

Monte-Carlo generator photon jets for the process $e^+e^- \rightarrow \gamma\gamma$

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Abstract Monte-Carlo generator with photon jets radiation in collinear regions for the process $e^+e^- \rightarrow \gamma\gamma$ is described in detail. Radiative corrections in the first order of α are treated exactly. Large leading logarithmic corrections coming from collinear regions are taken into account in all orders of α by applying the Structure Function approach. Theoretical precision of the cross section with radiative corrections is estimated to be 0.2%. This process is considered as an additional tool to measure luminosity in forthcoming experiments with the CMD-3 detector at the e^+e^- collider VEPP-2000.

1 Introduction

For the first time the process

$$e^+(p_+) + e^-(p_-) \rightarrow \gamma(k_1) + \gamma(k_2),$$

was considered in the classical papers by L. Brown and R. Feynman [1], I. Harris and L. Brown [2] in early 1950s and then revised in 1973 by F. Berends and R. Gastmans [3]. Due to the large magnitude of the cross section of this process, it can be exploited as an independent way to measure luminosity. Precise determination of luminosity is a key ingredient in all experiments which study hadronic cross sections at e^+e^- colliders. As a rule, a systematic error of luminosity measurements represents one of the largest sources of uncertainty which can cause significant reduction of an accuracy of the hadronic cross sections normalized to luminosity. The process of two-gamma-quantum annihilation has essential advantages for luminosity measurements with respect to those based on events of Bhabha scattering or annihilation into a muon pair. Indeed, the cross section value estimated for large angles is of the same order as that of Bhabha scattering. Events of this process have two collinear

photons at large angles providing a clean signature for their selection among other detected particles. The CLEO collaboration was the first to show in practice how the combined application of the processes $e^+e^- \rightarrow e^+e^-, \mu^+\mu^-$ and $\gamma\gamma$ helped to achieve a 1% accuracy of the luminosity measurement [4].

It is worth noting that the dependence of the Born cross section on the photon polar angle θ is not so steep as that of Bhabha scattering and it is symmetric under the transformation $\theta \rightarrow \pi - \theta$ facilitating a study of the systematic uncertainties on the detector acceptance. In addition, it is free of difficulties related to both radiation and Coulomb interaction of the final state particles. It is also of utmost importance that corresponding Feynman graphs do not contain photon propagators affected by vacuum polarization effects, Fig. 1.

Therefore, knowledge of this cross section with radiative corrections (RC) at the level of per mill accuracy is urgently needed. On the other hand, it is a purely quantum electrodynamics (QED) process giving large background while studying hadronic processes with neutral particles in the final state. These reasons are the main motivation to consider this process with precise radiative corrections and create a Monte-Carlo generator to simulate $e^+e^- \rightarrow \gamma\gamma$ events.

2 Cross section of the process $e^+e^- \rightarrow \gamma\gamma$ with the first-order α corrections

The differential Born cross section for the two-gamma-quantum annihilation in the Born approximation reads

$$\frac{d\sigma_B}{d\Omega_1} = \frac{\alpha^2}{s\beta} \left[\frac{1 + \beta^2 c_1^2}{1 - \beta^2 c_1^2} + 2\beta^2(1 - \beta^2) \frac{1 - c_1^2}{(1 - \beta^2 c_1^2)^2} \right], \quad (1)$$

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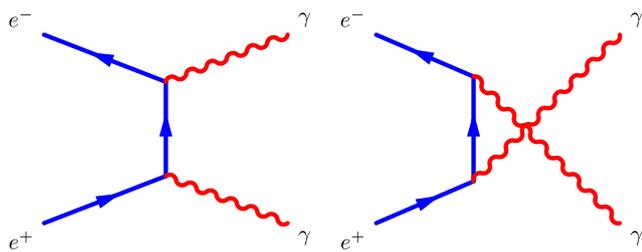


Fig. 1 Feynman diagrams for two-gamma-quantum annihilation

where $s = (p_+ + p_-)^2 = 4\varepsilon^2$, ε is the beam energy and $\beta = v/c = \sqrt{1 - m_e^2/\varepsilon^2}$, $c_1 = \cos\theta_1$, $\theta_1 = \widehat{\mathbf{k}_1 \mathbf{p}_-}$. It is assumed that both final photons are detected and their polar angles with respect to the beam directions are not too small ($\theta_{1,2} \gg m_e/\varepsilon$).

Following the well-known results obtained in [5] and considering the RC due to emission of virtual and soft real photons, this part of the cross section can be written as

$$\begin{aligned}
 d\sigma_{B+S+V} &= d\sigma_B \left\{ 1 + \frac{\alpha}{\pi} \left[(L-1) \left(2 \ln \frac{\Delta\varepsilon}{\varepsilon} + \frac{3}{2} \right) \right. \right. \\
 &\quad \left. \left. + K_{SV} \right] \right\}, \\
 K_{SV} &= \frac{\pi^2}{3} + \frac{1-c_1^2}{2(1+c_1^2)} \left[\left(1 + \frac{3}{2} \frac{1+c_1}{1-c_1} \right) \ln \frac{1-c_1}{2} \right. \\
 &\quad \left. + \left(1 + \frac{1-c_1}{1+c_1} + \frac{1}{2} \frac{1+c_1}{1-c_1} \right) \cdot \ln^2 \frac{1-c_1}{2} + (c_1 \rightarrow -c_1) \right], \tag{2}
 \end{aligned}$$

where $L = \ln(s/m_e^2)$ is a large logarithm. For $s \sim 1 \text{ GeV}^2$, $L \approx 15$. The energy of the radiated soft photons is implied to be sufficiently low so that they are not detected and its value does not exceed some small quantity $\Delta\varepsilon \ll \varepsilon$.

Consider now the process of three-gamma-quantum annihilation which can be treated as a radiative correction for two-gamma-quantum annihilation:

$$e^+(p_+) + e^-(p_-) \rightarrow \gamma(k_1) + \gamma(k_2) + \gamma(k_3).$$

For the first time the analytic expression for this process was obtained by M.V. Terentjev in [6]. A much simpler way to obtain the same expression was suggested in [7, 8], based on the chiral amplitude method, when all three hard photons are emitted in the so called non-collinear region (outside narrow cones). The differential cross section is given by

$$d\sigma^{e^+e^- \rightarrow 3\gamma} = \frac{\alpha^3}{8\pi^2 s} R_{3\gamma} d\Gamma, \tag{3}$$

$$\begin{aligned}
 R_{3\gamma} &= s \frac{\chi_3^2 + (\chi'_3)^2}{\chi_1 \chi_2 \chi'_1 \chi'_2} \\
 &\quad - 2m_e^2 \left[\frac{\chi_1^2 + \chi_2^2}{\chi_1 \chi_2 (\chi'_3)^2} + \frac{(\chi'_1)^2 + (\chi'_2)^2}{\chi'_1 \chi'_2 \chi_3^2} \right] \\
 &\quad + \text{two cyclic permutations,}
 \end{aligned}$$

$$\begin{aligned}
 d\Gamma &= \frac{d^3k_1 d^3k_2 d^3k_3}{k_1^0 k_2^0 k_3^0} \delta^{(4)}(p_+ + p_- \\
 &\quad - k_1 - k_2 - k_3),
 \end{aligned}$$

$$d\Gamma = \frac{s}{2} \frac{d\Omega_1 d\Omega_2 d\Omega_3 dx_3}{[2 - x_3(1 - c_3)]},$$

where $\chi_i = k_i p_-$, $\chi'_i = k_i p_+$, $i = 1, 2, 3$, $x_i = k_i^0/\varepsilon$, $c_i = \cos(\theta_i)$, $\theta_i = \widehat{\mathbf{p}_- \mathbf{k}_i}$.

Energy-momentum conservation allows to determine kinematics of the final photons:

$$\begin{aligned}
 x_1 + x_2 + x_3 &= 2, & x_1 c_1 + x_2 c_2 + x_3 c_3 &= 0, \\
 x_1 &= \frac{1 - x_3}{1 - x_3 \sin^2 \frac{\psi}{2}}, \\
 x_2 &= \frac{\cos^2 \frac{\psi}{2} + (1 - x_3)^2 \sin^2 \frac{\psi}{2}}{1 - x_3 \sin^2 \frac{\psi}{2}}, \\
 c_2 &= -\frac{x_1 c_1 + x_3 c_3}{x_2}, & \psi &= \widehat{\mathbf{k}_1 \mathbf{k}_3}. \tag{4}
 \end{aligned}$$

The sum of this cross section integrated over all variables with those describing radiation of *soft* and *virtual* photons does not depend on inner parameters. It allows to construct a MC event generator to simulate three-photon events and to take into account proper selection criteria of a given experiment as well as include specific detector imperfections.

2.1 Matching NLO and higher-order (HO) corrections

It is known that photon jets are radiated in collinear regions inside narrow cones along the motion of electrons and positrons give the dominant contribution to the cross section. So, in order to achieve a theoretical precision of about per mill, all enhanced HO corrections must be taken into account and combined with NLO corrections. The opening angle of these narrow cones is small and obeys the restrictions: $1/\gamma \ll \theta_0 \ll 1$. As a rule, its value is chosen as $\theta_0 = 1/\sqrt{\gamma}$. Since the photon radiation outside these cones is not enhanced, it is sufficient to consider only radiation of one photon at large angles [9, 10] to keep the theoretical accuracy at the per mill level.

For completeness, the cross section with one hard photon emission in the collinear region is presented below. It can be obtained using the method of quasireal electrons [11] similarly to [9]. The theorem of factorization [12] of *hard* and

soft photons permits to treat RC in the leading logarithmic approximation in all orders of perturbation theory in terms of Structure Functions of electron and positron, $\mathcal{D}(z, L)$. This fact allows one to write the differential cross section for the process of two-gamma-quantum annihilation integrated inside the collinear region as follows:

$$d\sigma_{\text{coll}} = \frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx}{x} \left[\left(z + \frac{x^2}{2} \right) \left(L - 1 + \ln \frac{\theta_0^2}{4} \right) + \frac{x^2}{2} \right] [d\tilde{\sigma}_0(z, 1) + d\tilde{\sigma}_0(1, z)], \tag{5}$$

where $z = 1 - x$ is electron (positron) energy after emitting a photon with energy x .

The *shifted* Born cross section with reduced energies of the incoming electrons and positrons has the form

$$d\sigma_0(z_1, z_2) = \frac{2\alpha^2}{s_{z_1 z_2}} \cdot \frac{z_1^2(1 - c_1)^2 + z_2^2(1 + c_1)^2}{(1 - c_1^2)(z_1 + z_2 + (z_2 - z_1)c_1)^2} d\Omega_1, \tag{6}$$

where the scattering angle θ_1 is given for the original c.m. reference frame of the colliding beams, z_1 and z_2 are the energy fractions of electron and positron just before a collision after radiating photon jets. When z_1 and z_2 tend to unity, this cross section is transformed to (1). One can see that a part of this cross section has a term proportional to large logarithm $L = \ln(s/m_e^2)$, due to collinear photon emission, and it is already contained in Structure Functions. Therefore, to match NLO and HO corrections and exclude double counting, the term proportional to large logarithm must be removed. The remaining non-leading correction referred to as a *compensator* should be combined with the cross section describing three-photon production to cancel the dependence on the auxiliary parameter θ_0 .

3 Total cross section of the process $e^+e^- \rightarrow \gamma\gamma + n\gamma$

In the following, we summarize the main features of the matching procedure as implemented in the code MCGPJ [9]. Adding the higher-order RC in the leading logarithmic approximation to the complete one-loop result (NLO), the *master* formula for the resulting cross section can be represented as follows:

$$d\sigma^{e^+e^- \rightarrow \gamma\gamma + (n\gamma)} = \int_0^1 dz_1 \mathcal{D}(z_1) \int_0^1 dz_2 \mathcal{D}(z_2) \cdot d\tilde{\sigma}_0(z_1, z_2) \left(1 + \frac{\alpha}{\pi} K_{SV} \right) \Theta(\text{cut } s)$$

$$+ \frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx}{x} \left[\left(z + \frac{x^2}{2} \right) \ln \frac{\theta_0^2}{4} + \frac{x^2}{2} \right] \cdot [d\tilde{\sigma}_0(z, 1) + d\tilde{\sigma}_0(1, z)] \Theta(\text{cut } s) + \frac{1}{3} \int_{\pi - \theta_0 \geq \theta_i \geq \theta_0}^{x_i \geq \Delta} \frac{4\alpha^3}{\pi^2 s^2} \left[\frac{x_3^2(1 + c_3^2)}{x_1^2 x_2^2 (1 - c_1^2)(1 - c_2^2)} + \text{two cyclic permutations} \right] d\Gamma \Theta(\text{cut } s), \tag{7}$$

where $\mathcal{D}(z)$ is the smoothed representation for the Structure Functions according to [14]. A factor 1/3 in the last term takes into account the identity of the final photons. All variables have been defined above.

This expression contains the logarithmically enhanced contributions due to emission of photons at all powers of α in collinear regions and, as it will be shown later, provides the cross section accuracy of about 10^{-3} . The first term describes radiation of photon jets which is approximated by the convolution of the Structure Functions with the *shifted* Born cross section ($s' = s_{z_1 z_2}$). The step functions $\Theta(\text{cut } s)$ stand for particular cuts applied. The sum of the last two terms provides cancellation of the auxiliary parameters Δ and θ_0 . A detailed comparison was performed between the results obtained with MCGPJ and the MC generator based on [13] for the cuts modeling the CMD-2 event selection criteria at the c.m. energy $\sqrt{s} = 900$ MeV. These cuts are:

- The polar angles of the two most energetic photons must be inside a range $1.1 < \theta_{1,2} < \pi - 1.1$
- Acollinearity must obey $|\theta_1 + \theta_2 - \pi| < 0.25$ and $||\phi_1 - \phi_2| - \pi| < 0.15$
- The energies of the two most energetic photons must be larger than half of beam energy.

In Fig. 2 the cross section as a function of the $\Delta\varepsilon$ is shown when other parameters are fixed according to selection criteria pointed above. It can be seen that there is a broad plateau, where the cross section deviations lie within a band with a width of $\sim 0.2\%$, whereas $\Delta\varepsilon$ runs by more than two orders of magnitude. Only for large values of $\Delta\varepsilon$ some trend appears which can be explained by the omitted terms proportional to $\Delta\varepsilon/\varepsilon$. Similar dependence is seen in Fig. 3, where the cross section variations with the auxiliary parameter θ_0 are presented. Only for extremely small values of θ_0 , when a condition $1/\gamma \ll \theta_0$ is not valid, the cross section falls down by $\sim 0.1\%$ only. It is an important result, which confirms that the cross section does not depend on the auxiliary parameters $\Delta\varepsilon$ and θ_0 . It is important to reliably estimate the total theoretical precision of this cross section with RC. In order to quantify the theoretical accuracy, an independent comparison has been performed with the MC event generator based on [13], where only first-order α corrections are treated exactly. It was found that the relative difference of the cross

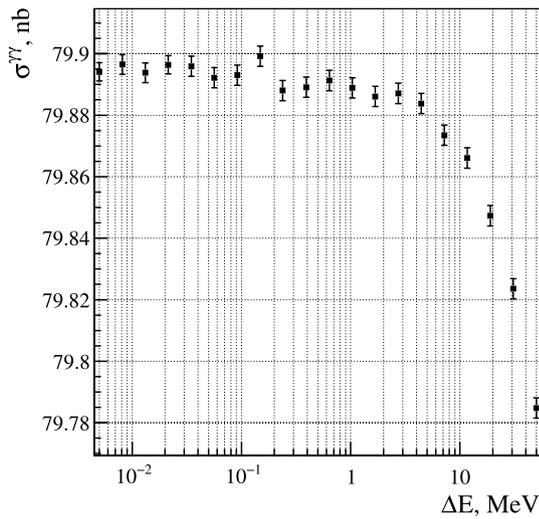


Fig. 2 Cross section dependence on the auxiliary parameter $\Delta\varepsilon$, $\sqrt{s} = 900$ MeV

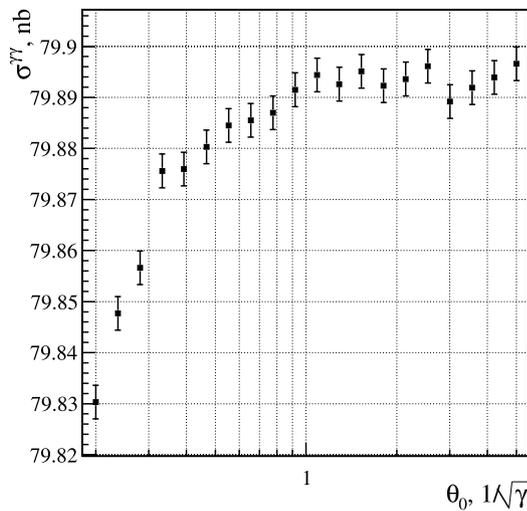


Fig. 3 Cross section dependence on the auxiliary parameter θ_0 , $\sqrt{s} = 900$ MeV

sections is larger than 1% for small angles $\Delta\theta < 0.1$ radians and it is about $\sim 0.6\%$ for an acollinearity angle ~ 0.25 radians. The simulation results are presented in Fig. 4. It is seen that the difference practically does not depend on the choice of the value $\Delta\theta$ with accuracy $\pm 0.1\%$. The same difference was studied as a function of beam energy when the acollinearity angle $\Delta\theta$ was fixed to 0.25 radians. The results are shown in Fig. 5. In this case the difference slowly increases with energy from 0.3% to 0.7%. It is an important fact, which means that for this energy band the radiation of two and more photons (jets) in the collinear region contributes to the cross section by amount $\sim 0.5\%$ only. As the uncertainty of this correction is at least a few times smaller than the magnitude of the correction itself, we can conclude: a theoretical precision of the cross section with RC is certainly better than $\sim 0.2\%$.

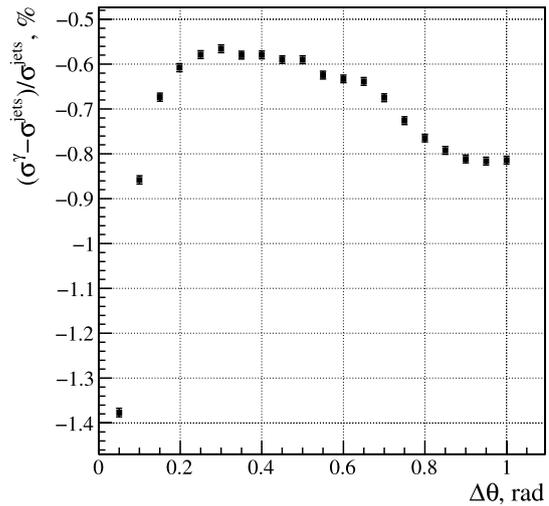


Fig. 4 Relative cross section difference calculated with the MCGPJ code and generator based on Ref. [5] versus the angle $|\Delta\theta|$

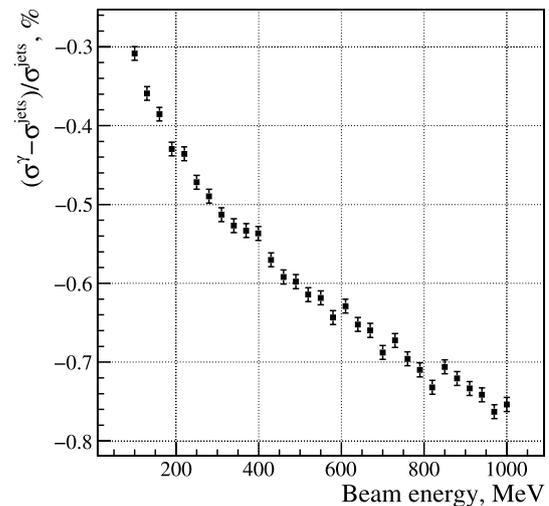


Fig. 5 Relative cross section difference calculated with the MCGPJ code and generator based on Ref. [5]

4 Conclusion

The main features of the Monte-Carlo generator to simulate events of the process $e^+e^- \rightarrow \gamma\gamma$ are described. The theoretical precision of the cross section with RC is estimated to be 0.2% if soft selection criteria are applied. This accuracy was achieved due to the application of the Structure Function approach. In frame of this scheme one can combine the higher-order terms proportional to $(\alpha \log(s/m_e^2))^n$ with the Born and first-order matrix element, if conditions for matching described in [9, 10] are met. A similar precision was quoted for other processes considered in [9].

It is proposed to use the generator as a complementary tool to measure the collider luminosity compared to the

regularly used processes of Bhabha scattering or $e^+e^- \rightarrow \mu^+\mu^-$. The precision of collider luminosity determination represents one of the largest sources of systematic errors which can cause significant reduction of the accuracy of hadronic cross sections, which are, as a rule, normalized to luminosity.

The considered cross section is rather large, events have a clean signature in the detector and can be easily recognized and selected. From the theoretical point of view, the main advantage of using the process $e^+e^- \rightarrow \gamma\gamma$ compared to others is the following: the cross section does not contain the first order of α corrections due to the vacuum polarization effects and there is no FSR and Coulomb interaction of particles.

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References

1. L. Brown, R. Feynman, Phys. Rep. **85**, 231 (1952)
2. I. Harris, L. Brown, Phys. Rep. **105**, 1656 (1957)
3. F.A. Berends, R. Gastmans, Nucl. Phys. B **61**, 414 (1973)
4. G. Grawford et al., Nucl. Instrum. Methods A **345**, 429 (1994)
5. S.I. Eidelman, E.A. Kuraev, Nucl. Phys. B **143**, 353 (1978)
6. M.V. Terentjev, Yad. Fiz. **9**, 1212 (1969)
7. G. Altarelli, R. Parisi, Nucl. Phys. B **126**, 298 (1977)
8. L. Lipatov, Yad. Fiz. **20**, 181 (1974)
9. A.B. Arbuzov et al., Eur. Phys. J. C **46**, 689 (2006)
10. S. Actis et al., Eur. Phys. J. C **66**, 585 (2010)
11. V.N. Baier, V.S. Fadin, V.A. Khoze, Nucl. Phys. B **65**, 381 (1973)
12. S. Eidelman, E. Kuraev, V. Panin, Nucl. Phys. B **148**, 245 (1979)
13. F.A. Berends et al., Nucl. Phys. B **206**, 61 (1982)
14. E.A. Kuraev, V.S. Fadin, Sov. J. Nucl. Phys. **41**, 466 (1985)