n} \equiv g^\rho{}_\mu n^{\nu\,(4)}R_{\rho\nu} = D_\rho K^\rho{}_\mu - D_\mu K, \tag{2.32} $$

$$ {}^{(4)}_{\perp}R_{n\nu} \equiv g^\rho{}_\nu n^{\mu\,(4)}R_{\mu\rho} = D_\rho K^\rho{}_\nu - D_\nu K, \tag{2.33} $$

$$ \begin{aligned} {}^{(4)}R_{nn} &\equiv {}^{(4)}R_{\mu\nu}n^\mu n^\nu = \frac{1}{2}\left(K^2 - K_{ij}K^{ij} + R - {}^{(4)}R\right) \\ &= K^2 - K_{ij}K^{ij} - \nabla_\mu\left(n^\mu K\right) + \frac{1}{N}D^i D_i N. \end{aligned} \tag{2.34} $$

In (2.31) $R_{\mu\nu}$ is the Ricci tensor of the hypersurface Σ_t and $\mathcal{L}_{Nn}$ denotes the Lie derivative along Nn^μ. Note that for any tensor field T that is tangent to Σ_t, $\mathcal{L}_{Nn}T$ is also tangent to Σ_t. In (2.31)–(2.34) we have used the *Gauss relation*, the *Ricci equation* and the *Codazzi relation*, and in (2.34) the decomposition of ${}^{(4)}R$ from (2.25) was also used. Hence the Einstein tensor can be decomposed:

$$ \begin{aligned} {}^{(4)}G_{\mu\nu} &\equiv {}^{(4)}R_{\mu\nu} - \frac{1}{2}{}^{(4)}g_{\mu\nu}{}^{(4)}R \\ &= R_{\mu\nu} + KK_{\mu\nu} - 2K_{\mu\rho}K^\rho{}_\nu \\ &\quad - \frac{1}{N}D_\mu D_\nu N + \frac{1}{N}\mathcal{L}_{Nn}K_{\mu\nu} \\ &\quad - \frac{1}{2}g_{\mu\nu}\bigg(R + K_{ij}K^{ij} + K^2 + 2\nabla_n K \\ &\quad - \frac{2}{N}D^i D_i N\bigg) \\ &\quad + \left(D_\mu K - D_\rho K^\rho{}_\mu\right)n_\nu + n_\mu\left(D_\nu K - D_\rho K^\rho{}_\nu\right) \\ &\quad + \frac{1}{2}n_\mu n_\nu\left(K^2 - K_{ij}K^{ij} + R\right), \end{aligned} \tag{2.35} $$

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where we have also used (2.29). Thus in the actions (2.13) and (2.14) we have

$$\begin{aligned}\partial^\mu\phi\partial^\nu\phi^{(4)}G_{\mu\nu} = D^i\phi D^j\phi\bigg[&R_{ij}+KK_{ij}-2K_{ik}K^k{}_j\\&-\frac{1}{N}D_iD_jN+\frac{1}{N}\mathcal{L}_{Nn}K_{ij}\\&-\frac{1}{2}g_{ij}\bigg(R+K_{kl}K^{kl}+K^2\\&+2\nabla_nK-\frac{2}{N}D^kD_kN\bigg)\bigg]\\&+2(\nabla_n\phi)D^i\phi\big(D_iK-D^jK_{ji}\big)\\&+\frac{1}{2}(\nabla_n\phi)^2\big(K^2-K_{ij}K^{ij}+R\big).\end{aligned}\quad(2.36)$$

In the Hamiltonian analysis of Sect. 3. *Hamiltonian formalism*, the variables N_i should be replaced by N^i, which are related by $N_i = g_{ij}N^j$. Consequently the following changes to the constraints of the theory are required.

For the case $z = 3$ in Sect. 3.1, the momenta conjugate to the shift variables N^i are written as p_i, which belong to the primary constraints (3.1) of the theory. The momentum constraint $\mathcal{H}_3^i$ is replaced with $\mathcal{H}_i^3 = g_{ij}\mathcal{H}_3^j$ and accordingly redefined in (3.11) as

$$\mathcal{H}_i^3 = -2g_{ij}D_kp^{jk} + D_i\zeta_2 p_{\zeta_2}. \quad (3.11)$$

The smeared momentum constraint $\Phi_3^S(\chi_i)$ is replaced with $\Phi_S^3(\chi^i)$ and redefined in (3.21) as

$$\Phi_S^3\big(\chi^i\big) = \int \mathrm{d}^3\boldsymbol{x}\,\chi^i\mathcal{H}_i^3 \approx 0. \quad (3.21)$$

The Poisson bracket in (3.24) is corrected as

$$\big\{\Phi_S^3\big(\chi^i\big), \Phi_S^3\big(\psi^i\big)\big\} = \Phi_S^3\big(\chi^j\partial_j\psi^i - \psi^j\partial_j\chi^i\big) \approx 0. \quad (3.24)$$

Otherwise the presented algebra of constraints corresponds to the choice of variables N^i and the associated momentum constraint $\mathcal{H}_i^3$. Indeed, if one uses $\mathcal{H}_3^i$ the Poisson bracket $\{\Phi_3^S(\chi_k), p^{ij}\}$ contains an additional term $-\chi^{(i}\mathcal{H}^{j)}$ compared to $\{\Phi_S^3(\chi^k), p^{ij}\}$, which was not intended.

Similar correction is applied to the case $z = 4$ in Sect. 3.2.