Letter

AdS gravity and the scalar glueball spectrum

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Abstract. The scalar glueball spectrum has attracted much attention since the formulation of Quantum Chromodynamics. Different approaches give very different results for the glueball masses. We revisit the problem from the perspective of the AdS/CFT correspondence.

1 Introduction

Glueballs have been a matter of study and experimental search since the formulation of the theory of the strong interaction Quantum Chromodynamics (QCD) [1,2]. QCD sum rules [3,4], QCD-based models [5] and lattice QCD computations both with sea quarks [6] and in the pure glue theory [7–10] have been used to determine their spectra and properties. However, glueballs have not been an easy subject to study due to the lack of phenomenological support and therefore much debate has been associated with their properties [5]. In particular, the lightest scalar glueball has been a matter of discussion because depending on the approaches its calculated mass ranges from $m \sim 700\,\text{GeV}$ [11–14] to that of lattice calculations $m > 1500\,\text{MeV}$ [6,7,9,10]. Our idea here is to discuss what the AdS/CFT correspondence has to add to this controversy. In this letter we center our analysis in the spectrum in gluodynamics obviating the difficult problem of mixing.

2 The glueball spectrum

We recall the glueball spectrum of lattice QCD in table 1. A phenomenological analysis of the lattice spectrum at the light of the f_0 spectrum leads to the conclusion that a scalar glueball should exist in quenched QCD in the mass range 1650–1750 MeV [15]. Let us isolate the quenched lattice glueball spectrum as shown in table 2. We note several features. The lightest glueball is a scalar 0^{++} with a mass on average $\sim 1600\,\text{MeV}$ with errors at the level of 100 MeV [7,9,10], precisely $1638 \pm 119\,\text{MeV}$. The two closest excitations are a 2^{++} and 0^{++} with masses 2331 ± 142 and $2712 \pm 244\,\text{MeV}$, respectively. Given their large errors

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we could in principle consider the latter two degenerate in first approximation. In summary we will demand from our spectrum to have a low mass 0^{++} scalar and some 2^{++}, 0^{++} degenerate pair. One should also notice that lattice calculations miss particles. For example if we look at table 1 we see in the second column that the excited 0^{++} is missing and in the fourth column the 0^{-+} and the 1^{+-} have lower masses than the 0^{++}, an indication that some states might be missing. Also we recall that very sophisticated lattice calculations are required to obtain low mass particles if almost Goldstone bosons are involved. With these caveats we proceed to our analysis of the AdS/CFT spectrum aiming at reproducing these characteristics.

3 The AdS/CFT glueball spectrum

The AdS/CFT correspondence provides new techniques to deal with non Abelian gauge theories. The Maldacena duality conjecture [16] and subsequent developments [17–19] lead to a geometrical picture for QCD, which is based on an AdS_7 soliton whose metric is [20–22]

$$\mathrm{d}s^2 = \left(r^2 - \frac{1}{r^4}\right)\mathrm{d}\tau^2 + r^2\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \left(r^2 - \frac{1}{r^4}\right)^{-1}\mathrm{d}r^2,$$

(1)

where $\eta_{\mu\nu}$ is the Minkowski metric in five dimensions.

The strong coupling glueball calculation consists in finding the normal modes for the supergraviton multiplet. The supergravity modes represent excitations of QCD operators which posses a mass spectrum. One has to find all quadratic fluctuations in the background that survive for QCD in the scaling limit. The result in appropriate units is given in table 3. Note that in the calculation there are two sources of 0^{++} states, one is associated with the dilaton field and the other with the scalar component of the

Table 1. Lattice glueball spectrum obtained from refs. MP [7], CA [9], MT [10] and GI [6].

	Quenched lattice			Unquenched lattice
	MP	CA	MT	GI
0^{++}	1730 ± 130	1710 ± 130	1475 ± 95	1795 ± 60
2^{++}	2400 ± 145	2390 ± 150	2150 ± 130	2620 ± 50
0^{++}	2670 ± 310		2755 ± 150	3760 ± 240
0^{-+}	2590 ± 170	2560 ± 155	2250 ± 160	2887 ± 180
1^{+-}	2940 ± 170	2980 ± 170	2670 ± 185	3730 ± 233
1^{--}	3850 ± 240	3830 ± 230	3240 ± 480	4658 ± 291

Table 2. Glueball masses with J^{PC} assignments. The columns are obtained from refs. MP [7], KY [9] and MT [10].

J^{PC}	MP	CA	MT
0^{++}	1730 ± 130	1710 ± 130	1475 ± 95
2^{++}	2400 ± 145	2390 ± 150	2150 ± 130
0^{++}	2670 ± 310		2755 ± 150

Table 3. The mode spectrum of the supergraviton, m_n^2 for QCD glueballs from ref. [22].

J^{PC}	0^{++}	$2^{++}/1^{++}/0^{++}$	$1^{-+}/0^{-+}$	$1^{+-}/0^{+-}$	$1^{--}/1^{--}$
$n=1$	7.308	22.097	31.985	53.376	83.046
$n=2$	46.986	55.584	72.489	109.446	143.582

Table 4. Gueball spectrum for two parameterizations of the AdS modes of ref. [22] obtained by fixing the scale as indicated in the text. The * signals the data used to fix the scale.

	AdS1	AdS2
0^{++}	1323–1553	834–979
2^{++}	2300–2700*	1450–1702
0^{++}	2300–2700*	1450–1702
0^{-+}	2767–3248	1744–2048
0^{++}	3353–3937	2114–2482
1^{+-}	3575–4196	2253–2645
2^{++}	3648–4282	2300–2700*
0^{++}	3648–4282	2300–2700*
0^{-+}	4165–4890	2626–3083
1^{--}	4449–5234	2811–3300

AdS graviton. The latter is the lightest one. Moreover, the former, the dilaton, is degenerate with the tensor component of the AdS graviton [20,22]. In order to move from the AdS modes to the glueball spectrum we need a scale. To fix the scale we use the assumed approximate degeneracy between the 2^{++} and the 0^{++} described above and which arises naturally in the AdS/CFT result as seen in table 3. We thus assume that in the spectrum the 2^{++} and the first excited 0^{++} are degenerate with a mass between 2300–2700 MeV and we choose these degenerate pair fo fix the scale. We study two schemes: the first scheme assumes that a mass of 2300–2700 MeV corresponds to the first degenerate pair of the AdS/CFT spectrum; the second scheme assumes that this mass corresponds to the second degenerate pair of the AdS/CFT spectrum. The result of that study is shown in table 4.

The first fit is called AdS1 in table 4 and its lightest resonance is a scalar glueball whose mass is between 1300–1500 MeV, which is low compared with the lattice average. Other resonances are: the 0^{-+} in good agreement with the lattice; the 1^{+-} and 1^{--} which are too massive; additional 0^{++}, 2^{++}, 0^{++}, 0^{-+} states in the intermediate range which do not appear in the lattice spectrum. A caveat, the second 0^{++} state of the unquenched calculation [6] has a mass, which is close to that of the additional 0^{++} state, and may hint missing resonances in the quenched calculations. Finally, if we consider that the AdS calculation is a large N approximation and we look

into lattice calculations dealing with large N as shown in table 5 [23,24], we notice that to compare with the $SU(3)$ lattice calculations we must increase the scalar masses by 10%, while the tensors do not change with N. This would fix the light scalars and tensors but not the high lying resonances, and moreover the AdS spectrum remains too crowded unless our caveat regarding missing resonances is confirmed. In any case we conclude from this analysis that given the simplicity of the AdS model this fit supports the QCD lattice spectrum.

The second fit called AdS2 is based on fixing the masses to the degeneracy of the second pair. It provides new ingredients with respect to the lattice results: a low mass scalar ($m \sim 900$ MeV) not seen in lattice calculations; the seen scalar at $m \sim 1600$ MeV, but in this fit degenerate with an unseen tensor; many states close to the fitted degenerate pair which pile up closer than in lattice calculations; the masses of the higher lying resonances are in reasonable agreement with lattice results. This fit supports the sum rule approaches, which tend to predict a low mass scalar glueball. Given the crudeness of the AdS model we argue that the most crucial detectable signature of the fit is the doubling of the ~ 1600 glueball with the tensor. If this doubling is found in lattice calculations then necessarily the low mass glueball should be looked for. The corrections for large N do not alter these statements.

Table 5. Ratios of glueball masses for $N = 3$ and very large N as shown in refs. [23,24].

	0^{++}	0^{++*}	2^{++}
	Continuum		
$m(SU(3))/m(SU(\infty))$	1.07 ± 0.04	0.94 ± 0.04	1.00 ± 0.03
	Smallest lattice		
	1.17 ± 0.05	1.00 ± 0.04	0.99 ± 0.04
	Average		
	1.12 ± 0.03	0.97 ± 0.03	1.00 ± 0.03

Probably the lowest state, which corresponds to the dilaton of spontaneous broken dilatation symmetry in QCD, is the $0^{++}(1600)$ since in lattice calculations given the right currents the lowest mass particle is difficult to miss. Maybe more sophisticated lattice calculations could get its mass down to the sum rule mass, then at the mass of the present $0^{++}(1600)$ an excitation would appear, which is the one we are using for the fit. However, consistency with AdS/CFT will require a degenerate tensor resonance $2^{++}/0^{++}(1600)$.

The naive AdS model used might not be precise in getting the mass levels and orderings but it seems consistent with the labelling of quantum numbers. Therefore the existence of a closely lying 2^{++} tensor to the glueball would be a signature of AdS2.

4 Concluding remarks

We have revisited the AdS/CFT glueball spectrum in the light of new phenomenological analyses of the scalar glueballs and QCD lattice calculations with the aim of clarifying the controversy regarding the mass of the lightest glueball. Our analysis has been based on several assumptions. We have taken the working assumption that not sophisticated lattice calculations might fail to obtain the correct mass of almost Goldstone bosons and might miss some excited states. Moreover, we also have assumed, as shown in some lattice calculations, that unquenched lattice calculations [6] and lattice calculations with mixing [8] cannot lower the mass from the quenched values dramatically.

With these assumptions we have used the results of a well-known model of the AdS/CFT glueball spectrum taking the liberty of fixing the scale to analyze two possible scenarios which reproduce the two conflicting views, namely the lattice and sum rule spectra.

Under our working assumptions according to AdS/CFT a light scalar glueball carries unavoidably to an almost degenerate 2^{++} resonance to the $0^{++}(1600)$ glueball. Despite the simplicity of the model used, which might affect the precise values of the masses, the labelling of the quantum numbers of the states and their degeneracies seem to be quiet consistent with the QCD spectrum. In summary AdS/CFT provides a perfect characterization of the possible scenarios. A non-doubling of the $0^{++}(1600)$ would imply that the AdS1 scenario is realized in nature and we would have to revisit our sum rule calculations.

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