

# Lorentz violation in high-energy ions

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Received: 20 June 2010 / Revised: 28 July 2010 / Published online: 15 September 2010  
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**Abstract** The theoretical interest in small Lorentz violations has motivated experiments that investigate it by measuring deviations in the time dilation predicted by special relativity (SR) using high-energy ions. The main contribution of this article is to show that including the Doppler effect in the emission (which is of the same order as the time dilation effect) in the analysis leads to differences between experimental and theoretical predictions that indicate potential Lorentz violation.

## 1 Introduction

Recent theoretical efforts allow for small Lorentz violations, see, e.g., [1–6]. For example, Doppler shift experiments such as the Ives–Stillwell experiment are sensitive to Lorentz violating terms in standard model extension [1, 2]. This renews the interest in experiments on high-speed ions that check for Lorentz violations by experimentally measuring potential deviations in the time dilation predicted by special relativity [7–11].

The time dilation predicted by special relativity (SR) has been verified at low speeds with a number of experiments starting with the classical experiment using hydrogen canal rays by Ives and Stilwell [12] at speed  $v = 0.005c$  where  $c$  is the speed of light. More recent experiments [7–11] have evaluated the time-dilation effect by using Doppler-shifted lasers to excite transitions in high-energy neon (Ne) and lithium (Li) ions and then observe the emissions at high ion speeds—as high as  $v = 0.338c$  in [11].

The main contribution of this article is to show that including the Doppler effect on the emission from high-energy ions (which is of the same order as the time dilation effect) in the analysis leads to differences between experimental and theoretical predictions that indicate potential Lorentz violation.

## 2 High-energy ion spectroscopy

### 2.1 Doppler effect on laser and emission frequencies

The main idea is to use Doppler-shifted lasers to excite transitions in high-energy ions and then observe the resulting emissions to evaluate time dilation. In particular, an ion moving with speed  $v = \beta c$  with respect to a laboratory frame  $F_L$  can be excited by using parallel (co-propagating) or anti-parallel (counter-propagating) lasers as in Fig. 1.

The relations between associated laser frequencies  $\nu_p, \nu_a$  (parallel and anti-parallel to the ion velocity with respect to the laboratory frame  $F_L$ ) and  $\nu_p^*, \nu_a^*$  (parallel and anti-parallel to the ion velocity with respect to a frame  $F_I$  attached to the moving ion) are given by relativistic Doppler expressions as (see, e.g., [10, 13])

$$\nu_p^* = \nu_p \gamma (1 - \beta), \quad (1)$$

$$\nu_a^* = \nu_a \gamma (1 + \beta), \quad (2)$$

where the time dilation term is

$$\gamma = 1/\sqrt{1 - \beta^2}. \quad (3)$$

Similarly, the emission frequency  $\nu_e^*$  of photons from the ions (with respect to the ion frame  $F_I$ ) and the detection frequency  $\nu_e$  perpendicular to the moving ions (with respect to the laboratory frame  $F_L$ ) are related by (see, e.g., [13])

$$\nu_e^* = \nu_e \gamma, \quad (4)$$

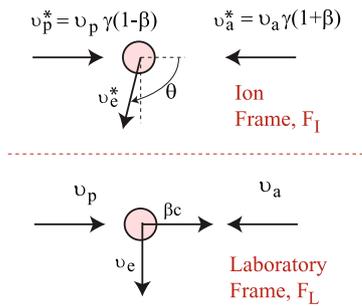
with angle  $\cos \theta = -\beta$  in the ion frame  $F_I$  as shown in Fig. 1.

### 2.2 Evaluating potential Lorentz violation

The ratio  $R$ , given by

$$R = \frac{\nu_p \nu_a}{\nu_p^* \nu_a^*} = \frac{1}{\gamma^2 (1 - \beta^2)} = 1, \quad (5)$$

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**Fig. 1** Laser frequencies  $\nu_p, \nu_a$  in the laboratory frame  $F_L$  are Doppler shifted to frequencies  $\nu_p^*, \nu_a^*$  in a frame  $F_I$  that is moving with the ion (circle) at speed  $v = \beta c$ . Photons emitted at frequency  $\nu_e^*$  in the ion frame  $F_I$  are observed at frequency  $\nu_e$  in the laboratory frame  $F_L$  perpendicular to the moving ions

is independent of the speed of the ions. Potential dependence of the measured ratio  $R$  on speed  $\beta$  is used to evaluate the time dilation predicted by SR and thereby, to evaluate potential Lorentz violation.

The frequency terms  $\nu_p^*, \nu_a^*$  needed to evaluate the expression for  $R$  (in (5)) are not directly measurable. This inability to measure  $\nu_p^*, \nu_a^*$  can be avoided if the lasers excite a known transition, say at frequency  $\nu_e^*$  in the ion frame  $F_I$ . For example, in saturation spectroscopy [10, 11], one of the laser frequencies is kept constant and the other frequency is varied to observe the Lamb dip in the fluorescence spectrum, which indicates that both lasers are acting on ions with the same speed, i.e.,

$$\nu_p^* = \nu_e^* \quad \text{and} \quad \nu_a^* = \nu_e^*.$$

Then, the ratio  $R$  (in (5)) can be rewritten as

$$R = \frac{\nu_p \nu_a}{(\nu_e^*)^2} = \frac{1}{\gamma^2(1 - \beta^2)} = 1. \tag{6}$$

### 2.3 Transition frequency shift

Ideally, the transition frequency  $\nu_e^*$  in the moving ion frame  $F_I$  should be the same as the transition frequency  $\nu_o$  for ions that are stationary in the laboratory frame  $F_L$ , and is therefore known—the transition frequency  $\nu_o$  can be determined with high accuracy using stationary ions in the laboratory frame  $F_L$ . However, the transition frequency for the moving ions can get shifted (i.e.,  $\nu_e^* \neq \nu_o$ ) due to external fields and charged particles causing Stark and Zeeman effects, e.g., [11].

The potential shift in the transition frequency implies that the transition frequency  $\nu_e^*$  excited in the moving ion cannot be assumed to be exactly the same as the transition frequency  $\nu_o$  measured for stationary ions under different experimental conditions. Moreover, this shifted frequency  $\nu_e^*$  is not directly measurable (in the moving ion frame  $F_I$ )—although the transition frequency  $\nu_o$  of the stationary ions

(in laboratory frame  $F_L$ ) is known. Therefore, the ratio  $R$  (in (6)) cannot be evaluated directly from the measurements, and an expression in terms of the directly measurable frequencies ( $\nu_a, \nu_p, \nu_o$ ) is sought.

### 2.4 Effect of PMT pre-filters

Emissions from the ions are used to identify when the Doppler-shifted laser frequencies match the transition frequencies in the moving ions, e.g., using the Lamb dip in the observed fluorescence spectrum and to optimize the experiment [8–10].

In general, measurements of the number of photons emitted by the moving ions (to determine excitation of the ion transition) using photomultiplier tubes (PMTs) should be independent of the photon frequency. However, the measurements will not be independent of photon frequency if optical pre-filters are used before the PMTs. For example, the emission is observed using an interference filter (before the PMT) centered at the transition frequency  $\nu_o$  (of stationary ions) with a narrow (10 nm) halfwidth in [14] to precisely detect the laser frequency where the Lamb dip occurs. In this case, the observed Lamb dip corresponds to emitted photons of a specific frequency,

$$\nu_e = \nu_o, \tag{7}$$

with respect to the laboratory frame  $F_L$ , which is related to the emitted photon frequency  $\nu_e^*$  in the ion frame  $F_I$  by (from (4))

$$\nu_e^* = \gamma \nu_e = \gamma \nu_o. \tag{8}$$

Therefore, a theoretical expression  $R_o$  that can be evaluated in terms of measurable frequencies  $\nu_o, \nu_p, \nu_a$  (in the laboratory frame  $F_L$ ) can be found from (6), (8) as

$$R_o = \frac{\nu_p \nu_a}{(\nu_o)^2} = \frac{\gamma^2 \nu_p \nu_a}{(\nu_e^*)^2} = \frac{1}{1 - \beta^2} \approx 1 + \beta^2. \tag{9}$$

### 3 Lorentz violation

Experimental observations find this ratio  $R_{o,exp}$  of the product of the laser frequencies  $\nu_{p,exp} \nu_{a,exp}$  to the transition frequency  $\nu_{o,exp}$  of the stationary ion to be a constant

$$R_{o,exp} = \frac{\nu_{p,exp} \nu_{a,exp}}{(\nu_{o,exp})^2} = 1 \tag{10}$$

that is independent of the speed  $\beta$  [11] where the subscript exp indicates an experimentally obtained value. This results in a difference between theoretical ( $R_o$ ) and experimental ( $R_{o,exp}$ ) predictions of this ratio (from (9), (10))

$$R_o \neq R_{o,exp}. \tag{11}$$

This  $\beta^2$ -order deviation in  $R_o$  using SR time dilation expressions (between (9) and (10)) indicates potential Lorentz violation in high-energy ions.

The difference between the theoretical ratio ( $R_o$  in (9)) and the experimental ratio ( $R_{o,\text{exp}}$  in (10)) should be experimentally discernible as the speed  $\beta$  increases and, therefore, the difference  $\beta^2$  increases. For example, at speed  $\beta = 0.064$ , the experimental accuracy of  $R_{o,\text{exp}}$  (in (10)) is estimated in [10] as

$$|R_{o,\text{exp}} - 1| \leq 2\beta^2\hat{\alpha} = 1.8 \times 10^{-9} \quad (12)$$

with  $\hat{\alpha} = 2.2 \times 10^{-7}$  [10] whereas the theoretical ratio ( $R_o$  in (9)) yields

$$\|R_o - 1\| \approx \beta^2 = 0.0041. \quad (13)$$

### 3.1 Analysis without Doppler shift in emission

If the Doppler shift in the photon emission is neglected, i.e., emission frequency  $\nu_e^*$  (in the ion frame  $F_I$ ) is approximated by the emission frequency  $\nu_e = \nu_o$  (in the laboratory frame  $F_L$ ), i.e.,  $\nu_e^* \approx \nu_e = \nu_o$  then the constant  $R_o$  (in (9)) can be approximated as

$$R_o = \frac{\nu_p \nu_a}{(\nu_o)^2} \approx \frac{\nu_p \nu_a}{(\nu_e^*)^2} = 1, \quad (14)$$

which would match the experimental ratio  $R_{o,\text{exp}}$ .

The SR predictions are matched exactly by the experimental results only if the transverse Doppler effect in the observed emissions from the ions are neglected in the analysis. However, such an approximation ( $\nu_e^* \approx \nu_e = \nu_o$ ) is not reasonable because the Doppler effect in the emission is of the same order ( $\gamma$ ) as the time dilation being measured and is, therefore, not negligible.

### 3.2 Other effects

It is possible that other effects (such as variations in the observation angle) might explain or reduce the apparent Lorentz violation; further work is needed to investigate such effects. For example, previous work has shown that resonance fluorescence can be affected by the observational angle [15]. Therefore, the experimental results would be affected if the PMT is not measuring emissions that are exactly perpendicular to the ion beam. Further analysis would be needed to evaluate such angle-deviation effects.

Future experiments could include the Doppler shift in the emissions in their design. For example, it might be possible to place filters, centered at the frequency  $\nu_o/\gamma$ , before the PMT. In this case the emissions measured in the laboratory frame  $F_L$  at frequency  $\nu_o/\gamma$  (and the associated Lamb dip) would correspond to  $\nu_o$  (a known value) in the ion frame. This would guarantee that there is no shift in the transition

frequency in the ion frame  $F_I$ , i.e., emissions measured correspond to frequency  $\nu_o$  in the ion frame, which is the known value for stationary ions. Such experimental efforts could clarify and quantify, better, the potential Lorentz violation identified in this article and its potential effect on systematic errors reported in other related experiments, e.g., when the ions are moving perpendicular to the lasers [8].

## 4 Conclusions

This article analyzed experiments that investigate Lorentz violation by measuring deviations in the time dilation predicted by special relativity (SR) using high-energy ions. It was shown that including the Doppler effect on the emission (which is of the same order as the time dilation effect) in the analysis leads to differences between experimental and theoretical predictions that indicate potential Lorentz violation in high-energy ions.

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