

The enhancement of v_4 in nuclear collisions at the highest densities signals a first-order phase transition

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Received: 14 September 2018

Published online: 6 November 2018

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Communicated by G. Torrieri

Abstract. The beam energy dependence of v_4 (the quadrupole moment of the transverse radial flow) is sensitive to the nuclear equation of state (EoS) in mid-central Au + Au collisions at the energy range of $3 < \sqrt{s_{NN}} < 30$ GeV, which is investigated within the hadronic transport model JAM. Different equations of state, namely, a free hadron gas, a first-order phase transition and a crossover are compared. An enhancement of v_4 at $\sqrt{s_{NN}} \approx 6$ GeV is predicted for an EoS with a first-order phase transition. This enhanced v_4 flow is driven by both the enhancement of v_2 as well as the positive contribution to v_4 from the squeeze-out of spectator particles which turn into participants due to the admixture of the strong collective flow in the shocked, compressed nuclear matter.

The azimuthal distribution of particles emitted, in high energy heavy-ion collisions, contains important information about the bulk properties of strongly interacting matter [1–8]. The azimuthal momentum distribution of particles can be expressed as a Fourier series [9–11],

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left(1 + \sum_{n=1}^{\infty} 2v_n \cos(n[\phi - \Phi_n]) \right), \quad (1)$$

where ϕ is the azimuthal angle with respect to the event plane Φ_n , which is estimated experimentally in various ways. The harmonic flow coefficients

$$v_n = \langle \cos(n[\phi - \Phi_n]) \rangle \quad (2)$$

measure the strength of the system response to the initial coordinate space anisotropy and fluctuations in the collision zone.

Anisotropic flow is generated by the participant pressure [1,2] during the early stages of the collisions, therefore, it is considered a sensitive messenger of the equation of state (EoS) [1–8]. A large elliptic flow has been observed in RHIC and LHC experiments, and is in good agreement with hydrodynamical simulations [12–18]. Hydrodynamical predictions revealed that the study of v_4 contains important information about the collision dynamics [19–23].

Recently, higher order coefficients v_n have been measured at RHIC and LHC [24–26].

To investigate the phase structure of QCD, both the beam energy-, centrality-, and system size- dependence are studied to access the different regions of T - μ_B phase diagram [27]. In particular, the search for a first-order phase transition and the critical end point at high baryon density is a challenging goal of high energy heavy-ion collisions [28–31].

At lower beam energies ($\sqrt{s_{NN}} < 10$ GeV), the strength of the elliptic flow is determined by the interplay between out-of-plane (squeeze-out) and in-plane emission [4,32]. In a previous work we predicted a first-order phase transition [33,34] will cause an enhancement of the elliptic flow v_2 as function of the beam energy by the suppression of the squeeze-out due to the softening of EoS [35].

Does this enhancement of v_2 suggest that v_4 is also enhanced in the vicinity of a first-order phase transition? This letter presents the beam energy dependence of v_4 as calculated with the microscopic transport model JAM [36], using the modified scattering style method [37, 38] and confirms our conjecture. In JAM, particle production is modeled by the excitations of hadronic resonances and strings, and their decays in a similar way as in the RQMD and UrQMD models [39–41]. Secondary products are allowed to scatter again, which generates collective

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effects within our approach. In the standard cascade version of the model, one usually chooses the azimuthal scattering angle randomly for any two-body scattering. (The effects of a preserved two-body reaction plane have been studied in ref. [42]). Thus, cascade simulations yield the free-hadronic gas EoS in equilibrium, as then two-body scatterings, on average, do not generate additional pressure. In our approach, the pressure of the system is controlled by changing the scattering style in the two-body collision terms. It is well known that an attractive orbit reduces the pressure, while repulsive orbit enhances the pressure [43,44]. Thus, the pressure is controlled by appropriately choosing the azimuthal angle in the two-body scatterings. Specifically, the pressure difference from the free streaming pressure ΔP is obtained by the following constraints [45]:

$$\Delta P = \frac{\rho}{3(\delta\tau_i + \delta\tau_j)} (\mathbf{p}'_i - \mathbf{p}_i) \cdot (\mathbf{r}_i - \mathbf{r}_j), \quad (3)$$

where ρ is the local particle density and $\delta\tau_i$ is the proper time interval of the i -th particle between successive collisions, $(\mathbf{p}'_i - \mathbf{p}_i)$ is the momentum change and \mathbf{r}_i is the coordinate of the i -th particle. Momenta and coordinates in eq. (3) refer to the values in the c.m. frame of the respective binary collisions. We had demonstrated that a given EoS can be simulated by choosing the azimuthal angle according to the constraint in eq. (3) in the two-body scattering process [38]. We note that the total cross section and scattering angle of the two-body scattering are not changed by this method; the only modification is the choice of the azimuthal angle.

In this work, we use the same EoS as developed and used in ref. [38] to simulate both the conjectured first-order phase transition (1OPT) and also the alternative crossover transition (X-over). The EoS with a first-order phase transition (EoS-Q) [19,46] is constructed by matching a free, massless quark-gluon phase with the bag constant $B^{1/4} = 220$ MeV with the hadron gas EoS. In the hadronic gas phase, hadron resonances with mass up to 2 GeV are included, with a repulsive, baryon density ρ_B dependent mean field potential $V(\rho_B) = \frac{1}{2}K\rho_B^2$, with $K = 0.45$ GeV fm³. For the crossover EoS, we use the chiral model EoS from ref. [47–49], where the EoS at vanishing and at finite baryon density is consistent with a smooth crossover transition, *i.e.* this EoS is consistent with recent lattice QCD results.

For all presented results we compute v_4 with respect to the reaction plane $\Phi_n = \Phi_{RP}$, where Φ_{RP} is the reaction plane angle of the collision. As usual, the reaction plane anisotropies in the even-order Fourier coefficients are in good agreement with the anisotropies taken with respect to the event plane, while odd-order Fourier coefficients are generated by event-by-event fluctuations.

Figure 1 shows the beam energy dependence of v_4 , for charged particles at mid-rapidity $|\eta| < 1.0$ in mid-central Au + Au collisions from the JAM model with the cascade mode, JAM with the first-order EoS (JAM/1OPT), and a crossover EoS (JAM/X-over). The effects of our three different EoS on the v_4 at higher beam energy

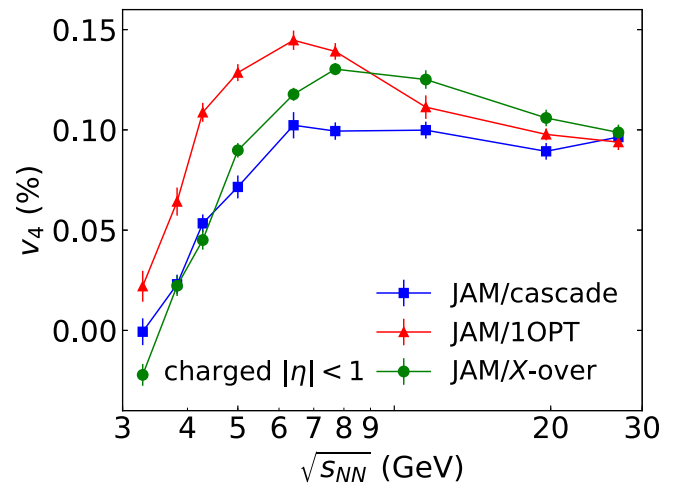


Fig. 1. Beam energy dependence of the v_4 for charged hadrons at $|\eta| < 1.0$ in mid-central Au+Au collisions ($4.6 \leq b \leq 9.4$ fm) from the JAM cascade mode (squares), JAM with first-order EoS (triangles), and crossover EoS (circles).

$\sqrt{s_{NN}} > 10$ GeV are quite similar, in contrast to the high baryon density, *i.e.* at $\sqrt{s_{NN}} < 10$ GeV, where the effect of the EoS is very strong. The cascade mode results do not show any clear maximum or bump in the beam dependence of the v_4 . The calculations using an EoS with a first-order phase transition and those with a crossover transition exhibit an enhancement of v_4 relative to the cascade result at 5 GeV, a factor of two for the 1OPT case, and an inversion of sign of v_4 at 3 GeV for the X-over case. JAM/1OPT shows a strong bump around the beam energy of $\sqrt{s_{NN}} \approx 6$ GeV. A similar enhancement was observed in the case of v_2 for the 1OPT mode [34].

To understand the collision dynamics which enhances both the v_2 and the v_4 , we consider the effects of spectator interactions: out-of-plane emission (squeeze-out) is mainly driven by the pressure release perpendicular to the spectator plane, which yields the negative $v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$ at lower beam energies. In the beam energy range of $3 < \sqrt{s_{NN}} < 10$ GeV, the cancellation between the in-plane flow (p_x) and the out-of-plane flow (p_y) determines the final value of v_2 . Thus, if the spectator-matter interaction is neglected, the elliptic flow is strongly positive. To see the effects of spectator interactions on the flows quantitatively, we perform the calculations in which interactions with “spectator nucleons” are disabled, where “spectator nucleons” are defined as the nucleons which are not in the list of initial collisions; collisions of nucleons which are initially located outside the overlap region of the two colliding nuclei therefore are excluded in the calculations without spectator matter.

Figure 2 compares the calculations of flow with and without “spectator nucleons”. If the EoS with the first-order phase transition is employed, the effect of spectator shadowing is smaller than in the cascade mode, as the pressure is significantly smaller and, hence, the acceleration of the stopped matter is less for this softest equation

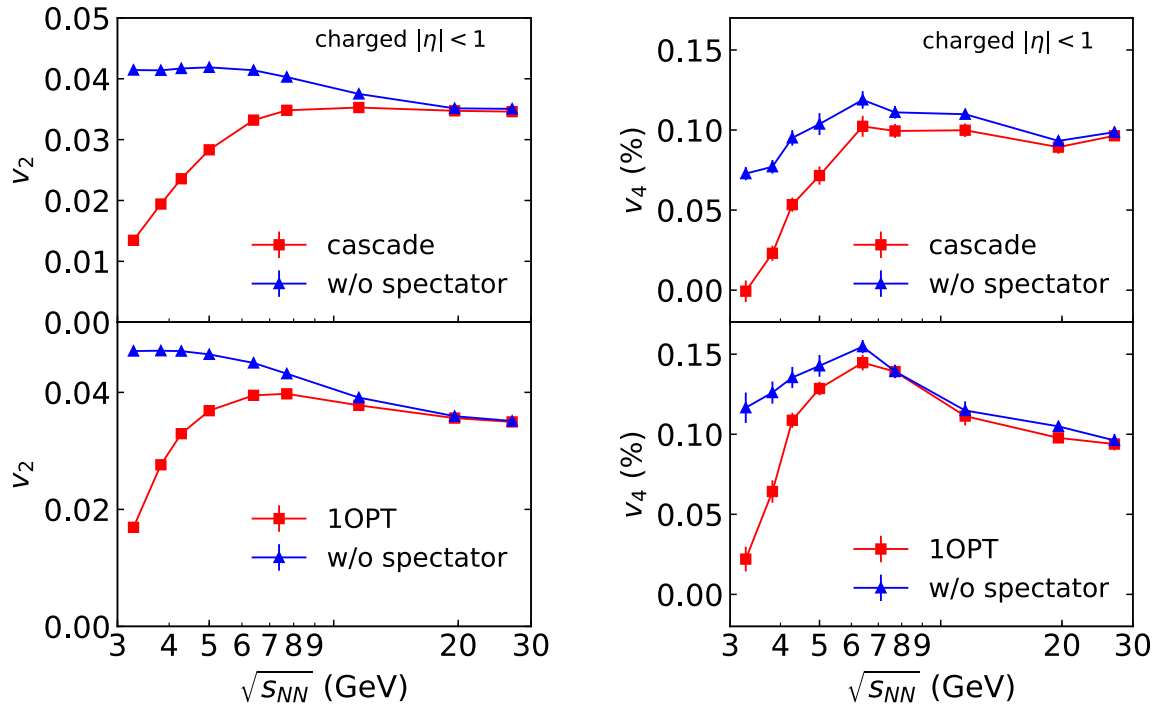


Fig. 2. Beam energy dependence of the v_2 - (left panel) and the v_4 (right panel) coefficients in mid-central Au + Au collisions, with- and without spectator interactions, are compared to the different EoS in JAM simulations.

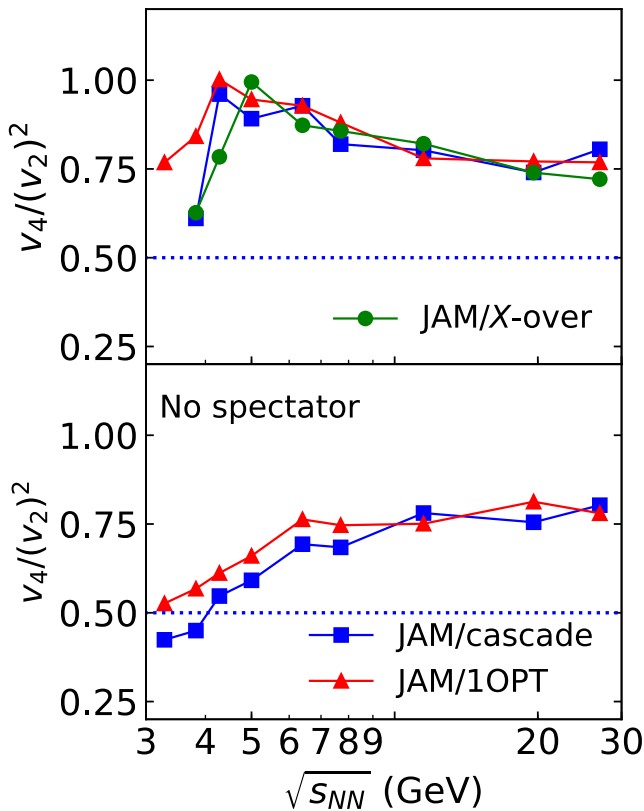


Fig. 3. Same as fig. 1, but for the $v_4/(v_2)^2$ ratio of charged particles with (upper panel) and without spectator matter interactions (lower panel).

of state —then the system remains in this low pressure region for a long time reached in the system. This is the origin of the enhancement of v_2 if there is a first-order phase transition.

In the following we will discuss the effects of the spectator matter on v_4 : the elliptic flow v_2 is positive in the case of stronger in-plane emission, see the left-hand side of fig. 2, while v_2 is negative for predominant out-of-plane emission at $\sqrt{s_{NN}} < 3$ GeV. On the other hand, v_4 is positive, and large for both, in-plane and out-of-plane emission. Thus, spectator shadowing will enhance the v_4 value. Thus, it is indeed seen in fig. 2 (right panel) if the spectator interactions are neglected, v_4 is not suppressed, up to $\sqrt{s_{NN}} = 6$ GeV. In the case of a first-order phase transition, v_4 does increase, both with and without spectator interactions. At the lower beam energies, v_4 decreases for both calculations, with and without spectator interactions, in contrast to v_2 , which increases at lower energies if the spectator interactions are neglected. Here, particle emission is not so strongly directed to the in-plane direction, which decreases the v_4 at lower beam energies $\sqrt{s_{NN}} < 5$ GeV. One should note that at even lower beam energies $\sqrt{s_{NN}} \leq 4$ GeV the effects of nuclear potentials need to be taken into account for quantitative predictions on the v_4 .

The harmonic v_4 is generated both by the intrinsic v_2 and by the forth-order moment of the collective flow [21–23]. Within ideal fluid dynamics (and without any fluctuations), the elliptic flow contribution to v_4 is simply given by $v_4 = 0.5(v_2)^2$ [21–23]. Hence, the ratio $v_4/(v_2)^2$ contains valuable information about the intrinsic collision dynamics. Experimental data show that v_4 is about dou-

ble the ideal hydro values, $v_4 \approx (v_2)^2$ at RHIC [24, 50–52]. Note that the PHSD results show a fourfold higher value, $v_4/(v_2)^2 \approx 2$ [53], for a wide range of beam energies in min-bias Au + Au collisions.

Figure 3 shows the beam energy dependence of the $v_4/(v_2)^2$ ratio, it stands close to 0.75 at 3 GeV, then rises, and flattens to a constant value of 0.75 at beam energies of $\sqrt{s_{NN}} > 10$ GeV, with a slight increase around 6 GeV. Calculations where spectator matter interactions are neglected yield smaller values $v_4/(v_2)^2 \sim 0.5$ at moderate energies, but also approach 0.75 at ~ 6 GeV, and above. This indicates that the v_4 is dominated by the v_2 component as without spectator shadowing there exists no squeeze-out effect. Actually, the beam energy dependence of v_2 exhibits a similar dependence as v_4 , in the simulations without spectator matter, as can be seen in fig. 2.

In summary, we have studied the beam energy dependence of the fourth harmonics v_4 for charged particles in mid-central Au + Au collisions at $3 < \sqrt{s_{NN}} < 30$ GeV. An enhancement of v_4 around beam energies of 6 GeV is predicted if and only if a first-order phase transition is present—hence, this can serve as a clean signal. The enhancement of v_2 is caused by the weaker squeeze-out effects exerted by the spectator matter, due to the soft EoS. An enhancement of v_4 comes from the enhancement of v_2 itself as well as from the positive contributions from the squeeze-out.

Predicted v_4 signal can be studied experimentally at future experiments such as RHIC-BESII [54], FAIR [55, 56] NICA [57], and J-PARC-HI [58, 59], which offer the best opportunities to explore the compressed baryonic matter, and reveal the phase structure of QCD.

YN thanks the team of the Frankfurt Institute of Advanced Studies where part of this work was done for their splendid hospitality. This work was supported in part by the Grants-in-Aid for Scientific Research from JSPS (JP17K05448). HS appreciates the generous endowment of the Judah M. Eisenberg Laureatus professorship. JS appreciates the support of the SAMSON AG and the C.W. Fueck-Stiftungs Prize 2018. Computational resources have been provided by the Center for Scientific Computing (CSC) at the J. W. Goethe-University, Frankfurt, and GSI, Darmstadt.

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References

1. H. Stoecker, J.A. Maruhn, W. Greiner, Phys. Rev. Lett. **44**, 725 (1980).
2. H. Stoecker, L.P. Csernai, G. Graebner, G. Buchwald, H. Kruse, R.Y. Cusson, J.A. Maruhn, W. Greiner, Phys. Rev. C **25**, 1873 (1982).
3. G. Buchwald, G. Graebner, J. Theis, J. Maruhn, W. Greiner, H. Stoecker, K.A. Frankel, M. Gyulassy, Phys. Rev. C **28**, 2349 (1983).
4. H. Stoecker, W. Greiner, Phys. Rep. **137**, 277 (1986).
5. C. Hartnack, J. Aichelin, H. Stoecker, W. Greiner, Phys. Lett. B **336**, 131 (1994).
6. J.-Y. Ollitrault, Phys. Rev. D **46**, 229 (1992).
7. P. Danielewicz, R. Lacey, W.G. Lynch, Science **298**, 1592 (2002).
8. H. Stoecker, Nucl. Phys. A **750**, 121 (2005).
9. S. Voloshin, Y. Zhang, Z. Phys. C **70**, 665 (1996).
10. A.M. Poskanzer, S.A. Voloshin, Phys. Rev. C **58**, 1671 (1998).
11. S.A. Voloshin, A.M. Poskanzer, R. Snellings, in *Relativistic Heavy Ion Physics* (Springer, 2010) pp. 293–333, arXiv:0809.2949.
12. U. Heinz, R. Snellings, Annu. Rev. Nucl. Part. Sci. **63**, 123 (2013).
13. C. Gale, S. Jeon, B. Schenke, Int. J. Mod. Phys. A **28**, 1340011 (2013).
14. P. Huovinen, Int. J. Mod. Phys. E **22**, 1330029 (2013).
15. T. Hirano, P. Huovinen, K. Murase, Y. Nara, Prog. Part. Nucl. Phys. **70**, 108 (2013).
16. S. Jeon, U. Heinz, Int. J. Mod. Phys. E **24**, 1530010 (2015).
17. A. Jaiswal, V. Roy, Adv. High Energy Phys. **2016**, 9623034 (2016).
18. P. Romatschke, U. Romatschke, arXiv:1712.05815 [nucl-th].
19. P.F. Kolb, J. Sollfrank, U.W. Heinz, Phys. Lett. B **459**, 667 (1999).
20. P.F. Kolb, Phys. Rev. C **68**, 031902 (2003).
21. N. Borghini, J.Y. Ollitrault, Phys. Lett. B **642**, 227 (2006).
22. C. Gombeaud, J.Y. Ollitrault, Phys. Rev. C **81**, 014901 (2010).
23. M. Luzum, C. Gombeaud, J.Y. Ollitrault, Phys. Rev. C **81**, 054910 (2010).
24. STAR Collaboration (J. Adams *et al.*), Phys. Rev. Lett. **92**, 062301 (2004) arXiv:nucl-ex/0310029.
25. ALICE Collaboration (S. Acharya *et al.*), arXiv:1805.04390 [nucl-ex].
26. ALICE Collaboration (S. Acharya *et al.*), JHEP **07**, 103 (2018) arXiv:1804.02944 [nucl-ex].
27. B. Friman, C. Hohne, J. Knoll, S. Leupold, J. Randrup, R. Rapp, P. Senger (Editors), *The CBM Physics Book*, Lect. Notes Phys., Vol. **814** (Springer, 2011).
28. M. Asakawa, K. Yazaki, Nucl. Phys. A **504**, 668 (1989).
29. D.H. Rischke, Prog. Part. Nucl. Phys. **52**, 197 (2004).
30. M.A. Stephanov, Prog. Theor. Phys. Suppl. **153**, 139 (2004) (Int. J. Mod. Phys. A **20**, 4387 (2005)).
31. K. Fukushima, C. Sasaki, Prog. Part. Nucl. Phys. **72**, 99 (2013).
32. H. Sorge, Phys. Rev. Lett. **78**, 2309 (1997).
33. J. Chen, X. Luo, F. Liu, Y. Nara, Chin. Phys. C **42**, 024001 (2018).
34. Y. Nara, H. Niemi, A. Ohnishi, J. Steinheimer, X. Luo, H. Stoecker, Eur. Phys. J. A **54**, 18 (2018).
35. C. Zhang, J. Chen, X. Luo, F. Liu, Y. Nara, Phys. Rev. C **97**, 064913 (2018).
36. Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, Phys. Rev. C **61**, 024901 (2000).
37. Y. Nara, H. Niemi, A. Ohnishi, H. Stoecker, Phys. Rev. C **94**, 034906 (2016) arXiv:1601.07692 [hep-ph].

38. Y. Nara, H. Niemi, J. Steinheimer, H. Stoecker, Phys. Lett. B **769**, 543 (2017).
39. H. Sorge, Phys. Rev. C **52**, 3291 (1995).
40. S.A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998).
41. M. Bleicher *et al.*, J. Phys. G **25**, 1859 (1999).
42. D.E. Kahana, D. Keane, Y. Pang, T. Schlagel, S. Wang, Phys. Rev. Lett. **74**, 4404 (1995) arXiv:nucl-th/9405017.
43. M. Gyulassy, K.A. Frankel, H. Stoecker, Phys. Lett. B **110**, 185 (1982).
44. P. Danielewicz, S. Pratt, Phys. Rev. C **53**, 249 (1996).
45. H. Sorge, Phys. Rev. Lett. **82**, 2048 (1999).
46. J. Sollfrank, P. Huovinen, M. Kataja, P.V. Ruuskanen, M. Prakash, R. Venugopalan, Phys. Rev. C **55**, 392 (1997).
47. J. Steinheimer, S. Schramm, H. Stoecker, J. Phys. G **38**, 035001 (2011).
48. J. Steinheimer, S. Schramm, H. Stoecker, Phys. Rev. C **84**, 045208 (2011).
49. P. Rau, J. Steinheimer, S. Schramm, H. Stoecker, Phys. Rev. C **85**, 025204 (2012).
50. PHENIX Collaboration (H. Masui), Nucl. Phys. A **774**, 511 (2006).
51. STAR Collaboration (B.I. Abelev *et al.*), Phys. Rev. C **75**, 054906 (2007).
52. PHENIX Collaboration (S. Huang), J. Phys. G **35**, 104105 (2008).
53. V.P. Konchakovski, E.L. Bratkovskaya, W. Cassing, V.D. Toneev, S.A. Voloshin, V. Voronyuk, Phys. Rev. C **85**, 044922 (2012).
54. G. Odyniec, EPJ Web of Conferences **95**, 03027 (2015).
55. CBM Collaboration (T. Ablyazimov *et al.*), Eur. Phys. J. A **53**, 60 (2017).
56. C. Sturm, B. Sharkov, H. Stoecker, Nucl. Phys. A **834**, 682c (2010).
57. V. Kekelidze, A. Kovalenko, R. Lednicky, V. Matveev, I. Meshkov, A. Sorin, G. Trubnikov, Nucl. Phys. A **956**, 846 (2016).
58. J-PARC Heavy-Ion Collaboration (H. Sako *et al.*), Nucl. Phys. A **931**, 1158 (2014).
59. J-PARC Heavy-Ion Collaboration (H. Sako *et al.*), Nucl. Phys. A **956**, 850 (2016).