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Letter

New approach to GUTs

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Abstract We introduce a new string-inspired approach to the subject of grand unification which allows the GUT scale to be small, $\lesssim$200 TeV, so that it is within the reach of *conceivable* laboratory accelerated colliding beam devices. The key ingredient is a novel use of the heterotic string symmetry group physics ideas to render baryon number violating effects small enough to have escaped detection to date. This part of the approach involves new unknown parameters to be tested experimentally. A possible hint at the existence of these new parameters may already exist in the EW precision data comparisons with the SM expectations.

The success and structure of the Standard Model (SM) [1–10] suggest that all forces associated with the gauge interactions therein may be unified into a single gauge principle associated with a larger group $\mathcal{G}$ which contains $SU(2)_L \times U(1)_Y \times SU(3)^c$ as a subgroup, where we use a standard notation for the SM gauge group. This idea was originally introduced in the modern context in Refs. [11, 12] and continues to be a fashionable area of investigation today, where approaches which unify the SM gauge forces with that of quantum gravity are now in very much vogue via the superstring theory [13–19] and its various low energy reductions and morphisms [19]. In this paper, we focus only on the unification of the SM gauge forces themselves, candidates for which we call as usual GUTs, leaving aside any possible unification with quantum gravity until a later study [20].

We need to admit at the outset that a part of our motivation is the recent progress in approaches to the Einstein-Hilbert theory for quantum gravity in which improved treatments of perturbation theory via resummation methods, the asymptotic safety approach [21–34], the resummed quantum gravity approach [35–38] or the Hopf-algebraic Dyson–Schwinger equation renormalization theory approach [39, 40], and the introduction of an underlying loop-space at Planck scales, loop quantum gravity [41–44], have shown that the apparently bad unrenormalizable behavior of that theory may be cured by the dynamical interactions or modifications within the theory itself, as first anticipated by Weinberg [21–34]. Such progress would suggest that the unification of all other forces can be a separate problem from the problem of treating the apparently bad UV behavior of quantum gravity. We explore this suggestion in what follows.

Our idea is to try to formulate GUTs so that they are accessible to very high energy colliding beam devices such as the VLHC, which has been discussed elsewhere [45–50] with cms energies in the 100–200 TeV regime. We will show that we can achieve such GUTs that satisfy the usual requirements: no anomalies, unified SM couplings, baryon stability, absence/suppression of other unwanted transitions and naturalness requirements (this may just mean $N = 1$ SUSY here [51–55]). Here, we add the new condition that the theory will live in 4-dimensional Minkowski space. We call this our *known physical reality condition*. The most demanding requirement will be seen to be baryon stability.

Indeed, let us just illustrate why the most difficult aspect of a GUT with a (several) hundred TeV unification scale is the issue of baryon number stability: the proton must be stable to $\sim 10^{29-33}$ yr, depending on the mode, whereas the natural lifetime for physics with a 100 TeV scale for a dimension 6 transition in a state with the size and mass of the proton is $\sim$0.01 yr for example. Evidently, some new mechanism is needed to suppress the proton decay process here.

Rather than to move the GUT scale to $\sim 10^{13}$ TeV as is usually done [56, 57], or invoke hitherto unknown phenomena, such as extra dimensions [56–60], extra vector representations of the gauge group [61–68], etc., we will try to rely on well-tested ideas used in a novel way—we will use what is sometimes called a radically conservative approach. We look at the fundamental structure of a GUT theory. We notice that it is organized by gauge sector, by family sector and by Higgs sector for spontaneous symmetry breaking. Let us look at the family and gauge sectors.

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Fig. 1 $SU(5)$ decomposition of six SM families with the prediction of six new heavy leptons and six new heavy quarks, all possibly accessible at the next set of high energy colliders. The superscript c denotes charge conjugation as usual and the color index runs from 1 to 3 here. When the effective high energy gauge symmetry is $E_8 \times E_8 \times E_8 \times E_8 \equiv E_{8,1} \times E_{8,2} \times E_{8,3} \times E_{8,4}$, the structure in this figure occurs twice, but with new quarks and leptons as yet unseen

$$\begin{bmatrix} d'^c_{\ell 1} \\ d'^c_{\ell 2} \\ d'^c_{\ell 3} \\ \ell^- \\ -\nu_\ell \end{bmatrix}_{\rm L} \oplus \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & u'^c_{\ell 3} & -u'^c_{\ell 2} & -u'^1_\ell & -d'^1_\ell \\ -u'^c_{\ell 3} & 0 & u'^c_{\ell 1} & -u'^2_\ell & -d'^2_\ell \\ u'^c_{\ell 2} & -u'^c_{\ell 1} & 0 & -u'^3_\ell & -d'^3_\ell \\ u'^1_\ell & u'^2_\ell & u'^3_\ell & 0 & -\ell^+ \\ d'^1_\ell & d'^2_\ell & d'^3_\ell & \ell^+ & 0 \end{bmatrix}_{\rm L} \oplus \quad [\nu^c_\ell]_{\rm L}\,, \ \ell = e, \mu, \tau,$$

$$\begin{bmatrix} d^c_{\ell' 1} \\ d^c_{\ell' 2} \\ d^c_{\ell' 3} \\ \ell'^- \\ -\nu_{\ell'} \end{bmatrix}_{\rm L} \oplus \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & u^c_{\ell' 3} & -u^c_{\ell' 2} & -u_{\ell'}{}^1 & -d_{\ell'}{}^1 \\ -u^c_{\ell' 3} & 0 & u^c_{\ell' 1} & -u_{\ell'}{}^2 & -d_{\ell'}{}^2 \\ u^c_{\ell' 2} & -u^c_{\ell' 1} & 0 & -u_{\ell'}{}^3 & -d_{\ell'}{}^3 \\ u_{\ell'}{}^1 & u_{\ell'}{}^2 & u_{\ell'}{}^3 & 0 & -\ell'^+ \\ d_{\ell'}{}^1 & d_{\ell'}{}^2 & d_{\ell'}{}^3 & \ell'^+ & 0 \end{bmatrix}_{\rm L} \oplus \quad [\nu^c_{\ell'}]_{\rm L}\,, \ \ell' = e', \mu', \tau',$$

$$u = u_{e'},\ d = d_{e'},\ c = u_{\mu'},\ s = d_{\mu'},\ t = u_{\tau'},\ b = d_{\tau'}$$

In Ref. [12], the $\mathbf{10} + \bar{\mathbf{5}}$ of $SU(5)$ was advocated and shown to accommodate the SM family with a massless neutrino. Recently, with advent of neutrino masses [69–71], we need to extend this to a sixteen dimensional representation. We will use the **16** of $SO(10)$ [72], as it decomposes as $\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$ under an inclusion of $SU(5)$ into $SO(10)$. We know from the heterotic string formalism [13–19] (we view here modern string theory as an extension of quantum field theory which can be used to abstract dynamical relationships which would hold in the real world even if the string theory itself is in detail only an approximate, mathematical treatment of that reality, just as the old strong interaction string theory [73] could be used to abstract properties of QCD [8–10], such as Regge trajectories, even before QCD was discovered) that in the only known and accepted unification of the SM and gravity, the gauge group $E_8 \times E_8$ is singled out when all known dualities [19] are taken into account to relate equivalent superstring theories. A standard breakdown of this symmetry to the SM gauge group and family structure is [19]

$$\begin{aligned} E_8 &\to SU(3) \times E_6 \to SU(3) \times SO(10) \times U'(1) \\ &\to SU(3) \times SU(5) \times U''(1) \times U'(1) \\ &\to SU(3) \times SU(3)^c \times SU(2)_L \times U(1)_Y \\ &\quad \times U''(1) \times U'(1) \end{aligned} \tag{1}$$

where the SM gauge group is now called out as $SU(3)^c \times SU(2)_L \times U(1)_Y$. It can be shown that the **248** of E_8 then splits under this breaking into $(\mathbf{8}, \mathbf{1}) + (\mathbf{1}, \mathbf{78}) + (\mathbf{3}, \mathbf{27}) + (\bar{\mathbf{3}}, \overline{\mathbf{27}})$ under $SU(3) \times E_6$ and that each **27** under E_6 contains exactly one SM family 16-plet with 11 other states that are paired with their anti-particles in helicity via real representations so that they would be expected to become massive at the GUT scale. Let us consider that we have succeeded with the heterotic string breaking scenario to get three families [56, 57] under the first E_8 factor in the $E_8 \times E_8$ gauge group. They are singlets under the second E_8. We now repeat the same pattern for the second factor as well. This gives us six families, one set of "family triplets" transforming non-trivially only under E_{8a} and the other set of "family triplets" transforming non-trivially only under E_{8b}, where $E_8 \times E_8 \equiv E_{8a} \times E_{8b}$. To stop baryon instability, we identify the light quarks as those from E_{8a} and the light leptons as those from E_{8b}—here, light means light on the scale of $M_{\rm GUT}$, the grand unified theory (GUT) scale. The remaining particles in each sector are then at the respective scales $M_{\rm LM}$ between their current experimental limits and the GUT scale. The proton cannot decay because the leptons to which it could transform via (leptoquark) bosons are all at too high a scale.

Already, let us note that, while our approach to proton stability is very much related to the approaches in Refs. [61–68], it differs from the standpoint of radical conservatism—we only use the family structure that has been seen in Nature in the standard SM families: $\{\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}\}_j$, where we now have six of them instead of the usual three so that the family index j now runs from $j = 1$ to $j = 6$ and we do not have any vector representations, as we illustrate in Fig. 1. For this reason we have no problem with such issues as charge quantization. We are predicting the discovery of the *equivalent* of three new families of quarks and leptons at the next very high energy machines with gauge quantum numbers entirely the same as those that have already been seen in Nature but with significantly larger masses. In particular, we note that just a single Higgs $SU(2)_L$ doublet, as originally proposed by Weinberg and Salam [13–18], is enough to give all particles their masses in a large region of our parameter space. Of course, this does not exclude more than one such doublet.

It is also important to stress that we are only abstracting the family and gauge structure of the breaking pattern in (1) (we discard everything else), very much in the spirit of Gell-Mann's abstraction of the algebra of currents from free field theory in Ref. [74] for the strong interaction, without claiming that the details in the breaking are themselves also relevant. Indeed, our entire point is that these details, whether they are from the string theoretic perspective or the usual GUT perspective, may not be relevant at all.

Since we are entering into some discussion about hitherto unexplored phenomena, we have to be open about the framework in the most basic ways. For example, the heterotic superstring Lagrangian has the action, using the language of conformal field theory for definiteness [19], for the matter sector (as opposed to the ghost sector) given by

$$S(X,\tilde{\psi},\lambda) = \frac{1}{4\pi}\int d^2z\left(\frac{2}{\alpha'}\partial X^\mu\bar{\partial}X_\mu + \lambda^A\bar{\partial}\lambda^A + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu\right) \tag{2}$$

where we have introduced the fields $X^\mu(z,\bar{z})$, $\tilde{\psi}^\mu(\bar{z})$, $\mu = 0,\dots,9$ for the left-moving part of the bosonic string and the right-moving part of the type II superstring, respectively, as well as the 32 left-moving spin-$\frac{1}{2}$ fields λ^A, $A = 1,\dots,32$, with the boundary conditions $\lambda^A(w+2\pi) = \eta\lambda^A(w)$, $A = 1,\dots,16$, $\lambda^A(w+2\pi) = \eta'\lambda^A(w)$, $A = 17,\dots,32$, where η and η' each are ± 1, $e^{-iw} = z$ and $\frac{1}{2\pi\alpha'}$ is the usual string tension. The λ^A are needed to complete the cancellation of central charge when all ghosts are taken into account and the boundary conditions, with the attendant GSO projections, just give us the $E_8\times E_8$ heterotic superstring theory [13–18] as is well known [19]. Here, we extend this to the possibility that we have two such contributions to the world action, two strings, that for the moment will be non-interacting copies of each other: $S_{\text{world}} = S(X(1),\tilde{\psi}(1),\lambda(1)) + S(X(2),\tilde{\psi}(2),\lambda(2))$ where each $\{X(j),\tilde{\psi}(j),\lambda(j)\}$ is an independent copy of the heterotic string fields in (2). The gauge group of the world is then two copies of $E_8\times E_8$.[1] If we repeat the model construction above, we have the possibility of making 6 light families, three of which we have not seen, so we take it that they may be at any scale above what has been eliminated up to the GUT scale. They may appear at LHC, for example.

The ordinary electroweak and strong interaction gauge bosons are now an unknown mixture of the two copies of two sets of such bosons from the two $E_8's$ associated to a given string Lagrangian: when we break the four E_8's each to a product group $SU(3)\times E_6$ and then subsequently break each of the four E_6's to get four copies of $SU(3)^c \times SU(2)_L\times U(1)_Y$, for the initially massless gauge bosons for $SU(3)^c_i\times SU(2)_{Li}\times U(1)_{Yi}\in E_{8,i}$, G_i^a, $a=1,\dots,8$, $A_i^{i'}$, $i'=1,\dots,3$, B_i, $i=1,\dots,4$, in a standard notation, we assume a further breaking at the GUT scale so that the following linear combinations are massless at the GUT scale M_{GUT} while the orthogonal linear combinations acquire masses $\mathcal{O}(M_{\text{GUT}})$

$$A_f^{i'} = \sum_{i=1}^{4}\eta_{2i}A_i^{i'}, \qquad B_f = \sum_{i=1}^{4}\eta_{1i}B_i. \tag{3}$$

The mixing coefficients $\{\eta_{aj}\}$ satisfy $\sum_{i=1}^4\eta_{ai}^2 = 1$, $a = 1,2$. By the discrete symmetry that obtains if the strings are identical copies of each other, all color and electroweak gauge couplings at the scale M_{GUT} satisfy the usual GUT relations as first given by Georgi and Glashow in Ref. [12].

For the strong interaction, we take the minimal view that the quarks in each set of three families from the four E_8's are confined. By our discrete symmetry all four strong interaction gauge couplings to be equal at the GUT scale. This means that for the known quarks, we have gluons G_1^a. Of course, experiments may ultimately force us to break the as yet unseen color groups. This is straightforward to do following Ref. [75].

For the low energy EW bosons, we have quite a bit of freedom in (3). We note the following values [76, 77] of the known gauge couplings at scale M_Z:

$$\begin{aligned}
\alpha_s(M_Z)|_{\overline{\text{MS}}} &= 0.1184\pm 0.0007\\
\alpha_W(M_Z)|_{\overline{\text{MS}}} &= 0.033812\pm 0.000021\\
\alpha_{\text{EM}}(M_Z)|_{\overline{\text{MS}}} &= 0.00781708\pm 0.00000098
\end{aligned} \tag{4}$$

It is well known [79] that the factor of almost 4 between $\alpha_s(M_Z)$ and $\alpha_W(M_Z)$ and between $\alpha_W(M_Z)$ and $\alpha_{\text{EM}}(M_Z)$ when the respective unified values are 1 and 2.67 require $M_{\text{GUT}}\sim 10^{13}$–$10^{12}$ TeV. Here, with the use of the $\{\eta_{kj}\}$ we can absorb most of the discrepancy between the unification and observed values of the coupling ratios so that the GUT scale is not beyond current technology for accelerated colliding beam devices.

More precisely, we can set

$$\begin{aligned}
\eta_{21} = \eta_{22} &\cong \frac{1}{\sqrt{2.032}}\\
\eta_{11} = \eta_{12} &\cong \frac{1}{\sqrt{3.341}}
\end{aligned} \tag{5}$$

and this will leave a "small" amount of evolution do be done between the scale M_Z and M_{GUT}.

Indeed, with the choices in (5), and the use of the one-loop beta functions [8–10], if we use continuity of the gauge coupling constants at mass thresholds with one such threshold at $m_H\cong 120$ GeV and a second one at $m_t = 171.2$ GeV

[1] If one wants to avoid any reference to superstring theory, one can just postulate our symmetry and families as needed, obviously, with the effective GUT gauge group $SO(10)\times SO(10)\times SO(10)\times SO(10)$ with discrete symmetry used to achieve equality of the gauge couplings at the GUT scale and the textbook [72] symmetry breaking to the respective SM gauge group factors; we leave this to the discretion of the reader.

for definiteness to illustrate our approach,[2] then the GUT scale can be easily evaluated to be $M_{\mathrm{GUT}} \cong 100$ TeV, as advertised. For we get

$$b_0^{U(1)_Y} = \frac{1}{12\pi^2}\begin{cases}4.385, & M_Z \le \mu \le m_H \cong 120\ \mathrm{GeV}\\ 4.417, & m_H < \mu \le m_t\\ 5.125, & m_t < \mu \le M_{\mathrm{GUT}}\end{cases} \tag{6}$$

from the standard formula [8–10]

$$b_0^{U(1)_Y} = \frac{1}{12\pi^2}\left(\sum_j n_j\left(\frac{Y_j}{2}\right)^2\right) \tag{7}$$

where $b_0^{U(1)_Y}$ is the coefficient of g'^3 in the beta function for the $U(1)_Y$ coupling constant g' in the $SU(2)_L \times U(1)_Y$ EW theory of Glashow, Salam and Weinberg [1–7], n_j is the effective number of Dirac fermion degrees of freedom, i.e., a left-handed Dirac fermion counts as $1/2$, a complex scalar counts as $1/4$, and so on. Similarly, for the QCD and $SU(2)_L$ theories, we get the analogous

$$b_0^{SU(2)_L} = \frac{-1}{16\pi^2}\begin{cases}3.708, & M_Z \le \mu \le m_H \cong 120\ \mathrm{GeV}\\ 3.667, & m_H < \mu \le m_t\\ 3.167, & m_t < \mu \le M_{\mathrm{GUT}}\end{cases} \tag{8}$$

$$b_0^{\mathrm{QCD}} = \frac{-1}{16\pi^2}\begin{cases}7.667, & M_Z \le \mu \le m_t\\ 7, & m_t < \mu \le M_{\mathrm{GUT}}\end{cases} \tag{9}$$

from the standard formula [8–10]

$$b_0^{\mathcal{H}} = \frac{-1}{16\pi^2}\left(\frac{11}{3}C_2(\mathcal{H}) - \frac{4}{3}\sum_j n_j T(R_j)\right) \tag{10}$$

where $T(R_j)$ sets the normalization of the generators $\{\tau_a^{R_j}\}$ of the group $\mathcal{H}$ in the representation R_j via $\mathrm{tr}\,\tau_a^{R_j}\tau_b^{R_j} = T(R_j)\delta_{ab}$ where δ_{ab} is the Kronecker delta and $C_2(\mathcal{H})$ is the quadratic Casimir invariant eigenvalue for the adjoined representation of $\mathcal{H}$. These results (6)–(10) together with the standard one-loop solution [8–10]

$$g_{\mathcal{H}}^2(\mu) = \frac{g_{\mathcal{H}}^2(\mu_0)}{1 - 2b_0^{\mathcal{H}} g_{\mathcal{H}}^2(\mu_0)\ln(\mu/\mu_0)} \tag{11}$$

allow us to compute the value $M_{\mathrm{GUT}} \cong 100$ TeV for the values of η_{aj} given in (5). Here, we use standard notation that $g_{\mathcal{H}}^2(\mu)$ is the squared running coupling constant at scale μ for $\mathcal{H} = U(1)_Y$, $SU(2)_L$, $\mathrm{QCD} \equiv SU(3)^c$ and we note as well that the parameters η_{aj} modify the usual unification conditions at the GUT scale, for $\eta_{a1} = \eta_{a2}$, $a = 1, 2$, via

$$\begin{aligned}\alpha_{\mathrm{QCD}}(M_{\mathrm{GUT}}) &= \frac{1}{\eta_{21}^2}\alpha_{SU(2)_L}(M_{\mathrm{GUT}}),\\ \alpha_{\mathrm{QCD}}(M_{\mathrm{GUT}}) &= \frac{5}{3\eta_{11}^2}\alpha_{U(1)_Y}(M_{\mathrm{GUT}}),\end{aligned} \tag{12}$$

where as usual $\alpha_{\mathcal{H}}(\mu) \equiv g_{\mathcal{H}}^2(\mu)/(4\pi)$. The remaining parameters η_{aj}, $a = 1, 2$, $j = 3, 4$ are such that the conditions $\sum_j \eta_{aj}^2 = 1$, $a = 1, 2$ hold and would be subject to investigations of the higher energy multiplets/massive gauge bosons that have yet to be discovered according to the model we present here. We note the value $\alpha_{\mathrm{QCD}}(M_{\mathrm{GUT}}) = 0.0613$ for the case presented here, for reference. Its dependence on the η_{aj} can be seen, in the current example, from the result

$$\begin{aligned}&\alpha_{\mathrm{QCD}}(M_{\mathrm{GUT}})\\ &\quad= \frac{\alpha_{\mathrm{QCD}}(m_t)}{1 + \frac{b_0^{\mathrm{QCD}}}{b_0^{\mathrm{QCD}} - \eta_{21}^2 b_0^{SU(2)_L}}\left(\frac{\eta_{21}^2\alpha_{\mathrm{QCD}}(m_t)}{\alpha_{SU(2)_L}(m_t)} - 1\right)},\end{aligned} \tag{13}$$

from which one can see why M_{GUT} is significantly lowered by the values of η_{aj} that we use. As usual, $\alpha_s \equiv \alpha_{\mathrm{QCD}}$, $\alpha_W = \alpha_{SU(2)_L}$.

We note that the value of 100 TeV for the unification scale has been chosen for illustration, as in principle any value between the TeV scale and the Planck scale is allowed in our approach. Experiment would tell us what the true value is.

In principle the problem with baryon stability could reappear if the leptoquark bosons from different $E_{8,i}$ would mix. To prevent this, it is enough that the B–L charge from each $E_{8,i}$ is separately conserved, so that leptoquarks from different $E_{8,i}$ cannot mix.

We sum up with an interesting possible application of our approach. We recall the very precise values of the EW parameter $\sin^2\theta_{W,\mathrm{eff}}^{\mathrm{lept}}$ from the lepton sector via the A_{LR} and from the precision hadronic observable A_{FB}^b as summarized in Ref. [78]. These two measurements, arguably two of the most precise measurements at SLC and LEP, disagree by 3.2σ, where the two respective values are [78] 0.23098 ± 0.00026 and 0.23221 ± 0.00029. We see above that just a small change in the mixing coefficients for gauge bosons attendant to the families with the light quarks versus those for gauge bosons attendant to the families with light leptons easily accommodates any actual difference in these two measurements. A more precise set of EW measurements, such as those possible at an ILC/CLIC high energy e^+e^- colliding beam device, would eventually clarify the situation, presumably. More importantly, we propose here a "green pasture" instead of the traditional "desert" [12, 79].

[2] Here, we take the limit that M_{LM} is near M_{GUT} for the illustration—the case where it is a few TeV is done in the first Note-Added for completeness.

Notes added

- In the text, we took the intermediate scale M_{LM} to the limiting value M_{GUT} so that the new quarks and leptons we predict are all at the GUT scale and do not enter into our running coupling constant analyses for simplicity to illustrate the basic ideas of the discussion. It is straightforward to redo the analyses to allow the more interesting case, where for example we put one set of new leptons and new quarks at $M_{\mathrm{LM}} = 2$ TeV, so that they would be accessible at the LHC. Then by evolving our coupling constants first from $\mu = m_t$ to $\mu = 2$ TeV using the results given in the text and then from $\mu = 2$ TeV to $\mu = M_{\mathrm{GUT}} = 100$ TeV with the new values $(b_0^{U(1)_Y}, b_0^{SU(2)_L}, b_0^{\mathrm{QCD}}) = (10.1875/(12\pi^2), (5/6)/(16\pi^2), (-3/(16\pi^2)))$, we find that the required values of the η_{ij} change to $\eta_{21} = \eta_{22} \cong \frac{1}{\sqrt{2.218}}$ and $\eta_{11} = \eta_{12} \cong \frac{1}{\sqrt{3.760}}$. LHC may therefore very well discover some of our new states.
- It is possible that the 3.2σ effect discussed above is just a statistical fluctuation. Without this effect, we can simplify our approach as follows. We take a six light family string compactification [19, 56, 57] in the first E_{8a} breaking and we leave open what number of light families we get from the breaking of the second E_{8b} factor for a single heterotic string. We then only have two sets of SM gauge bosons at M_{GUT}. Again, with the three families with the known light quarks we associate heavy leptons at scale M_{LL} and with the three families with the known light leptons we associate heavy quarks at the scale M_{QL} where in the text we set generically $M_{\mathrm{LL}} \sim M_{\mathrm{QL}} \sim M_{\mathrm{LM}}$. The mixing formulas (3) now just involve $\{\eta_{jk}, j = 1, 2, k = 1, 2\}$ and the known light quarks and leptons have the 'same' leptonic effective weak mixing angle. The same calculations as we presented above in the text still obtain: for illustration with $M_{\mathrm{LM}} \sim M_{\mathrm{GUT}}$, if we take now $\eta_{21} = 1/\sqrt{2.000}$, $\eta_{11} = 1/\sqrt{3.260}$, we get $M_{\mathrm{GUT}} \cong 136$ TeV which is again in the 100–200 TeV regime. Our 'broken family' hypothesis again realizes a 'green pasture' instead of the traditional 'desert'.

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