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The Giant Pairing Vibration in heavy nuclei*

Present status and future studies

M. Assié[1], C.H. Dasso[2], R.J. Liotta[3], A.O. Macchiavelli[4,a], and A. Vitturi[5]

[1] Institut de Physique Nucléaire, CNRS/IN2P3, Université de Paris Sud, Université de Paris-Saclay, 91406 Orsay, France
[2] Departamento de Física Atómica, Molecular y Nuclear, Facultad de Física, Apartado 1065, E-41080 Sevilla, Spain
[3] KTH (Royal Institute of Technology), Alba Nova University Center, S-10691 Stockholm, Sweden
[4] Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley CA 94720, USA
[5] Università degli Studi di Padova and INFN, I-35131 Padova, Italy

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Abstract. The Giant Pairing Vibration, a two-nucleon collective mode originating from the second shell above the Fermi surface, has long been predicted and expected to be strongly populated in two-nucleon transfer reactions with cross sections similar to those of the low-lying Pairing Vibration. Recent experiments have provided evidence for this mode in ^{14,15}C but, despite sensitive studies, it has not been definitively identified in Sn or Pb nuclei where pairing correlations are known to play a crucial role near their ground states. In this paper we review the basic theoretical concepts of this "elusive" state and the status of experimental searches in heavy nuclei. We discuss the hindrance effects due to Q-value mismatch and the use of weakly-bound projectiles as a way to overcome the limitations of the $(\mathrm{p,t})$ and $(\mathrm{t,p})$ reactions. We also discuss the role of the continuum and conclude with some possible future developments.

Pier Francesco Bortignon was a giant in the studies of Giant Collective Modes in Atomic Nuclei. His seminal works in this and many other aspects of nuclear structure leave a strong legacy in the field and provide a guiding light for future developments. We are honored to contribute to this special issue of *EPJ A*, in memory of Pier Francesco, on a subject of the highest interest to him. In fact, the last paper he wrote in collaboration with Ricardo Broglia was entitled *Elastic response of the atomic nucleus in gauge space: Giant Pairing Vibrations* [1] and focused on the implications of the (rather unexpected) discovery of the Giant Pairing Vibration in light nuclei to further our knowledge of the basic mechanisms —Landau, doorway, and compound damping— through which giant resonances acquire a finite lifetime.

This paper is dedicated to the memory of Pier Francesco and Hugo Sofia, another friend and colleague who made important contributions to this topic and also left us too early. They are sorely missed.

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[a] e-mail: aom@lbl.gov

1 Introduction

Pairing correlations provided a key to understand the excitation spectra of even-A nuclei, odd-even mass differences, rotational moments of inertia, and a variety of other phenomena [2–4]. An early approach to describing pair correlations in nuclei was the derivation of a collective Hamiltonian by Bès and co-workers in formal analogy to the Bohr collective Hamiltonian, which describes the quadrupole degree of freedom for the nuclear shape [5].

The analogy between particle-hole (shape) and particle-particle (pairing) excitations became well established and thoroughly explored by Broglia and co-workers [6]. The key concept in the treatment of pair correlations as a collective mode is the pairing field [7]. The pair correlations produce an average potential, acting on the nucleons, of the form

$$U_{pair} = \Delta \sum_j a_j^\dagger a_{\bar{j}}^\dagger, \tag{1}$$

which creates two nucleons in time reversed orbits. This potential is analogous to the deformed potential associated with distortions of the nuclear shape, proportional to $a_j^\dagger a_{\bar{j}}$.

Considering a constant force, the pairing gap parameter is

$$\Delta = G \sum_j a_j a_{\bar{j}} \tag{2}$$

and represents the average value of the pairing density, from which a deformation parameter of the field can be introduced, $\beta_{pair} \approx \Delta/G$. This is a measure of the available levels for scattering of the pairs, Ω.

In the same way as a deformed potential violates angular momentum (I) conservation, the pairing potential in eq. (1) violates particle number (N), thus there is a clear correspondence between N and I. In analogy to the 5-dimensional oscillator of the Bohr collective Hamiltonian, we have here a two-dimensional oscillator in "gauge" space with $\beta_{pair} \leftrightarrow (\beta, \gamma)$ and a "gauge" angle $\leftrightarrow$ the Euler angles.

On general grounds, we expect that nuclei with two identical particles added or removed from a closed-shell configuration should be close to a quantum fluid limit, since the pairing correlations are not strong enough to overcome the large single-particle energy required to add a pair. In this limit, there is no static deformation of the pair field which fluctuates about $\beta_{pair} = 0$, and gives rise to a vibrational spectrum [8]. The ground state energies, referenced to that of the doubly-magic core, will follow a linear dependence as a function of the number of pairs added, *i.e.* $\Delta E_{gs}(N) \propto (N - N_0)$. Low-lying pair-vibrational structures have been observed around ^{208}Pb by using conventional pair-transfer reactions such as (p, t) and (t, p) [6, 7].

Nuclei with many particles (pair quanta) outside of a closed-shell configuration correspond to a superconducting limit, where there is a static deformation of the pair field and rotational behavior results, *i.e.* $\Delta E_{gs}(N) \propto (N - N_0)^2$. A beautiful example would be the pair-rotational sequence comprising the ground states of the even-even Sn isotopes around ^{116}Sn [9]. In ref. [10], simple analytical approximations to the pairing collective hamiltonian were used to describe the transition from normal to superfluid behavior.

Taking the analogy even further, it has long been predicted that there should be a concentration of strength, with $L = 0$ character, in the high-energy region (10–15 MeV) of the pair-transfer spectrum. This is called the Giant Pairing Vibration (GPV) and is understood microscopically as the coherent superposition of 2p (addition mode) or 2h (removal mode) states in the next major shell $2\hbar\omega$ above (below) the Fermi surface [11]. Similar to the well-known pairing vibrational mode (PV) [6–8], which involves spin-zero-coupled pair excitations across a single major shell gap.

Consider again a schematic Hamiltonian describing the motion of independent particles interacting by a (constant) pairing force:

$$H = \sum_j e_j \left(a_j^\dagger a_j + a_{\bar{j}}^\dagger a_{\bar{j}} \right) - G \sum_{j,k} a_j^\dagger a_{\bar{j}}^\dagger a_k a_{\bar{k}}, \tag{3}$$

where the single-particle energies, e_j, are measured from the Fermi surface, and the single-particle creation operators, $a_j^\dagger$, introduced in eq. (1). The nature of the GPV is schematically illustrated in fig. 1, that shows the solution of the dispersion relation obtained in the harmonic approximation of the Hamiltonian in eq. (3) [11]

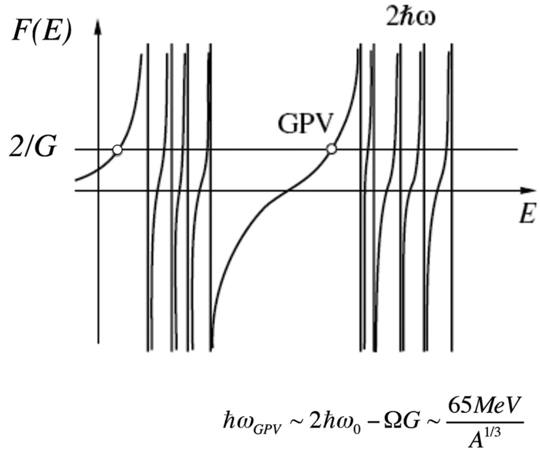

Fig. 1. Schematic of the dispersion relation, eq. (4), showing the appearance of the collective GPV state and its estimated energy. The lowest solution corresponds to the PV.

$$F(E) = \sum_j \frac{(2j+1)}{E - 2e_j} = \frac{2}{G}\,. \tag{4}$$

The two bunches of vertical lines represent the unperturbed energy of a pair of particles placed in a given potential. The GPV is the collective state relative to the second major shell. It is analogous to the giant resonances of nuclear shapes which involve the coherent superposition of particle-hole intrinsic excitations.

As in the case of the low-lying PV, the GPV should be populated through (p, t) or (t, p) reactions but it has never been identified so far [12–14].

Very recently refs. [15, 16] reported on experiments to investigate the GPV mode in light nuclei, using heavy-ion-induced two-neutron transfer reactions. The reactions ^{12}C(^{18}O,^{16}O)^{14}C and ^{13}C(^{18}O,^{16}O)^{15}C were studied at 84 MeV incident laboratory energy. "Bump" structures in the excitation energy spectra were identified as the GPV states in ^{14}C and ^{15}C nuclei at excitation energies of ≈ 20 MeV. Their energies and $L = 0$ nature, as well as the extracted transfer probabilities are consistent with the GPV population. It still remains as an intriguing puzzle that this mode has not been observed in heavier systems like Sn and Pb isotopes, where the collective effects are expected to be much stronger and for which the low-lying pair excitations are well described by pairing rotations in the Sn's and by pairing fluctuations near the critical point in the Pb's [4, 10].

The goal of this work is to provide an overview of both theoretical and experimental studies of this collective pairing mode in heavy nuclei. The article is organized as follows: In sect. 2 we give an overview of the theory of the GPV; in sect. 3 we review the status of experimental searches, and in sect. 4 we discuss the effects of Q-value mismatch on the cross-sections and the anticipated advantage of using weakly bound projectiles. We conclude our manuscript by addressing some open questions and speculating on possible future studies.

The observation and characterization of the GPV in light nuclei is being discussed in detail by Cavallaro *et al.* in this issue of *EPJ A* [17].

2 Overview of the theory of the GPV

Collective excitations in nuclei were recognized at the very beginning of nuclear studies with the introduction of the liquid drop model by Gamow in 1930 [18]. This model had a great success, particularly in the study of nuclear masses [19], neutron capture [20] and nuclear fission [21]. But the influence of the liquid drop model went beyond these outstanding applications. One can assert that both the rotational and the vibrational models were inspired by the liquid drop model. From the viewpoint of this paper the important outcome of the liquid model is the vibrational motion. Before the appearance of the shell model vibrations were envisioned as a macroscopic motion of the nucleus vibrating along an equilibrium liquid drop surface. The first microscopic calculations of surface vibrational states were performed in terms of particle-hole excitations using a harmonic oscillator representation and a separable force. One thus obtained the collective vibration as the lowest correlated particle-hole state. Here collective means that the strength of the electromagnetic transition probability, $B(E\lambda)$ is large, compared to a single-particle estimate, due to the coherent contribution of all particle-hole configurations [22]. Soon afterwards it was found that by including high lying shell model configurations the $B(E1)$ strength corresponding to dipole excitations concentrates in the uppermost state (instead of in the lowest one mentioned above). This state, which accounts for most of the electromagnetic energy weighted sum rule, is the giant dipole resonance [23].

This type of shell model calculations was enlarged by including correlations in the ground state. This was performed in the framework of the particle-hole Green function. Using the ladder approximation this treatment turned out to be equivalent to the Random Phase Approximation (RPA) [24]. One thus obtains the correlated (vibrational) gound state, with hole-particle (backward RPA) configurations which in some cases are rather large. One important feature of the RPA treatment is that there is a value of the strength of the interaction large enough which yields complex values for the energies, *i.e.* the RPA eigenvalues. At this point there is a phase transition in the nucleus from a spherical to a deformed shape [25].

As discussed in the introduction, there is a formal equivalence between particle-hole and two-particle excitations [6]. Studies performed within the two-particle Green function using the ladder approximation lead to the two-particle RPA that gives eigenvalues corresponding to states of both $A+2$ and $A-2$ systems, where A is the nucleon number in the spherical normal core [26]. As in the particle-hole case, it is found that if the interaction strength is large enough then there is a phase transition, in this case from a normal to a superconducting state.

The collective character of the particle-hole (surface) vibration is probed by inelastic scattering reactions. In the same fashion two-particle transfer reactions provide much of our knowledge of pairing correlations. For excitations to 0^+ states these reactions are important probes of collective pairing excitations in nuclei. This has the same origin as the collectivity of surface vibrations in inelastic scattering. Namely all configurations contribute with the same phase to the two-particle transfer form factor leading to the collective pairing state (a vibration in gauge space [7]). As we will show later, the cross section corresponding to pairing vibrations is much larger than those corresponding to other 0^+ states.

The analogy between the surface and pairing modes goes even farther. In ref. [11] it was predicted that a collective pairing vibration induced by excitations of pairs of particles and holes across major shells should exist at an energy of $\approx 65/A^{1/3}$ MeV carrying a cross section which is 20%–100% of the ground state cross section. However, it is important to point out in the context of this paper that the (absolute) cross section leading to the GPV as predicted above is not as large as the one leading to particle-hole giant resonances in inelastic scattering.

Since the GPV was not observed experimentally, the subject gradually lost its interest from a theoretical perspective. However, it revived independently and in a completely different framework more than a decade later, in relation to alpha-decay as we discuss below.

One standing problem in alpha-decay is the evaluation of the absolute decay width, *i.e.* of the half life of the decaying state. The decay process takes place in two steps. First the alpha-particle is formed on the surface of the mother nucleus and in a second step the alpha-particle, thus formed, penetrates the centrifugal and Coulomb barriers. The calculation corresponding to this second step is relatively easy to perform since it is just the penetrability introduced by Gamow in his seminal paper of 1928 [27]. The great difficulty is to evaluate the alpha formation probability. In the beginning one expected that this calculation was feasible within the framework of the shell model, that provides an excellent representation to describe nuclear properties [28]. In the first calculation only one shell-model configuration was used [29] due to the inadequate computing facilities at that time. The results were discouraging since the theoretical decay rates were smaller than the corresponding experimental values by many orders of magnitude. It was eventually found that the reason of this huge discrepancy was due to the lack of configurations in the shell model basis [30]. It was soon realized that the physical feature behind the configuration mixing was that the clustering of the two neutrons and two protons that eventually become the alpha particle proceeds through the high-lying configurations [31]. That was shown in spherical normal systems, but even in deformed and superfluid nuclei that property is valid [32]. Moreover, it was also found that the same feature holds for the particle and the hole that constitute the surface vibration [33].

Calculations performed to evaluate the half life of the ground state of ^{212}Po, with two neutrons and two protons outside the ^{208}Pb core, including a large number of neutron-neutron and proton-proton configurations were still in disagreement with experimental data by about one order of magnitude [30, 31]. It was realized that this disagreement was due to the lack of any neutron-proton interaction. That is, that calculation included the neutron-neutron and proton-proton clusterizations but not the neutron-proton one. This was done in ref. [34], where the wave function of ^{212}Po(gs) was assumed to be

$$|^{212}\mathrm{Po(gs)}\rangle = A|^{210}\mathrm{Pb(gs)} \otimes {}^{210}\mathrm{Po(gs)}\rangle + B|^{210}\mathrm{Bi}(0_1^+) \otimes {}^{210}\mathrm{Bi}(0_1^+)\rangle \tag{5}$$

where A and B are constants to be determined (for details see ref. [34]). One sees in this equation that the first term corresponds to the clustering of the neutrons through the isovector pairing state ^{210}Pb(gs), with $T = 1$, $T_z = 1$ and the protons through the isovector pairing state ^{210}Po(gs), with $T = 1, T_z = -1$ while the second term corresponds to the neutron-proton clustering through the isovector pairing state $^{210}\mathrm{Bi}(0_1^+)$, with $T = 1, T_z = 0$. But this last state was not measured at that time (and it is not at present either). Since in $^{210}\mathrm{Bi}(0_1^+)$ both neutrons and protons move in the shells above $N = 126$, which implies that the proton moves in an excited major shell, it was assumed that this $T_z = 0$ pairing state lies at 5 MeV, which is about the energy difference between two major shells in the lead region. Using appropriate values of A and B one found a very strong clustering of the four nucleons that constitute the alpha-particle, and a ^{212}Po(gs) half life which was in perfect agreement with experiment. This showed the importance of the neutron-proton clustering, but unfortunately the values of A and B thus used were unrealistic.

The feature that has to be underlined for the purpose of this paper is that the states ^{210}Pb(gs) and $^{210}\mathrm{Bi}(0_1^+)$ are isoanalogous, and that the third component of these three $T = 1$ states, with $T = 1$, $T_z = -1$ should be a collective pairing state lying at about 10 MeV. This is the state $^{210}\mathrm{Po}(0^+_{GPV})$. Thus in the lead isotope the equivalent of $^{210}\mathrm{Bi}(0_1^+)$ is ^{210}Pb(gs). But there should also exist the state $^{210}\mathrm{Pb}(0^+_{GPV})$. Unaware of the work of ref. [11], in ref. [35] one evaluated again $^{210}\mathrm{Pb}(0^+_{GPV})$ finding a large neutron-neutron clustering and a large two-neutron cross section leading to the GPV.

3 Status of experimental searches for the GPV

3.1 The population of the GPV in two-nucleon transfer reactions

As discussed in the previous Section, an important consideration in the observability of the GPV is the coherence in the mixed wave functions. This is expected to enhance the observed cross sections as the different amplitudes for the two-particle transfer operator have the same sign and add coherently [36]. As a measure of the collectivity, we then look at the transfer operator, realizing that a realistic estimate should take into account the kinematic features of the two nucleon transfer cross sections to 0^+ states, by considering a DWBA calculation. The 2-nucleon transfer operator plays a similar role to the $B(E\lambda)$ for surface modes.

Given a set of single particle orbits $|n\ell j\rangle \equiv |j\rangle$, the wave function of the GPV state can be written:

$$|GPV\rangle = \sum_j \alpha_j |j^2\rangle.$$

The matrix element for the transfer of a pair of $L = 0$ neutrons to the GPV in nucleus $|A_0 + 2\rangle$ from the ground state of $|A_0\rangle$ is

$$\langle GPV|T|A_0\rangle = \sum_j \alpha_j \langle j^2|T|0\rangle,$$

and the cross section

$$\sigma(GPV) \propto \langle GPV|T|A_0\rangle^2 = \left(\sum_j \alpha_j\right)^2 \sigma_{sp},$$

with the further assumption that the single particle matrix elements are all approximately equal, $\langle j^2|T|0\rangle^2 \approx \sigma_{sp}$. As we will discuss later, this simplification is not always realistic.

The limiting case of Ω degenerate levels provides an estimate of the maximum enhancement (collectivity). Here we have $\alpha_j \approx \frac{1}{\sqrt{\Omega}}$ and thus

$$EF = \frac{\sigma(GPV)}{\sigma_{sp}} \sim \Omega, \tag{6}$$

which in the harmonic oscillator should scale with mass number as $\sim A^{2/3}$. A realistic example of the enhancement in the population of collective pairing modes is illustrated in fig. 2, comparing the pairing strength $\langle GPV|T|A_0\rangle$ for the addition modes in ^{208}Pb calculated in the Tamm-Dancoff (TDA) and RPA approximations with the unperturbed results.

3.2 Search for GPV through (p, t) reactions

The simple estimate in eq. (6) shows that the collectivity of the GPV increases with the mass of the nucleus. Therefore, the pair strength is expected to be maximum for the heaviest nuclei, such as Sn and Pb isotopes, where numerous nucleons may contribute coherently. Two candidate regions of the nuclear chart have been envisaged: in Pb (closed-shell, normal nuclei) and the Sn's (mid-shell, superfluid nuclei). In these nuclei, the GPV is supposed to be typically located around 12 MeV and 14 MeV, respectively. So far, the GPV in those nuclei has not been found, although a great experimental effort was devoted to it using (p, t) and (t, p) reactions in various conditions.

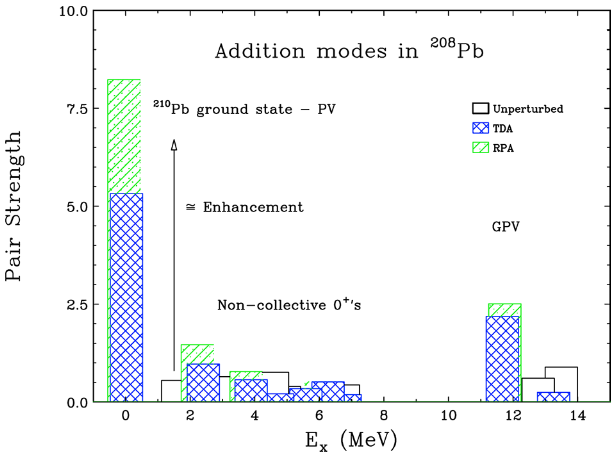

Fig. 2. (Color online) TDA and RPA results for the pair strength for the addition of two neutrons on the gs of ^{208}Pb. The enhancement (with respect to the unperturbed results) in the population of the PV and the GPV is clearly seen.

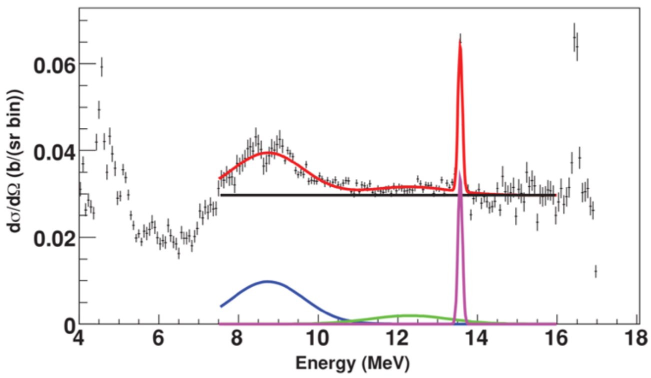

Fig. 3. (Color online) Excitation energy spectrum of ^{118}Sn for the 0 degree measurement at $Ep = 50$ MeV. The individual fits for a linear background (black), deep-hole states (blue), a possible GPV around 12 MeV (green), and oxygen contaminant (magenta) are shown together with the total fitting function (red). The bin width is 67 keV/bin. From ref. [13].

In the 60s and 70s, the searches for the GPV focused on (p, t) reactions at high energy for both Pb and Sn isotopes. However they remained unsuccessful. There could be several reasons as mentioned in ref. [13]:

- The L matching conditions are of great importance. The proton incident energy should be high enough to excite a 14 MeV mode but not too high in order not to hinder the $L = 0$ transfer. The smaller the proton energy the larger the cross section for $L = 0$ modes.
- The use of a spectrometer is decisive in order to precisely measure the triton in the exit channel. The only reported search for the GPV with $Ep \approx 50$ MeV used Si detectors, and was plagued by a strong background [12].
- As the $L = 0$ cross sections are known to exponentially increase when approaching 0 degree, the measurement has to be performed at small angles and is even better if it includes 0 degree.

There was a revival of the experimental GPV searches in the 2000s with several experiments aiming at improving the three experimental conditions mentioned above. All used a spectrometer for the triton measurement to improve the measurement at 0 degree. Several attempts with different proton energies were performed. The first attempt used a 60 MeV proton beam produced at the iThemba LABS facility in South Africa impinging on ^{208}Pb and ^{120}Sn targets [13]. The tritons were measured at 7 degrees with the $K = 600$ QDD magnetic spectrometer. The strong deuteron background was removed thanks to their different optical characteristics. No evidence for the GPV was found in the region of interest in neither of both targets.

The measurement was repeated with 50 MeV and a 60 MeV proton beams and the $K = 600$ QDD magnetic spectrometer in zero degree mode to combine the best experimental conditions to probe the GPV. In this case the beam stopper, placed midway between the two dipole magnets of the spectrometer, produced a strong proton background with a rate ~ 500 times higher than that of the tritons of interest. This background consisted of protons scattering off the beam stop with the combinations of angles and magnetic rigidities so that their trajectories reached the focal plane detectors. The time of flight between the SSC radio-frequency (RF) signal and the scintillator (from the spectrometer focal plane detection) trigger allowed for the triton identification and removed most of the background. The excitation energy spectrum obtained for ^{118}Sn is shown in fig. 3. The deep holes contribution between 8 and 10 MeV is stronger in the 0 degree spectrum than at 7 degrees indicating a possible low L composition of this region of the spectrum. Assuming a linear dependence, obtained by averaging the background between 14 and 16 MeV, a fit of the different components assuming a width between 600 keV and 1 MeV for the GPV was performed. It leads to a higher limit on the cross-section for populating the GPV between 0.13 and 0.19 mb over the angular acceptance of the spectrometer (± 2 degrees).

The last attempt with the (p, t) reaction was performed at LNS Catania with a proton beam produced by the cyclotron accelerator at $Ep = 35$ MeV impinging on a ^{120}Sn target [37]. The lower proton energy was supposed to enhance the $L = 0$ cross-sections and favor the population of the GPV. The measurement was performed with the MAGNEX large acceptance spectrometer. Tritons with energies between 12 and 18 MeV are expected for a GPV between 10 and 16 MeV. The MAGNEX energy acceptance is $\pm 25\%$, which allows to cover a range of about 7 MeV in the expected GPV energy region. The excitation energy function obtained for ^{118}Sn is shown in fig. 4 for the six magnetic settings of the spectrometer. The tritons were identified from their energy loss as a function of their position in the focal plane so that a very small background contribution remains. The spectrum zoomed in the region of interest for the GPV shows a small bump over the background in the same energy region as the previous measurements at 50 and 60 MeV. The width was fitted to 1.5 ± 0.4 MeV. No clear evidence for a GPV mode has been found from the searches through (p, t) reactions.

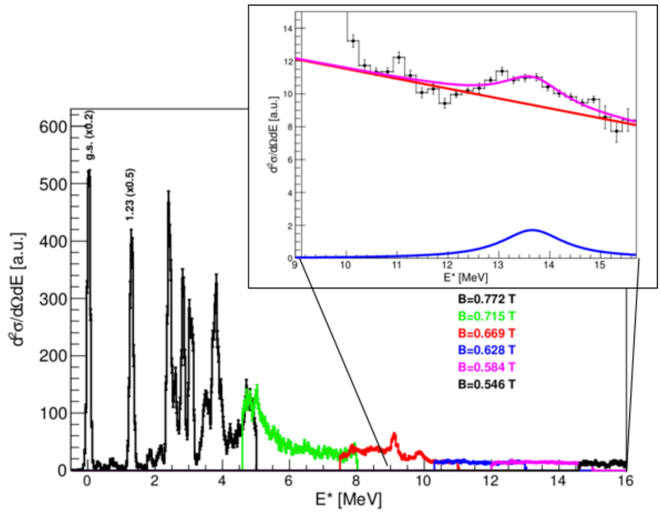

Fig. 4. (Color online) Excitation energy spectrum of ^{118}Sn for the six spectrometer settings at $Ep = 35\,\mathrm{MeV}$. A zoom on the GPV region is shown in the insert where a Lorentzian fit of the GPV was performed (blue). From ref. [37].

Improved experiments with (t, p) transfer reactions should be revisited to rule out any difference between two-neutron stripping and two-neutron pick-up reactions.

4 Q-value effects on the reaction cross sections

As it is well known from the theory of 2-nucleon transfer reactions, there is an optimum Q-window for the transfer to occur [36, 38]. The two-particle form factor, entering in the cross-section calculations, includes the overlap between the distorted scattering waves in the entrance and exit channels. If these waves are very different, then the overlap is small, and the cross section itself will be small. A measure of the differences between those scattering waves is the reaction Q-value. A large Q-value means small overlap. This translates into an exponential quenching of the cross-section outside the optimum Q-value window,

$$\sigma \sim \exp\left[-\frac{(Q - Q_{\mathrm{opt}})^2}{2\hbar^2 \kappa \ddot{r}_0}\right] \qquad (7)$$

where κ is the slope of the two-particle transfer effective form factor and $\ddot{r}_0$ the acceleration at the distance of closest approach r_0.

In what follows we focus our attention mostly on the role of the Q-value matching represented by the numerator of the exponent in eq. (7). There is, however, a very important aspect of these considerations that is associated to the width of the gaussian distribution related, in turn, to an effective nuclear collision time τ. There are several ways to estimate this quantity and its connection with the bombarding energy. For example, in ref. [38] the authors give the following expressions for the collision time and the acceleration at the barrier

$$\tau \approx \sqrt{a/\ddot{r}_o},$$
$$\ddot{r}_o \approx \frac{(2E - E_b)}{m r_o},$$

where a is the difuseness parameter and E and E_b are the bombarding and barrier energies, respectively.

As the bombarding energy increases, the effective collision time tends to decrease. This has two important consequences: The width of the distribution gets larger and larger and eventually it becomes irrelevant once it encompasses the value of Q_{opt}. Also, the increasingly shorter interaction times tend to reduce the importance of multistep processes (multiple-pair transfer) that, in principle, could also be quite relevant in connection with the transfer of correlated pairs. For this reason we only consider direct single-pair transitions.

Thus, a plausible explanation as to why the GVP has not been seen experimentally might rely on the fact that both (p, t) and (t, p) reactions are well matched for gs to gs transitions, but the large excitation energy of the GPV hinders the cross-section more than it is enhanced by the coherence in the wavefunction. References [39–41] have studied in detail the problem of exciting high-energy collective pairing modes in two-neutron transfer reactions. Relying on the analogy with the surface modes, they used a collective form factor [42]

$$\left(\frac{\beta_{pair}}{3A}\right) R_0 \frac{\partial U(r)}{\partial r}, \qquad (8)$$

with β_{pair} the deformation parameter of the pairing field, as input to the DWBA calculations. The results confirmed that, using conventional reactions with standard beams, one is faced with a large energy mismatch that favors the transition to the ground state over the population of the high-lying states. Instead, the Q-values in a stripping reaction involving weakly bound nuclei are much closer to the optimum for the transition to excited states in the 10–15 MeV range.

Figure 5 shows a survey map for possible projectiles (^{A}X), for which the cross-sections to populate the GPV in ^{210}Pb are anticipated to be larger than that to the gs. An inspection of the figure suggests the use of ^{6}He and ^{11}Li beams. Transfer strengths and cross-section are compared to the case of ^{18}O induced reactions in fig. 6. The effect on the cross-section due to the Q-value mismatch is clearly seen.

4.1 Search for GPV through (^{6}He,^{4} He) reactions

Following from the discussions above, the $^{208}\mathrm{Pb}(^6\mathrm{He}, \alpha)$ reaction has been investigated at GANIL [43] with the ^{6}He beam produced by the Spiral1 facility at $20\,\mathrm{MeV}/A$ with an intensity of 10^7 pps. The detection system was composed of an annular Silicon detector. The background due to the various channels of two-neutron emission from ^{6}He into $^4\mathrm{He} + 2\mathrm{n}$ and also to the channeling in the detector of the elastically scattered ^{6}He beam was large and

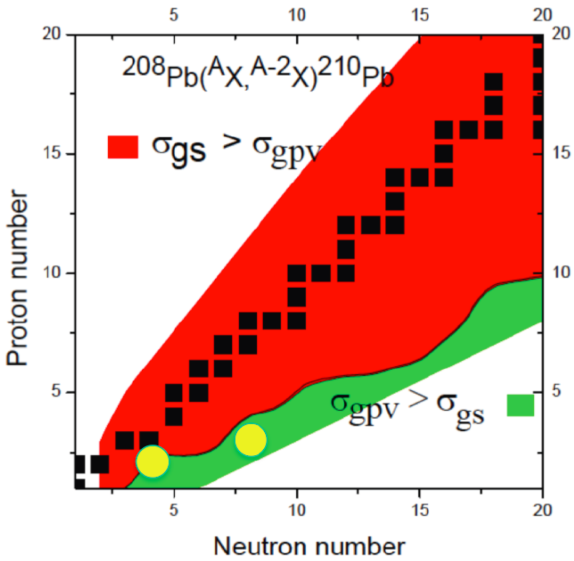

Fig. 5. (Color online) Survey map for the ^{208}Pb(^{A}X,$^{A-2}$X)^{210}Pb reaction indicating the cases for which the transfer cross-section to the GPV is larger than to the ground state (green). From ref. [41]. ^{6}He and ^{11}Li beams are indicated with yellow circles.

no indication of the GPV was found in this experiment. The results of ref. [44] show a similar situation. More recently, the reaction ^{116}Sn(^{6}He, α) at 8 MeV/A was studied at TRIUMF [45] with the IRIS Array [46]. The analysis of these data is still in progress but due to the breakup background is too early to make any conclusions.

5 Open questions

5.1 The 2n-transfer form factor and cross-sections

As discussed in ref. [47], the two-nucleon transfer cross sections to 0^+ states depend not only on the coherence of the wave functions but also on the specific amplitudes for transfer of angular momentum zero-coupled pairs for different single-particle states entering in the form factor. Basically, this reflects the probability for finding a $^1S_0$ 2n pair in the configurations $|(n\ell j)^2, L=0\rangle$ [48]. These amplitudes depend strongly on the orbital angular momentum ℓ, and the transfer probability could drop by order(s) of magnitude for each increase $\Delta\ell = 2$. Hence the bare cross section at the first maximum of the angular distributions for, say, two nucleons in an $i_{13/2}$ orbit, will be about 4 orders of magnitude less than that for the transfer of two $s_{1/2}$ particles. This effect is likely to be more important in the final cross sections than the detailed collectivity of the final states. The selectivity of different two-particle transfer reactions, such as (t, p), (^{18}O,^{16} O), and (^{14}C,^{12} C), with respect to detailed microscopic configurations in initial and final target states has recently been investigated in ref. [49].

5.2 Weak binding and continuum effects

The influence of the continuum on the properties of the giant pairing resonances was also motivated by alpha decay studies. As stated in ref. [50], it is necessary to include the continuum in *e.g.* the formation of the alpha-particle in alpha decay and in the building up of resonant states lying high in nuclear spectra. In ref. [50] a representation was used consisting of bound states, resonances and the proper continuum (composed by scattering waves) in the complex energy plane. This is the Berggren representation [51], in which the scalar product between two vectors, *i.e.* the metric, is the product of one vector times the other (instead of the complex conjugate of the other). But this affects only the radial part of the wave functions. The angular and spin parts are treated as usual. Since the radial parts of the wave functions can be chosen to be real quantities (*e.g.* harmonic oscillator functions for bound states, sine and cosine functions for scattering states) the Berggren scalar product coincides with the Hilbert one on the real energy axis. Therefore the space spanned by the Berggren representation (the Berggren space) can be considered a generalization of the Hilbert space. It has been shown that this representation is indeed a representation, that is to say, it can describe any process in the complex energy plane [52].

Within the Berggren representation the energies may be complex. Gamow showed that in a time independent context a resonance can be understood as having complex energy [27]. The real part of this energy corresponds to the energy of the resonance and the imaginary part is, in absolute value, half the width. The resonances entering in the Berggren representation are Gamow resonances. Due to the metric of the Berggren space, not only energies but also transition probabilities related to the evaluated states can be complex. A many-particle state lying on the complex energy plane may be considered a resonance, *i.e.* a measurable state appearing in the continuum part of the spectrum, if the wave function is localized within the nuclear system. This usually happens if the imaginary part of the energy (*i.e.* the width) is small [53]. Otherwise the state is just a part of the continuum background. This property will be important in the analysis of the giant pairing vibration.

By using the Berggren representation the shell model was extended to the complex energy plane given rise to the complex shell model and the Gamow shell model. A review on this can be found in [54].

The Berggren representation was used to analyze particle-hole resonances within a RPA formalism [55]. It was thus found that in ^{208}Pb the escaping widths of the giant resonances, which lie well above the neutron escape threshold, are small because the particle moves on bound shells or narrow Gamow resonances, while the hole states are all bound. But from the viewpoint of this paper the important outcome of calculations in the complex energy plane was the study of giant pairing vibrations performed in ref. [56]. Before this, one used bound (*e.g.* harmonic oscillator) representations, which did no consider the decaying nature of the resonances. Instead, it was found that

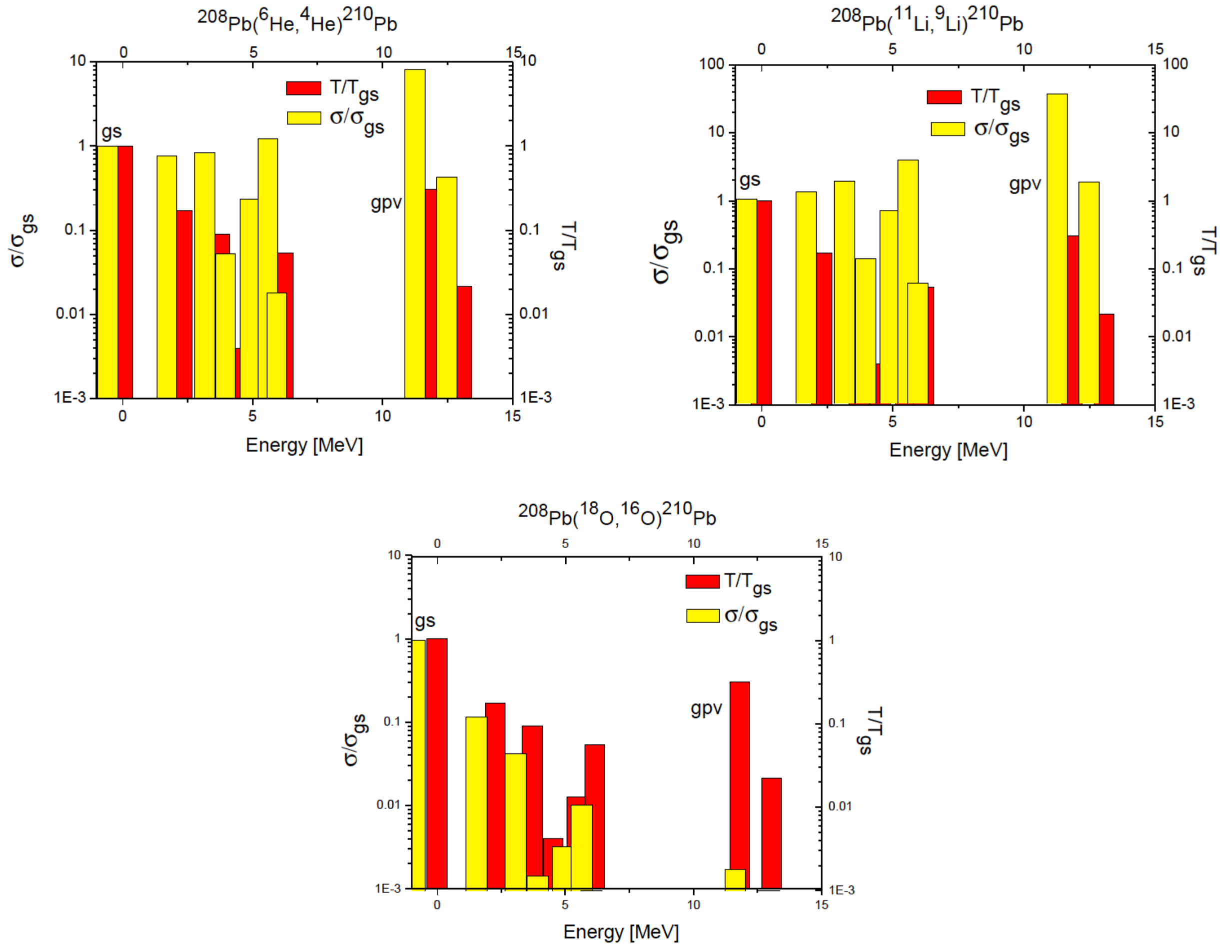

Fig. 6. (Color online) Pair addition strength distribution, normalized to the ground state, to all 0^+ states in ^{210}Pb (red bars) compared to the normalized pair transfer cross section (yellow bars) in the case of ^{6}He (upper left), ^{11}Li (upper right) and ^{18}O (lower frame). From ref. [41].

within the Berggren representation the two-neutron GPV in ^{210}Pb is very wide and is not a physically relevant state but a part of the continuum background. The proton-neutron state ^{210}Bi(0^+_1) was found to be a meaningful state only if it is not a resonance but a bound state lying below 7 MeV of excitation. As this energy approaches the continuum threshold, then the collectivity of the state gradually disappears. Above the threshold not only does the collectivity vanish altogether but also the resulting resonance is very wide. Instead, the state ^{210}Po(GPV) was found to be a meaningful resonances [56].

In ref. [47] the formalism developed by von Brentano, Weidenmuller and collaborators for mixing of bound and unbound levels [57, 58] was applied to the study of simple toy-model and realistic calculations to asses the effects of weak binding and continuum coupling on the non-observation of the GPV. It was found that the mixing in the presence of weak binding was a minor contributor to the weak population. Rather, the main reason was attributed to the melting of the GPV peak due to the width it acquires from the low orbital angular momentum single particle states playing a dominant role in two-nucleon transfer amplitudes. This effect, in addition to the Q-value mismatch, may account for the elusive nature of this mode in (t, p) and (p, t) reactions.

It is important to mention the work in ref. [59], where the nuclear response to the pair transfer in $^{18-22}$O was investigated in the framework of the continuum quasiparticle random phase approximation (cQRPA), which allows for a consistent determination of the residual interaction and an exact treatment of the continuum coupling. It was found that a significant part of the transfer amplitude corresponds to a GPV built entirely upon continuum quasiparticle states, and pointed to the possibility of finding this mode in light systems.

In summary, the continuum part of the nuclear spectrum appears to be more important for the pairing vibration mode than for the surface vibration one. This is because the two particles in the pairing mode at high energy escape easily into the continuum. Instead the only particle lying in the continuum in the particle-hole mode moves largely in narrow resonances, thus been trapped within the nucleus during a time long enough for the resonance to be seen. Citing Bortignon and Broglia [1], "... the fact that the GPV have likely been serendipitously observed in these light nuclei when it has failed to show up in more propitious nuclei like Pb, provides unexpected and fundamental insight into the relation existing between basic mechanisms —Landau, doorway, compound damping— through which giant resonances acquire a finite lifetime, let alone the radical difference regarding these phenomena displayed by correlated (ph) and (pp) modes."

6 Future studies and conclusions

In spite of several experimental efforts, the elusive nature of the GPV in heavy nuclei remains as an intriguing puzzle. Severe Q-value quenching of the cross-sections for $(\mathrm{t,p})$ and $(\mathrm{p,t})$ reactions has suggested the use of weakly bound projectiles, such as ^{6}He and ^{11}Li, to overcome those limitations. Unfortunately the large 2n breakup probability conspires to mask the GPV signal with a large background. Nevertheless, further exclusive measurements should be carried-out in order to either rule out the population of the GPV or establish a firm limit that could be compared to theory. The availability of state-of-the art instrumentation, tritium targets, and possibly tritium beams also suggests that the $(\mathrm{t,p})$ reaction should be revisited.

As discussed before, another independent way of probing the GPV is by exploiting the $T = 1$ isobaric character of the states ^{210}Po(GPV), and ^{210}Pb(gs). The state ^{210}Pb(gs) (a typical isovector pairing vibration) is mainly built by the nn pair moving in the major shell above the magic number 126, *i.e.* the $N = 7$ major shell and, correspondingly, the state ^{210}Po(GPV) is built by the pp pair outside the $Z = 82$, $N = 126$ core also moving in the high lying $N = 7$ proton major shell, *i.e.* a proton GPV. The $T_z = 1$ member of this multiplet is a 0^+_1 state in ^{210}Bi built by a neutron-proton pair moving in the $N = 7$ major shell, expected at $\lessapprox 7\,\mathrm{MeV}$. All these three $T = 1$ states are expected to have similar pairing collective properties. Since from the point of view of the continuum effects both the proton-neutron state in ^{210}Bi and the proton-proton GPV in ^{210}Po are anticipated to be meaningful states, a search for these resonances using for example the $(^3\mathrm{He,p})$ and $(^3\mathrm{He,n})$ reactions should be pursued. One could even speculate on using the (α,d) reaction combining particle and gamma-spectroscopy to tag on the ^{2}H $2.2\,\mathrm{MeV}$ gamma to select the transfer of an isovector np pair.

Furthermore, the proton-neutron state in ^{210}Bi could be populated by means of charge-exchange reactions like $(\mathrm{p,n})$ or $(^3\mathrm{He,t})$ on a ^{210}Pb (radioactive) target or in inverse kinematics using a ^{210}Pb (radioactive) beam.

Finally, given the fact that recent theoretical efforts have pointed out the important effects of weak-binding and continuum coupling, realistic estimates of the total damping width (Escape width, Landau and doorway damping [60]) of the GPV in Sn and Pd isotopes will be extremely valuable.

To conclude, after more than fifty years since the analogy between atomic nuclei and the superconducting state in metals was pointed out in ref. [2], the role of pairing correlations in nuclear structure continues to be a topic of much interest and excitement in the field of nuclear physics [61]. The discovery of the GPV in light nuclei opens up a unique opportunity to advance our knowledge of high-lying pairing resonances but, at the same time, the non-observation of these modes in heavy nuclei remains as an open question that needs to be further addressed both by theory and experiment.

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