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Erratum

Optimization of robustness of complex networks

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We predicted that the network design which maximizes the robustness of networks to both random failure and intentional attack while keeping the cost of the network constant is one in which all but one of the nodes have the same degree, k_1 (close to the average number of links per node), and one node is of very large degree, $k_2 \sim N^{2/3}$, where N is the number of nodes in the network. This prediction was based on the use of an equation for f_c, the fraction of nodes which are randomly removed before a network loses global connnectivity [1]:

$$f_c^{\rm rand} = 1 - \frac{1}{\kappa_0 - 1}, \qquad (1)$$

where $\kappa_0 \equiv \langle k^2 \rangle / \langle k \rangle$. This equation is valid only for random networks without multiple edges and self loops [2] – i.e. for *simple* networks – a fact we did not initially consider. There is no loss of generality in restricting the networks we consider to be simple because neither multiple edges nor self loops add to robustness against node removal. The requirement that the network be simple is reflected in the constraint that the largest degree k_{max} with non-zero probability k_{max} must obey [3–6]

$$k_{max} < \sqrt{\langle k \rangle N}. \qquad (2)$$

The fraction of high degree nodes r is found by enforcing this constraint by setting $k_2 = \sqrt{\langle k \rangle N}$ and using the relation

$$k_2 \sim \left\{ 2\langle k \rangle^2 (\frac{\langle k \rangle - 1)^2}{2\langle k \rangle - 1} \right\}^{1/3} r^{-2/3} \equiv A r^{-2/3}. \qquad (3)$$

We find

$$r = \left(\frac{A^2}{\langle k \rangle N} \right)^{3/4}. \qquad (4)$$

Thus the optimal network is one in which $rN \sim N^{1/4}$ of the nodes have degree $k_2 = \sqrt{\langle k \rangle N}$ and the remaining nodes have degree k_1 (close to the average number of links per node).

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