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Chiralspin symmetry and QCD at high temperature

L.Ya. Glozman[a]

Institute of Physics, University of Graz, A–8010 Graz, Austria

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Abstract. It has been found very recently on the lattice that at high temperature at vanishing chemical potential QCD is increasingly $SU(2)_{CS}$ and $SU(2N_F)$ symmetric. We demonstrate that the chemical potential term in the QCD Lagrangian has precisely the same symmetry. Consequently the QCD matter beyond the chiral restoration line at high temperature on the T-μ plane is at least approximately $SU(2)_{CS}$ and $SU(2N_F)$ symmetric.

1 Introduction

The structure of the QCD phase diagram as well as the nature of the strongly interacting matter in different regimes attract enormous experimental and theoretical interest. It is established in QCD calculations on the lattice that there is a transition to the chirally symmetric regime at large temperatures and low densities, where the quark condensate, an order parameter of $SU(N_F)_L \times SU(N_F)_R$ chiral symmetry, vanishes. In addition there is a strong evidence that above the critical temperature also the $U(1)_A$ symmetry gets restored [1–3]. Very recently $N_F = 2$ lattice simulations with the domain-wall Dirac operator have demonstrated emergence of $SU(2)_{CS}$ and $SU(4)$ symmetries [4,5] at increasing temperature [6]. These symmetries have been observed earlier in dynamical lattice simulations upon artificial truncation of the near-zero modes of the Dirac operator at zero temperature [7–10]. The $SU(2)_{CS} \supset U(1)_A$ and $SU(2N_F) \supset SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ symmetries are symmetries of the chromo-electric interaction in QCD. In addition to the chiral transformations the $SU(2)_{CS}$ and $SU(2N_F)$ rotations mix the left- and right-handed components of quark fields. The chromo-magnetic interaction as well as the quark kinetic term break these symmetries down to $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$.

Here we demonstrate that the quark chemical potential term in the QCD Lagrangian is $SU(2)_{CS}$ and $SU(2N_F)$ symmetric, *i.e.* it has the same symmetry as the confining chromo-electric interaction. Consequently the quark chemical potential term can only impose the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of confinement. This means that QCD at high temperature beyond the chiral symmetry restoration line on the μ-T plane, where the quark condensate vanishes, should have approximate $SU(2)_{CS}$ and $SU(2N_F)$ symmetries with increasing accuracy with temperature and chemical potential.

2 $\mathrm{SU(2)_{CS}}$ and $\mathrm{SU(2N_F)}$ symmetries

The $SU(2)_{CS}$ chiralspin transformations, defined in the Dirac spinor space are

$$\Psi \rightarrow \Psi' = e^{i\boldsymbol{\varepsilon}\cdot\boldsymbol{\Sigma}/2}\Psi, \tag{1}$$

with the following generators:

$$\boldsymbol{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}, \tag{2}$$

$k = 1, 2, 3, 4$. Different k define different irreducible representations of dim $= 2$. $U(1)_A$ is a subgroup of $SU(2)_{CS}$. The $su(2)$ algebra $[\Sigma_\alpha, \Sigma_\beta] = 2i\epsilon^{\alpha\beta\gamma}\Sigma_\gamma$ is satisfied with any Euclidean gamma-matrix, obeying the following anticommutation relations:

$$\gamma_i\gamma_j + \gamma_j\gamma_i = 2\delta^{ij}; \qquad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4. \tag{3}$$

The $SU(2)_{CS}$ transformations mix the left- and right-handed fermions. The free massless quark Lagrangian does not have this symmetry.

An extension of the $SU(2)_{CS} \times SU(N_F)$ product leads to a $SU(2N_F)$ group. This group has the chiral symmetry of QCD $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ as a subgroup. Its transformations and generators are given by

$$\Psi \rightarrow \Psi' = e^{i\boldsymbol{\epsilon}\cdot\mathbf{T}/2}\Psi, \tag{4}$$

$$\{(\tau_a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma_i), (\tau_a \otimes \Sigma_i)\}, \tag{5}$$

[a] e-mail: `leonid.glozman@uni-graz.at`

where τ are flavour generators with flavour index a and $i = 1, 2, 3$ is the $SU(2)_{CS}$ index.

The fundamental vector of $SU(2N_F)$ at $N_F = 2$ is

$$\Psi = \begin{pmatrix} u_{\rm L} \\ u_{\rm R} \\ d_{\rm L} \\ d_{\rm R} \end{pmatrix}. \tag{6}$$

The $SU(2N_F)$ transformations mix both flavour and chirality.

3 Symmetries of different parts of the QCD Lagrangian and $\mathrm{SU(2)_{CS}}$, $\mathrm{SU(2N_F)}$ emergence at high temperatures

The interaction of quarks with the gluon field in Minkowski space-time can be splitted into a temporal and a spatial part,

$$\overline{\Psi}\gamma^\mu D_\mu \Psi = \overline{\Psi}\gamma^0 D_0 \Psi + \overline{\Psi}\gamma^i D_i \Psi. \tag{7}$$

The first (temporal) term includes an interaction of the color-octet quark charge density $\bar{\Psi}(x)\gamma^0 \boldsymbol{\lambda}\Psi(x) = \Psi(x)^\dagger \boldsymbol{\lambda}\Psi(x)$ with the chromo-electric part of the gluonic field ($\boldsymbol{\lambda}$ are color Gell-Mann matrices). It is invariant with respect to any unitary transformation that can be defined in the Dirac spinor space, in particular it is invariant under the chiral transformations, the $SU(2)_{CS}$ transformations (1) as well as the transformations (4).

The spatial part contains a quark kinetic term and an interaction of the chromo-magnetic field with the color-octet spatial current density. This spatial part is invariant only under chiral $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ transformations and does not admit higher $SU(2)_{CS}$ and $SU(2N_F)$ symmetries. Consequently the QCD Lagrangian has, in the chiral limit, only the $U(N_F)_L \times U(N_F)_R$ chiral symmetry.

It was found on the lattice with chirally invariant fermions in $N_F = 2$ dynamical simulations that truncation of the near-zero modes of the Dirac operator results in emergence of the $SU(2)_{CS}$ and $SU(4)$ symmetries in hadrons [7–10].

The emergence of the $SU(2)_{CS}$ and $SU(4)$ symmetries upon truncation of the lowest modes of the Dirac operator means that the effect of the chromo-magnetic interaction in QCD is located exclusively in the near-zero modes. At the same time the confining chromo-electric interaction, which is $SU(2)_{CS}$- and $SU(4)$-symmetric, is distributed among all modes of the Dirac operator.

To conclude, the low-lying modes of the Dirac operator are responsible not only for chiral symmetry breaking, as is seen from the Banks-Casher relation [11], but also for the $SU(2)_{CS}$ and $SU(4)$ breaking via the magnetic effects. The magnetic effects are linked exclusively to the near-zero modes.

Given this insight one could expect emergence of the $SU(2)_{CS}$ and $SU(4)$ symmetries at high temperatures,

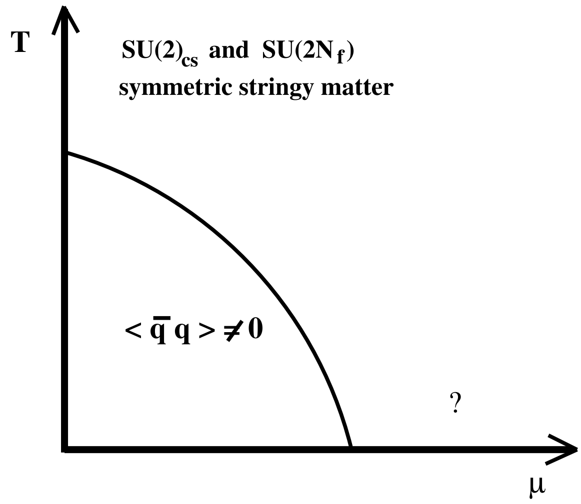

Fig. 1. A sketch of the QCD phase diagram at high temperatures and chemical potentials.

because at high temperature the near-zero modes of the Dirac operator are suppressed. This expectation has been confirmed very recently in lattice simulations with chiral fermions [6]. It was found that indeed above the critical temperature at vanishing chemical potential the approximate $SU(2)_{CS}$ and $SU(4)$ symmetries are seen in spatial correlation functions and by increasing the temperature the $SU(2)_{CS}$ and $SU(4)$ breaking effects decrease rapidly; at the highest available temperature 380 MeV these breaking effects are at the level of 5%.

4 Symmetries of the quark chemical potential

Will a non-zero chemical potential break this symmetry? Consider the quark part of the Euclidean QCD action at a temperature $T = 1/\beta$ in a medium with the quark chemical potential μ

$$S = \int_0^\beta {\rm d}\tau \int {\rm d}^3x \overline{\Psi}[\gamma_\mu D_\mu + \mu\gamma_4]\Psi, \tag{8}$$

where Ψ and $\overline{\Psi}$ are independent integration variables. The field $\overline{\Psi}$ is defined such that it transforms like $\Psi^\dagger\gamma_4$, *i.e.* like Minkowskian $\overline{\Psi}$.

This means that the quark chemical potential term

$$\mu\overline{\Psi}\gamma_4\Psi \tag{9}$$

transforms under chiral, $SU(2)_{CS}$ and $SU(2N_F)$ transformations as

$$\mu\Psi^\dagger\Psi, \tag{10}$$

i.e. it is invariant under all these unitary groups. In other words, the dense QCD matter not only does not break the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries, but in a sense imposes them since the chemical potential μ is an external parameter that can be arbitrarily large. The chemical potential term is a color-singlet. Consequently this term can only reinforce the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries at high temperature and zero chemical potential arising from the chromo-electric color-octet term and a compensation is impossible.

We conclude that at high temperature $T \sim 400\,\mathrm{MeV}$ at any chemical potential the QCD matter is approximately $SU(2)_{CS}$- and $SU(4)$-symmetric. The $SU(2)_{CS}$ and $SU(4)$ symmetries emerge due to yet unknown microscopic dynamics. This dynamics suppresses (screens) the chromo-magnetic field while the chromo-electric interaction between quarks is still active[1].

The elementary objects in the high temperature QCD matter are chiral quarks connected by the chromo-electric field, without any magnetic effects, a kind of a string [12]. These objects cannot be described as bound states in some nonrelativistic potential. With the nonrelativistic Schrödinger equation, appearance of chiral as well as of $SU(2)_{CS}$ and $SU(4)$ symmetries is impossible. Consequently the QCD matter at high temperature and low chemical potential could be named a "stringy matter", see fig. 1.

5 Conclusions

The main new insight of this short note is that the approximate $SU(2)_{CS}$ and $SU(2N_F)$ symmetries emerge at a temperature $\sim 2T_c$ on the T-μ phase diagram and their breaking decreases with increased chemical potential. So we can consider the QCD matter at these temperatures as at least approximately $SU(2)_{CS}$- and $SU(2N_F)$-symmetric. These symmetries rule out the asymptotically free deconfined quarks: free quarks are incompatible with these symmetries. Note that these symmetries cannot be obtained in perturbation theory, which relies on a symmetry of a free Dirac equation, *i.e.* on chiral symmetry. The elementary objects in the QCD matter at these temperatures are chiral quarks connected by the chromo-electric field. Such a matter is not a quark-gluon plasma (the plasma notion is defined in physics as a system of free charges with Debye screening of the electric field) and could be more adequately named as a stringy fluid.

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[1] A plausible microscopic explanation of this phenomenon could be related to suppression at high T of the local topological fluctuations of the gluonic field, like instantons, monopols, etc. According to the Atiyah-Singer theorem the difference of the number of the left- and right-handed zero modes of the Dirac operator is related to the topological charge Q of the gauge configuration. Consequently with $|Q| \geq 1$ the amount of the right- and left-handed zero modes is not equal, which manifestly breaks the $SU(2)_{CS}$ symmetry since the $SU(2)_{CS}$ transformations mix the left- and right-handed components of quarks. The topological configurations contain the chromo-magnetic field. What would be exact zero modes become the near-zero modes of the Dirac operator in the global gauge configuration that contain local topological fluctuations, like in the Shuryak-Diakonov-Petrov theory of chiral symmetry breaking in the instanton liquid. Consequently all effects of the chromo-magnetic field are localised in the near-zero modes, while confining chromo-electric field is distributed among all modes. At $T > T_C$ the local toplogical fluctuations are melt, which leads first to restoration of chiral symmetry and then to $SU(2)_{CS}$ emergence.