



High-resolution radioactive beam study of the $^{26}\text{Al}(d, p)$ reaction and measurements of single-particle spectroscopic factors

G. Lotay^{1,a}, P. J. Woods², M. Moukaddam^{1,4}, M. Aliotta², G. Christian^{3,5}, B. Davids³, T. Davinson³, D. T. Doherty¹, D. Howell³, V. Margerin³, C. Ruiz³

¹ Department of Physics, University of Surrey, Guildford GU2 7XH, UK

² School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, UK

³ TRIUMF, Vancouver, BC V6T 2A3, Canada

⁴ Present address: Université de Strasbourg, IPHC, 23 rue du Loess, 67037 Strasbourg, France

⁵ Present address: Department of Astronomy and Physics, Saint Mary's University, Halifax NS B3H 3C3, Canada

Received: 20 August 2019 / Accepted: 21 October 2019

© The Author(s) 2020

Communicated by C. Brogini

Abstract We present a detailed comparison of shell model calculations with inverse kinematic transfer reaction data, obtained using a radioactive beam. Experimentally extracted spectroscopic factors from the $^{26}\text{Al}(d, p)^{27}\text{Al}$ reaction for both even and odd parity states are found to be exceptionally well reproduced by the shell model and a high level of consistency is observed between bound isobaric analog states in ^{27}Al and ^{27}Si , populated via (d, p) and (d, n) transfer, respectively. Furthermore, an evaluation of key resonances in the astrophysical $^{26}\text{Al}(p, \gamma)^{27}\text{Si}$ reaction indicates that shell model calculations provide relatively accurate predictions for the existence of strong resonances and mirror nucleus comparisons appear to hold exceptionally well for proton-unbound levels. Consequently, we expect that the utilization of both techniques will likely be a very effective tool in the investigation of stellar processes outside the current reach of experiment.

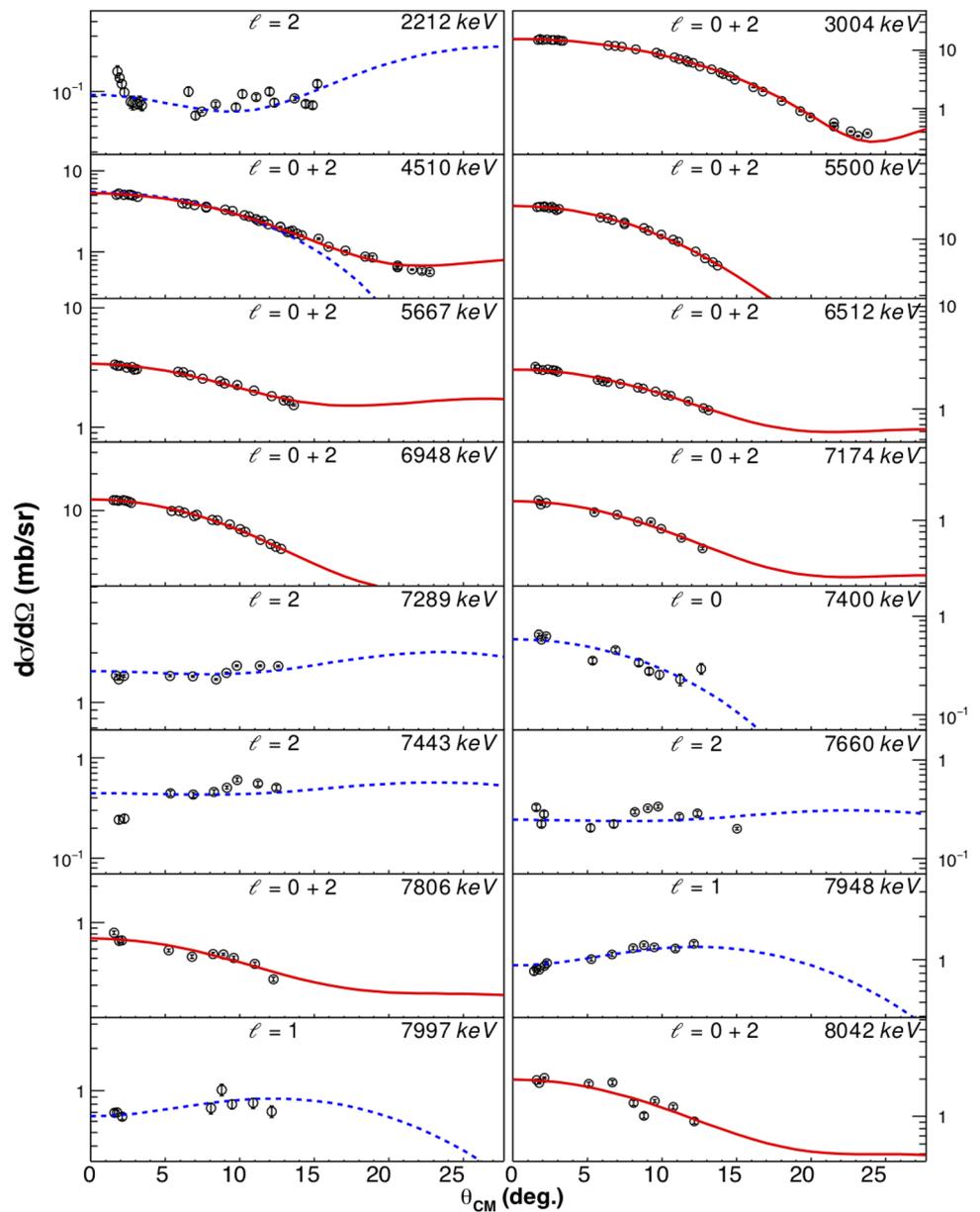
In analogy with electrons in atoms, certain numbers of protons and neutrons in nuclei lead to particularly stable configurations. These so-called “magic” numbers were first reproduced in their entirety, by Goeppert-Mayer, Jensen and Suess, in 1949 [1, 2], and established the nuclear shell model as a bedrock of nuclear physics research. Since then, the shell model description of the quantum many-body problem has been developed to become the principal mechanism for understanding nuclear structure across the chart of isotopes [3]. In particular, by considering only those particles that exist outside of an “inert” doubly-magic core to be active, shell model calculations have been able to act as powerful tools for predicting the spins, parities and nuclear matrix ele-

ments of excited states in the sd shell [4]. However, in order to extend and explicate the predictive power of the shell model for nuclei far from stability, there is a need to perform detailed studies of shell model wave functions up to regions of high excitation energy and weak binding.

Single-particle spectroscopic factors, C^2S , obtained from transfer reactions provide one of the most fundamental tests of wave functions incorporated in the nuclear shell model. Specifically, they quantify the occupancy of single-particle orbits in a nucleus and, as such, represent an essential check of model spaces used in current calculations. Detailed, high-resolution measurements of spectroscopic factors, and precise comparisons with shell model predictions, have largely involved reactions with stable target nuclei [5]. In this Letter, we present such an analysis for a study of the (d, p) reaction using a beam of radioactive ^{26}Al nuclei in inverse kinematics. This represents a benchmark for comprehensive, high-resolution transfer reaction studies using radioactive beams and, as such, provides key information for the broader development of worldwide initiatives in this area [6–11]. Spectroscopic factors are reported for 17 excited states and excellent agreement is obtained with nuclear shell model wave function predictions. Furthermore, we explore the robustness of the mirror symmetry in the $T = 1/2$, $A = 27$ system, for the first time, through a comparison of the present data with more limited information obtained for analog levels in ^{27}Si via $^{26}\text{Al}(d, n)$ transfer [12]. Isospin symmetry is observed to hold extremely well up to high excitation energies. Moreover, shell model calculations and transfer reaction estimates of resonance strengths for the $^{26}\text{Al}(p, \gamma)^{27}\text{Si}$ reaction, which destroys the cosmic γ -ray emitter ^{26}Al in stars [13], are found

^a e-mail: g.lotay@surrey.ac.uk

Fig. 1 (Color online) Angular distributions together with ADWA fits for excited states in ^{27}Al . Pure distributions are displayed by dashed blue lines, while mixed distributions are shown as solid red lines. The effect of mixing on angular distributions has been highlighted for the 4510 keV level, in which the $\ell = 0$ component is given by a blue dashed line. Data points have been enlarged for ease of viewing



to be consistent with direct measurement data (where available).

The experimental setup used for this study has already been reported in an earlier paper [14] and, as such, we will provide a brief summary of the key features here. An intense, ~ 1 pnA beam of radioactive ^{26}Al at 6 MeV was used to bombard a thin $\sim 60 \mu\text{g}/\text{cm}^2$ thick $(\text{CD}_2)_n$ target in order to populate excited states in ^{27}Al via the $^{26}\text{Al}(d, p)$ transfer reaction—it is expected that the population of excited states is purely via the ground state component of the beam (the ground to isomeric state ratio was measured to be $\sim 17000:1$). Resulting protons were detected with three MSL type S2 silicon strip detectors [16], placed upstream of the target position, covering an angular range $\theta_{CM} \sim 0.5^\circ\text{--}24^\circ$ (although it should be noted that only the most strongly populated states

could be analysed for angles $>12^\circ$). Using such a configuration allowed for the state of the art in resolution performance for an inverse kinematic transfer reaction study using a radioactive beam to be achieved (~ 40 keV FWHM—see Fig. 1 of Ref. [14]). Figure 1 illustrates the angular distributions obtained for excited states observed in the current study. Levels in ^{27}Al [15] corresponding to low orbital angular momentum, ℓ , transfers are observed to be selectively populated at the forward center-of mass angular range covered in the present experimental set-up.

For the determination of spectroscopic factors, C^2S , measured differential cross sections were compared with theoretical distributions obtained from adiabatic distorted wave approximation (ADWA) calculations using the code TWOFNR [17]. Here, the deuteron wave function of the

Table 1 Properties of excited states in ^{27}Al in comparison with shell model calculations [4] and mirror states in ^{27}Si . Also included are spectroscopic factors for excited levels in ^{27}Si obtained from a previous (d, n) reaction study [12]. Excitation energies for states in ^{27}Al and ^{27}Si are adopted from Ref. [15] and excitation energies listed for SM are for states in ^{27}Si

$E_{x,^{27}\text{Al}}$ (keV)	$E_{x,^{27}\text{Si}}$ (keV)	$E_{x,SM}$ (keV)	J^π	ℓ	$C^2S_{(d,n)}$ [12]	$C^2S_{(d,p)}$	$C^2S_{(SM)}$	Ratio $_{(d,p)/(SM)}$
2212	2164	2313	$7/2_1^+$	2		0.23(4)	0.10	2.30(40)
3004	2910	2949	$9/2_1^+$	0		0.49(9)	0.50	0.98(18)
				2		0.17(3)	0.04	4.25(75)
4510	4448	4437	$11/2_1^+$	0		0.11(2)	0.13	0.85(15)
				2		0.31(6)	0.26	1.19(23)
5500	5283	5382	$11/2_2^+$	0		0.37(7)	0.34	1.09(21)
				2		0.49(9)	0.13	3.77(69)
5667	5547	5726	$9/2_3^+$	0		0.05(1)	0.095	0.53(11)
				2	0.61(13)	0.74(14)	0.42	1.76 (33)
6512	6344	6492	$9/2_4^+$	0		0.04(1)	0.022	1.82(45)
				2		0.22(4)	0.31	0.71(13)
6948	6734	7118	$11/2_3^+$	0		0.17(3)	0.16	1.06(19)
				2	0.35(8)	0.57(11)	0.50	1.14(22)
7174	7001	7349	$9/2_7^+$	0		0.022(4)	0.017	1.29(24)
				2		0.10(2)	0.052	1.92(38)
7289	7129	6677	$13/2_1^+$	2	0.50(10)	0.45(9)	0.74	0.61(12)
7400	7245	7623	$11/2_4^+$	0		0.009(2)	0.004	2.25(50)
7443	7222	7201	$13/2_2^+$	2		0.12(2)	0.13	0.92(15)
7660	7428	7302	$7/2_9^+$	2		0.10(2)	0.15	0.67(13)
7790	7532	7852	$5/2_{12}^+$	2		<0.016	0.0088	<1.82
7806	7590	7737	$9/2_8^+$	0	$\leq 0.1^a$	0.0093(19)	0.011	0.85(17)
				2	$\leq 0.05^a$	0.068(14)	0.011	6.2(13)
7948	7652	7630	$11/2_1^-$	1	0.22(5)	0.04(1)	0.067	0.60(15)
7997	7739	7781	$9/2_2^-$	1	0.07(4)	0.04(1)	0.038	1.05(26)
8043	7831	8364	$9/2_{10}^+$	0		0.027(5)	0.014	1.93(36)
				2		0.12(2)	0.028	4.14(71)

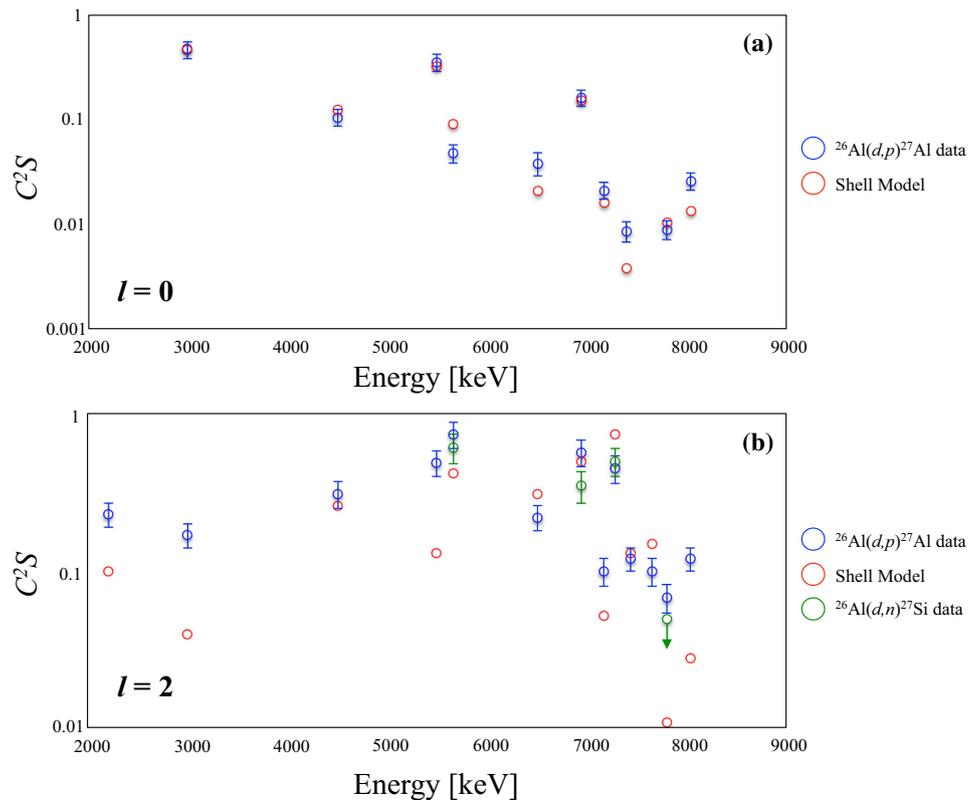
^a C^2S was extracted assuming pure, single ℓ -transfer

Argonne AV18 np interaction [18], as well as the Koning–Delaroche global optical model parameterization [19], were used to calculate the distorting potential [20]. Radii were obtained by the Hartree–Fock (HF) method of Ref. [21], except in the case HF-unbound, p -wave orbitals, for which radius and diffuseness parameters of 1.25 and 0.65 fm, respectively, were used. Table 1 presents a summary of extracted spectroscopic factors for excited levels in ^{27}Al , together with a comparison to shell model calculations and mirror analogs in ^{27}Si populated via (d, n) transfer [12]. The information obtained for $\ell = 0$ and $\ell = 2$ transfers is also shown in graphical form in Fig. 2. Shell model calculations of spectroscopic factors for positive parity states were performed using the USDB-cdpn interaction [4], in which cdpn refers to the addition of the Coulomb, charge-dependent and charge asymmetric nuclear Hamiltonian obtained by Ormand and Brown in a proton–neutron basis [22]. For negative par-

ity states, we adopt the shell model calculations of Ref. [12] that used the $1\hbar\omega$ basis introduced by Brown [23] to allow for the excitation of one nucleon from $0p$ to $1s-0d$ or the excitation of one nucleon from $1s-0d$ to $0p-1f$.

In considering the observed even-parity states, it is clear from Table 1 and Fig. 2 that there is striking agreement between spectroscopic factors extracted from the $^{26}\text{Al}(d, p)$ reaction and shell model calculations (to within a factor ~ 2) up to high excitation energy. Somewhat surprisingly, very good agreement between experiment and shell model predictions is obtained even for relatively small values down to $C^2S \sim 0.01$. The only significant systematic discrepancy found is for some states (e.g. $E_x = 3004$ and 5500 keV), where experiment gives a significantly larger $\ell = 2$ strength than predicted by the shell model, as illustrated in Fig. 2 (b). However, the $\ell = 0$ spectroscopic factors for these states are still in excellent agreement with theoretical predictions.

Fig. 2 Color online) A comparison between experimentally extracted spectroscopic factors and shell model calculations for even-parity states in ^{27}Al as a function of excitation energy for (a) $\ell = 0$ transfer and (b) $\ell = 2$ transfer



A possible explanation for the rather impressive predictive power of the shell model may relate to the nature of the 5^+ ground state of ^{26}Al . In particular, the addition of a neutron, via low- ℓ transfers, to this $\pi(1d_{5/2})^{-1} \otimes \nu(1d_{5/2})^{-1}$ shell model configuration state, will result in relatively pure, high-spin states in the residual nucleus, ^{27}Al . These states are likely to exhibit a strong single-particle character and, as such, may be well represented in the shell model. Furthermore, while the present (d, p) reaction study is the first experimental investigation of neutron spectroscopic factors in ^{27}Al , other known properties such as excitation energies and spin-parity assignments will have been used as input data to generate the sd -shell model interaction.

Examining negative parity states populated in $^{26}\text{Al}(d, p)$ transfer, we find that only the 7948 and 7997 keV levels have angular distributions characteristic of odd- ℓ transfers. The 7997 keV excited state in ^{27}Al can be described by pure $\ell = 1$ transfer with $C^2S = 0.04(1)$, in excellent agreement with the nuclear shell model and supporting an assignment of $9/2^-$ for this level [24]. Similarly, the angular distribution of the $11/2^-$, 7948 keV excited state can be well-fitted by pure $\ell = 1$ transfer with $C^2S = 0.04(1)$.

For the majority of states, there is very good agreement for spectroscopic factors between mirror levels in ^{27}Al and ^{27}Si and with shell model calculations. In particular, several bound excited states in ^{27}Si at 5547, 6734 and 7129 keV, as well as a proton-unbound level at 7739 keV, which

were strongly populated in the (d, n) reaction [12], are found to have spectroscopic factors in almost complete agreement with their mirror analogs in ^{27}Al at 5667, 6948, 7289 and 7997 keV, respectively. The one exception to this observed agreement, between mirror states, is the proton-unbound level in ^{27}Si at 7652 keV. The previously reported $C^2S_{(\ell=1)}$ value of 0.22(5) [12] is not consistent with the presently extracted neutron spectroscopic factor of the mirror analog at 7948 keV in ^{27}Al . However, in Ref. [12], spectroscopic factors were obtained from an angle-integrated measurement of the cross section, which does not allow for the discrimination between different ℓ -components of the wave function. Shell model calculations predict an additional, strong $C^2S_{(\ell=3)} = 0.48$ component for the 7652 keV state and, in an earlier ($^3\text{He}, d$) study, the angular distribution for this level can be almost entirely fit with $\ell = 3$ transfer [28]. Intriguingly, a slightly better fit to the present data for the mirror state in ^{27}Al at 7948 keV can also be achieved by including a large $\ell = 3$ component. However, we draw attention to the fact that the current work is not very sensitive to such ℓ -transfers, due to the limited angular range, and we refrain from making any definitive statements about $\ell = 3$ spectroscopic factors.

Considering states above the proton-emission threshold in ^{27}Si at 7643.25(16) keV [29], a key application of the present study relates to nuclear astrophysics. In particular, shell model calculations and information on mirror states can

Table 2 Parameters of $^{26}\text{Al} + p$ resonances determined from different methodologies. Total widths have been determined from lifetimes reported in Ref. [24], unless otherwise indicated, and resonance strengths for the (d, p) reaction and shell model have been evaluated

$E_{x,^{27}\text{Al}}$ (keV)	$E_{x,^{27}\text{Si}}$ (keV)	E_r (keV)	J^π	C^2S	Γ_p	Γ	$\omega\gamma_{(d,p)}$	$\omega\gamma_{(SM)}$	$\omega\gamma_{(p,\gamma)}$ [25–27]
7790	7532	68	$5/2^+$	< 0.016	$< 1.3 \times 10^{-12}$	> 110	$< 3.6 \times 10^{-13}$	2×10^{-13}	
7806	7590	127	$9/2^+$	0.0093(7)	$1.3(1) \times 10^{-4}$	33_{-4}^{+6}	$5.7(4) \times 10^{-5}$	6.7×10^{-5}	
7948	7652	189	$11/2^-$	0.04(1)	0.05(1)	55_{-11}^{+18}	0.026(7)	0.046	0.045(11) ^b
7997	7739	276	$9/2^-$	0.04(1)	5.9(15)	30_{-5}^{+7}	2.2(5)	2.1	3.4(10) ^c
8043	7831	369	$9/2^+$	0.027(5)	484(90)	$\leq 658^a$	≤ 82	≤ 71	67(19) ^c

^a The published lifetime measurement of 8(5) fs for the 369-keV resonance [24] is incompatible with the predicted proton width. As such, a lower limit of 1 fs has been assumed for this state

^b Taken as the average of Refs. [26, 27]

^c Taken as the average of Refs. [25, 26]

be used to determine the properties of experimentally inaccessible astrophysical resonances in (p, γ) capture reactions, as separately illustrated in Refs. [30–32] and Refs. [14, 33], respectively. Here, high-lying excited levels in ^{27}Al represent analogs of proton-unbound resonant states in ^{27}Si that dominate the $^{26}\text{Al}(p, \gamma)$ reaction in explosive stellar phenomena. This reaction governs the destruction of the cosmic γ -ray emitting nucleus ^{26}Al and, as such, directly affects a distinctive astronomical observable [13]. A subset of states in ^{27}Al of relevance for this reaction have already been considered in earlier work by our collaboration [14]. Here, a summary of $^{26}\text{Al}(p, \gamma)$ resonance strengths obtained from different methodologies is presented in Table 2. It should be noted that the $11/2^-$ resonance at 189 keV was previously reported as having $C^2S_{(\ell=1)} = 0.14(3)$ [14]. However, it has since been determined that incorrect nodes in TWOFNR were used in the earlier analysis of negative parity levels [14] (transfer to the $0p$ orbital was used instead of $1p$). This does not affect any of the conclusions made in Ref. [14] and brings the extracted spectroscopic factor into more consistent agreement with shell model calculations.

We find that resonance strengths determined from the current (d, p) reaction data and shell model calculations are in excellent agreement with direct measurements, highlighting the usefulness of indirect studies for the evaluation of stellar reaction rates. Specifically, we have definitively shown that, for three key resonances in the $^{26}\text{Al}(p, \gamma)$ reaction, neutron spectroscopic factors in ^{27}Al may be successfully used to obtain the proton partial widths of particle-unbound levels in the mirror nucleus ^{27}Si . It may seem somewhat surprising that isospin symmetry holds for unbound levels in the $T = 1/2, A = 27$ system. However, it should be noted that, in this case, the states of astrophysical interest lie very close to the particle-emission threshold and the height of Coulomb barrier is such that they are strongly shielded against proton decay. Consequently, they act as though they are effectively

using spectroscopic factors presented in Table 1. For states with multiple ℓ -transfers, only the lowest ℓ -value has any influence on the overall resonance strength. Particle widths and resonance strengths are given in meV

bound and both the shell model and the (d, p) data are able to accurately reproduce their properties. A limitation of the mirror analysis technique relates to proton-unbound states with $\Gamma_p > \Gamma_\gamma$. Here, the lifetime of the excited level must be known precisely in order to accurately estimate the resonance strength. Furthermore, for shell model calculations, both the energy and the level ordering of all resonant states must be determined experimentally, prior to an evaluation of the stellar reaction rate. This is illustrated in Table 1, in which shell model energies can differ from experimental values by up to ~ 500 keV.

In summary, a detailed spectroscopic study of the $^{26}\text{Al}(d, p)$ transfer reaction with a radioactive beam has been used to perform a precise test of the nuclear shell model. Extracted spectroscopic factors are found to be exceptionally well reproduced by shell model calculations, even for C^2S values as small as 0.01, and a comparison with the mirror nucleus ^{27}Si suggests strong isospin symmetry in the $T = 1/2, A = 27$ system. Furthermore, $^{26}\text{Al} + p$ resonance strengths determined from neutron spectroscopic factors and shell model calculations are found to be in excellent agreement with direct measurements [25–27]. This highlights the significant accuracy of shell model predictions and opens up fascinating opportunities for investigating astrophysical reactions outside the reach of current experimental technology. However, it is possible that the $T = 1/2, A = 27$ system represents a special case. In particular, the preferential population of relatively pure configuration, high-spin states from both proton and neutron transfer on the 5^+ ground state of ^{26}Al , could be viewed as rather exotic. Consequently, we strongly encourage further investigation to elucidate how general the present results are across the chart of the nuclides. For example, in the specific case of negative parity states in the $T = 1/2, A = 31$ system, values of C^2S approximately 10 times smaller than those predicted by the shell model have recently been reported for proton-unbound levels in ^{31}S [34].

Acknowledgements The authors wish to thank the ISAC operations and technical staff at TRIUMF. The U.K. authors acknowledge support by STFC. The Canadian authors acknowledge support from NSERC. TRIUMF receives federal funding via a contribution agreement through the National Research Council of Canada.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Data may be made available upon request.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

1. M.G. Mayer, Phys. Rev. **75**, 1969 (1949)
2. O. Haxel, J.H.D. Jensen, H.E. Suess, Phys. Rev. **75**, 1766 (1949)
3. B.A. Brown, B.H. Wildenthal, Annu. Rev. Nucl. Part. Sci. **38**, 29 (1988)
4. B.A. Brown, W.A. Richter, Phys. Rev. C **74**, 034315 (2006)
5. B.P. Kay, J.P. Schiffer, S.J. Freeman, Phys. Rev. Lett. **111**, 042502 (2013)
6. S.D. Pain et al., Phys. Proc. **90**, 455 (2017)
7. A.H. Wuosmaa et al., Phys. Rev. Lett. **105**, 132501 (2010)
8. G.L. Wilson et al., Phys. Lett. B **759**, 417 (2016)
9. K. Smidt et al., Nucl. Instrum. Methods Res. A **911**, 1 (2018)
10. J. Chen et al., Phys. Lett. B **781**, 412 (2018)
11. M.J.G. Borge, Nucl. Instrum. Methods Res. B **376**, 408 (2016)
12. A. Kankainen et al., Eur. Phys. J A **52**, 6 (2016)
13. R. Diehl et al., Nature **439**, 45 (2006)
14. V. Margerin et al., Phys. Rev. Lett. **115**, 062701 (2015)
15. M. Shamsuzzoha Basunia, Nuclear Data Sheets **112**, 1875 (2011)
16. <http://www.micronsemiconductor.co.uk/>
17. J.A. Tostevin, University of Surrey version of the code TWOFNR (of M. Toyama, M. Igarashi and N. Kishida) and code FRONT (private communication)
18. R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C **51**, 38 (1995). <https://www.phy.anl.gov/theory/research/av18/>
19. A.J. Koning, J.P. Delaroche, Nucl. Phys. A **713**, 231 (2003)
20. R.C. Johnson, P.C. Tandy, Nucl. Phys. A **235**, 56 (1974)
21. A. Gade et al., Phys. Rev. C **77**, 044306 (2008)
22. W.E. Ormand, B.A. Brown, Nucl. Phys. A **491**, 1 (1989)
23. B.A. Brown, W.A. Richter, C. Wrede, Phys. Rev. C **89**, 062801(R) (2014)
24. G. Lotay et al., Phys. Rev. C **84**, 035802 (2011)
25. L. Buchmann et al., Nucl. Phys. A **415**, 93 (1984)
26. R.B. Vogelaar, Ph.D. thesis, California Institute of Technology (1989)
27. C. Ruiz et al., Phys. Rev. Lett. **96**, 252501 (2006)
28. R.B. Vogelaar et al., Phys. Rev. C **53**, 4 (1996)
29. M. Wang et al., Chin. Phys. C **36**, 1603 (2012)
30. W.A. Richter, B. Alex Brown, A. Signoracci, M. Wiescher, Phys. Rev. C **83**, 065803 (2011)
31. W.A. Richter, B.A. Brown, Phys. Rev. C **85**, 045806 (2012)
32. W.A. Richter, B. Alex Brown, Phys. Rev. C **87**, 065803 (2013)
33. S.D. Pain et al., Phys. Rev. Lett. **114**, 212501 (2015)
34. A. Kankainen et al., Phys. Lett. B **769**, 543 (2017)