Eur. Phys. J. A (2018) **54**: 152
DOI 10.1140/epja/i2018-12599-9

THE EUROPEAN PHYSICAL JOURNAL A

Letter to the Editor

On the eikonal approach to nuclear diffraction dissociation

Angela Bonaccorso[1,a] and David M. Brink[2]

[1] Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Largo Bruno Pontecorvo 3, 56124 Pisa, Italy
[2] Rudolf Peierls Centre of Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

Received: 25 July 2018 / Revised: 16 August 2018
Published online: 18 September 2018

Communicated by N. Alamanos

Abstract. The study of nuclear breakup of halo and weakly bound particles has been one of the key ingredients in the understanding of exotic nuclei during the last thirty years. One of the most used methods to analyse data, in particular absolute breakup cross sections, has been the eikonal approximation. Here we revise critically the formalisms used for calculating the diffraction dissociation part of nuclear breakup and show that there is a formula that can be applied to breakup on any target, while a most commonly used formula must be restricted to light targets as it contains also the effect of Coulomb breakup calculated to first order in the sudden approximation which is well known for not being accurate.

1 Introduction

The study of nuclear breakup of halo and weakly bound particles has been one of the key ingredients in the understanding of exotic nuclei during the last thirty years [1]. Here we discuss and compare formalisms used to calculate the nuclear elastic breakup. Following refs. [2–4] we consider a single-particle model for a halo nucleus and introduce the eikonal approximation to study its scattering on another target nucleus. The ground state is described by a wave function $\phi_0(\mathbf{r})$ which depends on the relative coordinate $\mathbf{r}$ between the nucleon and the core. After interacting with the target the eikonal wave function of the halo nucleus in its rest frame has the form

$$\Psi(\mathbf{r},\mathbf{R}) = S_n(\mathbf{b}_n)\, S(\mathbf{b}_c)\, \phi_0(\mathbf{r}), \tag{1}$$

where $\mathbf{R}$ and $\mathbf{r}$ are the coordinates of the center of mass of the projectile consisting of the core plus one neutron, and of the neutron with respect to the core, respectively, see fig. 1. The vectors

$$\mathbf{b}_n = \mathbf{R}_\perp + \beta_2\mathbf{r}_\perp \quad \text{and} \quad \mathbf{b}_c = \mathbf{R}_\perp - \beta_1\mathbf{r}_\perp \tag{2}$$

are the impact parameters of the neutron and the core with respect to the target nucleus. Thus $\beta_1 = m_n/m_p$, $\beta_2 = m_c/m_p = 1-\beta_1$, where m_n is the neutron mass, m_c is the mass of the projectile core and $m_p = m_n + m_c$ is the projectile mass. The two profile functions S_n and S_c are defined in terms of the corresponding potentials by

$$S(\mathbf{b}) = \exp\left(-\frac{i}{\hbar v}\int \mathrm{d}z V(\mathbf{b},z)\right), \tag{3}$$

where v is the beam velocity. The breakup amplitude generated from the eikonal wave function (1) has a direct contribution from the neutron-target optical potential V_{nT} and represented by neutron-target profile function S_n and a core recoil contribution from the core-target interaction V_{cT} represented by the profile function S_c [5,6]. The recoil contribution depends on the ratio β_1 of the neutron mass to the projectile mass and goes to zero in the limit $\beta_1 \to 0$. The potential V_{cT} includes the core-target Coulomb potential and the real and imaginary parts of the nuclear potential. The Coulomb part of V_{cT} is responsible for Coulomb breakup. Using the approximate form of the wave function (1) with (3) implies the "frozen halo" approximation; the neutron velocity relative to the core in the projectile and in the final state is slow compared with the incident velocity v.

The eikonal breakup amplitude is defined by [3]

$$A(\mathbf{K},\mathbf{k}) = \int \mathrm{d}^2\mathbf{R}_\perp\, e^{-i\mathbf{K}_\perp\cdot\mathbf{R}_\perp} \times \int \mathrm{d}^3\mathbf{r}\, \phi^*_{\mathbf{k}}(\mathbf{r})\, (S_c(\mathbf{b}_c)\, S_n(\mathbf{b}_n) - 1)\, \phi_0(\mathbf{r}). \tag{4}$$

The impact parameters $\mathbf{b}_n$ and $\mathbf{b}_c$ are defined in eq. (2). The quantities $(\mathbf{K},\mathbf{k})$ are the momenta conjugate to the coordinates $(\mathbf{R},\mathbf{r})$. They are related to the final momenta of the core, neutron and target by

$$\mathbf{k}_c = -\mathbf{k} + \beta_2\mathbf{K}, \qquad \mathbf{k}_n = \mathbf{k} + \beta_1\mathbf{K}, \qquad \mathbf{k}_T = -\mathbf{K}. \tag{5}$$

The wave function $\phi_{\mathbf{k}}(\mathbf{r})$ is the final continuum wave function of the neutron relative to the core. The complete dif-

[a] e-mail: bonac@df.unipi.it

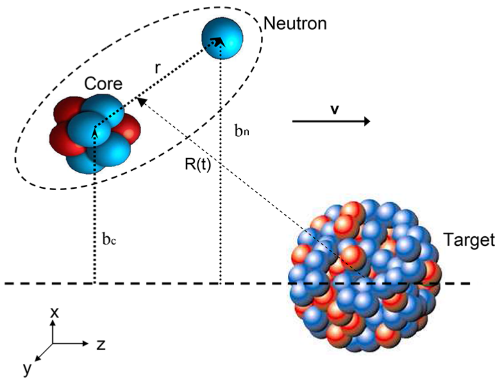

Fig. 1. Coordinate system.

ferential cross-section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{K}\,\mathrm{d}^3\mathbf{k}} = \frac{1}{(2\pi)^5}\left|A(\mathbf{K},\mathbf{k})\right|^2. \tag{6}$$

Equation (4) can also be written as

$$A(\mathbf{K},\mathbf{k}) = \int \mathrm{d}^2\mathbf{R}_\perp\, e^{-i\mathbf{K}_\perp\cdot\mathbf{R}_\perp} \times \int \mathrm{d}^3\mathbf{r}\,\phi^*_{\mathbf{k}}(\mathbf{r}) S_c(\mathbf{b}_c)\, S_n(\mathbf{b}_n)\,\phi_0(\mathbf{r}) \tag{7}$$

because of the orthogonality of $\phi_{\mathbf{k}}(\mathbf{r})$ and $\phi_0(\mathbf{r})$ (cf. eq. (8) of ref. [3]). This form is convenient for the developments made in the next paragraph. Equation (7) is a general eikonal expressions which has been used in [3] and by many other authors.

Now we change the integration variable $\mathbf{R}_\perp$ in eq. (7) to $\mathbf{b}_c$ using eq. (2) and then the amplitude (7) can also be written as

$$A(\mathbf{K},\mathbf{k}) = \int \mathrm{d}^2\mathbf{b}_c\, e^{-i\mathbf{K}_\perp\cdot\mathbf{b}_c} S_c(\mathbf{b}_c) \times \int \mathrm{d}^3\mathbf{r}\,\phi^*_{\mathbf{k}}(\mathbf{r}) e^{(-i\beta_1\mathbf{K}_\perp\cdot\mathbf{r}_\perp)} S_n(\mathbf{b}_n)\,\phi_0(\mathbf{r}), \tag{8}$$

where $\mathbf{b}_n = \mathbf{b}_c + \mathbf{r}_\perp$. The scattering amplitude (8) is a full 3-body eikonal amplitude. It is exactly equivalent to (4) and (7). Both $\mathbf{K}$ and $\mathbf{k}$ are observables which can, in principle, be measured. The next step is to write eq. (8) as

$$A(\mathbf{K},\mathbf{k}) = \int \mathrm{d}^2\mathbf{b}_c\, e^{-i\mathbf{K}_\perp\cdot\mathbf{b}_c} S_c(\mathbf{b}_c) \times \int \mathrm{d}^3\mathbf{r}\phi^*_{\mathbf{k}}(\mathbf{r})\left(e^{(-i\beta_1\mathbf{K}_\perp\cdot\mathbf{r}_\perp)} S_n(\mathbf{b}_n) - 1\right)\phi_0(\mathbf{r}), \tag{9}$$

where we have again used the orthogonality of $\phi_{\mathbf{k}}(\mathbf{r})$ and $\phi_0(\mathbf{r})$. Provided that there are no core neutron resonances in the final state we can make the approximation to neglect the final state interaction of the neutron with the projectile core and replace the final continuum state $\phi_{\mathbf{k}}(\mathbf{r})$ in eq. (9) by a plane wave $e^{i\mathbf{k}\cdot\mathbf{r}}$. Then we obtain $\mathbf{k}$ from the definition of $\mathbf{k}_n$ in eq. (5) and the transverse component of the final neutron momentum is $\mathbf{k}_{n\perp} = \mathbf{k}_\perp + \beta_1\mathbf{K}_\perp$.

Thus the amplitude (9) becomes

$$A(\mathbf{K},\mathbf{k}) = \int \mathrm{d}^2\mathbf{b}_c\, e^{-i\mathbf{K}_\perp\cdot\mathbf{b}_c} S_c(\mathbf{b}_c) \times \int \mathrm{d}^3\mathbf{r} e^{-i(\mathbf{k}_n-\beta_1\mathbf{K})\cdot\mathbf{r}}\left(e^{(-i\beta_1\mathbf{K}_\perp\cdot\mathbf{r}_\perp)} S_n(\mathbf{b}_n) - 1\right) \times\phi_0(\mathbf{r}). \tag{10}$$

which can be further written as

$$A(\mathbf{K},\mathbf{k}) = -\int \mathrm{d}^2\mathbf{b}_c\, e^{-i\mathbf{K}_\perp\cdot\mathbf{b}_c} S_c(\mathbf{b}_c)\, g(\mathbf{k}_n\mathbf{b}_c), \tag{11}$$

where

$$g(\mathbf{k}_n,\mathbf{b}_c) \approx \int \mathrm{d}^3\mathbf{r}\, e^{-i\mathbf{k}_n\cdot\mathbf{r}}\left(e^{(i\beta_1\mathbf{K}_\perp\cdot\mathbf{r}_\perp)} - S_n(\mathbf{b}_n)\right)\phi_0(\mathbf{r}). \tag{12}$$

Then we write the total breakup amplitude as a sum,

$$g(\mathbf{k}_n,\mathbf{b}_c) = g_n(\mathbf{k}_n,\mathbf{b}_c) + g_c(\mathbf{k}_n,\mathbf{b}_c), \tag{13}$$

where

$$g_n(\mathbf{k}_n,\mathbf{b}_c) = \int \mathrm{d}^2\mathbf{r}_\perp\, e^{-i\mathbf{k}_n\cdot\mathbf{r}_\perp}\left(1 - S_n(\mathbf{b}_n)\right)\tilde{\phi}_0(\mathbf{r}_\perp, k_z) \tag{14}$$

$$g_c(\mathbf{k}_n,\mathbf{K}_\perp,\mathbf{b}_c) = \int \mathrm{d}^2\mathbf{r}_\perp\, e^{-i\mathbf{k}_n\cdot\mathbf{r}_\perp}\left(e^{(i\beta_1\mathbf{K}_\perp\cdot\mathbf{r}_\perp)} - 1\right)\tilde{\phi}_0(\mathbf{r}_\perp, k_z). \tag{15}$$

Here k_z is the z-component of the final neutron momentum and $\tilde{\phi}_0(\mathbf{r}_\perp, k_z)$ is the one-dimensional Fourier transform of the initial wave function with respect to the z-coordinate. The amplitude g_n is just the eikonal amplitude used in ref. [5]. It depends on the target neutron interaction through the profile function $S_n(\mathbf{b}_n)$. The second integral g_c is the recoil breakup amplitude. It depends on the recoil momentum $\mathbf{K}_\perp$.

In the following paragraph we simplify eq. (15) by making a semi-classical approximation. If the core-target profile function $S_c(\mathbf{b}_c)$ is smooth and $K_\perp$ is large enough ($K_\perp b_c \gg 1$) then the integral over $\mathbf{b}_c$ in (11) can be estimated by the method of stationary phase. The dominant contribution comes from $\mathbf{K}_\perp$ parallel to $\mathbf{b}_c$ and at the stationary point it will be approximated by the classical momentum transfer

$$\mathbf{K}_\perp \approx \mathcal{K}_\perp(\mathbf{b}_c) = \frac{1}{\hbar}\int \mathbf{F}_{cT}(\mathbf{b}_c, vt)\,\mathrm{d}t, \tag{16}$$

where $\mathbf{F}_{cT} = -\nabla V_{cT}$ is the classical force on the projectile core due to the core-target interaction and the integral is calculated along the path with impact parameter $\mathbf{b}_c$. For a full semi-classical evaluation of the $\mathbf{b}_c$ integral in eq. (11) we have to assume that for each value of $\mathbf{K}_\perp$ there is a unique core-target impact parameter which satisfies eq. (16). At this stage we do not do this but instead approximate $\mathbf{K}_\perp$ in the integral in eq. (15) by its semi-classical value. For each value of $\mathbf{b}_c$ there is a unique $\mathcal{K}_\perp(\mathbf{b}_c)$ given by eq. (16). This approximation results in a decoupling of the two integrals in (11) where now the recoil amplitude

$$g_c(\mathbf{k}_n, \mathbf{b}_c) = \int \mathrm{d}^2\mathbf{r}_\perp\, e^{-i\mathbf{k}_n\cdot\mathbf{r}_\perp} \left(e^{(i\beta_1 \mathcal{K}_\perp(\mathbf{b}_c)\cdot\mathbf{r}_\perp)} - 1\right) \phi_0\left(\mathbf{r}_\perp, k_z\right) \quad (17)$$

is a function of $\mathbf{k}_n$ and $\mathbf{b}_c$.

With this approximation the breakup cross-section as a function of the neutron momentum $\mathbf{k}_n$ when $\mathbf{K}_\perp$ is not observed is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^3\mathbf{k}_n} = \int |A(\mathbf{K},\mathbf{k})|^2 \mathrm{d}^2\mathbf{K}_\perp = \int \mathrm{d}^2\mathbf{b}_c P_{el}\left(b_c\right) |g(\mathbf{k}_n, \mathbf{b}_c)|^2. \quad (18)$$

Here $P_{el}(b_c) = |S_c(\mathbf{b}_c)|^2$ is the probability that the core remains in its ground state during the collision. Different ways of calculating it and their respective accuracies have been recently revised in ref. [7].

The Coulomb breakup of an odd-neutron nucleus like ^{11}Be is due to the core-target interaction. Its contribution is included in the recoil amplitude (15) or (17). When the recoil effect is small enough the exponential factor in eq. (17) can be expanded to first order in β_1 and the recoil amplitude reduces to the standard dipole form in the eikonal limit

$$g_c(\mathbf{k}_n, \mathbf{b}_c) = i\beta_1 \int \mathrm{d}^2\mathbf{r}_\perp\, e^{-i\mathbf{k}_n\cdot\mathbf{r}_\perp}\, \mathcal{K}_\perp\left(\mathbf{b}_c\right)\cdot\mathbf{r}_\perp\, \tilde{\phi}_0\left(\mathbf{r}_\perp, k_z\right). \quad (19)$$

The explicit expression for the momentum transfer (16) is

$$\mathcal{K}_\perp\left(\mathbf{b}_c\right) = \frac{2Z_P Z_T e^2}{\hbar v b_c^2}\mathbf{b}_c. \quad (20)$$

Choosing the x-axis in the direction of $\mathbf{b}_c$, eq. (19) reduces to

$$g_c(\mathbf{k}_n, \mathbf{b}_c) = \beta_1 \frac{2Z_P Z_T e^2}{\hbar v b_c} \frac{\partial}{\partial k_x}\tilde{\phi}_0\left(\mathbf{k}\right). \quad (21)$$

The Coulomb amplitude calculated in the standard dipole approximation by time dependent perturbation theory [5] reads

$$g_c(\mathbf{k}, \mathbf{b}_c) = \beta_1 \frac{2Z_P Z_T e^2}{\hbar v b_c}\left(\bar{\omega} K_1\left(\bar{\omega}\right)\frac{\partial}{\partial k_x} + i\bar{\omega} K_0\left(\bar{\omega}\right)\frac{\partial}{\partial k_z}\right)\tilde{\phi}_0(\mathbf{k}). \quad (22)$$

Thus eq. (21) is just the sudden limit of the usual dipole Born approximation. In fact when the adiabaticity parameter

$$\bar{\omega} = \frac{\varepsilon_k - \varepsilon_0}{\hbar v} b_c \quad (23)$$

is small the sudden limit $\bar{\omega} \to 0$ applies and $\bar{\omega} K_1(\bar{\omega}) \to 1$ and $\bar{\omega} K_0(\bar{\omega}) \to 0$.

If we consider only the nuclear part of the amplitude eq. (14) and integrate over the neutron momentum $\mathrm{d}^3\mathbf{k}_n$ the total nuclear diffraction cross section becomes [5,6]

$$\sigma_{-n} = \int \mathrm{d}^2\mathbf{b}_c |S_{ct}\left(\mathbf{b}_c\right)|^2 \int \mathrm{d}^2\mathbf{r}_\perp\, |1 - S_n\left(\mathbf{b}_n\right)|^2\, |\tilde{\phi}_0\left(\mathbf{r}_\perp\right)|^2. \quad (24)$$

The general eikonal expressions eqs. (7) and (12) have been used instead in [2–4] and by many other authors, without the steps discussed above to separate the nuclear and Coulomb parts thus leading to eq. (2.19) of [2], eq. (10) of [3] and eq. (13) of [4] which are all equivalent and read

$$\sigma_{-n} = \int \mathrm{d}^2\mathbf{b}_c \int \mathrm{d}^3\mathbf{r}\ |S_{ct}\left(\mathbf{b}_c\right) S_n\left(\mathbf{b}_n\right)|^2\, |\phi_0\left(\mathbf{r}\right)|^2 - \int \mathrm{d}^2\mathbf{b}_c \left|\int\!\!\int \mathrm{d}^3\mathbf{r}\, S_{ct}(\mathbf{b}_c) S_n\left(\mathbf{b}_n\right)\, |\phi_0\left(\mathbf{r}\right)|^2\right|^2. \quad (25)$$

On the basis of the previous equations if one goes from eq. (7) to eq. (18) and then to eq. (25) the effect of recoil is automatically included, which corresponds to what is also usually called Coulomb breakup calculated in the sudden limit. It is well known that the sudden approximation to the Coulomb breakup gives too large cross sections [5,6, 8]. Therefore when applied to breakup on a heavy target results from eq. (25) will be larger than what one would get from eq. (24) where only the nuclear part of the amplitude has been used.

We conclude therefore that eq. (25) cannot be defined as the equation representing only the nuclear elastic breakup part. From the point of view of the formalism it contains also the Coulomb breakup calculated in the sudden approximation. On light targets it gives close results to eq. (24), within other numerical incertitudes. On the other hand on heavy targets it definitely gives too large cross sections, also because it contains an interference term and it would be particularly unreliable to make predictions on the nuclear part of elastic breakup while eq. (24) provides a safer way. Finally we propose to follow the method introduced in [5] and [6] to calculate consistently nuclear and Coulomb breakup for a neutron, while for proton breakup we propose to follow [9,10]. In these references Coulomb breakup is treated to all orders and all multipolarities.

Comparisons between numerical results of eqs. (24), (25) will be presented elsewhere.

We thank Ravinder Kumar for drawing the figure.

Author contribution statement

All the authors were involved in the preparation of the manuscript. All the authors have read and approved the final manuscript.

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