Eur. Phys. J. A (2024) 60:170
https://doi.org/10.1140/epja/s10050-024-01387-3

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

On interpretation of fluctuations of conserved charges at high T

T. D. Cohen[1], **L. Ya. Glozman**[2,a]

[1] Department of Physics and Maryland Center for Fundamental Physics, University of Maryland, College Park, MD 20742, USA
[2] Institute of Physics, University of Graz, Graz 8010, Austria

Received: 12 April 2024 / Accepted: 26 July 2024

Communicated by Giorgio Torrieri

Abstract Fluctuations of conserved charges calculated on the lattice which can be measured experimentally, are well reproduced by a hadron resonanse gas model at temperatures below $T_{ch} \sim 155$ MeV and radically deviate from the hadron resonance gas predictions above the chiral restoration crossover. This behaviour is typically interpreted as an indication of deconfinement in the quark-gluon plasma regime. We present an argument that this interpretation may be too simple. The argument is based on the scaling of quantities with the number of colors: demonstration of deconfinement and QGP requires observable that is sensitive to $\sim N_c^2$ gluons while the conserved charges are sensitive only to quarks and above T_{ch} scale as N_c^1. The latter scaling is consistent with the existence of an intermediate regime characterized by restored chiral symmetry and by approximate chiral spin symmetry which is a symmetry of confining interaction. In this regime the energy density, pressure and entropy density scale as N_c^1. In the large N_c limit this regime might become a distinct phase separated from the hadron gas and from QGP by phase transitions. A natural observable that associates with deconfinement and is directly sensitive to deconfined $N_c^2 - 1$ gluons is the Polyakov loop; in the $N_c = 3$ world it remains very close to 0 at temperatures well above chiral crossover, reaches the value ~ 0.5 around $\sim 3T_{ch}$ and the value close to 1 at temperatures ~ 1 GeV.

1 Introduction

It was widely believed for a long time that in QCD upon heating there are two qualitatively different regimes connected by a fast but smooth crossover: a hadron gas and a deconfined quark-gluon plasma. The hadron gas, which is a dilute system of hadrons at temperatures below a critical one, is characterized by a spontaneously broken chiral symmetry while the QGP is a system of effectively deconfined (quasi)quarks and (quasi)gluons with restored chiral symmetry. The pseudo-critical temperature was believed to be a common temperature of the chiral restoration and of the deconfinement. In the real world $N_c = 3$ and with small but nonzero quark masses the pseudocritical temperature of chiral restoration was established to be around $T_{ch} \sim 155$ Mev [1–3] while in the chiral limit the critical temperature of the second order chiral restoration phase transition was determined to be around 130 MeV [4]. In the large N_c limit it was believed that the common deconfinement and chiral restoration phase transition is of the first order [5].

Lattice studies on artificial truncation of the near-zero modes of the Dirac operator at T = 0 [6–9] have suggested approximate emergent symmetries associated with the symmertries of the color charge and of the electric part of the QCD Lagrangian [10,11]: the $SU(2)_{CS}$ chiral spin symmetry that includes as a subgroup the $U(1)_A$ symmetry and its flavor extension $SU(2N_F)$. The latter contains as a subgroup the full chiral symmetry of QCD $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$. Neither chiral spin symmetry nor its flavor extension are symmetries of the Dirac Lagrangian but are symmetries of only the electric part of the QCD Lagrangian; they are explicitly violated by the magnetic interactions and by the quark kinetic terms. For a review on symmetries and their implications for hot QCD see Ref. [12].

Above the chiral symmetry restoration crossover one might expect emergence of the approximate $SU(2)_{CS}$ and $SU(4)$ symmetries [13], which would suggest that the system is still in the confining regime. Lattice studies appear to be consistent with this [14–17]. Namely at temperatures roughly $T_{ch} < T <\sim 3T_{ch}$ one observes correlators consistent with approximate chiral spin symmetry and its flavor extension; this behaviour smoothly disappears at higher temperatures. This suggests that in QCD upon heating above the hadron gas regime but below the quark-gluon plasma regime there exists an intermediate confining regime with restored chiral symmetry and approximate chiral spin symmetry.

[a] e-mail: leonid.glozman@uni-graz.at (corresponding author)

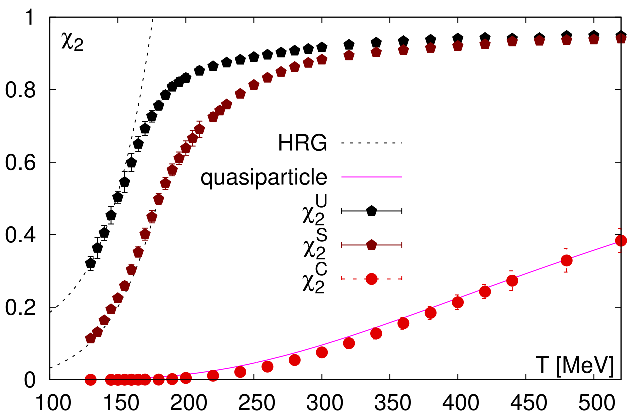

Fig. 1 Fluctuations of conserved net u ($\chi_2^U = \chi_{2,0,0}^{u,d,s}$) and strange ($\chi_2^S = \chi_{0,0,2}^{u,d,s}$) quark numbers in 2+1 QCD at physical quark masses. χ_2^C is irrelevant to our discussion and can be ignored. Source: Ref. [25]

There are arguments about the dynamics about how this intermediate regime behaves and how its effective degrees of freedom arise [12] but for the purpose of this paper the validity of these arguments is not relevant.

The center symmetry of the gauge action is explicitly broken by quark loops. However, it becomes exact in the large N_c limit[1]: QCD in the combined large N_c and chiral limit has two distinct symmetries: chiral symmetry and center symmetry. This allows one to define unambiguously possible phases with confinement or deconfinement and with spontaneously broken or restored chiral symmetry. The order parameters for the two (Polyakov loop, quark condensate) are independent. It was suggested in Ref. [19] that three regimes of QCD connected by smooth crossovers in the real world $N_c = 3$ and with small but nonzero quark mass might become distinct phases separated by phase transitions once the number of colors increases to infinity with the quarks kept massless. Standard large N_c scaling analysis implies that the energy density, pressure and entropy density in the hadron gas phase scale as N_c^0, in the intermediate phase they scale as N_c^1 and in the QGP phase as N_c^2. The intermediate phase, which is chirally symmetric and confined, has a temperature range $\sim N_c^0$ and should be at least approximately chiral spin symmetric. It should contain a gas of noninteracting glueballs (as is seen in the low temperature hadron gas phase).

The confined and chirally symmetric phase at $T = 0$ and large density was discussed in Ref. [5]. In that case the chiral symmetry restoration can be attributed to a large quark Fermi sphere. The mechanism of chiral symmetry restoration at large T and vanishing chemical potential is clearly different; Ref. [20] argues that it is due to Pauli blocking of the quark levels, necessary for the formation of the quark condensate, by the thermal excitations of quarks and antiquarks.

There are models - both quark-based and meson-based, that effectively imitate confinement in thermodynamics, via a coupling of the light quark sector to the effective Polyakov loop potential [21–24]. Within these models, at vanishing chemical potential the chiral restoration phase transition at large N_c typically coincides or very close with the "deconfinement" phase transition. This behaviour is notably different from what is proposed here based on large N_c considerations.

The above overview of developments of the field introduces the main subject of this paper, that will be discussed in the following sections: interpretation of the fluctuations of conserved charges related to quark bilinears. We stress that the purpose of the paper is not to calculate something new but rather to give a correct interpretation to already existing results that are rather influential for the understanding of the physics at high temperatures.

2 Conserved charges and their fluctuations in hadron gas and at higher temperatures

The hadron resonance gas model of the QCD matter at low temperatures assumes a dilute system of point-like structureless hadrons that do not interact. Consequently the number density of meson or baryon species k with spin S_k, isospin I_k and some strangeness at a temperature T is given by the Bose-Einstein distribution (minus sign in the denominator of the equation below) for mesons and Fermi–Dirac (plus sign) for baryons

$$n_k(T) = (2S_k + 1)(2I_k + 1) \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\sqrt{p^2+m_k^2}/T} \pm 1} . \quad (1)$$

The fluctuations of the numbers of the various quark flavors are then obtained from the standard results for the Bose- and Fermi-gases at the given T assuming hadron masses from PDG. It is well known that for $T < T_{ch}$ the hadron resonance gas model reproduces fluctuations of conserved charges calculated on the lattice, see e.g. [25] and references therein. At larger temperatures the lattice results for these observables radically deviate from the HRG model predictions. This is often taken as evidence of deconfinement and of transition to QGP. However as it will become evident below such interpretation is erroneous.

The success of the HRG model below T_{ch} indicates that at these temperatures the hadron structure is not yet resolved and its internal degrees of freedom are frozen. This is the rea-

[1] There is a subtlety in that the large N_c limit should to be taken at the end of the analysis in order to maintain hierarchies associated with factors N_c to various powers. If the large N_c limit is taken at the outset, as in Ref. [18], there is a possibility that explicit center-symmetry-breaking effects associated with quarks which could conceivably enter at order N_c in the action (i.e. relative order $1/N_c$) would be missed. The scenario discussed in this paper assumes that it happens.

son for the N_c^0 scaling of the thermodynamical observables in the HRG regime. Once the density of hadrons increases so that they start to overlap the internal hadron structure gets relevant and a proper description of conserved charges should rely on quark degrees of freedom.

Here we focus on charges associated with the net number of up, down and strange quarks[2]:

$$N_q \equiv \int d^3x\, n_q(x) \text{ with } n_q(x) = \bar{q}(x)\gamma^0 q(x),\ q = u, d, s \tag{2}$$

Each quark can be in one of the N_c color states and contraction with respect to the color of quarks is assumed. This means that the conserved flavor charges N_q, scale as N_c^1. Then the fluctuations of the conserved quark charges also scale as N_c^1.

To see this note that the expectation value of the conserved quark number of a given flavor in volume V at temperature T can be obtained from the grand canonical partition function as

$$< N_i >= \frac{T\partial\,[log Z(T, V, \mu_u, \mu_d, ...)]}{\partial\mu_i}. \tag{3}$$

The fluctuations of conserved charges can be obtained as a derivative of these charges

$$\frac{\partial < N_i >}{\partial\mu_j} = \frac{T\partial^2\,[log Z(T, V, \mu_u, \mu_d, ...)]}{\partial\mu_j\partial\mu_i}. \tag{4}$$

The fluctuations and correlations of conserved charges can be expressed in terms of different cumulants

$$\chi^{u,d,s}_{i,j,k} = \frac{T\partial^{i+j+k}\left(P/T^4\right)}{(\partial\mu_u)^i(\partial\mu_d)^j(\partial\mu_s)^k}. \tag{5}$$

In Figs. 1 and 2 we show typical results for fluctuations of quark numbers of u, d, s quarks taken from Ref. [25] and their comparison with the HRG model. We see that the fluctuations of the u, d, s quark numbers deviate from the HRG just at the chiral restoration temperature 155 MeV.

The key point of this paper is that since the quark numbers scale as N_c^1 the above behaviour of fluctuations of conserved charges is consistent with the crossover from the hadron gas to the intermediate regime, as described in the introduction, where all main thermodynamical quantities scale as N_c^1. Consequently above T_{ch} the fluctuations of conserved charges in the large N_c limit would be expected to differ from the fluctuations in the HRG (of order N_c^0) by N_c, indicating a phase transition. In principle the scaling of the fluctuations should be seen on the lattice by providing calculations at $N_c > 3$. However,the N_c^1 scaling is a robust consequence of the definition of conserved charges (2) and hence there is no reason to doubt this scaling.

To demonstrate a transition to the QGP regime one needs an observable that is sensitive to presence of $\sim N_c^2$ deconfined gluons.

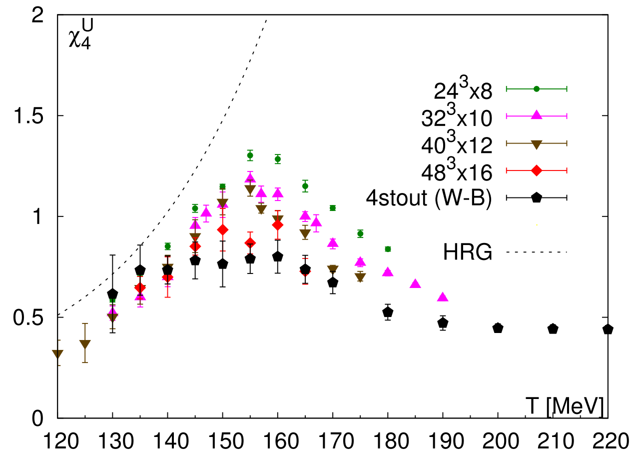

Fig. 2 Cumulant $\chi_4^U = \chi^{u,d,s}_{4,0,0}$ in 2+1 QCD at physical quark masses. Source: Ref. [25]

3 Polyakov loop and its evolution with temperature

The Polyakov loop is the trace of a Wilson line along a straight path in the compactified time direction

$$P_{N_c} = \frac{1}{N_c} Tr\left[T\exp\left(i\int_0^\beta d\tau A_0(\mathbf{x}, \tau)\right)\right]. \tag{6}$$

The expectation value of the Polyakov loop in the pure glue theory (or in quenched QCD and in QCD with infinitely heavy quarks) is the order parameter for center symmetry and for confinement. In the center symmetric confining phase at low temperatures the expectation value of the Polyakov loop identically vanishes while above the first order deconfinement phase transition the center symmetry gets spontaneously broken and the expectation value of the Polyakov loop takes a finite value.

It is important to clarify the N_c properties of the Polyakov loop. The factor $\frac{1}{N_c}$ in front of the trace normalises the trace of the $N_c \times N_c$ matrics. The absolute value of the normalised Polyakov loop varies in the interval [0, 1] and when it is nonvanishing scales as N_c^0.

[2] It is sensible to use these rather than the more traditional baryon number and electric charge as well as strangeness, since our goal is to make connection with the behavior at large N_c and the extrapolation to large N_c is straightforward and unambiguous with these and not with elecric charge or baryon number. There are two distinct "natural" ways to define the electric charge of quarks: $Q_u = \frac{2}{3}$ and $Q_d = Q_s = -\frac{1}{3}$ as at $N_c = 3$ or alternatively [26,27] $Q_u = \frac{1}{2N_c} + 1/2$ and $Q_d = Q_s = \frac{1}{2N_c} - 1/2$. Only the second definition gives the proton ($B = 1$, $I = 1/2$, $I_3 = 1/2$ and $S = 0$) electric charge of unity, while the first has the proton charge of order N_c. Similarly, the natural definition of baryon number has the baryon number of a quark equal to $1/N_c$ rather than $\frac{1}{3}$ which gives rise to suppression factors $1/N_c$ that can obscure the physics.

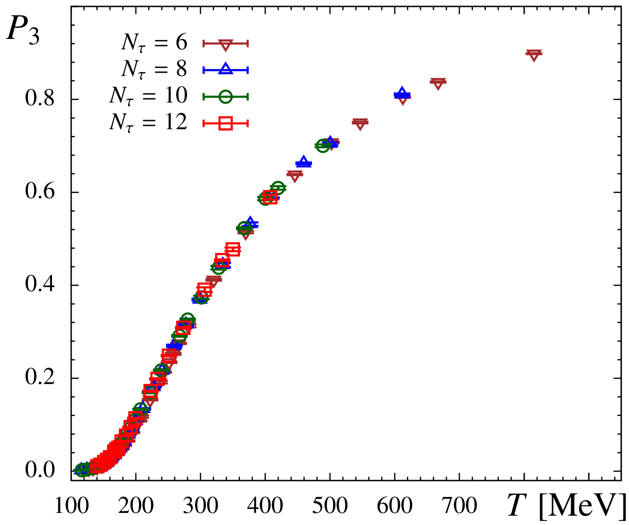

Fig. 3 Temperature evolution of the properly renormalized Polyakov loop in 2+1 QCD at physical quark masses. Source: Ref. [29]

The Polyakov loop in the deconfined phase, where it is not zero, is explicitly sensitive to $N_c^2 - 1$ gluons, which is the dimension of the adjoint representation, i.e. the number of independent gluons. The sensitivity should not be confused with the scaling.

It is known that the temperature of the first order deconfinement phase transition in pure Yang-Mills and in quenched QCD at $N_c = 3$ is at $T_d \sim 270$–300 MeV. At this temperature the Polyakov loop jumps from the zero value to the value around 0.5–0.6 and above the phase transition smoothly increases towards 1. It is also established that the deconfinement temperature in pure Yang-Mills theory is practically N_c-independent [28]. However, confining properties of QCD with light quarks and of pure Yang-Mills theory are identical in the large N_c limit. Then one expects a similar deconfinement temperature of QCD with light quarks at large N_c.

In the real world $N_c = 3$ in QCD with light quarks the center symmetry of the action is explicitly broken and the deconfinement first order phase transition is replaced by a very smooth crossover. The renormalized Polyakov loop informs us about the deconfinement crossover region.

The temperature evolution of the properly renormalized Polyakov loop, taken from Ref. [29], is shown in Fig. 3. We observe that above the chiral restoration temperature around $T_{ch} \sim 155$ MeV the Polyakov loop is very small which suggests that here QCD is in the confining regime. At the same time at these temperatures fluctuations of conserved charges demonstrate that the hadron gas picture does not work. Both these facts are consistent with the existence of the intermediate regime discussed in the introduction. The Polyakov loop reaches the value around 0.5 at a temperature roughly $3T_{ch}$, in agreement with the temperature of smooth disappearance of chiral spin symmetry.

4 Conclusions

In this paper we have considered fluctuations of conserved charges which are typically taken as evidence for transition from hadron gas to a QGP at the chiral restoration temperature. As the N_c scaling analysis makes manifest, the transition to the QGP requires, however, an observable that is sensitive to the presence of deconfined gluons. A natural observable is the Polyakov loop. The conserved charges associated with quark flavor scale as N_c and are not explicitly dependent on the presence of deconfined gluons. The fluctuations and correlations of conserved charges measured on the lattice indicate a transition from the hadron gas to a regime with the scaling N_c. At the same time the Polyakov loop just above the pseudocritical temperature of chiral restoration remains very small suggesting a confining regime. This data supports the previously found evidence that above the HRG regime QCD matter is in not the deconfined QGP regime, but in the confined regime with restored chiral symmetry and approximate chiral spin symmetry. A very smooth transition to the QGP regime from the intermediate regime takes place at essentially larger temperatures.

Acknowledgements The work of TDC was supported in part by the the U.S. Department of Energy, Office of Nuclear Physics under Award Number DE-FG02-93ER40762. This open access paper was published with the financial support of the University of Graz.

Funding Open access funding provided by University of Graz.

Data Availability Statement This manuscript has associated data. [Authors' comment: There are no related datasets.]

Code Availability Statement Code/software will be made available on reasonable request. [Author's comment: There are no related codes/software.]

References

1. Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S.D. Katz, S. Krieg, K.K. Szabo, JHEP **06**, 088 (2009). https://doi.org/10.1088/1126-6708/2009/06/088. arXiv:0903.4155 [hep-lat]

2. A. Bazavov et al., HotQCD. Phys. Lett. B **795**, 15–21 (2019). https://doi.org/10.1016/j.physletb.2019.05.013. arXiv:1812.08235 [hep-lat]
3. S. Borsanyi, Z. Fodor, J. N. Guenther, R. Kara, S. D. Katz, P. Parotto, A. Pasztor, C. Ratti, K. K. Szabo, Phys. Rev. Lett. **125**(5), 052001 (2020). https://doi.org/10.1103/PhysRevLett.125.052001. arXiv:2002.02821 [hep-lat]
4. H. T. Ding, *et al.* [HotQCD], Phys. Rev. Lett. **123**(6), 062002 (2019). https://doi.org/10.1103/PhysRevLett.123.062002. arXiv:1903.04801 [hep-lat]
5. L. McLerran, R.D. Pisarski, Nucl. Phys. A **796**, 83–100 (2007). https://doi.org/10.1016/j.nuclphysa.2007.08.013. arXiv:0706.2191 [hep-ph]
6. M. Denissenya, L. Y. Glozman, C. B. Lang, Phys. Rev. D **89**(7), 077502 (2014). https://doi.org/10.1103/PhysRevD.89.077502. arXiv:1402.1887 [hep-lat]
7. M. Denissenya, L. Y. Glozman, C. B. Lang, Phys. Rev. D **91**(3), 034505 (2015). https://doi.org/10.1103/PhysRevD.91.034505. arXiv:1410.8751 [hep-lat]
8. M. Denissenya, L. Y. Glozman, M. Pak, Phys. Rev. D **91**(11), 114512 (2015). https://doi.org/10.1103/PhysRevD.91.114512. arXiv:1505.03285 [hep-lat]
9. M. Denissenya, L. Y. Glozman, M. Pak, Phys. Rev. D **92**(7), 074508 (2015). [erratum: Phys. Rev. D **92** (2015) no.9, 099902]. https://doi.org/10.1103/PhysRevD.92.074508. arXiv:1508.01413 [hep-lat]
10. L. Y. Glozman, Eur. Phys. J. A **51**(3), 27 (2015). https://doi.org/10.1140/epja/i2015-15027-x. arXiv:1407.2798 [hep-ph]
11. L. Y. Glozman, M. Pak, Phys. Rev. D **92**(1), 016001 (2015). https://doi.org/10.1103/PhysRevD.92.016001. arXiv:1504.02323 [hep-lat]
12. L.Y. Glozman, Prog. Part. Nucl. Phys. **131**, 104049 (2023). https://doi.org/10.1016/j.ppnp.2023.104049. arXiv:2209.10235 [hep-lat]
13. L.Y. Glozman, Acta Phys. Polon. Supp. **10**, 583 (2017). https://doi.org/10.5506/APhysPolBSupp.10.583. arXiv:1610.00275 [hep-lat]
14. C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, L. Y. Glozman, S. Hashimoto, C. B. Lang, S. Prelovsek, Phys. Rev. D **96**(9), 094501 (2017). [erratum: Phys. Rev. D **99** (2019) no.3, 039901]. https://doi.org/10.1103/PhysRevD.96.094501. arXiv:1707.01881 [hep-lat]
15. C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer, L. Y. Glozman, S. Hashimoto, C. B. Lang, S. Prelovsek, Phys. Rev. D **100**(1), 014502 (2019). https://doi.org/10.1103/PhysRevD.100.014502. arXiv:1902.03191 [hep-lat]
16. C. Rohrhofer, Y. Aoki, L.Y. Glozman, S. Hashimoto, Phys. Lett. B **802**, 135245 (2020). https://doi.org/10.1016/j.physletb.2020.135245. arXiv:1909.00927 [hep-lat]
17. T. W. Chiu, Phys. Rev. D **107**(11), 114501 (2023). https://doi.org/10.1103/PhysRevD.107.114501. arXiv:2302.06073 [hep-lat]
18. P. Kovtun, M. Unsal, L.G. Yaffe, JHEP **06**, 019 (2007). https://doi.org/10.1088/1126-6708/2007/06/019. [hep-th/0702021]
19. T. D. Cohen, L. Y. Glozman. arXiv:2311.07333 [hep-ph]
20. L.Y. Glozman, A.V. Nefediev, R. Wagenbrunn, Phys. Lett. B **854**, 138707 (2024). https://doi.org/10.1016/j.physletb.2024.138707. arXiv:2404.02606 [hep-ph]
21. R.D. Pisarski, Phys. Rev. D **62**, 111501 (2000). https://doi.org/10.1103/PhysRevD.62.111501. arXiv:hep-ph/0006205 [hep-ph]
22. K. Fukushima, Phys. Lett. B **591**, 277–284 (2004). https://doi.org/10.1016/j.physletb.2004.04.027. arXiv:hep-ph/0310121 [hep-ph]
23. C. Ratti, M.A. Thaler, W. Weise, Phys. Rev. D **73**, 014019 (2006). https://doi.org/10.1103/PhysRevD.73.014019. arXiv:hep-ph/0506234 [hep-ph]
24. L. McLerran, K. Redlich, C. Sasaki, Nucl. Phys. A **824**, 86–100 (2009). https://doi.org/10.1016/j.nuclphysa.2009.04.001. arXiv:0812.3585 [hep-ph]
25. R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz, A. Pasztor, C. Ratti, K. K. Szabo, Phys. Rev. D **92** (2015) no.11, 114505 https://doi.org/10.1103/PhysRevD.92.114505. arXiv:1507.04627 [hep-lat]
26. J.M. Gerard, T. Lahna, Phys. Lett. B **356**, 381–385 (1995). https://doi.org/10.1016/0370-2693(95)00811-X. arXiv:hep-ph/9506255 [hep-ph]
27. O. Bar, U.J. Wiese, Nucl. Phys. B **609**, 225–246 (2001). https://doi.org/10.1016/S0550-3213(01)00288-7. arXiv:hep-ph/0105258 [hep-ph]
28. B. Lucini, M. Panero, Phys. Rept. **526**, 93–163 (2013). https://doi.org/10.1016/j.physrep.2013.01.001. arXiv:1210.4997 [hep-th]
29. P. Petreczky, H. P. Schadler, Phys. Rev. D **92** (2015) no.9, 094517 https://doi.org/10.1103/PhysRevD.92.094517. arXiv:1509.07874 [hep-lat]