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Quantum confined stark effect in wide parabolic quantum wells: real density matrix approach

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Abstract. We show how to compute the optical functions of wide parabolic quantum wells (WPQWs) exposed to uniform electric **F** applied in the growth direction, in the excitonic energy region. The effect of the coherence between the electron-hole pair and the electromagnetic field of the propagating wave including the electron-hole screened Coulomb potential is adopted, and the valence band structure is taken into account in the cylindrical approximation. The role of the interaction potential and of the applied electric field, which mix the energy states according to different quantum numbers and create symmetry forbidden transitions, is stressed. We use the real density matrix approach (RDMA) and an effective e-h potential, which enable to derive analytical expressions for the WPQWs electrooptical functions. Choosing the susceptibility, we performed numerical calculations appropriate to a GaAs/GaAlAs WPQWs. We have obtained a red shift of the absorption maxima (quantum confined Stark effect), asymmetric upon the change of the direction of the applied field ($\mathbf{F} \rightarrow -\mathbf{F}$), parabolic for the ground state and strongly dependent on the confinement parameters (the QWs sizes), changes in the oscillator strengths, and new peaks related to the states with different parity for electron and hole.

1 Introduction

The effects on optical spectra when an external electric field is applied, known in atomic physics as the Stark effect, evolved very rapidly with the invention and development of semiconductor nanostructures. The effects of confinement of carriers overlap with the interaction with the field giving rise to the new phenomenon known as the quantum confined Stark effect (QCSE). First reported for quantum wells by Miller et al. [1,2], the QCSE is is still the subject of a vivid interest [3,4]. References [1–38] are only a small collection of a very large number of papers studying the properties of various nanostructures (quantum wells, quantum dots, quantum rods, superlattices etc.) under electric field. In most of these nanostructures the applied electric field causes a red-shift of the positions of the lowest energy states, changes in the exciton binding energy, and lowering the oscillator strengths of the resonances. Here we consider QCSE in wide parabolic quantum wells (WPQWs), of thicknesses in the growth direction of the order of a few excitonic Bohr radii of the well material (see, for example [39–41,43], and references therein). The optical spectra of WPQWs show a large number of resonances, which are due to the transitions between confined states. The Coulomb e-h potential and different confinements for electrons and holes cause mixing of the states with different quantum (confinement) numbers. When additionally an electric field is applied, states symmetry forbidden appear in the spectra. The behavior of the positions of the resonances is more complicated than in the narrow QWs since the lower states show a red-shift, but some higher states show a blue-shift or a zig-zag shape, and their oscillator strengths decrease. As in the previous paper [43], to compute the electrooptical effects in WPQWs the RDMA will be used. As merely shown in the past, this approach is well suited for describing the electro- and magnetooptical properties of excitons in various systems, bulk and low dimensional. In particular, it was applied for electrooptic effects in superlattices [16,22,28], quantum wires [18,24], and quantum dots [20–22,24,27,31]. Even if the basic equations of the RDMA, the so-called constitutive equations, have the same form for the mentioned systems, the way of solution is different and accounts for the specific properties of the given system and of the phenomenon considered. In the case of SLs, treated in references [16,22], the solution accounts for the polaritonic aspect and the additional boundary conditions problem, and was obtained in terms of the appropriate Green function. The main effect discussed was the so-called Franz-Keldysh effect which consists in oscillatory behavior of the optical functions for excitation energy above the fundamental gap. A similar calculation method allowing for prediction of the Franz-Keldysh effect, was applied for the case of quantum wires [18] and quantum wells for an applied electric field parallel to the QWW axis or to the QW surface,

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respectively (for example [21]). In the case of shallow QWs and QDs, where one observes an energetic shift of the excitonic resonances, so-called Stark shift, the solution of the RDMA equations requires different approaches. The comparison of the effects in shallow and wide QWs will be given below. The Stark effect in WPQWs, which we wish to discuss in the following, is different from the electrooptic effects in the above mentioned low-dimensional systems and a different type of solution of the RDMA equations will be applied. Here, we extend the model described in our previous paper [43] including the electric field and this inclusion gives rise to new phenomena, a few examples of which will be presented. As an example, we consider a WPQW with GaAs as the optically active layer and $Ga_{1-x}Al_xAs$ as the barriers, where the active layer is of the extension of a few excitonic Bohr radii, and the constant electric field is applied in the growth direction, which we identify with the z axis.

Although our investigations deal with the theoretical model of WPQW exposed to uniform electric field it is believed that such systems are important due to their controllability and potential applications [44]. Those investigations are very promising for high-speed, low-power optical devices. Employing an external electrostatic field to quantum well allows one for steering the optical properties of the system. Together with the geometric characteristic of QW the external field is one of the strong modulating factor influencing the energy spectrum of charge carriers. Due to controllability of the field the optical properties of the nanostructures can be changed on demand. Performing the manipulations of the external interaction on WPQW gives one possibility of an effective processing of electrosusceptibility, which may in the future be exploited for constructing electrooptical modulators or optoelectronic processors operating on demand. The computed shift of the resonances, quadratic Stark shift, and changes in the oscillator strengths should be valuable for those experimentalists who try manipulate and optimize the properties of optoelectronic devices.

Our paper is organized as follows. In Section 2, we present the assumptions of considered model and solve the constitutive equation with effective electron-hole interaction potential and the applied field. Next, in Section 3, the derived solution of the constitutive equation is used to obtain the electrosusceptibility for two chosen $GaAs/Ga_{1-x}Al_xAs$ WPQWs. Finally, in Section 4, we present the conclusions.

2 The model

We will compute the linear optical response of a WPQW of thickness L to a plain electromagnetic wave

$$E_i(z,t) = E_{i0}\exp(ik_0 z - i\omega t), \qquad k_0 = \omega/c, \tag{1}$$

for energies $\hbar\omega$ near to the fundamental gap of the well material. A constant electric field $\mathbf{F}$ is applied parallel to the z-axis. In the calculations we use the real density matrix approach (RDMA), which also includes the effective mass approximation [24,45]. In this approach the linear optical response will be described by a set of coupled equations: constitutive equation for the coherent amplitude $Y_\nu(\mathbf{r}_e,\mathbf{r}_h)$ (ν labels the allowed interband transitions for the well material), and Maxwell's field equation. The movement of the carriers in the z direction is determined by one-dimensional parabolic potentials, characterized by the oscillator energies $\hbar\omega_e, \hbar\omega_h$, respectively. The amplitudes Y_ν determine the polarization which, inserted in Maxwell's field equations, gives the electric field of the wave propagating in the QW. Having the field we can determine the QW electroptical functions (reflectivity, transmission, and absorption).

Thus the next steps are the following: we formulate the constitutive equations. The equations will be then solved giving the coherent amplitudes Y. From the amplitudes we compute the polarization inside the quantum well, the electric field of the wave propagating in the QW, and the optical functions.

The constitutive equation for the coherent amplitude Y in a WPQW and with the applied homogeneous electric field $\mathbf{F} = F\mathbf{k}$ has the form (see, for example [24])

$$\left[E_g - \hbar\omega - \mathrm{i}\Gamma + \hat{H}_e + \hat{H}_h + \frac{\hat{\mathbf{p}}_\rho^2}{2\mu_\parallel} + \frac{\hat{\mathbf{p}}_\parallel^2}{2M_\parallel} + V_{eh}(\rho, z_e, z_h)\right] Y = \mathbf{M}(\mathbf{r})\mathbf{E}(\mathbf{R}), \tag{2}$$

where $H_{e,h}$ are Hamilton operators for the one-dimensional harmonic oscillator including the effects of the applied electric field

$$\hat{H}_{eh} = \frac{\hat{p}_{e,hz}^2}{2m_{e,hz}} + \frac{1}{2}m_{e,hz}\omega_{e,h}^2 z_e^2 \pm eFz_{e,h}, \tag{3}$$

$m_{e,hz}$ are the electron and hole effective masses in the z-direction, $\rho = \sqrt{(x_e - x_h)^2 + (y_e - y_h)^2}$ is the two-dimensional e-h distance, $V_{eh}(\rho, z_e, z_h)$ is the electron-hole interaction potential, $\mathbf{M}(\mathbf{r})$ is the transition dipole density, which form we have assumed as:

$$\mathbf{M}(\mathbf{r}) = \mathbf{M}(\rho, z, \phi) = \frac{\mathbf{M}_0}{2\pi\rho_0}\delta(z)\delta(\rho - \rho_0), \tag{4}$$

$z = z_e - z_h$ being the relative coordinate in the z direction, ρ_0 is the coherence radius (the physical meaning was explained, for example, in Refs. [45,46]), $\mathbf{R}$ is the excitonic center-of-mass coordinate, $\mathbf{E}(\mathbf{R})$ is the electric field vector of the wave propagating in the QW, and $\hat{\mathbf{p}}_\rho$, $\hat{\mathbf{p}}_\parallel$ are the momentum operators for the excitonic relative- and center-of-mass motion in the QW plane. In the consideration of narrow QWs (with extension less than one excitonic Bohr radius and arbitrary confinement shape) the following approximation was often used. The movement in the z-direction was decoupled from the movement in the xy plane, and the electron-hole interaction was assumed in

the 2-dimensional form

$$V_{eh} = -\frac{1}{4\pi\epsilon_0\epsilon_b}\frac{e^2}{\rho} \tag{5}$$

with the QW material dielectric constant ϵ_b. This approximation enabled to obtain analytical solutions for the electron and the hole wave functions, and thus the calculation of the optical properties (see for example, [24]). Such method cannot be used in the considered case of wide QWs (the extension of several excitonic Bohr radii) since the e-h interaction retains its 3-dimensional character. As was pointed in references [42,43], the direct numerical solution of the constitutive equation (2) is, at the moment, not available because of lack of the appropriate orthonormal basis to use in order to decrease the dimension of the 6-dimensional configuration space [47]. Therefore we use the following 3-dimensional form of the interaction potential

$$V_{eh} = -S\exp\left[-v\left(z_e - z_h\right)^2 - w\rho^2\right] \tag{6}$$

with parameters v, w appropriate for a given nanostructure, which enables to perform analytical calculations and reproduces the basic properties of the exciton [42,43].

In the following we assume that the propagating wave is linearly polarized in the x direction, and that the vector $\mathbf{M}$ has a non-vanishing component in the same direction. Such polarization, in the case of the considered below nanostructures, induces the heavy-hole and light-hole transitions. We look for a solutions Y in terms of the eigenfunctions ψ_{ej}, ψ_{hn} of the operators H_e, H_h:

$$\begin{aligned} Y\left(\rho, z_e, z_h\right) &= \psi_0(\rho)\sum_{j,n=0}^{N}\psi_{ej}(z_e)\psi_{nh}(z_h)Y_{jn} \\ &= \frac{\sqrt{2\lambda}}{\sqrt{2\pi}}e^{-\lambda\rho^2/2}\sum_{j,n=0}^{N}\psi_{ej}(z_e)\psi_{nh}(z_h)Y_{jn}, \end{aligned} \tag{7}$$

where $\psi_0(\rho,\phi)$ with the corresponding eigenvalue ϵ_0 stands for the excitonic ground state, and Y_{jn} are constant coefficients. This *Ansatz* for the amplitude was explained and justified in reference [43]. The eigenfunctions have the form

$$\psi_{ej}(\xi_e) = N_{ej}e^{-\xi_e^2/2}H_j(\xi_e) = |\mathrm{e}j\rangle, \tag{8}$$

$$\begin{aligned} \xi_e &= \alpha_e z_e - a_e, \\ \alpha_e &= \sqrt{\frac{m_{ez}\omega_e}{\hbar}}, \qquad a_e = -\frac{1}{2\alpha_e^3}\left(\frac{2m_e}{\hbar^2}eF\right) \end{aligned} \tag{9}$$

$$E_{je} = \frac{\hbar\omega_e}{2}(2j+1) - \frac{\hbar\omega_e}{8\alpha_e^6}\left(\frac{2m_e}{\hbar^2}eF\right)^2 \tag{10}$$

with analogous expressions for the hole, where

$$\begin{aligned} \xi_h &= \alpha_h z_h + a_h \\ \alpha_h &= \sqrt{\frac{m_{hz}\omega_h}{\hbar}}, \qquad a_h = -\frac{1}{2\alpha_h^3}\left(\frac{2m_h}{\hbar^2}eF\right). \end{aligned} \tag{11}$$

$H_j(x)$ are Hermite polynomials and N_j normalization constants. Assuming that in the QW under consideration the valence band splits into heavy- and light hole subbands, and the conduction band is isotropic with the effective mass m_e, we obtain

$$\begin{aligned} a_h \to a_{hH} &= -\frac{1}{2\alpha_{hH}^3}\left(\frac{2m_{hzH}}{\hbar^2}eF\right) \\ &= -\frac{1}{2\tilde{\alpha_{hH}}^3}\left(\frac{m_{hzH}}{\mu_{\|H}}\right)\frac{F}{F_{\mathrm{I}H}}, \\ a_e &= -\frac{1}{2\tilde{\alpha}_e^3}\left(\frac{m_e}{\mu_{\|H}}\right)\frac{F}{F_{\mathrm{I}H}}, \end{aligned} \tag{12}$$

where $\tilde{\alpha}_H = \alpha_H a_H^*$ and $F_{\mathrm{I}H}$ is the so-called ionization field

$$F_{\mathrm{I}H} = \frac{\hbar^2}{2\mu_{\|H}}ea_H^{*3} = \frac{R_H^*}{ea_H^*}. \tag{13}$$

Substituting (7) into equation (2), with the use of the potential (6), the dipole density (4), and neglecting the center-of-mass in plane motion, we obtain the following system of linear equations for the unknown coefficients Y_{jn} (see Ref. [38] for details of the calculation)

$$\begin{aligned} \left(k_{rs}^2 + \epsilon_0\right)Y_{rs} + v_{0000}\frac{\lambda}{\lambda+\varpi}Y_{rs} - \frac{\lambda}{\lambda+\varpi}\sum_{nj}v_{rsnj}Y_{nj} \\ = \frac{2\mu_\|}{\hbar^2}EM_0\langle er|\mathrm{h}s\rangle\psi_0\left(\rho_0\right), \end{aligned} \tag{14}$$

where

$$\begin{aligned} k_{rs}^2 &= \frac{1}{R^*}\left(E_g + E_{re} + E_{sh} - \hbar\omega - \mathrm{i}\Gamma\right), \\ v_{rsnj} &= \frac{1}{R^*}S\langle rs\left|\exp\left[-v\left(z_e - z_h\right)^2\right]\right|nj\rangle \\ \varpi &= wa^{*2}, \qquad r,s,= 0,1,2,\ldots \end{aligned} \tag{15}$$

with the oscillator eigenvalues E_{re}, E_{sh}, the effective Bohr radius a^*, and the damping Γ. Having the coefficients Y_{jn}, we determine the amplitudes $Y_{H,L}$, the polarization inside the quantum well, and the mean effective dielectric electrosusceptibility

$$\overline{\chi} = \frac{2M_0}{E}\psi_0(\rho_0)\sum_{j,n}Y_{jn}\int_{-L/2}^{L/2}\psi_{ej}(z)\psi_{hn}(z)\mathrm{d}z. \tag{16}$$

We assume the so-called long-wave approximation and consider $\mathbf{E}(\mathbf{R})$ in equation (2) as a constant vector with the components $(E,0,0)$.

When the electric field is absent, only states of the same parity will give non vanishing elements $\langle r|s\rangle$. For the field $F \neq 0$, due to the displacement between the electron and hole confinement eigenfunctions, all possible combinations, for example $|0e0h\rangle, |0e2h\rangle, |1e3h\rangle$, but also $|1e0h\rangle$ etc. have to be taken into account.

Table 1. Band parameter values for GaAs, AlAs, and $Ga_{0.7}Al_{0.3}As$, AlAs data from [48], for $Ga_{0.7}Al_{0.3}As$ by linear interpolation. Energies in meV, masses in free electron mass m_0, ionization energies in kV/cm, γ_1, γ_2 are Luttinger parameters.

Parameter	GaAs	AlAs	$Ga_{0.7}Al_{0.3}As$
E_g	1519.2	3130	2002
m_e	0.0665	0.124	0.084
γ_1	6.85	3.218	
γ_2	2.1	0.628	
$m_{h\|H}$	0.112	0.26	
$m_{h\|L}$	0.210	0.386	
$\mu_{\|H}$	0.042		
$\mu_{\|L}$	0.05		
m_{hzH}	0.38	0.51	0.39
m_{hzL}	0.09	0.22	0.13
R^*_H	3.64	13.32	
R^*_L	4.3	19.35	
R^*_e	5.76		
a^*_H	15.78	7.03	
a^*_L	13.265	4.84	
a^*_e	9.97		
ϵ_b	12.53	11.16	12.12
F_{IH}	2.318		
F_{IL}	3.286		

3 Results for $GaAs/Ga_{1-x}Al_xAs$ parabolic quantum well and discussion

The calculation of the WPQW electrooptical functions consists of several steps. First, we define the confinement energies $\hbar\omega_{e,h}$. To this end we choose a specific WPQW having in mind the experimental results of Miller et al. [49]. They obtained optical spectra for GaAs(well)/$Ga_{0.7}Al_{0.3}As$(barrier) QWs of three thicknesses: $L = 51 \pm 3.5$ nm, $L = 32.5 \pm 3.5$ nm, $L = 33.6 \pm 3.5$ nm. We have performed the calculations for the thicknesses $L = 51$ nm and $L = 32.5$ nm. The confinement parameters were obtained from the lowest energy levels of equivalent rectangular QWs with confinement potentials $V_{e,conf} = 410.38$ meV, $V_{h,conf} = 72.42$ meV [42], using the band parameters from Table 1. The values a^*, R^* are appropriate for electrons and holes for the QW material, and are defined as:

$$R^* = \frac{me^4}{2(4\pi\epsilon_0\epsilon_b)^2\hbar^2}, \quad a^* = \frac{\hbar^2(4\pi\epsilon_0\epsilon_b)}{me^2}. \tag{17}$$

The corresponding values, listed in Table 1, were obtained by using in equation (17) the appropriate effective masses: m_e for R^*_e, a^*_e, and $\mu_{\|H,L}$ for R^*_H, a^*_H and R^*_L, a^*_L; $\mu_{\|H,L}$ are the in-plane reduced masses for the electron-hole pair and for the heavy- and light-hole exciton data.

The results for the confinement energy states are displayed in Table 2. From this energies we obtained the confinement energies as:

$$\hbar\omega_e = 2E_{e0}, \qquad \hbar\omega_{hH,L} = 2E_{0zH,L}. \tag{18}$$

Table 2. Confinement parameters for the WPQWs from reference [49], dimensions in nm, energies in meV.

L	$\hbar\omega_e$	$\hbar\omega_{hH}$	$\hbar\omega_{hL}$	E_{e0}
E_{0zH}	E_{0zL}	$\alpha_e a^*_H$	$\alpha_{hH} a^*_H$	$\alpha_{hL} a^*_L$
32.5	81.66	19.74	108.4	40.83
9.87	54.2	4.21	4.95	4.76
51.5	43.56	8.46	34.4	21.78
4.23	17.2	3.07	3.08	2.68

Table 3. Confinement states accounted in computation.

$\|e0h0\rangle \to \|1\rangle$	$\|e0h1\rangle \to \|2\rangle$	$\|e0h2\rangle \to \|3\rangle$
$\|e0h3\rangle \to \|4\rangle$	$\|e0h4\rangle \to \|5\rangle$	$\|e1h0\rangle \to \|6\rangle$
$\|e1h1\rangle \to \|7\rangle$	$\|e1h2\rangle \to \|8\rangle$	$\|e1h3\rangle \to \|9\rangle$
$\|e1h4\rangle \to \|10\rangle$	$\|e2h0\rangle \to \|11\rangle$	$\|e2h1\rangle \to \|12\rangle$
$\|e2h2\rangle \to \|13\rangle$	$\|e2h3\rangle \to \|14\rangle$	$\|e2h4\rangle \to \|15\rangle$
$\|e3h0\rangle \to \|16\rangle$	$\|e3h1\rangle \to \|17\rangle$	$\|e3h2\rangle \to \|18\rangle$
$\|e3h3\rangle \to \|19\rangle$	$\|e3h4\rangle \to \|20\rangle$	$\|e4h0\rangle \to \|21\rangle$
$\|e4h1\rangle \to \|22\rangle$	$\|e4h2\rangle \to \|23\rangle$	$\|e4h3\rangle \to \|24\rangle$
$\|e4h4\rangle \to \|25\rangle$		

Having the confinement parameters, we have calculated the potential matrix elements (15). In particular, we have an analytical expression for the element $V_{0000} = v_0 R^*$

$$\begin{aligned} V_{0000} &= \frac{S\alpha_e^3\alpha_h}{\alpha_e^2+\alpha_h^2}\frac{\pi N_{e0}^2 N_{h0}^2}{\sqrt{c_1 c_3}} \exp\left(\frac{c_2^2}{4c_1}+\frac{c_4^2}{4c_3}-a_e^2-a_h^2\right) \\ &= \frac{S\alpha_e^3\alpha_h}{\alpha_e^2+\alpha_h^2}\frac{1}{\sqrt{c_1 c_3}} \exp\left(\frac{c_2^2}{4c_1}+\frac{c_4^2}{4c_3}-a_e^2-a_h^2\right), \end{aligned} \tag{19}$$

where

$$c_1 = \frac{\alpha_e^2\alpha_h^2}{\alpha_e^2+\alpha_h^2} + v, \quad c_2 = -2\frac{\alpha_e\alpha_h}{\alpha_e^2+\alpha_h^2}\left(a_e\alpha_h + a_h\alpha_e\right),$$
$$c_3 = \frac{\alpha_e^4}{\alpha_e^2+\alpha_h^2}, \qquad c_4 = -2\frac{\alpha_e^2}{\alpha_e^2+\alpha_h^2}\left(a_e\alpha_e - a_h\alpha_h\right).$$

The above expression, and also the remaining potential matrix elements, contain unknown parameters S and v. We used the values $S = 2.6, \varpi = 0.154$ and $v = 0.5$ determined with the procedure described in reference [43]. Using the above parameters and taking into account the lowest 25 confinement states (see Tab. 3) we have solved equation (14) and obtained the coefficients Y_{jn} from which we have determined the induced polarization inside the WPQW by the relation

$$P(z) = 2M_0\psi_0(\rho_0)\sum_{j,n=0}^{N}\psi_{ej}(z)\psi_{hn}(z)Y_{jn}, \tag{20}$$

and the mean dielectric susceptibility by equation (16). Then, having the mean susceptibility, one can compute, using the appropriate boundary conditions, the optical functions (reflectivity, transmission, and absorption).

The advantage of the RDMA is that we obtain simultaneously the real and the imaginary part of the susceptibility. The results for the the imaginary part of the mean

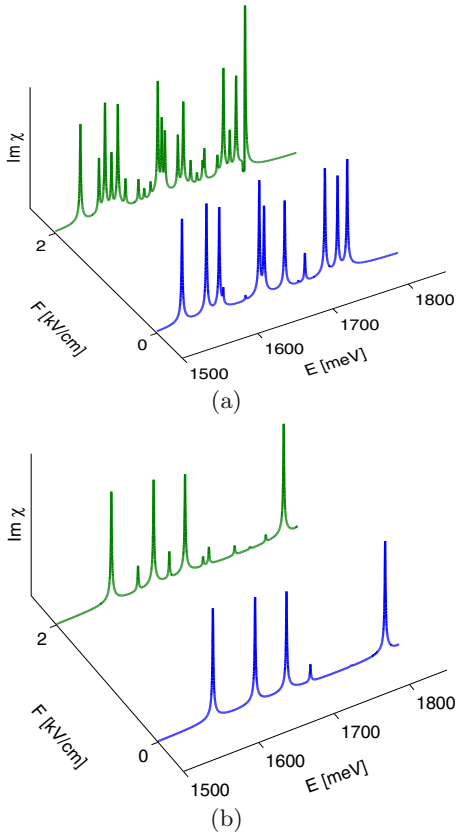

Fig. 1. The imaginary part of the mean susceptibility from equation (16), for two GaAs/$Ga_{0.7}Al_{0.3}As$ WPQWs, (a) of thickness 51 nm, (b) of thickness 32.5 nm.

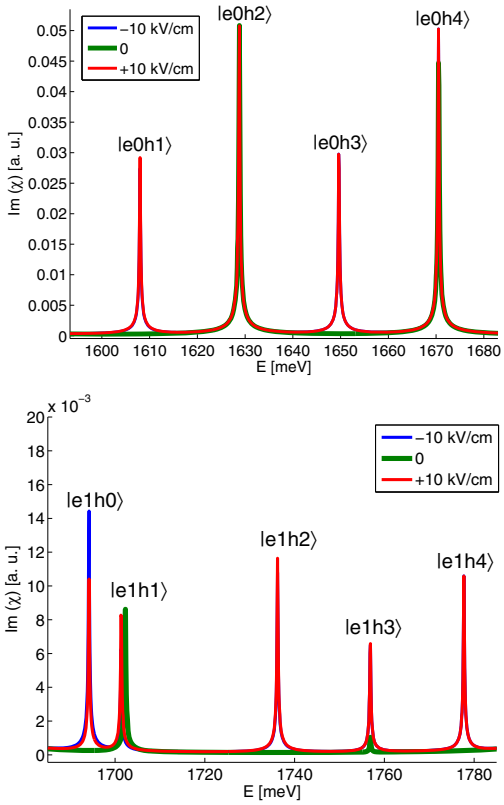

Fig. 2. The imaginary part of the mean electrosusceptibility for the GaAs/$Ga_{0.7}Al_{0.3}As$ WPQW of thickness 32.5 nm, for different energy intervals.

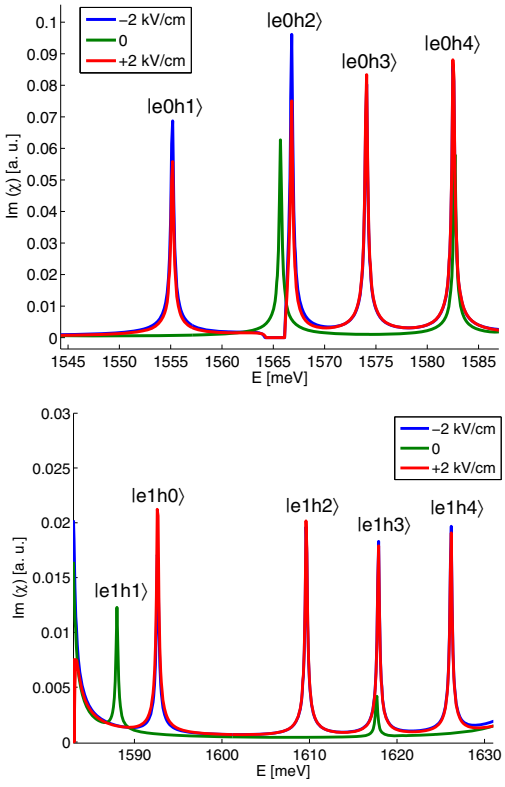

Fig. 3. The same as in Figure 2, for the WPQW of the thickness 51 nm.

susceptibility of the considered WPQWs are displayed in Figures 1−3. In Figure 1 we show the general effect of the applied electric field for two GaAs/$Ga_{0.7}Al_{0.3}As$ WPQws. We observe the red shift of the resonances, changes in the oscillator strengths, and the occurrence of new resonances due to the broken symmetry. The spectra for $F = 0$ agree well with the experimental results by Miller et al. [49] and our previous theoretical results [43]. In Figures 2 and 3 we show the obtained spectra in a more detailed form, as compared to Figure 1.

The calculated real part (which is related to the reflectivity) of the electrosusceptibility is displayed in Figures 4 and 5.

In all the cases we observe changes in the placement of resonances, and the occurrence of new peaks attributed to different symmetries for the electron and the hole confinement functions. Our method allows to determine the energy shift as a function of the applied field. We have computed the energy shift for the lowest confinement states. We observe the quadratic Stark shift for the lowest state and a more complicated field-dependence for higher states, as is displayed in Figure 6. The impact of high electric fields is displayed in Figure 7a, where we show the changes in the real part of the electrosusceptibility for the applied fields up to 60 kV/cm. For high values of the applied electric field the effects are smaller which is

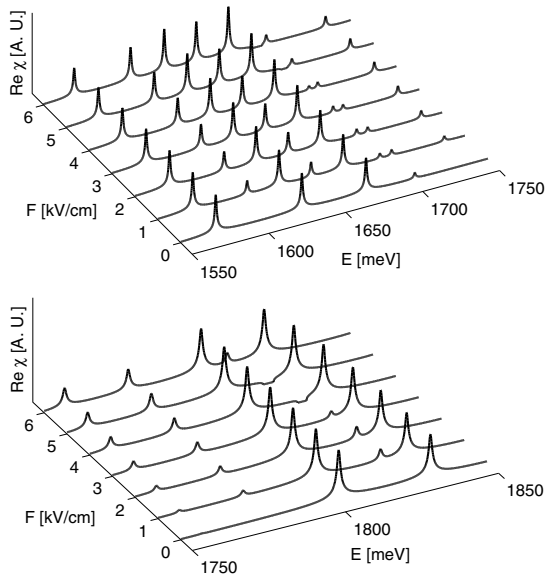

Fig. 4. The real part of the mean electrosusceptibility for the GaAs/$Ga_{0.7}Al_{0.3}As$ WPQWs of thicknesses 32.5 nm for different energy intervals and applied field strengths.

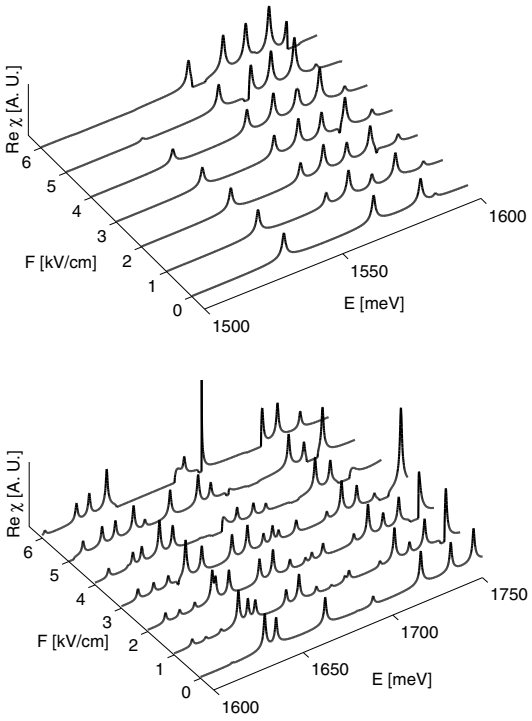

Fig. 5. The real part of the mean electrosusceptibility for the GaAs/$Ga_{0.7}Al_{0.3}As$ WPQWs of thickness 51 nm, for different energy intervals and applied field strengths.

due to the decreasing overlap of the electron and the hole confinement functions. We also observe that the shape of the spectra changes with the change of the direction of the applied field. Such effects were also observed in narrow QWs, not only based on the III-V compounds (see, for example, Refs. [15,29]). Finally, we show that the energy shift drastically depends on the thickness of the QW

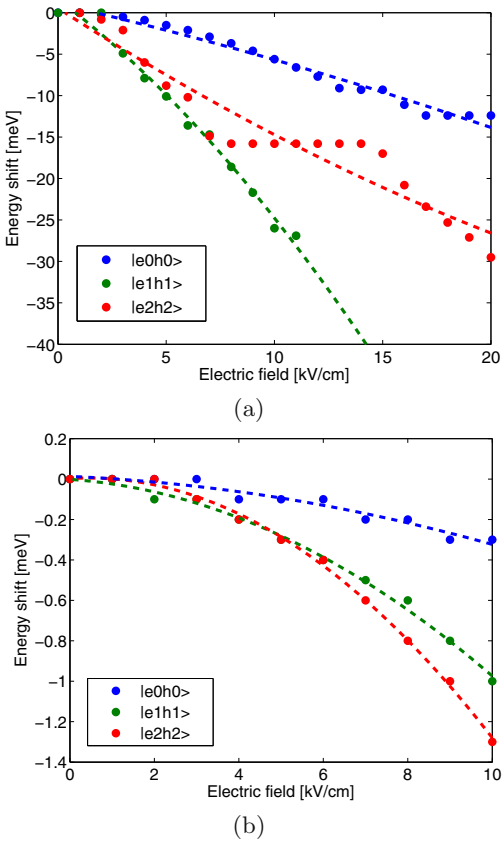

Fig. 6. The Stark energy shift for the lowest resonances: (a) for the GaAs/$Ga_{0.7}Al_{0.3}As$ WPQW of thickness 51 nm and (b) for the GaAs/$Ga_{0.7}Al_{0.3}As$ WPQW of thickness 32.5 nm.

(Fig. 7b), as was also observed for narrow QWs (see, for example [7,24], and references therein). Since the confinement energy depends on the QW thickness as L^{-2}, we see that for rectangular QWs $\Delta E = -CL^4$. The energy shift in the considered WPQW on the Figure 7b also closely follows this relation.

4 Conclusions

We have developed a simple mathematical procedure to calculate the electrooptical functions of wide parabolic quantum wells. Using the real density matrix approach and a model e-h interaction potential, we derived an analytical formula for the WPQW electrosusceptibility, from which another electrooptical functions can be obtained. The presented method has been used to investigate the electrooptical functions of GaAs/$Ga_{1-x}Al_xAs$ WPQWs for the case of radiation incidence parallel to the growth direction. We have obtained the shift of the resonances, which is the parabolic one for the ground state, but is more complicated for the higher states, changes in the oscillator strengths and new peaks related to electronic transitions forbidden for the case with absent electric field. We also observed the dependence of the spectra on the size of the QW and on the direction of the applied field.

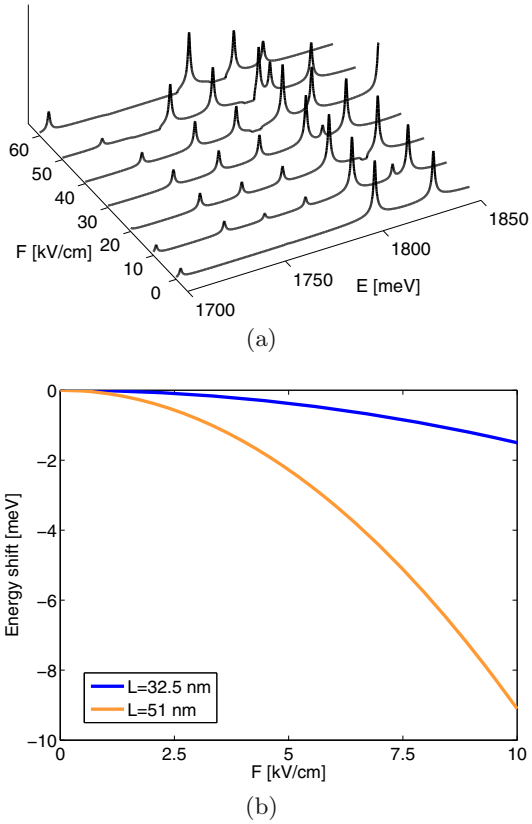

Fig. 7. (a) The impact of high electric fields on the real part of the electrosusceptibility (WPQW of thickness 32.5 nm). (b) The comparison of the Stark energy shift of the two considered WPQWs for the lowest resonance.

All the results are new and had not been contained in the previous papers on the RDMA approach to the electrooptical effects in low dimensional systems. For the cases where the experimental data were available (for example, for WPQWs with $\mathbf{F} = 0$), we obtained a good agreement of our theoretical results with experiment. WPQWs exposed to electric fields offer new possibilities of manipulating the optical properties of nanostructures and thus are the promising systems for advanced optoelectronics. We hope that our results may stimulate experiments on quantum confined Stark effect in WPQWs.

Author contribution statement

All authors designed the research and contributed equally to the development of the model and data interpretation.

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