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General operator form of the non-local three-nucleon force

K. Topolnicki[a]

M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30348, Kraków, Poland

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Abstract. This paper describes a procedure to obtain the general form of the three-nucleon force. The result is an operator form where the momentum space matrix element of the three-nucleon potential is written as a linear combination of 320 isospin-spin-momentum operators and scalar functions of momenta. Any spatial and isospin rotation invariant three-nucleon force can be written in this way and in order for the potential to be Hermitian, symmetric under parity inversion, time reversal and particle exchange, the scalar functions must have definite transformation properties under these discrete operations. A complete list of the isospin-spin-momentum operators and scalar function transformation properties is given.

1 Introduction

Three-nucleon (3N) forces are becoming an increasingly important ingredient in few-nucleon calculations. It is becoming clear that 3N potentials must be utilized in order to precisely describe experimental data. For instance, if only two-nucleon forces are used the binding energy of 3N systems is underestimated by 0.5–1.0 MeV and large discrepancies arise for the vector analyzing power in elastic neutron-deuteron scattering at low energies, for more details see, *e.g.*, [1] and references therein. The rich operator structure of 3N potentials [2–7] motivates considerations of the general structure of these forces. Furthermore, discrepancies between theory and experiment still exist and it is possible that this is caused by not utilizing the full structure of the 3N force.

In this paper, the general form of the three-nucleon potential, constructed to be invariant under spatial rotations, isospin rotations and discrete symmetry operations (parity inversion, time reversal, particle exchange, and Hermitian conjugation) is developed. This general form is compatible with any model of the 3N force that satisfies the appropriate symmetries making it useful for a verity of practical applications. I will follow the approach from [8,9], where a local 3N force was considered, and use the algorithm from [10], where the general form of the total momentum dependent two-nucleon potential was developed, to generate the spatial rotation invariant operator form of the 3N potential.

The general form is meant to become an important ingredient in the so called "three-dimensional" (3D) calculations. In this approach, the *three-dimensional*, vector, degrees of freedom of the nucleons are treated directly without resorting to angular momentum decomposition. The biggest advantage of the 3D formalism is the possibility to avoid the complicated numerics of partial wave representations at higher energies. Additionally, calculations performed within this formalism are flexible and allow different models of few-nucleon forces to be used. This is especially important since new models of few-nucleon potentials are constantly being derived from chiral effective field theory [2–7] in a form directly suitable for 3D calculations. A good overview of the 3D approach can be found in [1]. An introduction to these calculations can be found in earlier works, *e.g.* [11,12]. More detailed information about the 3D formalism, with emphasis on few-nucleon bound and scattering states, can be found in works by the Kraków, Bohum, Tehran, Ohio, and University of Iowa groups [13–28].

More traditional approaches that employ partial wave decomposition can also benefit from the possibility to represent different models of 3N forces using a common template. This useful property has important practical implications. It might result in numerical codes that are more general and can be applied to test a large verity of few-nucleon force models. Especially in the new effective methods of partial wave decomposition [29,30] used to obtain matrix elements in the 3N partial wave basis.

It should be emphasized that the discussion presented in this paper is applicable also to operators that depend on the total momentum of the 3N system. This opens the door for applications in calculations that include relativistic corrections.

The paper is organized as follows. Section 2 discusses the symmetry of the 3N force with respect to spatial ro-

[a] e-mail: kacper.topolnicki@uj.edu.pl

tations in spin space. Next, sect. 3 extends the potential to 3N isospin space and adds symmetry with respect to isospin rotations. Section 4 contains considerations related to discrete symmetries. Section 5 explicitly gives the final general operator form of the 3N potential. Finally, sect. 6 contains a summary and appendixes A, B, C, D.1, and D.2 contain additional materials necessary to construct the general form including a list of the 320 operators.

2 Invariance under spatial rotations

A modified version of the method from sect. 2 of ref. [10] is used to generate the general spatial rotation invariant form of the momentum space matrix element of the 3N potential operator $\check{V}$

$$\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle,$$

where $\boldsymbol{p}'$, $\boldsymbol{q}'$ and $\boldsymbol{p}$, $\boldsymbol{q}$ are Jacobi momenta in the final and initial state respectively. In the spin space of the 3N system (for a given isospin in the initial and final state) the operator form of $\check{V}$ is

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_{i=1}^{64} f_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q}) \left[\check{O}_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]^{8\times 8},$$

with square brackets being used (here and in the following) to denote a matrix representation, $[\check{O}_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})]$ being 8×8 matrices representing given spin operators (appendix A contains a complete list) and $f_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q})$ being scalar functions of momenta. Note that the $[\check{O}_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})]$ operators depend on only three of the four Jacobi momenta. Since the momentum vectors have three spatial dimensions the potential dependence of $[\check{O}]$ on some forth momentum vector $\boldsymbol{x}$ can be written entirely in terms of the angles $\boldsymbol{p}'\cdot\boldsymbol{x}$, $\boldsymbol{q}'\cdot\boldsymbol{x}$, $\boldsymbol{p}\cdot\boldsymbol{x}$ and $\boldsymbol{x}^2$. This results in the additional momentum dependence being separated out from $[\check{O}]$ and pushed into the scalar functions f. In general, it is possible to construct spatial rotation invariant operator forms with sets of 64 operators that depend on any combination of three of the four Jacobi momenta. The momentum dependence of the spin operators in the new sets will be the same as the momentum dependence in $[\check{O}_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})]$ except with $\boldsymbol{p}'$, $\boldsymbol{q}'$, $\boldsymbol{p}$ directly replaced by a different combination of three vectors. The choice of $\boldsymbol{p}'$, $\boldsymbol{q}'$, $\boldsymbol{p}$ used in this paper is arbitrary.

The algorithm given in [10] uses the observation that any scalar expression can be written as a product of two types of elements —a scalar product of two vectors ($\boldsymbol{a}\cdot\boldsymbol{b}$) and a scalar product of a vector and a vector product ($\boldsymbol{a}\cdot\boldsymbol{b}\times\boldsymbol{c}$). This observation can be verified using simple vector identities. In the present case $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ are the momentum vectors $\boldsymbol{p}'$, $\boldsymbol{q}'$, $\boldsymbol{p}$, $\boldsymbol{q}$ or vectors of spin operators $\boldsymbol{\sigma}(1)$, $\boldsymbol{\sigma}(2)$, $\boldsymbol{\sigma}(3)$ acting in the spaces of particles 1, 2, 3. As it turns out combining the two types of elements ($\boldsymbol{a}\cdot\boldsymbol{b}$, $\boldsymbol{a}\cdot\boldsymbol{b}\times\boldsymbol{c}$) results in only a finite number of independent operators —the 64 operators from appendix A. Independence means that none of the 64 operators can be expressed as a linear combination of the remaining 63 operators and scalar functions of momenta. Furthermore, a product of any two (or more) operators from this set can be expressed as a linear combination of the 64 operators and scalar functions making the set complete.

An additional observation can be made about the $[\check{O}_i]$ operators. If the matrix element of $\check{V}$ is allowed to depend also on the total momentum $\boldsymbol{K}$,

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K}\rangle\right], \tag{1}$$

then in the spatial rotation invariant, operator form in 3N spin space (for a given isospin in the initial and final state)

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}'\boldsymbol{K} \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\boldsymbol{K}\rangle\right] = \sum_{i=1}^{64} f_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q},\boldsymbol{K}) \left[\check{O}_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]^{8\times 8} \tag{2}$$

the set of $[\check{O}_{i=1,\ldots,64}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})]$ remains the same. The additional momentum dependence appears in the new scalar functions $f_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q},\boldsymbol{K})$. This is again a reflection of the three dimensional nature of space and the $[\check{O}_i(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})]$ operators being composed of only three of the momentum vectors. This property can be used to make the following discussion, where the dependence on $\boldsymbol{K}$ is omitted, more general.

3 Invariance under isospin rotations

In order to preserve symmetry with respect to isospin rotations one of the following five operators:

$$\left[\check{I}_1\right] = \check{1},\ \left[\check{I}_2\right] = \check{\boldsymbol{\tau}}(1)\cdot\check{\boldsymbol{\tau}}(2),\ \left[\check{I}_3\right] = \check{\boldsymbol{\tau}}(1)\cdot\check{\boldsymbol{\tau}}(3),$$
$$\left[\check{I}_4\right] = \check{\boldsymbol{\tau}}(2)\cdot\check{\boldsymbol{\tau}}(3),\ \left[\check{I}_5\right] = \check{\boldsymbol{\tau}}(1)\cdot(\check{\boldsymbol{\tau}}(2)\times\check{\boldsymbol{\tau}}(3)),$$

is appended to each $[\check{O}_{i=1,\ldots,64}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})]$ where $\boldsymbol{\tau}(i)$ is a vector isospin operator of particle $i=1,2,3$. This results in the following operator form:

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right]^{64\times 64} = \sum_{j=1}^{64}\sum_{i=1}^{5} g_{ij}\left[\check{I}_i\otimes\check{O}_j\right]^{64\times 64}$$
$$\equiv \sum_{k=1}^{320} g_k\left[\check{Q}_k\right]^{64\times 64}, \tag{3}$$

where $g_{k=5(j-1)+i}\equiv g_{i\,j}$ and $[\check{Q}_{k=5(j-1)+i}]\equiv[\check{I}_i\otimes\check{O}_j]$ are operators in the isospin-spin-momentum space of the 3N system with a 64×64 matrix representation. A list of these 320 operators is provided in appendix B.

In the following sections I will show that, after taking into account discrete symmetries, the general form of the 3N force will also consist of 320 operators. This number is much greater than the 80 operators in the local version of the 3N force [8,9]. The local potential depends only on two momentum transfer vectors which leads to a reduced number of operators in the rotation invariant form. This in turn translates into a reduced number of operators in the final form of the local 3N force.

4 Discrete symmetries

The potential is additionally required to be symmetric with respect to:

- time reflection ($\check{R}^t$)
- parity inversion ($\check{R}^s$)
- Hermitian conjugate ($\check{R}^h$)
- particle exchange ($\check{P} \in \mathbb{S}_3$)

All of these operations commute and the first three operations form simple cyclic groups. This results in the combined group being a direct product of three cyclic groups $\mathbb{Z}_2$ and $\mathbb{S}_3$

$$\mathbb{G} = \{\check{1}, \check{R}^t\} \times \{\check{1}, \check{R}^s\} \times \{\check{1}, \check{R}^h\} \times \mathbb{S}_3 = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{S}_3.$$

In order to enforce the discrete symmetries the method from [8,9], where a local 3N force was considered, is extended from $\mathbb{S}_3$ to $\mathbb{G}$. First the general operator form (3)

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_{k=1}^{320} g_k \left[\check{Q}_k\right]$$

is transformed into

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_{k=1}^{320} \sum_{r} \sum_{ij} h^r_{k;ij} G^r_{ij}\left(\left[\check{Q}_k\right]\right), \quad (4)$$

where the isospin-spin-momentum operators $G^r_{ij}([\check{Q}_k])$ are constructed from $[\check{Q}_k]$ in such a way as that they transform according to specific representations "r" of the group $\mathbb{G}$ and the indexes i, j take on the value 1 for one-dimensional representations "r" or 1, 2 for two-dimensional representations "r". Next, knowing the transformation properties of $G^r_{ij}([\check{Q}_k])$ under operations $\check{R} \in \mathbb{G}$, the scalar functions $h^r_{k;ij}$ are required to compensate for this behaviour and make the whole operator symmetric.

There are two representations for each of the three $\mathbb{Z}_2$ groups and three representations for $\mathbb{S}_3$ (given, *e.g.*, in [8, 9]). The notation $r = (r_t, r_s, r_h, r_p)$ will be used with the value $r_t, r_s, r_h = 1, 2$ denoting the representations of the three $\mathbb{Z}_2$ groups (for time reversal, parity inversion, and the Hermitian conjugate) and $r_p = 1, 2, 3$ denoting the representations for $\mathbb{S}_3$ (particle exchange). This gives a total of 24 representations of $\mathbb{G}$. Finally, the function G is defined as

$$G^r_{ij}\left(\left[\check{Q}_k\right]\right) = \sum_{\check{R} \in \mathbb{G}} D^r_{ij}(\check{R})\check{R}\left(\left[\check{Q}_k\right]\right), \quad (5)$$

where $D^r_{ij}(\check{R})$ is the matrix representation (or just a single number for one-dimensional representations) of the group element $\check{R} \in \mathbb{G}$ for a given representation "r" and $\check{R}([\check{Q}_k])$ is the action of the discrete operation $\check{R}$ on $[\check{Q}_k]$. The new operators, constructed according to (5), will transform under symmetry operations $\check{R} \in \mathbb{G}$ as (see, *e.g.*, [8,9]):

$$\check{R}(G^r_{ij}\left(\left[\check{Q}_k\right]\right)) = \sum_{l} G^r_{lj}\left(\left[\check{Q}_k\right]\right) D^r_{li}(\check{R}). \quad (6)$$

It is easy to work out that if the scalar functions $h^r_{k;ij}$ satisfy

$$h^r_{k;ij} = \sum_{l} D^r_{il}(\check{R})\check{R}(h^r_{k;lj}) \quad (7)$$

for all $\check{R} \in \mathbb{G}$, then they compensate for the transformations of the $G^r_{lj}([\check{Q}_k])$ operators and make (4) invariant under the discrete symmetry operations.

The two following subsections discuss the matrix representations and the implementation of discrete symmetries in more detail.

4.1 Matrix representations of $\mathbb{G}$

There are two irreducible linear representations for the cyclic group $\mathbb{Z}_2$. Both are 1×1 dimensional matrices and the notation $D^{1,2}_{\mathbb{Z}_2}$ will be used to denote these matrices for the two representations. The first representation is trivial,

$$D^1_{\mathbb{Z}_2}(\check{1}) = (1),$$
$$D^1_{\mathbb{Z}_2}(\check{R}^t) = D^1_{\mathbb{Z}_2}(\check{R}^s) = D^1_{\mathbb{Z}_2}(\check{R}^h) = (-1)$$

and the second one changes the sign,

$$D^2_{\mathbb{Z}_2}(\check{1}) = (1),$$
$$D^2_{\mathbb{Z}_2}(\check{R}^t) = D^2_{\mathbb{Z}_2}(\check{R}^s) = D^2_{\mathbb{Z}_2}(\check{R}^h) = (-1).$$

Next, there are three representations for the $\mathbb{S}_3$ group of particle permutations in the 3N system [8,9]. The cycle representation for permutations will be used with (ij) being a permutation exchanging particles $i, j = 1, 2, 3$: $i \to j$, $j \to i$ and (ijk) being a permutation changing particles $i, j, k = 1, 2, 3$: $i \to j$, $j \to k$, $k \to i$. Two representations are one-dimensional; $D^{1,2}_{\mathbb{S}_3}$ is used to denote 1×1 matrices belonging to these two representations. The first representation is trivial,

$$D^1_{\mathbb{S}_3}((1)) = (1),$$
$$D^1_{\mathbb{S}_3}((12)) = (1),$$
$$D^1_{\mathbb{S}_3}((23)) = (1),$$
$$D^1_{\mathbb{S}_3}((13)) = (1),$$
$$D^1_{\mathbb{S}_3}((132)) = (1),$$
$$D^1_{\mathbb{S}_3}((123)) = (1).$$

and the second representation changes the sign,

$$D^2_{\mathbb{S}_3}((1)) = (1),$$
$$D^2_{\mathbb{S}_3}((12)) = (-1),$$
$$D^2_{\mathbb{S}_3}((23)) = (-1),$$
$$D^2_{\mathbb{S}_3}((13)) = (-1),$$
$$D^2_{\mathbb{S}_3}((132)) = (1),$$
$$D^2_{\mathbb{S}_3}((123)) = (1).$$

The third representation is two-dimensional. The 2×2 matrices $D^3_{\mathbb{S}_3}$ for this representation are

$$D^3_{\mathbb{S}_3}((1)) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$D^3_{\mathbb{S}_3}((12)) = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix},$$
$$D^3_{\mathbb{S}_3}((23)) = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix},$$
$$D^3_{\mathbb{S}_3}((13)) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$D^3_{\mathbb{S}_3}((132)) = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix},$$
$$D^3_{\mathbb{S}_3}((123)) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}.$$

These four types of representations for the four discrete symmetries can be combined using the Kronecker product $\otimes$:

$$D^{r=(r_t,r_s,r_h,r_p)} = D^{r_t}_{\mathbb{Z}_2} \otimes D^{r_s}_{\mathbb{Z}_2} \otimes D^{r_h}_{\mathbb{Z}_2} \otimes D^{r_p}_{\mathbb{S}_3} \quad (8)$$

and in practice, since the first three D^{r_t}, D^{r_s}, D^{r_h} are 1×1 matrices, the Kronecker product can be replaced by a regular multiplication. The linear representation of $\mathbb{G}$, $D^{(r_t,r_s,r_h,r_p)}$, is a 1×1 matrix for all cases except when $r_p = 3$ that is when it is a 2×2 matrix.

4.2 Implementation of discrete transformations

In the proposed approach, the discrete symmetries are implemented as operations on the momentum space matrix element of an operator $\check{X}$. This element $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{X} \mid \boldsymbol{p}\boldsymbol{q}\rangle$ is a function of four Jacobi momenta and an operator in the isospin-spin-momentum space of the 3N system. In practice discrete symmetry operators are realized as operations on the 64×64 (2^3 isospin states and 2^3 spin states) matrix representation of the isospin-spin-momentum operator $\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{X} \mid \boldsymbol{p}\boldsymbol{q}\rangle \equiv [\check{X}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q})]$.

Time reversal is implemented using

$$\begin{aligned}&\check{R}^t\left(\left[\check{X}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q})\right]\right)\\ &= \Big(\left([\check{1}]\otimes[\check{1}]\otimes[\check{1}]\otimes[i\sigma^y]\otimes[i\sigma^y]\otimes[i\sigma^y]\right)\\ &\left[\check{X}(-\boldsymbol{p},-\boldsymbol{q},-\boldsymbol{p}',-\boldsymbol{q}')\right]\\ &\left([\check{1}]\otimes[\check{1}]\otimes[\check{1}]\otimes[i\sigma^y]\otimes[i\sigma^y]\otimes[i\sigma^y]\right)^\dagger\Big)^T, \end{aligned} \quad (9)$$

where the identity operators $[\check{1}]\otimes[\check{1}]\otimes[\check{1}]$ act in the isospin space of the 3N system and $[i\sigma^y]\otimes[i\sigma^y]\otimes[i\sigma^y]$ act in the spin space. If dependence on the total momentum of the 3N system $\boldsymbol{K}$ is considered then $-\boldsymbol{K}$ will appear in the momentum space matrix element after the application of time reversal. I would like to take this opportunity to correct a misprint, found in our paper [10]. The implementation of time reversal in equation (10) of [10] should, of course, be supplemented by a transposition.

Parity inversion is implemented as

$$\check{R}^s\left(\left[\check{X}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q})\right]\right) = \left[\check{X}(-\boldsymbol{p}',-\boldsymbol{q}',-\boldsymbol{p},-\boldsymbol{q})\right]. \quad (10)$$

Similarly as before, if dependence on the total momentum of the 3N system $\boldsymbol{K}$ is considered then $-\boldsymbol{K}$ will appear in the momentum space matrix element after the application of the spatial reflection.

Hermitian conjugation has a straightforward implementation

$$\check{R}^h\left(\left[\check{X}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q})\right]\right) = \left[\check{X}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{p}',\boldsymbol{q}')\right]^\dagger \quad (11)$$

and, if dependence on the total momentum of the 3N system $\boldsymbol{K}$ is considered, then the same vector $\boldsymbol{K}$ will appear in the momentum space matrix element after the application of the symmetry operation.

Particle exchange is more complicated since there are six operations to implement. In general for $\check{P} \in \mathbb{S}_3$

$$\begin{aligned}&\check{P}\left(\left[\check{X}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q})\right]\right) = [P]^T\\ &\left[\check{X}(J^{\check{P}}_A(\boldsymbol{p}',\boldsymbol{q}'), J^{\check{P}}_B(\boldsymbol{p}',\boldsymbol{q}'), J^{\check{P}}_A(\boldsymbol{p},\boldsymbol{q}), J^{\check{P}}_B(\boldsymbol{p},\boldsymbol{q}))\right][P] \end{aligned} \quad (12)$$

where $[P]$ is a 64×64 matrix performing a particle permutation in the isospin-spin-momentum space of the 3N system. $J^{\check{P}}_A$ and $J^{\check{P}}_B$ are functions that transform the Jacobi momenta to implement the appropriate particle permutation. The construction of $[P]$ and the functions $J^{\check{P}}_A$ and $J^{\check{P}}_B$ are given in appendix C. Again if dependence on the total momentum of the 3N system $\boldsymbol{K}$ is considered then the same vector $\boldsymbol{K}$ will appear in the momentum space matrix element after the application of the particle permutation.

5 Removing redundant operators

The above considerations show that there are potentially $320 \times 2 \times 2 \times 2 \times 2 \times 1 = 5120$ of $G^r_{i=1\,j=1}([\check{Q}_{k=1,\ldots,320}])$ operators that transform according to one-dimensional representations $r = (r_t = 1,2,\ r_s = 1,2,\ r_h = 1,2,\ r_p = 1,2)$ of $\mathbb{G}$ and $320 \times 2 \times 2 \times 2 \times 1 \times 4 = 10240$ of $G^r_{i=1,2\,j=1,2}$ $([\check{Q}_{k=1,\ldots,320}])$ operators that transform according to two-dimensional representations $r = (r_t = 1,2,\ r_s = 1,2,\ r_h = 1,2,\ r_p = 3)$ of $\mathbb{G}$. It was numerically verified that out of the 15360 possible $G^r_{ij}([\check{Q}_k])$ operators only 3507 (about 23%) are nonzero. This still leaves a number of redundant operators that should be removed from the final operator form (4) since only 320 operators are independent.

If any operator $\check{X}$ from the set of all nonzero $\{G^r_{ij}([\check{Q}_k]) \neq 0\}$ can be expressed as a linear combination of operators from $\{G^r_{ij}([\check{Q}_k]) \neq 0\} \setminus \check{X}$ and scalar functions of momenta then it is not independent and can be eliminated. It is not immediately obvious that this is true and to demonstrate this a situation where the operators $G^r_{i\,j}([\check{Q}_k])$ can be written as

$$G^r_{i\,j}\left([\check{Q}_k]\right) = \sum_{k'}\sum_{r'}\sum_{i'\,j'} x^{i\,j\,k\,r}_{i'\,j'\,k'\,r'} G^{r'}_{i'\,j'}\left([\check{Q}_{k'}]\right) \quad (13)$$

will be considered with $x^{i\,j\,k\,r}_{i'\,j'\,k'\,r'}$ being scalar functions of momenta and $x^{i\,j\,k\,r}_{i\,j\,k\,r} = 0$ to ensure that the operators $G^{r'}_{i'\,j'}([\check{Q}_{k'}])$ are chosen from $\{G^{r'}_{i'j'}([\check{Q}_{k'}]) \neq 0\} \backslash G^{r}_{i\,j}([\check{Q}_k])$. The general operator form (4) now reads

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_k \sum_r \sum_{ij} h^r_{k;ij} \sum_{k'} \sum_{r'} \sum_{i'\,j'} x^{i\,j\,k\,r}_{i'\,j'\,k'\,r'} G^{r'}_{i'\,j'}\left(\left[\check{Q}_{k'}\right]\right). \quad (14)$$

Rearranging the terms in (14)

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_{k'} \sum_{r'} \sum_{i'\,j'} \left(\sum_k \sum_r \sum_{ij} h^r_{k;ij} x^{i\,j\,k\,r}_{i'\,j'\,k'\,r'}\right) G^{r'}_{i'\,j'}\left(\left[\check{Q}_{k'}\right]\right) \equiv \sum_{k'} \sum_{r'} \sum_{i'\,j'} h'^{r'}_{k';i'j'} G^{r'}_{i'\,j'}\left(\left[\check{Q}_{k'}\right]\right). \quad (15)$$

This equation defines new scalar functions

$$h'^{r'}_{k';i'j'} = \sum_k \sum_r \sum_{ij} h^r_{k;ij} x^{i\,j\,k\,r}_{i'\,j'\,k'\,r'} \quad (16)$$

and the invariance of the potential with respect to discrete symmetry operations implies that also the new scalar functions $h'^{r'}_{k';i'j'}$ satisfy (7). As a consequence of this, if (13) can be solved for a particular operator $G^r_{i\,j}([\check{Q}_k])$ such that $x^{i\,j\,k\,r}_{i\,j\,k\,r} = 0$ ($G^{r'}_{i'\,j'}([\check{Q}_{k'}])$ are chosen from $\{G^{r'}_{i'j'}\ ([\check{Q}_{k'}]) \neq 0\} \setminus G^r_{i\,j}([\check{Q}_k])$) then this operator is not independent and can safely be removed from the operator form (4) since it does not bring any new structures. In practice equation (13) is solved numerically by substituting random numbers for the momentum vector components.

There is another possibility to construct the set of 320 independent operators. Instead of eliminating non-independent operators from the set of 3507 non-zero $G^r_{ij}([\check{Q}_k])$ it is possible to start with an empty set and add, to this set, operators from $\{G^r_{ij}([\check{Q}_k]) \neq 0\}$ one by one or in small groups, checking each time if all newly added operators are independent (*i.e.* no solution to (13) with $x^{i\,j\,k\,r}_{i\,j\,k\,r} = 0$ exists). This process does not lead to a unique general form and the additional freedom allows the consideration of some practical issues related to the final set of 320 independent operators in (4). In particular, it is important to be able to easily work out the transformation properties of all the scalar functions. This is not a problem for one-dimensional representations of $\mathbb{G}$. For two-dimensional operators, however, the operators need to be added in groups of 2. This is a result of (7) and scalar functions $h^r_{k;ij}$ from a single column (with a given j) being transformed into scalar functions from the same column

$$h^r_{k;i1} = D^r_{i1}(\check{R})\check{R}(h^r_{k;11}) + D^r_{i2}(\check{R})\check{R}(h^r_{k;21}),$$
$$h^r_{k;i2} = D^r_{i1}(\check{R})\check{R}(h^r_{k;12}) + D^r_{i2}(\check{R})\check{R}(h^r_{k;22}).$$

As a consequence operators $G^r_{ij}(\left[\check{Q}_k\right])$ should be added in groups with a given j: $\{G^r_{11}([\check{Q}_k]),\ G^r_{21}([\check{Q}_k])\}$, $\{G^r_{12}([\check{Q}_k]),\ G^r_{22}([\check{Q}_k])\}$. This guarantees that the transformation properties of the scalar functions $\{h^r_{k;11}, h^r_{k;21}\}$, $\{h^r_{k;12}, h^r_{k;22}\}$ are easy to work out.

My choice of the 320 operators is listed in appendix D.1 and D.2. Appendix D.1 lists all operators that transform according to one-dimensional representations of $\mathbb{G}$. Appendix D.2 lists all operators that transform according to two-dimensional representations of $\mathbb{G}$, and this set is split into two additional categories. In the first one, there are all the operators from the first column $G^r_{ij=1}([\check{Q}_k])$ and in the second one all the operators from the second column $G^r_{ij=2}([\check{Q}_k])$. Together all these operators can be combined to the general form of the 3N force that is invariant with respect to spatial rotations, isospin rotations, and discrete symmetries

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_k \sum_r \sum_{i\,j} h^r_{k;i\,j} G^r_{ij}\left(\left[\check{Q}_k\right]\right) \equiv \sum_{k=1}^{320} h_k\left[\check{S}_k\right], \quad (17)$$

where the transformation properties of the scalar functions h_k and operators $[\check{S}_k]$ can be read off from appendix D.1 and D.2.

6 Summary

The construction began with the general spatial and isospin rotation symmetric form of the three nucleon potential

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right]^{64\times 64} = \sum_{k=1}^{320} g_k\left[\check{Q}_k\right]^{64\times 64},$$

with $[\check{Q}_k]$ being three-nucleon isospin-spin-momentum operators having 64×64 matrix representations and g_k being scalar functions of Jacobi momenta in the initial $\boldsymbol{p}$, $\boldsymbol{q}$ and final $\boldsymbol{p}'$, $\boldsymbol{q}'$ states.

Next, in order to take into account discrete symmetries, this operator form was transformed into

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_{k=1}^{320} \sum_r \sum_{ij} h^r_{k;ij} G^r_{ij}\left(\left[\check{Q}_k\right]\right),$$

where the operators $G^r_{ij}([\check{Q}_k])$ are constructed from $\left[\check{Q}_k\right]$

$$G^r_{ij}\left(\left[\check{Q}_k\right]\right) = \sum_{\check{R}\in\mathbb{G}} D^r_{ij}(\check{R})\check{R}\left(\left[\check{Q}_k\right]\right)$$

using the matrix representation $D^r_{ij}(\check{R})$ of the symmetry group transformations $\check{R}$ for a given representation "r" and the indices i, j take on a single value 1 for one-dimensional representations and 1, 2 for two-dimensional representations. The $G^r_{ij}([\check{Q}_k])$ operators have simple, known transformation properties with respect to time reversal, parity, Hermitian conjugation and particle exchange, that are determined by one of the 24 representations "r" of the symmetry group

$$\check{R}(G^r_{ij}\left([\check{Q}_k]\right)) = \sum_l G^r_{lj}\left([\check{Q}_k]\right) D^r_{li}(\check{R}).$$

The knowledge of these transformation properties leads to symmetry constraints on the scalar functions $h^r_{k;ij}$

$$\check{R}(G^r_{ij}\left([\check{Q}_k]\right)) = \sum_l G^r_{lj}\left([\check{Q}_k]\right) D^r_{li}(\check{R}).$$

These constraints compensate for the behavior of $G^r_{ij}([\check{Q}_k])$ under symmetry transformations and make the whole operator invariant.

Finally, knowing that there are only 320 operators $G^r_{ij}([\check{Q}_k])$ that are independent —they cannot be expressed as linear combinations of each other and scalar functions— a subset of 320 operators from $\{G^r_{ij}([\check{Q}_k]) \neq 0\}$ is chosen. The choice is dictated by practical considerations, namely, it is important that the transformation properties of the scalar functions h_k in the final operator form

$$\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] = \sum_{k=1}^{320} h_k\left[\check{S}_k\right].$$

are easy to work out. These transformation properties, together with all the $[\check{S}_k]$ operators are listed in the appendixes.

As mentioned in the beginning of this paper, the general form of the three-nucleon force can easily be extended to operators that depend on the total momentum of the system by adding new arguments to the scalar functions. This opens the door for applications in calculations that include relativistic corrections. The general form has potential to become an important ingredient in the, so called, "three-dimensional" formalism, where instead of relying on angular momentum decomposition, the three-dimensional degrees of freedom of the nucleons are used directly. Additionally, being able to represent different models of three-nucleon forces using the same template is a very useful property which might also be utilized in more traditional, partial wave based, calculations.

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Appendix A. Operators in the general form invariant under spatial rotations

Below is a list of the 64 operators that make the spatial rotation invariant form of the 3N potential (they also appear in the general form of the 3N scattering amplitude [31] but with the names of vectors changed). In the 3N spin space (for a given isospin in the initial and final state) the momentum space matrix element of the 3N potential between an initial state with Jacobi momenta $\boldsymbol{p}$, $\boldsymbol{q}$ and a final state with Jacobi momenta $\boldsymbol{p}'$, $\boldsymbol{q}'$ it has an 8×8 matrix representation and can be written as

$$\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle = \sum_{k=1}^{64} f_k(\boldsymbol{p}',\boldsymbol{q},\boldsymbol{p},\boldsymbol{q})\left[\check{Q}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})\right]^{8\times 8},$$

where $f_k(\boldsymbol{p}',\boldsymbol{q},\boldsymbol{p},\boldsymbol{q})$ are scalar functions and the $[\check{O}_k(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p})]$ operators only depend on three of the four momenta with the additional momentum dependence transferred to the scalar functions. In the list below $\boldsymbol{\sigma}(i)$ are spin operators acting in the spaces of particles $i = 1,2,3$. An electronic version of these operators is available upon request from `kacper.topolnicki@uj.edu.pl`.

$$\left[\check{O}_1\right] = 1$$
$$\left[\check{O}_2\right] = \boldsymbol{p}'\cdot\boldsymbol{\sigma}(1)$$
$$\left[\check{O}_3\right] = \boldsymbol{p}'\cdot\boldsymbol{\sigma}(2)$$
$$\left[\check{O}_4\right] = \boldsymbol{p}'\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_5\right] = \boldsymbol{q}'\cdot\boldsymbol{\sigma}(1)$$
$$\left[\check{O}_6\right] = \boldsymbol{q}'\cdot\boldsymbol{\sigma}(2)$$
$$\left[\check{O}_7\right] = \boldsymbol{q}'\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_8\right] = \boldsymbol{p}\cdot\boldsymbol{\sigma}(1)$$
$$\left[\check{O}_9\right] = \boldsymbol{p}\cdot\boldsymbol{\sigma}(2)$$
$$\left[\check{O}_{10}\right] = \boldsymbol{p}\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{11}\right] = \boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$\left[\check{O}_{12}\right] = \boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{13}\right] = \boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{14}\right] = \boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$\left[\check{O}_{15}\right] = \boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{16}\right] = \boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{17}\right] = \boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$\left[\check{O}_{18}\right] = \boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{19}\right] = \boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{20}\right] = \boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$\left[\check{O}_{21}\right] = \boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{22}\right] = \boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{23}\right] = \boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{O}_{24}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{O}_{25}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{26}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$

$$\left[\check{O}_{27}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{28}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(2))$$
$$\left[\check{O}_{29}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{30}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{31}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p}' \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{32}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{33}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p} \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{34}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1) \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{35}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{36}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{37}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{38}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{39}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p}' \times \boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{40}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{q}' \times \boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{41}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p} \times \boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{42}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))$$
$$\left[\check{O}_{43}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{44}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(2))$$
$$\left[\check{O}_{45}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{46}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{47}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p}' \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{48}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{49}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p} \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{50}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1) \times \boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{51}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{52}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{53}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{54}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p}' \times \boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{55}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{q}' \times \boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{56}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p} \times \boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{57}\right] = (\boldsymbol{p} \cdot \boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2) \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{58}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{59}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{60}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{61}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{62}\right] = (\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{63}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{O}_{64}\right] = (\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(2))(\boldsymbol{p} \cdot \boldsymbol{\sigma}(3))$$

Appendix B. Operators in general form invariant under spatial and isospin rotations

Below is a list of the 320 operators that make up the spatial and isospin rotation invariant form of the 3N potential

$$\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle = \sum_{k=1}^{320} g_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q}) \left[\check{Q}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p})\right]^{64\times 64},$$

where $\boldsymbol{p}'$, $\boldsymbol{q}'$, $\boldsymbol{p}$, $\boldsymbol{q}$ are Jacobi momenta in the initial and final state, $f_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p}, \boldsymbol{q})$ are scalar functions and square brackets are used to mark a matrix representation in the isospin-spin-momentum space of the 3N system. The $[\check{Q}_k(\boldsymbol{p}', \boldsymbol{q}', \boldsymbol{p})]$ operators only depend on three of the four momenta with the additional momentum dependence transfered to the scalar functions. In the list below $\boldsymbol{\tau}(i)$, $\boldsymbol{\sigma}(i)$ are isospin and spin operators acting in the spaces of particles $i = 1, 2, 3$.

$$\left[\check{Q}_{1}\right] = 1$$
$$\left[\check{Q}_{2}\right] = \boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2)$$
$$\left[\check{Q}_{3}\right] = \boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(3)$$
$$\left[\check{Q}_{4}\right] = \boldsymbol{\tau}(2) \cdot \boldsymbol{\tau}(3)$$
$$\left[\check{Q}_{5}\right] = \boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2) \times \boldsymbol{\tau}(3)$$
$$\left[\check{Q}_{6}\right] = \boldsymbol{p}' \cdot \boldsymbol{\sigma}(1)$$
$$\left[\check{Q}_{7}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{8}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{9}\right] = (\boldsymbol{\tau}(2) \cdot \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{10}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2) \times \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{11}\right] = \boldsymbol{p}' \cdot \boldsymbol{\sigma}(2)$$
$$\left[\check{Q}_{12}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{13}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{14}\right] = (\boldsymbol{\tau}(2) \cdot \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{15}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2) \times \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{16}\right] = \boldsymbol{p}' \cdot \boldsymbol{\sigma}(3)$$
$$\left[\check{Q}_{17}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{18}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{19}\right] = (\boldsymbol{\tau}(2) \cdot \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{20}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2) \times \boldsymbol{\tau}(3))(\boldsymbol{p}' \cdot \boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{21}\right] = \boldsymbol{q}' \cdot \boldsymbol{\sigma}(1)$$
$$\left[\check{Q}_{22}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{23}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(3))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{24}\right] = (\boldsymbol{\tau}(2) \cdot \boldsymbol{\tau}(3))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{25}\right] = (\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2) \times \boldsymbol{\tau}(3))(\boldsymbol{q}' \cdot \boldsymbol{\sigma}(1))$$
$$\left[\check{Q}_{26}\right] = \boldsymbol{q}' \cdot \boldsymbol{\sigma}(2)$$

$$[\check{Q}_{27}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{28}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{29}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{30}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{31}] = \boldsymbol{q}'\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{32}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{33}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{34}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{35}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{36}] = \boldsymbol{p}\cdot\boldsymbol{\sigma}(1)$$
$$[\check{Q}_{37}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))$$
$$[\check{Q}_{38}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))$$
$$[\check{Q}_{39}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))$$
$$[\check{Q}_{40}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))$$
$$[\check{Q}_{41}] = \boldsymbol{p}\cdot\boldsymbol{\sigma}(2)$$
$$[\check{Q}_{42}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{43}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{44}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{45}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{46}] = \boldsymbol{p}\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{47}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{48}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{49}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{50}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{51}] = \boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$[\check{Q}_{52}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{53}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{54}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{55}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{56}] = \boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{57}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{58}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{59}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{60}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{61}] = \boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{62}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{63}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{64}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{65}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{66}] = \boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$[\check{Q}_{67}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{68}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{69}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{70}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{71}] = \boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{72}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{73}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{74}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{75}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{76}] = \boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{77}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{78}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{79}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{80}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{81}] = \boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$[\check{Q}_{82}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{83}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{84}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{85}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{86}] = \boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{87}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{88}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{89}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{90}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{91}] = \boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{92}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{93}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{94}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{95}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{96}] = \boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$$
$$[\check{Q}_{97}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{98}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{99}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{100}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{101}] = \boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{102}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{103}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{104}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{105}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{106}] = \boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$[\check{Q}_{107}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$

$$\left[\check{Q}_{108}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{109}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{110}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{111}\right] = \boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3)$$
$$\left[\check{Q}_{112}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{113}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{114}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{115}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{116}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{117}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{118}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{119}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{120}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{121}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{122}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{123}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{124}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{125}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{126}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{127}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{128}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{129}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{130}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{131}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{132}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{133}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{134}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{135}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{136}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{137}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{138}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{139}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{140}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$\left[\check{Q}_{141}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{142}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{143}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{144}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{145}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{146}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{147}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{148}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{149}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{150}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{151}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{152}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{153}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{154}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{155}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3)) (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{156}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{157}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{158}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{159}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{160}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3)) (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{161}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{162}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{163}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{164}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{165}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3)) (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{166}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{167}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{168}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{169}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{170}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3)) (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{171}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{172}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{173}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{174}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{175}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{176}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{177}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{178}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{179}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{180}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{181}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{182}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{183}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{184}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$

$$[\check{Q}_{185}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{186}] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{187}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{188}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{189}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{190}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{191}] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{192}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{193}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{194}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{195}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{196}] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{197}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{198}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{199}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{200}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{201}] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{202}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{203}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{204}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{205}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{206}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{207}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{208}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{209}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{210}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{211}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{212}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{213}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{214}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{215}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{216}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{217}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{218}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{219}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{220}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(2))$$
$$[\check{Q}_{221}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{222}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{223}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{224}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{225}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{226}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{227}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{228}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{229}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{230}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{231}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{232}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{233}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{234}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{235}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{236}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{237}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{238}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{239}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{240}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{241}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{242}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{243}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{244}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{245}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{246}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{247}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{248}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{249}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{250}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{251}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{252}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{253}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{254}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{255}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{256}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{257}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{258}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{259}] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{260}] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$[\check{Q}_{261}] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$

$$\left[\check{Q}_{262}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{263}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{264}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{265}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{266}\right] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{267}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{268}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{269}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{270}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{271}\right] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{272}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{273}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{274}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{275}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{276}\right] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{277}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{278}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{279}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{280}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\times\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{281}\right] = (\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{282}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{283}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{284}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{285}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(1))(\boldsymbol{\sigma}(2)\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{286}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{287}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{288}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{289}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{290}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{291}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{292}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{293}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{294}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{295}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{296}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{297}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{298}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{299}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{300}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{301}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{302}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{303}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{304}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{305}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{306}\right] = (\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{307}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{308}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{309}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{310}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{p}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{311}\right] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{312}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{313}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{314}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{315}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{316}\right] = (\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{317}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{318}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{319}\right] = (\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(3))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3))$$
$$\left[\check{Q}_{320}\right] = (\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\times\boldsymbol{\tau}(3))\,(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(1))(\boldsymbol{q}'\cdot\boldsymbol{\sigma}(2))(\boldsymbol{p}\cdot\boldsymbol{\sigma}(3)).$$

Appendix C. Particle permutations

For $\check{P} \in \mathbb{S}_3$, the matrix performing the appropriate particle permutation in the isospin spin space of the 3N system is a Kronecker product of two permutation matrices,

$$[P] = [P]^{\mathrm{isospin}} \otimes [P]^{\mathrm{spin}}. \qquad \text{(C.1)}$$

The matrices $[P]^{\mathrm{isospin}} = [P]^{\mathrm{spin}} \equiv [P]^{\mathrm{is}}$ are listed below for all operators in $\mathbb{S}_3$

$$[(1)]^{\mathrm{is}} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$[(12)]^{\text{is}} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$[(23)]^{\text{is}} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$[(13)]^{\text{is}} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$[(132)]^{\text{is}} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$[(123)]^{\text{is}} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

and work in a basis of 8 spin (isospin) states $\mid i = 1,\ldots,8\rangle$ such that $\mid i = 4(\frac{1}{2}-m_1)+2(\frac{1}{2}-m_2)+(\frac{1}{2}-m_3)+1\rangle = \mid \frac{1}{2}\, m_1\rangle \otimes \mid \frac{1}{2}\, m_2\rangle \otimes \mid \frac{1}{2}\, m_3\rangle$ where $m_{1,2,3} = \pm\frac{1}{2}$ are single particle spin (isospin) $\frac{1}{2}$ projections.

Finally, the functions that are used to permute the momentum vectors have the form

$$\begin{aligned}
J_A^{(1)}(\boldsymbol{p},\boldsymbol{q}) &= \boldsymbol{p}\\
J_A^{(12)}(\boldsymbol{p},\boldsymbol{q}) &= \frac{1}{4}(2\boldsymbol{p}+3\boldsymbol{q})\\
J_A^{(23)}(\boldsymbol{p},\boldsymbol{q}) &= -\boldsymbol{p}\\
J_A^{(13)}(\boldsymbol{p},\boldsymbol{q}) &= \frac{1}{4}(2\boldsymbol{p}-3\boldsymbol{q})\\
J_A^{(132)}(\boldsymbol{p},\boldsymbol{q}) &= \frac{1}{4}(3\boldsymbol{q}-2\boldsymbol{p})\\
J_A^{(123)}(\boldsymbol{p},\boldsymbol{q}) &= \frac{1}{4}(-2\boldsymbol{p}-3\boldsymbol{q})\\
J_B^{(1)}(\boldsymbol{p},\boldsymbol{q}) &= \boldsymbol{q}\\
J_B^{(12)}(\boldsymbol{p},\boldsymbol{q}) &= \boldsymbol{p}-\frac{\boldsymbol{q}}{2}\\
J_B^{(23)}(\boldsymbol{p},\boldsymbol{q}) &= \boldsymbol{q}\\
J_B^{(13)}(\boldsymbol{p},\boldsymbol{q}) &= -\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\\
J_B^{(132)}(\boldsymbol{p},\boldsymbol{q}) &= -\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\\
J_B^{(123)}(\boldsymbol{p},\boldsymbol{q}) &= \boldsymbol{p}-\frac{\boldsymbol{q}}{2}.
\end{aligned}$$

Appendix D. Operators in general form

The general, Hermitian 3N potential that is symmetric with respect to spatial and isospin rotations, parity, time reversal, particle exchange can be written as

$$\begin{aligned}
\left[\langle \boldsymbol{p}'\boldsymbol{q}' \mid \check{V} \mid \boldsymbol{p}\boldsymbol{q}\rangle\right] &= \sum_k \sum_r \sum_{i\,j} h^r_{k;i\,j} G^r_{ij}\left([\check{Q}_k]\right)\\
&\equiv \sum_{k=1}^{320} h_k\left[\check{S}_k\right],
\end{aligned}$$

where $G^r_{ij}([\check{Q}_k])$ depends on the representation "r" and takes one of the operators from appendix B creating operators that transform according to one-dimensional ($i,j = 1$) or two-dimensional ($i,j = 1,2$) representations of $\mathbb{G}$. Invariance under symmetry transformations is preserved if for every $\check{R} \in \mathbb{G}$ the scalar functions $h^{r;k}_{i\,j}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q})$ and $u^{r;k}_{i\,j}(\boldsymbol{p}',\boldsymbol{q}',\boldsymbol{p},\boldsymbol{q},\boldsymbol{K})$ satisfy

$$h^r_{k;ij} = \sum_l D^r_{il}(\check{R})\check{R}(h^r_{k;lj}).$$

Below is a list with a choice for the final 320 operators $[\check{S}_k]$. An electronic version of this list is available upon request from `kacper.topolnicki@uj.edu.pl`.

Appendix D.1. Operators transforming according to one-dimensional representations

r				
$(1,1,1,1)$	$[\check{S}_1] = G^r_{1\,1}([\check{Q}_1])$	$[\check{S}_2] = G^r_{1\,1}([\check{Q}_2])$	$[\check{S}_3] = G^r_{1\,1}([\check{Q}_{58}])$	$[\check{S}_4] = G^r_{1\,1}([\check{Q}_{61}])$
	$[\check{S}_5] = G^r_{1\,1}([\check{Q}_{62}])$	$[\check{S}_6] = G^r_{1\,1}([\check{Q}_{141}])$	$[\check{S}_7] = G^r_{1\,1}([\check{Q}_{200}])$	$[\check{S}_8] = G^r_{1\,1}([\check{Q}_{213}])$
$(1,1,1,2)$	$[\check{S}_9] = G^r_{1\,1}([\check{Q}_{209}])$			
$(1,2,1,2)$	$[\check{S}_{10}] = G^r_{1\,1}([\check{Q}_{34}])$	$[\check{S}_{11}] = G^r_{1\,1}([\check{Q}_{36}])$	$[\check{S}_{12}] = G^r_{1\,1}([\check{Q}_{39}])$	$[\check{S}_{13}] = G^r_{1\,1}([\check{Q}_{44}])$
$(1,2,2,1)$	$[\check{S}_{14}] = G^r_{1\,1}([\check{Q}_{30}])$			
$(2,1,1,2)$	$[\check{S}_{15}] = G^r_{1\,1}([\check{Q}_5])$	$[\check{S}_{16}] = G^r_{1\,1}([\check{Q}_{60}])$		
$(2,2,1,1)$	$[\check{S}_{17}] = G^r_{1\,1}([\check{Q}_{35}])$			
$(2,2,1,2)$	$[\check{S}_{18}] = G^r_{1\,1}([\check{Q}_{45}])$			
$(2,2,2,1)$	$[\check{S}_{19}] = G^r_{1\,1}([\check{Q}_{11}])$	$[\check{S}_{20}] = G^r_{1\,1}([\check{Q}_{43}])$		
$(2,2,2,2)$	$[\check{S}_{21}] = G^r_{1\,1}([\check{Q}_{12}])$	$[\check{S}_{22}] = G^r_{1\,1}([\check{Q}_{13}])$	$[\check{S}_{23}] = G^r_{1\,1}([\check{Q}_{29}])$	$[\check{S}_{24}] = G^r_{1\,1}([\check{Q}_{31}])$

Appendix D.2. Operators transforming according to two-dimensional representations

Operators from the first column ($j = 1$):

r				
$(1,1,1,3)$	$[\check{S}_{25}] = G^r_{2\,1}([\check{Q}_{59}])$	$[\check{S}_{26}] = G^r_{1\,1}([\check{Q}_{59}])$	$[\check{S}_{27}] = G^r_{2\,1}([\check{Q}_{121}])$	$[\check{S}_{28}] = G^r_{1\,1}([\check{Q}_{121}])$
	$[\check{S}_{29}] = G^r_{2\,1}([\check{Q}_{129}])$	$[\check{S}_{30}] = G^r_{1\,1}([\check{Q}_{129}])$	$[\check{S}_{31}] = G^r_{2\,1}([\check{Q}_{138}])$	$[\check{S}_{32}] = G^r_{1\,1}([\check{Q}_{138}])$
	$[\check{S}_{33}] = G^r_{2\,1}([\check{Q}_{143}])$	$[\check{S}_{34}] = G^r_{1\,1}([\check{Q}_{143}])$	$[\check{S}_{35}] = G^r_{2\,1}([\check{Q}_{177}])$	$[\check{S}_{36}] = G^r_{1\,1}([\check{Q}_{177}])$
	$[\check{S}_{37}] = G^r_{2\,1}([\check{Q}_{178}])$	$[\check{S}_{38}] = G^r_{1\,1}([\check{Q}_{178}])$	$[\check{S}_{39}] = G^r_{2\,1}([\check{Q}_{182}])$	$[\check{S}_{40}] = G^r_{1\,1}([\check{Q}_{182}])$
	$[\check{S}_{41}] = G^r_{2\,1}([\check{Q}_{207}])$	$[\check{S}_{42}] = G^r_{1\,1}([\check{Q}_{207}])$	$[\check{S}_{43}] = G^r_{2\,1}([\check{Q}_{216}])$	$[\check{S}_{44}] = G^r_{1\,1}([\check{Q}_{216}])$
	$[\check{S}_{45}] = G^r_{2\,1}([\check{Q}_{218}])$	$[\check{S}_{46}] = G^r_{1\,1}([\check{Q}_{218}])$	$[\check{S}_{47}] = G^r_{2\,1}([\check{Q}_{224}])$	$[\check{S}_{48}] = G^r_{1\,1}([\check{Q}_{224}])$
	$[\check{S}_{49}] = G^r_{2\,1}([\check{Q}_{252}])$	$[\check{S}_{50}] = G^r_{1\,1}([\check{Q}_{252}])$	$[\check{S}_{51}] = G^r_{2\,1}([\check{Q}_{270}])$	$[\check{S}_{52}] = G^r_{1\,1}([\check{Q}_{270}])$
	$[\check{S}_{53}] = G^r_{2\,1}([\check{Q}_{275}])$	$[\check{S}_{54}] = G^r_{1\,1}([\check{Q}_{275}])$		
$(1,2,1,3)$	$[\check{S}_{55}] = G^r_{2\,1}([\check{Q}_{48}])$	$[\check{S}_{56}] = G^r_{1\,1}([\check{Q}_{48}])$	$[\check{S}_{57}] = G^r_{2\,1}([\check{Q}_{306}])$	$[\check{S}_{58}] = G^r_{1\,1}([\check{Q}_{306}])$
	$[\check{S}_{59}] = G^r_{2\,1}([\check{Q}_{309}])$	$[\check{S}_{60}] = G^r_{1\,1}([\check{Q}_{309}])$	$[\check{S}_{61}] = G^r_{2\,1}([\check{Q}_{317}])$	$[\check{S}_{62}] = G^r_{1\,1}([\check{Q}_{317}])$
	$[\check{S}_{63}] = G^r_{2\,1}([\check{Q}_{319}])$	$[\check{S}_{64}] = G^r_{1\,1}([\check{Q}_{319}])$		
$(2,1,1,3)$	$[\check{S}_{65}] = G^r_{2\,1}([\check{Q}_{215}])$	$[\check{S}_{66}] = G^r_{1\,1}([\check{Q}_{215}])$	$[\check{S}_{67}] = G^r_{2\,1}([\check{Q}_{260}])$	$[\check{S}_{68}] = G^r_{1\,1}([\check{Q}_{260}])$
$(2,1,2,3)$	$[\check{S}_{69}] = G^r_{2\,1}([\check{Q}_{137}])$	$[\check{S}_{70}] = G^r_{1\,1}([\check{Q}_{137}])$	$[\check{S}_{71}] = G^r_{2\,1}([\check{Q}_{144}])$	$[\check{S}_{72}] = G^r_{1\,1}([\check{Q}_{144}])$
	$[\check{S}_{73}] = G^r_{2\,1}([\check{Q}_{221}])$	$[\check{S}_{74}] = G^r_{1\,1}([\check{Q}_{221}])$	$[\check{S}_{75}] = G^r_{2\,1}([\check{Q}_{222}])$	$[\check{S}_{76}] = G^r_{1\,1}([\check{Q}_{222}])$
	$[\check{S}_{77}] = G^r_{2\,1}([\check{Q}_{253}])$	$[\check{S}_{78}] = G^r_{1\,1}([\check{Q}_{253}])$		
$(2,2,1,3)$	$[\check{S}_{79}] = G^r_{2\,1}([\check{Q}_{45}])$	$[\check{S}_{80}] = G^r_{1\,1}([\check{Q}_{45}])$		
$(2,2,2,3)$	$[\check{S}_{81}] = G^r_{2\,1}([\check{Q}_8])$	$[\check{S}_{82}] = G^r_{1\,1}([\check{Q}_8])$	$[\check{S}_{83}] = G^r_{2\,1}([\check{Q}_{17}])$	$[\check{S}_{84}] = G^r_{1\,1}([\check{Q}_{17}])$
	$[\check{S}_{85}] = G^r_{2\,1}([\check{Q}_{42}])$	$[\check{S}_{86}] = G^r_{1\,1}([\check{Q}_{42}])$	$[\check{S}_{87}] = G^r_{2\,1}([\check{Q}_{70}])$	$[\check{S}_{88}] = G^r_{1\,1}([\check{Q}_{70}])$
	$[\check{S}_{89}] = G^r_{2\,1}([\check{Q}_{188}])$	$[\check{S}_{90}] = G^r_{1\,1}([\check{Q}_{188}])$	$[\check{S}_{91}] = G^r_{2\,1}([\check{Q}_{227}])$	$[\check{S}_{92}] = G^r_{1\,1}([\check{Q}_{227}])$
	$[\check{S}_{93}] = G^r_{2\,1}([\check{Q}_{282}])$	$[\check{S}_{94}] = G^r_{1\,1}([\check{Q}_{282}])$	$[\check{S}_{95}] = G^r_{2\,1}([\check{Q}_{292}])$	$[\check{S}_{96}] = G^r_{1\,1}([\check{Q}_{292}])$
	$[\check{S}_{97}] = G^r_{2\,1}([\check{Q}_{293}])$	$[\check{S}_{98}] = G^r_{1\,1}([\check{Q}_{293}])$	$[\check{S}_{99}] = G^r_{2\,1}([\check{Q}_{296}])$	$[\check{S}_{100}] = G^r_{1\,1}([\check{Q}_{296}])$
	$[\check{S}_{101}] = G^r_{2\,1}([\check{Q}_{297}])$	$[\check{S}_{102}] = G^r_{1\,1}([\check{Q}_{297}])$	$[\check{S}_{103}] = G^r_{2\,1}([\check{Q}_{298}])$	$[\check{S}_{104}] = G^r_{1\,1}([\check{Q}_{298}])$
	$[\check{S}_{105}] = G^r_{2\,1}([\check{Q}_{299}])$	$[\check{S}_{106}] = G^r_{1\,1}([\check{Q}_{299}])$	$[\check{S}_{107}] = G^r_{2\,1}([\check{Q}_{303}])$	$[\check{S}_{108}] = G^r_{1\,1}([\check{Q}_{303}])$
	$[\check{S}_{109}] = G^r_{2\,1}([\check{Q}_{306}])$	$[\check{S}_{110}] = G^r_{1\,1}([\check{Q}_{306}])$	$[\check{S}_{111}] = G^r_{2\,1}([\check{Q}_{307}])$	$[\check{S}_{112}] = G^r_{1\,1}([\check{Q}_{307}])$
	$[\check{S}_{113}] = G^r_{2\,1}([\check{Q}_{308}])$	$[\check{S}_{114}] = G^r_{1\,1}([\check{Q}_{308}])$	$[\check{S}_{115}] = G^r_{2\,1}([\check{Q}_{309}])$	$[\check{S}_{116}] = G^r_{1\,1}([\check{Q}_{309}])$
	$[\check{S}_{117}] = G^r_{2\,1}([\check{Q}_{313}])$	$[\check{S}_{118}] = G^r_{1\,1}([\check{Q}_{313}])$	$[\check{S}_{119}] = G^r_{2\,1}([\check{Q}_{316}])$	$[\check{S}_{120}] = G^r_{1\,1}([\check{Q}_{316}])$
	$[\check{S}_{121}] = G^r_{2\,1}([\check{Q}_{317}])$	$[\check{S}_{122}] = G^r_{1\,1}([\check{Q}_{317}])$	$[\check{S}_{123}] = G^r_{2\,1}([\check{Q}_{318}])$	$[\check{S}_{124}] = G^r_{1\,1}([\check{Q}_{318}])$
	$[\check{S}_{125}] = G^r_{2\,1}([\check{Q}_{319}])$	$[\check{S}_{126}] = G^r_{1\,1}([\check{Q}_{319}])$		

Operators from the second column ($j = 2$):

r				
$(1,1,1,3)$	$[\check{S}_{127}] = G^r_{2\,2}([\check{Q}_{3}])$	$[\check{S}_{128}] = G^r_{1\,2}([\check{Q}_{3}])$	$[\check{S}_{129}] = G^r_{2\,2}([\check{Q}_{59}])$	$[\check{S}_{130}] = G^r_{1\,2}([\check{Q}_{59}])$
	$[\check{S}_{131}] = G^r_{2\,2}([\check{Q}_{61}])$	$[\check{S}_{132}] = G^r_{1\,2}([\check{Q}_{61}])$	$[\check{S}_{133}] = G^r_{2\,2}([\check{Q}_{64}])$	$[\check{S}_{134}] = G^r_{1\,2}([\check{Q}_{64}])$
	$[\check{S}_{135}] = G^r_{2\,2}([\check{Q}_{117}])$	$[\check{S}_{136}] = G^r_{1\,2}([\check{Q}_{117}])$	$[\check{S}_{137}] = G^r_{2\,2}([\check{Q}_{121}])$	$[\check{S}_{138}] = G^r_{1\,2}([\check{Q}_{121}])$
	$[\check{S}_{139}] = G^r_{2\,2}([\check{Q}_{129}])$	$[\check{S}_{140}] = G^r_{1\,2}([\check{Q}_{129}])$	$[\check{S}_{141}] = G^r_{2\,2}([\check{Q}_{131}])$	$[\check{S}_{142}] = G^r_{1\,2}([\check{Q}_{131}])$
	$[\check{S}_{143}] = G^r_{2\,2}([\check{Q}_{133}])$	$[\check{S}_{144}] = G^r_{1\,2}([\check{Q}_{133}])$	$[\check{S}_{145}] = G^r_{2\,2}([\check{Q}_{136}])$	$[\check{S}_{146}] = G^r_{1\,2}([\check{Q}_{136}])$
	$[\check{S}_{147}] = G^r_{2\,2}([\check{Q}_{138}])$	$[\check{S}_{148}] = G^r_{1\,2}([\check{Q}_{138}])$	$[\check{S}_{149}] = G^r_{2\,2}([\check{Q}_{143}])$	$[\check{S}_{150}] = G^r_{1\,2}([\check{Q}_{143}])$
	$[\check{S}_{151}] = G^r_{2\,2}([\check{Q}_{177}])$	$[\check{S}_{152}] = G^r_{1\,2}([\check{Q}_{177}])$	$[\check{S}_{153}] = G^r_{2\,2}([\check{Q}_{178}])$	$[\check{S}_{154}] = G^r_{1\,2}([\check{Q}_{178}])$
	$[\check{S}_{155}] = G^r_{2\,2}([\check{Q}_{182}])$	$[\check{S}_{156}] = G^r_{1\,2}([\check{Q}_{182}])$	$[\check{S}_{157}] = G^r_{2\,2}([\check{Q}_{184}])$	$[\check{S}_{158}] = G^r_{1\,2}([\check{Q}_{184}])$
	$[\check{S}_{159}] = G^r_{2\,2}([\check{Q}_{205}])$	$[\check{S}_{160}] = G^r_{1\,2}([\check{Q}_{205}])$	$[\check{S}_{161}] = G^r_{2\,2}([\check{Q}_{206}])$	$[\check{S}_{162}] = G^r_{1\,2}([\check{Q}_{206}])$
	$[\check{S}_{163}] = G^r_{2\,2}([\check{Q}_{207}])$	$[\check{S}_{164}] = G^r_{1\,2}([\check{Q}_{207}])$	$[\check{S}_{165}] = G^r_{2\,2}([\check{Q}_{216}])$	$[\check{S}_{166}] = G^r_{1\,2}([\check{Q}_{216}])$
	$[\check{S}_{167}] = G^r_{2\,2}([\check{Q}_{218}])$	$[\check{S}_{168}] = G^r_{1\,2}([\check{Q}_{218}])$	$[\check{S}_{169}] = G^r_{2\,2}([\check{Q}_{224}])$	$[\check{S}_{170}] = G^r_{1\,2}([\check{Q}_{224}])$
	$[\check{S}_{171}] = G^r_{2\,2}([\check{Q}_{245}])$	$[\check{S}_{172}] = G^r_{1\,2}([\check{Q}_{245}])$	$[\check{S}_{173}] = G^r_{2\,2}([\check{Q}_{252}])$	$[\check{S}_{174}] = G^r_{1\,2}([\check{Q}_{252}])$
	$[\check{S}_{175}] = G^r_{2\,2}([\check{Q}_{256}])$	$[\check{S}_{176}] = G^r_{1\,2}([\check{Q}_{256}])$	$[\check{S}_{177}] = G^r_{2\,2}([\check{Q}_{259}])$	$[\check{S}_{178}] = G^r_{1\,2}([\check{Q}_{259}])$
	$[\check{S}_{179}] = G^r_{2\,2}([\check{Q}_{275}])$	$[\check{S}_{180}] = G^r_{1\,2}([\check{Q}_{275}])$		
$(1,1,2,3)$	$[\check{S}_{181}] = G^r_{2\,2}([\check{Q}_{180}])$	$[\check{S}_{182}] = G^r_{1\,2}([\check{Q}_{180}])$	$[\check{S}_{183}] = G^r_{2\,2}([\check{Q}_{255}])$	$[\check{S}_{184}] = G^r_{1\,2}([\check{Q}_{255}])$
$(1,2,1,3)$	$[\check{S}_{185}] = G^r_{2\,2}([\check{Q}_{8}])$	$[\check{S}_{186}] = G^r_{1\,2}([\check{Q}_{8}])$	$[\check{S}_{187}] = G^r_{2\,2}([\check{Q}_{41}])$	$[\check{S}_{188}] = G^r_{1\,2}([\check{Q}_{41}])$
	$[\check{S}_{189}] = G^r_{2\,2}([\check{Q}_{43}])$	$[\check{S}_{190}] = G^r_{1\,2}([\check{Q}_{43}])$	$[\check{S}_{191}] = G^r_{2\,2}([\check{Q}_{48}])$	$[\check{S}_{192}] = G^r_{1\,2}([\check{Q}_{48}])$
	$[\check{S}_{193}] = G^r_{2\,2}([\check{Q}_{75}])$	$[\check{S}_{194}] = G^r_{1\,2}([\check{Q}_{75}])$	$[\check{S}_{195}] = G^r_{2\,2}([\check{Q}_{250}])$	$[\check{S}_{196}] = G^r_{1\,2}([\check{Q}_{250}])$
	$[\check{S}_{197}] = G^r_{2\,2}([\check{Q}_{284}])$	$[\check{S}_{198}] = G^r_{1\,2}([\check{Q}_{284}])$	$[\check{S}_{199}] = G^r_{2\,2}([\check{Q}_{299}])$	$[\check{S}_{200}] = G^r_{1\,2}([\check{Q}_{299}])$
	$[\check{S}_{201}] = G^r_{2\,2}([\check{Q}_{306}])$	$[\check{S}_{202}] = G^r_{1\,2}([\check{Q}_{306}])$	$[\check{S}_{203}] = G^r_{2\,2}([\check{Q}_{309}])$	$[\check{S}_{204}] = G^r_{1\,2}([\check{Q}_{309}])$
	$[\check{S}_{205}] = G^r_{2\,2}([\check{Q}_{311}])$	$[\check{S}_{206}] = G^r_{1\,2}([\check{Q}_{311}])$	$[\check{S}_{207}] = G^r_{2\,2}([\check{Q}_{317}])$	$[\check{S}_{208}] = G^r_{1\,2}([\check{Q}_{317}])$
	$[\check{S}_{209}] = G^r_{2\,2}([\check{Q}_{319}])$	$[\check{S}_{210}] = G^r_{1\,2}([\check{Q}_{319}])$		
$(1,2,2,3)$	$[\check{S}_{211}] = G^r_{2\,2}([\check{Q}_{25}])$	$[\check{S}_{212}] = G^r_{1\,2}([\check{Q}_{25}])$	$[\check{S}_{213}] = G^r_{2\,2}([\check{Q}_{166}])$	$[\check{S}_{214}] = G^r_{1\,2}([\check{Q}_{166}])$
	$[\check{S}_{215}] = G^r_{2\,2}([\check{Q}_{249}])$	$[\check{S}_{216}] = G^r_{1\,2}([\check{Q}_{249}])$	$[\check{S}_{217}] = G^r_{2\,2}([\check{Q}_{315}])$	$[\check{S}_{218}] = G^r_{1\,2}([\check{Q}_{315}])$
$(2,1,1,3)$	$[\check{S}_{219}] = G^r_{2\,2}([\check{Q}_{55}])$	$[\check{S}_{220}] = G^r_{1\,2}([\check{Q}_{55}])$	$[\check{S}_{221}] = G^r_{2\,2}([\check{Q}_{175}])$	$[\check{S}_{222}] = G^r_{1\,2}([\check{Q}_{175}])$
	$[\check{S}_{223}] = G^r_{2\,2}([\check{Q}_{185}])$	$[\check{S}_{224}] = G^r_{1\,2}([\check{Q}_{185}])$	$[\check{S}_{225}] = G^r_{2\,2}([\check{Q}_{215}])$	$[\check{S}_{226}] = G^r_{1\,2}([\check{Q}_{215}])$
	$[\check{S}_{227}] = G^r_{2\,2}([\check{Q}_{220}])$	$[\check{S}_{228}] = G^r_{1\,2}([\check{Q}_{220}])$	$[\check{S}_{229}] = G^r_{2\,2}([\check{Q}_{260}])$	$[\check{S}_{230}] = G^r_{1\,2}([\check{Q}_{260}])$
$(2,1,2,3)$	$[\check{S}_{231}] = G^r_{2\,2}([\check{Q}_{129}])$	$[\check{S}_{232}] = G^r_{1\,2}([\check{Q}_{129}])$	$[\check{S}_{233}] = G^r_{2\,2}([\check{Q}_{137}])$	$[\check{S}_{234}] = G^r_{1\,2}([\check{Q}_{137}])$
	$[\check{S}_{235}] = G^r_{2\,2}([\check{Q}_{144}])$	$[\check{S}_{236}] = G^r_{1\,2}([\check{Q}_{144}])$	$[\check{S}_{237}] = G^r_{2\,2}([\check{Q}_{181}])$	$[\check{S}_{238}] = G^r_{1\,2}([\check{Q}_{181}])$
	$[\check{S}_{239}] = G^r_{2\,2}([\check{Q}_{221}])$	$[\check{S}_{240}] = G^r_{1\,2}([\check{Q}_{221}])$	$[\check{S}_{241}] = G^r_{2\,2}([\check{Q}_{222}])$	$[\check{S}_{242}] = G^r_{1\,2}([\check{Q}_{222}])$
	$[\check{S}_{243}] = G^r_{2\,2}([\check{Q}_{253}])$	$[\check{S}_{244}] = G^r_{1\,2}([\check{Q}_{253}])$	$[\check{S}_{245}] = G^r_{2\,2}([\check{Q}_{259}])$	$[\check{S}_{246}] = G^r_{1\,2}([\check{Q}_{259}])$
	$[\check{S}_{247}] = G^r_{2\,2}([\check{Q}_{275}])$	$[\check{S}_{248}] = G^r_{1\,2}([\check{Q}_{275}])$		
$(2,2,1,3)$	$[\check{S}_{249}] = G^r_{2\,2}([\check{Q}_{10}])$	$[\check{S}_{250}] = G^r_{1\,2}([\check{Q}_{10}])$	$[\check{S}_{251}] = G^r_{2\,2}([\check{Q}_{190}])$	$[\check{S}_{252}] = G^r_{1\,2}([\check{Q}_{190}])$
	$[\check{S}_{253}] = G^r_{2\,2}([\check{Q}_{247}])$	$[\check{S}_{254}] = G^r_{1\,2}([\check{Q}_{247}])$	$[\check{S}_{255}] = G^r_{2\,2}([\check{Q}_{300}])$	$[\check{S}_{256}] = G^r_{1\,2}([\check{Q}_{300}])$
	$[\check{S}_{257}] = G^r_{2\,2}([\check{Q}_{305}])$	$[\check{S}_{258}] = G^r_{1\,2}([\check{Q}_{305}])$	$[\check{S}_{259}] = G^r_{2\,2}([\check{Q}_{310}])$	$[\check{S}_{260}] = G^r_{1\,2}([\check{Q}_{310}])$
$(2,2,2,3)$	$[\check{S}_{261}] = G^r_{2\,2}([\check{Q}_{8}])$	$[\check{S}_{262}] = G^r_{1\,2}([\check{Q}_{8}])$	$[\check{S}_{263}] = G^r_{2\,2}([\check{Q}_{11}])$	$[\check{S}_{264}] = G^r_{1\,2}([\check{Q}_{11}])$
	$[\check{S}_{265}] = G^r_{2\,2}([\check{Q}_{17}])$	$[\check{S}_{266}] = G^r_{1\,2}([\check{Q}_{17}])$	$[\check{S}_{267}] = G^r_{2\,2}([\check{Q}_{36}])$	$[\check{S}_{268}] = G^r_{1\,2}([\check{Q}_{36}])$
	$[\check{S}_{269}] = G^r_{2\,2}([\check{Q}_{42}])$	$[\check{S}_{270}] = G^r_{1\,2}([\check{Q}_{42}])$	$[\check{S}_{271}] = G^r_{2\,2}([\check{Q}_{70}])$	$[\check{S}_{272}] = G^r_{1\,2}([\check{Q}_{70}])$
	$[\check{S}_{273}] = G^r_{2\,2}([\check{Q}_{170}])$	$[\check{S}_{274}] = G^r_{1\,2}([\check{Q}_{170}])$	$[\check{S}_{275}] = G^r_{2\,2}([\check{Q}_{188}])$	$[\check{S}_{276}] = G^r_{1\,2}([\check{Q}_{188}])$
	$[\check{S}_{277}] = G^r_{2\,2}([\check{Q}_{226}])$	$[\check{S}_{278}] = G^r_{1\,2}([\check{Q}_{226}])$	$[\check{S}_{279}] = G^r_{2\,2}([\check{Q}_{227}])$	$[\check{S}_{280}] = G^r_{1\,2}([\check{Q}_{227}])$
	$[\check{S}_{281}] = G^r_{2\,2}([\check{Q}_{281}])$	$[\check{S}_{282}] = G^r_{1\,2}([\check{Q}_{281}])$	$[\check{S}_{283}] = G^r_{2\,2}([\check{Q}_{282}])$	$[\check{S}_{284}] = G^r_{1\,2}([\check{Q}_{282}])$
	$[\check{S}_{285}] = G^r_{2\,2}([\check{Q}_{291}])$	$[\check{S}_{286}] = G^r_{1\,2}([\check{Q}_{291}])$	$[\check{S}_{287}] = G^r_{2\,2}([\check{Q}_{292}])$	$[\check{S}_{288}] = G^r_{1\,2}([\check{Q}_{292}])$
	$[\check{S}_{289}] = G^r_{2\,2}([\check{Q}_{293}])$	$[\check{S}_{290}] = G^r_{1\,2}([\check{Q}_{293}])$	$[\check{S}_{291}] = G^r_{2\,2}([\check{Q}_{296}])$	$[\check{S}_{292}] = G^r_{1\,2}([\check{Q}_{296}])$
	$[\check{S}_{293}] = G^r_{2\,2}([\check{Q}_{297}])$	$[\check{S}_{294}] = G^r_{1\,2}([\check{Q}_{297}])$	$[\check{S}_{295}] = G^r_{2\,2}([\check{Q}_{298}])$	$[\check{S}_{296}] = G^r_{1\,2}([\check{Q}_{298}])$
	$[\check{S}_{297}] = G^r_{2\,2}([\check{Q}_{299}])$	$[\check{S}_{298}] = G^r_{1\,2}([\check{Q}_{299}])$	$[\check{S}_{299}] = G^r_{2\,2}([\check{Q}_{303}])$	$[\check{S}_{300}] = G^r_{1\,2}([\check{Q}_{303}])$
	$[\check{S}_{301}] = G^r_{2\,2}([\check{Q}_{306}])$	$[\check{S}_{302}] = G^r_{1\,2}([\check{Q}_{306}])$	$[\check{S}_{303}] = G^r_{2\,2}([\check{Q}_{307}])$	$[\check{S}_{304}] = G^r_{1\,2}([\check{Q}_{307}])$
	$[\check{S}_{305}] = G^r_{2\,2}([\check{Q}_{308}])$	$[\check{S}_{306}] = G^r_{1\,2}([\check{Q}_{308}])$	$[\check{S}_{307}] = G^r_{2\,2}([\check{Q}_{309}])$	$[\check{S}_{308}] = G^r_{1\,2}([\check{Q}_{309}])$
	$[\check{S}_{309}] = G^r_{2\,2}([\check{Q}_{311}])$	$[\check{S}_{310}] = G^r_{1\,2}([\check{Q}_{311}])$	$[\check{S}_{311}] = G^r_{2\,2}([\check{Q}_{313}])$	$[\check{S}_{312}] = G^r_{1\,2}([\check{Q}_{313}])$
	$[\check{S}_{313}] = G^r_{2\,2}([\check{Q}_{316}])$	$[\check{S}_{314}] = G^r_{1\,2}([\check{Q}_{316}])$	$[\check{S}_{315}] = G^r_{2\,2}([\check{Q}_{317}])$	$[\check{S}_{316}] = G^r_{1\,2}([\check{Q}_{317}])$
	$[\check{S}_{317}] = G^r_{2\,2}([\check{Q}_{318}])$	$[\check{S}_{318}] = G^r_{1\,2}([\check{Q}_{318}])$	$[\check{S}_{319}] = G^r_{2\,2}([\check{Q}_{319}])$	$[\check{S}_{320}] = G^r_{1\,2}([\check{Q}_{319}])$

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