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Erratum

Erratum to: Covariant tensor formalism for partial-wave analyses of ψ decays into $\gamma B\bar{B}$, $\gamma\gamma V$ and $\psi(2S) \to \gamma\chi_{c0,1,2}$ with $\chi_{c0,1,2} \to K\bar{K}\pi^+\pi^-$ and $2\pi^+2\pi^-$

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In the section 4 of our original paper, Eur. Phys. J. A 26, 125-134 (2005), for the PWA formulae of $\psi \to \gamma R \to \gamma\gamma V$, we labelled the two photons as one from the first step and another one from the second step as distinguishable. But in reality the two photons are as identical particles indistinguishable. In this Erratum, we provide the corrected section 4 of the paper by taking into account the two photons in the final state as identical indistinguishable particles. For completeness of this Erratum, we provide for the process under consideration, our notations and then the correct PWA formulae.

4 Covariant tensor formalism for ψ decay into $\gamma\gamma V$

We are considering the double radiative decays of the ψ meson,

$$\psi(p, m_\psi) \to \gamma\ R \to \gamma(q_1, m_\gamma)\ \gamma(q_2, m'_\gamma)\ V(p_V, m_V), \tag{1}$$

where p, q_1, q_2 and p_V are the four momenta of the ψ, two photons and the vector particle $V(\rho, \phi, \omega)$, respectively; m_ψ, m_γ, m'_γ and m_V denote the third components of each particle's spin. The transition amplitude can be written as follows by using the polarization four-vectors of the initial and final state particles,

$$\begin{aligned} A &= \psi_\mu(p, m_\psi)\ \varepsilon^*_\nu(q_1, m_\gamma)\ \varepsilon^*_\alpha(q_2, m'_\gamma)\ A^{\mu\nu\alpha} \\ &= \psi_\mu(p, m_\psi)\ \varepsilon^*_\nu(q_1, m_\gamma)\ \varepsilon^*_\alpha(q_2, m'_\gamma)\ \sum_i \Lambda_i A_i^{\mu\nu\alpha}\ , \end{aligned} \tag{2}$$

where $\psi(p, m_\psi)$ is the polarization four vector of the ψ-mesons; $\varepsilon_\nu(q_i, m_\gamma)$, $i = 1, 2$ are the polarization vectors of the two photons. The sum over the two physical polarization states of a photon is given by,

$$\begin{aligned} \sum_{m_\gamma=1}^{2} \varepsilon^*_\mu(q_i, m_\gamma)\ \varepsilon_\nu(q_i, m_\gamma) &= -g_{\mu\nu} \\ &+ \frac{q_{i\mu}\ (p-q_i)_\nu + (p-q_i)_\mu\ q_{i\nu}}{q_i \cdot p} \\ &- \frac{(p-q_i)^2}{(q_i \cdot p)^2}\ q_{i\mu}\ q_{i\nu} \\ &= -g_{\mu\nu} + \frac{q_{i\mu}\ p_\nu + p_\mu\ q_{i\nu}}{q_i \cdot p} - \frac{p^2}{(q_i \cdot p)^2}\ q_{i\mu}\ q_{i\nu} \\ &\equiv -g^{(\perp)}_{\mu\nu}(q_i) \end{aligned} \tag{3}$$

where the relations $q_i^\nu\ \varepsilon_\nu(q_i, m_\gamma) = 0$ and $p^\nu\ \varepsilon_\nu(q_i, m_\gamma) = 0$ hold.

For ψ production from e^+e^- annihilation, the electrons are highly relativistic, with the result that $J_z = \pm 1$. If we

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take the beam direction to be the z-axis, this limits m_ψ to 1 and 2, *i.e.*, components along x and y. Note that for ψ at rest system with $p = (M_\psi, 0, 0, 0)$

$$\sum_{m_\psi=1}^{2} \psi_\mu(p, m_\psi)\psi^*_{\mu'}(p, m_\psi) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2}). \tag{4}$$

Then the differential decay width for the ψ radiative decay to an n-body final state is:

$$\begin{aligned}
\frac{d\Gamma}{d\Phi_n} &= \frac{(2\pi)^4}{2M_\psi} \cdot \frac{1}{2} \cdot \frac{1}{2} \sum_{m_\psi=1}^{2} \sum_{m'_\gamma, m_\gamma=1}^{2} \\
&\quad \times |\psi_\mu(p, m_\psi)\, \varepsilon^*_\nu(q_1, m_\gamma)\, \varepsilon^*_\alpha(q_2, m'_\gamma) A^{\mu\nu\alpha}|^2 \\
&= \frac{(2\pi)^4}{2M_\psi} \cdot \frac{1}{4} \sum_{m_\psi=1}^{2} \psi_\mu(p, m_\psi)\, \psi^*_{\mu'}(p, m_\psi)\, g^{(\perp)}_{\nu\nu'}(q_1)\, g^{(\perp)}_{\alpha\alpha'}(q_2)\, A^{\mu\nu\alpha}\, A^{*\mu'\nu'\alpha'} \\
&= \frac{(2\pi)^4}{2M_\psi} \cdot \frac{1}{4} \sum_{\mu=1}^{2} A^{\mu\nu\alpha}\, g^{(\perp)}_{\nu\nu'}(q_1)\, g^{(\perp)}_{\alpha\alpha'}(q_2)\, A^{*\mu\nu'\alpha'} \\
&= \frac{(2\pi)^4}{2M_\psi} \cdot \frac{1}{4} \sum_{i,j} \Lambda_i\, \Lambda^*_j \sum_{\mu=1}^{2} U^{\mu\nu\alpha}_i\, g^{(\perp)}_{\nu\nu'}(q_1)\, g^{(\perp)}_{\alpha\alpha'}(q_2) \\
&\quad \times U^{*\mu\nu'\alpha'}_j \equiv \sum_{i,j} P_{ij} \cdot F_{ij}\, ,
\end{aligned} \tag{5}$$

where

$$P_{ij} = P^*_{ji} = \frac{(2\pi)^4}{2M_\psi} \Lambda_i\, \Lambda^*_j, \tag{6}$$

$$F_{ij} = F^*_{ji} = \frac{1}{4} \sum_{\mu=1}^{2} U^{\mu\nu\alpha}_i\, g^{(\perp)}_{\nu\nu'}(q_1)\, g^{(\perp)}_{\alpha\alpha'}(q_2)\, U^{*\mu\nu'\alpha'}_j\, . \tag{7}$$

$d\Phi_n$ is the standard element of n-body phase space given by

$$d\Phi_n(p; p_1, \ldots p_n) = \delta^4(p - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\, . \tag{8}$$

4.1 Amplitudes for the doubly radiative decay $\psi \to \gamma\gamma V(\rho, \omega, \phi)$

This is a three step process: $\psi \to \gamma R$ with $R \to \gamma V(\rho, \omega, \phi)$ and $\rho \to \pi^+\pi^-$, $\omega \to \pi^0\pi^+\pi^-$, $\phi \to K^+K^-$. The intermediate resonance state R that may appear in the process with J^{PC} values are $0^{++}, 0^{-+}, 1^{++}, 1^{-+}, 2^{++}, 2^{-+}$, etc. Here J, P, C are the intrinsic spin, parity and C-parity of the R particle, respectively. For $\psi \to \gamma R$, we denote the spin-orbital angular momenta between the photon and ψ by S and L, respectively. The tensor describing the first and the second steps will be denoted by $\tilde{T}^{(L)}_{\mu_1 \ldots \mu_L}$ and $\tilde{t}^{(L_1)}_{\mu_1 \ldots \mu_{L_1}}$, respectively. The vector describing the third step will be denoted by V_μ, where $V(\rho, \phi)_\mu = p_{1\mu} - p_{2\mu}$, here we use the fact that π^+ and π^- (or K^+ and K^-) have equal masses, and

$$\begin{aligned}
V(\omega)_\mu = \epsilon^{\mu}{}_{\nu\lambda\sigma}\, p_1^\nu p_2^\lambda p_0^\sigma [&B_1(Q_{\omega\rho 0}) f^{(\rho)}_{(12)} B_1(Q_{\rho 12}) \\
&+ B_1(Q_{\omega\rho 2}) f^{(\rho)}_{(01)} B_1(Q_{\rho 10}) \\
&+ B_1(Q_{\omega\rho 1}) f^{(\rho)}_{(02)} B_1(Q_{\rho 20})].
\end{aligned}$$

Now we write the decay amplitude of the ψ into two photons and a vector in a general and compact form using the covariant tensor formalism. When writing the covariant tensor amplitude we have to keep in mind that there are two identical particles (photons) in the final state, due to Bose statistics decay amplitude is symmetric with respect to the exchange of photons. There is one independent covariant tensor amplitude for $\psi \to \gamma 0^{++} \to \gamma\gamma V(\rho, \omega, \phi)$

$$U^{\mu\nu\alpha} = g^{\mu\nu} \left(f^{(R)}_{V\gamma(q_1)} + f^{(R)}_{V\gamma(q_2)} \right) f^{(V)}\, V^\alpha, \tag{9}$$

where $f^{(V)}$ represents either $f^{(\rho,\phi)}_{(12)}$ or $f^{(\omega)}_{(012)}$,

$$\begin{aligned}
f^{(R)}_{V\gamma(q_i)} &= \frac{M_R\, \Gamma_R}{(p_V + q_i)^2 - M_R^2 + i M_R\, \Gamma_R}\, , \\
f^{(V)} &= \frac{M_V\, \Gamma_V}{p_V^2 - M_V^2 + i M_V\, \Gamma_V}\, ,
\end{aligned} \tag{10}$$

here M_R, M_V and Γ_R, Γ_V are the mass and width of each resonance.

There is also one independent covariant tensor amplitude for $\psi \to \gamma 0^{-+} \to \gamma\, \gamma\, V(\rho, \omega, \phi)$

$$\begin{aligned}
U^{\mu\nu\alpha}_{12} &= \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\delta}\, p_\lambda \\
&\quad \times \Big(\tilde{T}^{(1)}_\sigma(q_1)\, (p - q_1)_\rho\, t^{(1)}_\beta(q_2)\, f^{(R)}_{V\gamma(q_2)} \\
&\quad + \tilde{T}^{(1)}_\sigma(q_2)\, (p - q_2)_\rho\, t^{(1)}_\beta(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\delta\, f^{(V)}\, ,
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\tilde{T}^{(1)}_\mu(q_i) &= \tilde{g}_{\mu\nu}(p)\, (q_i - p_R)^\nu\, B_1(Q_{\psi\gamma(q_i)R})\, , \\
\tilde{t}^{(1)}_\mu(q_i) &= \tilde{g}_{\mu\nu}(p_R)\, (q_i - p_V)^\nu\, B_1(Q_{R\gamma(q_i)V})\, .
\end{aligned} \tag{12}$$

For the production reaction $\psi \to \gamma 1^{++}$ there are two independent covariant tensor amplitudes; there are also two amplitudes for the decay reaction $1^{++} \to \gamma V(\rho, \omega, \phi)$, all in all we have four amplitudes,

$$\begin{aligned}
U^{\mu\nu\alpha}_1 &= \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho}_{\sigma}\, p_\lambda \Big((p - q_1)_\rho\, f^{(R)}_{V\gamma(q_2)} \\
&\quad + (p - q_2)_\rho\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\, ,
\end{aligned} \tag{13}$$

$$\begin{aligned}
U^{\mu\nu\alpha}_2 &= \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\delta}\, p_\lambda \\
&\quad \times \Big(\tilde{T}^{(2)}_{\sigma\zeta}(q_1)\, (p - q_1)_\rho\, \tilde{t}^{(2)\zeta}_\delta(q_2)\, f^{(R)}_{V\gamma(q_2)} \\
&\quad + \tilde{T}^{(2)}_{\sigma\zeta}(q_2)\, (p - q_2)_\rho\, \tilde{t}^{(2)\zeta}_\delta(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\, ,
\end{aligned} \tag{14}$$

$$U_3^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\delta}\, p_\lambda \Big((p-q_1)_\rho\, \tilde{t}^{(2)}_{\sigma\delta}(q_2)\, f^{(R)}_{V\gamma(q_2)} + (p-q_2)_\rho\, \tilde{t}^{(2)}_{\sigma\delta}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{15}$$

$$U_4^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\delta}\, p_\lambda \Big(\tilde{T}^{(2)}_{\sigma\delta}(q_1)\, (p-q_1)_\rho\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(2)}_{\sigma\delta}(q_2)\, (p-q_2)_\rho\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{16}$$

$$\tilde{T}^{(2)}_{\sigma\gamma}(q_i) = [\tilde{r}_\sigma\, \tilde{r}_\gamma - \frac{1}{3}(\tilde{r}\cdot\tilde{r})\, \tilde{g}_{\sigma\gamma}(p_\psi)] B_2(Q_{\psi\gamma(q_i)R})\,, \tag{17}$$

$$\tilde{t}^{(2)}_{\sigma\gamma}(q_i) = [\tilde{r}_\sigma\, \tilde{r}_\gamma - \frac{1}{3}(\tilde{r}\cdot\tilde{r})\, \tilde{g}_{\sigma\gamma}(p_R)] B_2(Q_{R\gamma(q_i)V})\,, \tag{18}$$

where $r = p_b - p_c$ is the relative four momentum of the two decay products in the parent particle rest frame.

For the production reaction $\psi \to \gamma 1^{-+}$ there are two independent covariant tensor amplitudes; there are also two amplitudes for the decay reaction $1^{-+} \to \gamma V(\rho, \omega, \phi)$, all in all we have four amplitudes,

$$U_1^{\mu\nu\alpha} = g^{\mu\nu} \Big(\tilde{T}^{(1)}_\beta(q_1)\, \tilde{t}^{(1)\beta}(q_2)\, f^{(R)}_{V\gamma(q_2)} + T^{(1)}_\beta(q_2)\, \tilde{t}^{(1)\beta}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\alpha f^{(V)}\,, \tag{19}$$

$$U_2^{\mu\nu\alpha} = \Big(\tilde{T}^{(1)\mu}(q_1)\, \tilde{t}^{(1)\nu}(q_2)\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(1)\mu}(q_2)\, \tilde{t}^{(1)\nu}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\alpha\, f^{(V)}\,, \tag{20}$$

$$U_3^{\mu\nu\alpha} = g^{\mu\nu} \Big(\tilde{T}^{(1)\alpha}(q_1)\, \tilde{t}^{(1)\beta}(q_2)\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(1)\alpha}(q_2)\, \tilde{t}^{(1)\beta}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}, \tag{21}$$

$$U_4^{\mu\nu\alpha} = g^{\nu\alpha} \Big(\tilde{T}^{(1)\mu}(q_1)\, \tilde{t}^{(1)\beta}(q_2)\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(1)\mu}(q_2)\, \tilde{t}^{(1)\beta}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,. \tag{22}$$

For the production reaction $\psi \to \gamma 2^{++}$ there are three independent covariant tensor amplitudes; there are also three amplitudes for the decay reaction $2^{++} \to \gamma V(\rho, \omega, \phi)$, all in all we have nine amplitudes,

$$U_1^{\mu\nu\alpha} = \Big(P^{(2)\mu\nu\alpha\beta}(p-q_1)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\mu\nu\alpha\beta}(p-q_2)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{23}$$

$$U_2^{\mu\nu\alpha} = g^{\mu\nu} \Big(P^{(2)\lambda\sigma\rho\delta}(p-q_1)\, \tilde{T}^{(2)}_{\lambda\sigma}(q_1)\, \tilde{t}^{(2)}_{\rho\delta}(q_2)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\lambda\sigma\rho\delta}(p-q_2)\, \tilde{T}^{(2)}_{\lambda\sigma}(q_2)\, \tilde{t}^{(2)}_{\rho\delta}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\alpha\, f^{(V)}, \tag{24}$$

$$U_3^{\mu\nu\alpha} = \Big(P^{(2)\nu\sigma\alpha\lambda}(p-q_1)\, \tilde{T}^{(2)\mu}_{\sigma}(q_1)\, \tilde{t}^{(2)}_{\lambda\beta}(q_2)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\nu\sigma\alpha\lambda}(p-q_2)\, \tilde{T}^{(2)\mu}_{\sigma}(q_2)\, \tilde{t}^{(2)}_{\lambda\beta}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\beta\, f^{(V)}, \tag{25}$$

$$U_4^{\mu\nu\alpha} = \Big(P^{(2)\mu\nu\lambda\sigma}(p-q_1)\, \tilde{t}^{(2)}_{\lambda\sigma}(q_2)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\mu\nu\lambda\sigma}(p-q_2)\, \tilde{t}^{(2)}_{\lambda\sigma}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\alpha\, f^{(V)}, \tag{26}$$

$$U_5^{\mu\nu\alpha} = \Big(P^{(2)\mu\nu\alpha\lambda}(p-q_1)\, \tilde{t}^{(2)}_{\beta\lambda}(q_2)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\mu\nu\alpha\lambda}(p-q_2)\, \tilde{t}^{(2)}_{\beta\lambda}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\beta\, f^{(V)}, \tag{27}$$

$$U_6^{\mu\nu\alpha} = g^{\mu\nu} \Big(P^{(2)\lambda\sigma\alpha\beta}(p-q_1)\, \tilde{T}^{(2)}_{\lambda\sigma}(q_1)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\lambda\sigma\alpha\beta}(p-q_2)\, \tilde{T}^{(2)}_{\lambda\sigma}(q_2)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}, \tag{28}$$

$$U_7^{\mu\nu\alpha} = g^{\mu\nu} \Big(P^{(2)\lambda\sigma\alpha\delta}(p-q_1)\, \tilde{T}^{(2)}_{\lambda\sigma}(q_1)\, \tilde{t}^{(2)}_{\beta\delta}(q_2)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\lambda\sigma\alpha\delta}(p-q_2)\, \tilde{T}^{(2)}_{\lambda\sigma}(q_2)\, \tilde{t}^{(2)}_{\beta\delta}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\beta\, f^{(V)}\,, \tag{29}$$

$$U_8^{\mu\nu\alpha} = \Big(P^{(2)\nu\lambda\alpha\beta}(p-q_1)\, \tilde{T}^{(2)\mu}_{\lambda}(q_1)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\nu\lambda\alpha\beta}(p-q_2)\, \tilde{T}^{(2)\mu}_{\lambda}(q_2)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}, \tag{30}$$

$$U_9^{\mu\nu\alpha} = \Big(P^{(2)\nu\delta\lambda\sigma}(p-q_1)\, \tilde{T}^{(2)\mu}_{\delta}(q_1)\, \tilde{t}^{(2)}_{\lambda\sigma}(q_2)\, f^{(R)}_{V\gamma(q_2)} + P^{(2)\nu\delta\lambda\sigma}(p-q_2)\, \tilde{T}^{(2)\mu}_{\delta}(q_2)\, \tilde{t}^{(2)}_{\lambda\sigma}(q_1)\, f^{(R)}_{V\gamma(q_1)} \Big) V^\alpha\, f^{(V)}\,. \tag{31}$$

For the production reaction $\psi \to \gamma 2^{-+}$ there are three independent covariant tensor amplitudes; there are also three amplitudes for the decay reaction $2^{-+} \to \gamma V(\rho, \omega, \phi)$, all in all we have nine amplitudes,

$$U_1^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\xi}\, p_\sigma \Big(\tilde{T}^{(1)\zeta}(q_1)\, (p-q_1)_\xi\, \tilde{t}^{(1)\delta}(q_2) \times P^{(2)}_{\lambda\zeta\rho\delta}(p-q_1)\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(1)\zeta}(q_2)\, (p-q_2)_\xi\, \tilde{t}^{(1)\delta}(q_1) \times P^{(2)}_{\lambda\zeta\rho\delta}(p-q_2)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{32}$$

$$U_2^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\xi}\, p_\sigma \Big(\tilde{T}^{(3)}_{\lambda\zeta\delta}(q_1)\, (p-q_1)_\xi\, \tilde{t}^{(3)}_{\rho\zeta'\delta'}(q_2) \times P^{(2)\zeta\delta\zeta'\delta'}(p-q_1)\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(3)}_{\lambda\zeta\delta}(q_2)\, (p-q_2)_\xi\, \tilde{t}^{(3)}_{\rho\zeta'\delta'}(q_1) \times P^{(2)\zeta\delta\zeta'\delta'}(p-q_2)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{33}$$

$$U_3^{\mu\nu\alpha} = \epsilon^{\nu\lambda\sigma\zeta}\, \epsilon^{\beta\rho\delta\xi}\, p_\zeta \Big(\tilde{T}^{(3)\mu\lambda'}_{\sigma}(q_1)\, (p-q_1)_\xi\, \tilde{t}^{(3)\alpha\rho'}_{\delta}(q_2) \times P^{(2)}_{\lambda\lambda'\rho\rho'}(p-q_1)\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(3)\mu\lambda'}_{\sigma}(q_2)\, (p-q_2)_\xi\, \tilde{t}^{(3)\alpha\rho'}_{\delta}(q_1) \times P^{(2)}_{\lambda\lambda'\rho\rho'}(p-q_2)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{34}$$

$$U_4^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\xi}\, p_\sigma \Big(\tilde{T}^{(1)\zeta'}(q_1)\, (p-q_1)_\xi\, \tilde{t}^{(3)\delta\zeta}_{\rho}(q_2) \times P^{(2)}_{\lambda\zeta'\delta\zeta}(p-q_1)\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(1)\zeta'}(q_2)\, (p-q_2)_\xi\, \tilde{t}^{(3)\delta\zeta}_{\rho}(q_1) \times P^{(2)}_{\lambda\zeta'\delta\zeta}(p-q_2)\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{35}$$

$$U_5^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\beta\rho\delta\xi}\, p_\sigma \Big(\tilde{T}^{(1)\zeta'}(q_1)\, (p-q_1)_\xi\, \tilde{t}^{(3)\alpha\zeta}_{\delta}(q_2) \times P^{(2)}_{\lambda\zeta'\rho\zeta}\, f^{(R)}_{V\gamma(q_2)} + \tilde{T}^{(1)\zeta'}(q_2)\, (p-q_2)_\xi\, \tilde{t}^{(3)\alpha\zeta}_{\delta}(q_1) \times P^{(2)}_{\lambda\zeta'\rho\zeta}\, f^{(R)}_{V\gamma(q_1)} \Big) V_\beta\, f^{(V)}\,, \tag{36}$$

$$U_6^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}\, \epsilon^{\alpha\beta\rho\xi}\, p_\sigma \Big(\tilde{T}^{(3)\zeta'\delta}_{\lambda}(q_1)\, (p-q_1)_\xi\, \tilde{t}^{(1)\zeta}(q_2)$$

$$
\begin{aligned}
&\times P^{(2)}_{\zeta'\delta\rho\zeta}(p-q_1)\ f^{(R)}_{V\gamma(q_2)} \\
&+\tilde{T}^{(3)\zeta'\delta}_{\lambda}(q_2)\ (p-q_2)_\xi\ \tilde{t}^{(1)\zeta}(q_1) \\
&\times P^{(2)}_{\zeta'\delta\rho\zeta}(p-q_2)\ f^{(R)}_{V\gamma(q_1)}\Big) V^{\beta}\ f^{(V)}\ , \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
U_7^{\mu\nu\alpha} &= \epsilon^{\mu\nu\lambda\sigma}\ \epsilon^{\beta\tau\rho\xi}\ p_\sigma \Big(\tilde{T}^{(3)\zeta\delta}_{\lambda}(q_1)\ (p-q_1)_\xi\ \tilde{t}^{(3)\alpha\delta'}_{\rho}(q_2) \\
&\times P^{(2)}_{\zeta\delta\tau\delta'}(p-q_1)\ f^{(R)}_{V\gamma(q_2)} \\
&+\tilde{T}^{(3)\zeta\delta}_{\lambda}(q_2)\ (p-q_2)_\xi\ \tilde{t}^{(3)\alpha\delta'}_{\rho}(q_1) \\
&\times P^{(2)}_{\zeta\delta\tau\delta'}(p-q_1) f^{(R)}_{V\gamma(q_1)}\Big) V_\beta\ f^{(V)}\ , \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
U_8^{\mu\nu\alpha} &= \epsilon^{\nu\lambda\sigma\zeta'}\ \epsilon^{\alpha\beta\rho\xi}\ p'_\zeta \Big(\tilde{T}^{(3)\mu\zeta}_{\sigma}(q_1)\ (p-q_1)_\xi\ \tilde{t}^{(1)\delta}(q_2) \\
&\times P^{(2)}_{\lambda\zeta\rho\delta}(p-q_1)\ f^{(R)}_{V\gamma(q_2)} \\
&+\tilde{T}^{(3)\mu\zeta}_{\sigma}(q_2)\ (p-q_2)_\xi\ \tilde{t}^{(1)\delta}(q_1) \\
&\times P^{(2)}_{\lambda\zeta\rho\delta}(p-q_2)\ f^{(R)}_{V\gamma(q_1)}\Big) V_\beta\ f^{(V)}\ , \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
U_9^{\mu\nu\alpha} &= \epsilon^{\nu\lambda\sigma\zeta}\ \epsilon^{\alpha\beta\rho\xi}\ p_\zeta \Big(\tilde{T}^{(3)\mu\delta}_{\sigma}(q_1)\ (p-q_1)_\xi\ \tilde{t}^{(3)\lambda'\delta'}_{\rho}(q_2) \\
&\times P^{(2)}_{\lambda\delta\lambda'\delta'}(p-q_1)\ f^{(R)}_{V\gamma(q_2)} \\
&+\tilde{T}^{(3)\mu\delta}_{\sigma}(q_2)\ (p-q_2)_\xi\ \tilde{t}^{(3)\lambda'\delta'}_{\rho}(q_1) \\
&\times P^{(2)}_{\lambda\delta\lambda'\delta'}(p-q_2)\ f^{(R)}_{V\gamma(q_1)}\Big) V_\beta\ f^{(V)}\ . \qquad (40)
\end{aligned}
$$

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