

Time evolution of temperature fluctuation in a non-equilibrated system

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Abstract. The evolution equation for inhomogeneous and anisotropic temperature fluctuation inside a medium is derived within the ambit of Boltzmann Transport Equation (BTE) for a hot gas of massless particles. Also, specializing to a situation created after a heavy-ion collision (HIC), we analyze the Fourier space variation of temperature fluctuation of the medium using its temperature profile. The effect of viscosity on the variation of fluctuations in the latter case is investigated and possible implications for early universe cosmology, and its connection with HICs are also explored.

1 Introduction

Fluctuation is a normal occurrence in physical systems. Because of their stochastic nature, the value of certain observables deviates from their average value, which may be defined over a large time or over a large number of identically prepared systems (ensembles). The well-known phenomenon of critical opalescence, for example, is caused by fluctuations at all length scales during a second-order phase transition.

In high-energy collision experiments, the search for fluctuations of quantities (like net charge [1,2]) over large number of events is important for searching the critical point [3] or tri-critical point [4] in the quantum chromodynamic (QCD) phase diagram. The study of particle multiplicity ratio fluctuation [5] is another such example in this context.

Much in the same way as number of particles in a certain region of a system fluctuates, the everyday examples teach us that the temperature for physical systems can also fluctuate. Apart from the examples from high-energy collisions where particle yield has shown the signature of temperature fluctuation [6–16], there are numerous other situations (like cosmological perturbations in our expanding universe as sources of temperature fluctuation) where the concerned system is not in global thermal equilibrium. The temperature, on the contrary varies with time and space. The temperature fluctuation associated with such

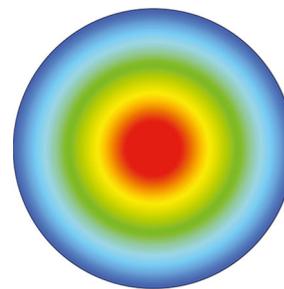


Fig. 1. (Color online) Pictorial representation of hotspots in a medium.

kinds of systems encodes transport properties like conductivity, shear viscosity, rates of chemical reactions etc. As the system evolves, dynamics dictates the temperature fluctuation until a state of minimum energy, or equilibrium is attained.

The evolution of the fluctuations has been investigated for systems concerned with high-energy collisions employing Boltzmann Transport Equation (BTE) [17, 18]. It is now important to study temperature fluctuation as well as its evolution in such systems as they can characterize the medium created after high-energy collisions [19, 20] or may give a hint to the QCD critical point [21].

A medium with spatially fluctuating temperature can be schematically represented by fig. 1, where, within a large system, we encounter subsystems with different temperature values and the temperature has radial variation only. The temperature profile for a system produced in

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one central event in Pb-Pb collisions at 2.76 TeV has been obtained from hydrodynamic calculations in refs. [19, 20] and the profile can very well be approximated to vary radially. In our present work, we model the time evolution of temperature fluctuation among these subsystems. At any certain time slice, we assume that the system comprises temperature hotspots or zones evolving with time. This is essentially the assumption of local thermodynamic equilibrium of matter, and the temperature hotspots are assumed to be weakly interacting. This assumption in turn implies that particles in a certain temperature zone hardly affect those in other temperature zones. In fact, (assuming zero chemical potential) such systems can be represented by a collection of canonical ensembles [22] with different temperature values. The probability that a certain member of the ensemble will be having a certain energy at some instant will depend on the fluctuating temperature values of the collection of subsystems.

In the present work, we try to find out an evolution equation of the fluctuation in Boltzmann parameter β ($= 1/T$) with the aid of Boltzmann Transport Equation in Relaxation Time Approximation (RTA) assuming a constraint that the observation time is much less than the relaxation time of the thermal bath. Later on we analyze the same problem for arbitrary observation time. This model may help us to understand the evolution of temperature fluctuation in the Quark-Gluon Plasma (QGP) medium created in Heavy-Ion Collision (HIC) experiments.

Hence, the manuscript is organized as follows. In sect. 2 we begin by considering the BTE and the evolution of β -fluctuation, followed by analysis specific to heavy-ion collisions with arbitrary observation time. We then discuss our results in sect. 3 where the relative variance of the Boltzmann β -parameter will be compared with the similar quantities extracted from experimental data. Lastly, we conclude by conjecturing possible connections with early universe cosmology.

2 The methodology

In order to gain qualitative insight into the evolution of temperature fluctuation, we consider an ansatz [23] of the particle distribution function f ,

$$f = e^{-\beta p(1+\Delta\beta)}, \quad (1)$$

where we consider a medium with Boltzmann distribution of massless particles ($p = |\mathbf{p}| = E$) with average inverse temperature $\beta(t)$, at some time slice. In the high-temperature regime ($\beta E \ll 1$), that we are interested in, quantum statistics tend towards Boltzmann distribution. The average inverse temperature of the system is calculated considering the arithmetic mean of the distribution of temperature hotspots, *i.e.* if there are n_i hotspots individually characterized by inverse temperatures β_i , then the average is calculated as $\frac{\sum n_i \beta_i}{\sum n_i}$. Generalizing this to the continuum limit, we add an anisotropic and inhomogeneous fluctuation function $\Delta\beta(\mathbf{r}, \hat{p}; t)$, where \hat{p} is a unit vector along the direction of motion of the particle. The \hat{p} dependence encodes the anisotropy of the fluctuation.

We now discuss the temporal evolution of the fluctuation with the help of BTE.

The generic form of BTE can be written as

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \mathbf{F} \cdot \nabla_p f = \mathcal{C}[f], \quad (2)$$

where \mathbf{v} is the particle velocity, \mathbf{F} is any external force (like gravity) and $\mathcal{C}[f]$ is the collision term encoding the information about interaction. ∇_p is the momentum-space gradient operator. For our present case, we assume that the system experiences no external force, and hence $\mathbf{F} = \mathbf{0}$. However, the inhomogeneity in $\Delta\beta$ still exists. Assuming the $|\Delta\beta| \ll 1$, we get

$$\begin{aligned} f &\approx e^{-p\beta - p\beta\Delta\beta} \Big|_{\Delta\beta=0} + \beta \frac{\partial}{\partial\beta} [e^{-p\beta - p\beta\Delta\beta}] \Big|_{\Delta\beta=0} \Delta\beta \\ &= e^{-p\beta} - p\beta e^{-p\beta} \Delta\beta \\ &= f^{(0)} - f^{(0)} p\beta \Delta\beta. \end{aligned} \quad (3)$$

Using $f^{(0)} = e^{-p\beta}$ and putting eq. (3) in eq. (2), we get

$$\begin{aligned} \frac{\partial}{\partial t} [e^{-p\beta} - p\beta e^{-p\beta} \Delta\beta] + \frac{p^i}{E} \frac{\partial}{\partial x^i} [e^{-p\beta} - p\beta e^{-p\beta} \Delta\beta] = \\ - \frac{f - f^{(0)}}{t_R}, \end{aligned} \quad (4)$$

where $v^i = p^i/E$, and we assume the relaxation time approximation for the collision term $\mathcal{C}[f]$, with t_R as the relaxation time. Since equilibrium distributions are stationary and (in absence of external force) homogeneous, the BTE for equilibrium distributions is identically satisfied. In the present scenario, we assume the equilibrium distribution function $f^{(0)}$ to be stationary for a time duration much longer than the observation time allowed by BTE (but this time should be much less than t_R , within which the distribution changes appreciably), then

$$\frac{\partial}{\partial t} f^{(0)} + \frac{p^i}{E} \frac{\partial}{\partial x^i} f^{(0)} = 0 \quad (5)$$

and hence, eq. (4) becomes

$$-p \frac{\partial \Delta\beta}{\partial t} \beta f^{(0)} - \frac{p^i}{E} \beta p f^{(0)} \frac{\partial \Delta\beta}{\partial x^i} = \frac{p\beta \Delta\beta f^{(0)}}{t_R} \quad (6)$$

if we assume the average inverse temperature of the hot zones to be changing very slowly with time.

Expressing $\Delta\beta(\mathbf{r}, \hat{p}; t)$ in terms of its Fourier Transform

$$\Delta\beta(\mathbf{r}, \hat{p}; t) = \int d^3k \Delta\beta_k(t) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (7)$$

where we denote $\Delta\beta(\mathbf{k}, \hat{p}; t) \equiv \Delta\beta_k(t)$ for simplicity. Equation (6) becomes

$$\begin{aligned} -p\beta \frac{\partial \Delta\beta_k(t)}{\partial t} - \frac{p\beta}{t_R} \Delta\beta_k(t) - i\beta \frac{|p|}{E} p k \mu \Delta\beta_k(t) = 0 \\ \frac{\partial \Delta\beta_k(t)}{\partial t} = - \left[i \frac{|p|}{E} k \mu + \frac{1}{t_R} \right] \Delta\beta_k(t), \end{aligned} \quad (8)$$

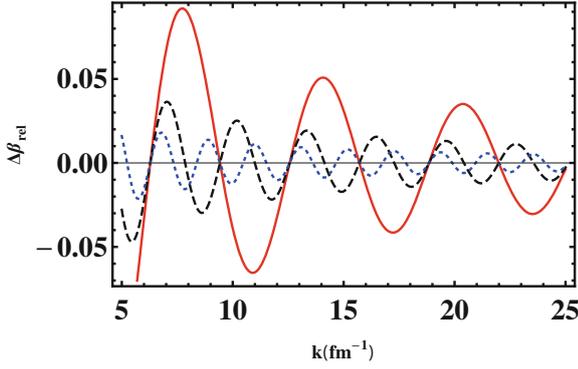


Fig. 2. (Color online) Variation of $\Delta\beta_{\text{rel}}(\mathbf{k}; t)$ with k . Red (solid): $(t - t^0) = 1$ fm, black (dashed): $(t - t^0) = 2$ fm, blue (dotted): $(t - t^0) = 3$ fm for $t_R = 3$ fm.

where $\hat{k} \cdot \hat{p} = \mu = \cos \theta$ (θ is the angle \hat{k} makes with \hat{p}). The solution of eq. (8) is then given by (see [24], for example, for similar equation in context of energy density fluctuation)

$$\Delta\beta(\mathbf{k}, \hat{p}; t) = \Delta\beta(\mathbf{k}, \hat{p}; t^0) e^{-i \frac{|p|}{E} k \mu (t - t^0)} e^{-\frac{t - t^0}{t_R}}. \quad (9)$$

We can simplify eq. (9) by assuming an isotropic fluctuation profile. Thus, assuming $|\mathbf{p}| = E$ we get a simplified expression for the temporal evolution of fluctuation for a medium of massless particles. We further average over the whole solid angle Ω subtended by \hat{p} . Here \mathbf{k} is a constant vector assumed to be directed along the z -axis. Hence, the averaged fluctuation becomes

$$\begin{aligned} \Delta\beta_{\text{av}}(\mathbf{k}; t) &= \Delta\beta(\mathbf{k}; t^0) e^{-\frac{t - t^0}{t_R}} \frac{1}{4\pi} \int_{\Omega} e^{-ik\mu(t - t^0)} d\Omega \\ &= \Delta\beta(\mathbf{k}; t^0) e^{-\frac{t - t^0}{t_R}} \frac{1}{4\pi} \int_{-1}^1 d\mu e^{-ik\mu(t - t^0)} \int_0^{2\pi} d\phi \\ \Delta\beta_{\text{rel}}(\mathbf{k}; t) &= \frac{\Delta\beta_{\text{av}}(\mathbf{k}; t)}{\Delta\beta(\mathbf{k}; t^0)} \\ &= e^{-\frac{t - t^0}{t_R}} \frac{\sin k(t - t^0)}{k(t - t^0)}. \end{aligned} \quad (10)$$

From eq. (10), we can infer that the relative fluctuation $\Delta\beta_{\text{rel}}(\mathbf{k}; t)$ is monotonically decreasing. In fig. 2 we provide the plot depicting the parametric Fourier space variation of the $\Delta\beta_{\text{rel}}(\mathbf{k}; t)$ with time $(t - t^0)$. The reliability of the variations shown in the figure is governed by the constraint that the observation time must be much less than the time taken by the distribution function to change appreciably [25], *i.e.* the relaxation time t_R ,

$$(t - t^0) \ll t_R. \quad (11)$$

According to our earlier assumption about very slow variation of β with time, $(t - t^0)$ should also be such a time-interval within which we can assume the temperature of the hotspots not to change appreciably.

We observe in fig. 2 that the relative fluctuations die down with time. Additionally, the soft modes of fluctuations, or in other words, fluctuations at larger distances towards the periphery of the medium, are large. We observe

no modification of fluctuation with increasing t_R when $(t - t^0) \ll t_R$.

We have thus solved the evolution equation for the β -fluctuation in the Fourier space using Boltzmann Transport Equation. However, the generality of our calculation is limited by the upper bound in eq. (11). Consequently, eq. (9), which assumes a very slow variation of temperature, cannot be applied to certain cases involving arbitrarily large observation times within which temperature changes appreciably. With the aim to study a more realistic situation, we can consider the temperature profiles of an evolving medium at different time slices which are arbitrarily separated. After quantifying the inverse temperature fluctuation, we can find out the inverse temperature fluctuation at every time-instant and will try to observe their variation at different stages.

As an example, we have chosen the radially varying temperature profile of a viscous medium created by heavy-ion collisions from ref. [26]. We can characterize the temperature profile of a viscous medium created in central HIC (and evolving hydrodynamically) shown in ref. [26] by the following function:

$$T_M(r, t) = \frac{T_0(t)}{e^{a(t)(\frac{r}{r_0} - 1)} + 1}, \quad (12)$$

where at $r = r_0$, $T_M(r) = T_0/2$; and $T_M(r) \approx T_0$ at $r = 0$ and $a(t)$ is a parameter which fixes how sharply the function drops down. From eq. (12), writing $\beta_M = 1/T_M$ we get

$$\beta_M(r; t) = \beta_0(t) \left(e^{a(t)(\frac{r}{r_0} - 1)} + 1 \right), \quad (13)$$

where r denotes the radial distances of the zones from the centre of the medium. Using eq. (13), we can generate $\{\beta_M\}$ — a collection of β_M values. Given the collection, we can now define an average β_M value $\langle \beta_M \rangle = \beta(t)$ at a certain instant t and can define a fluctuation $\Delta\beta(r, t)$ as

$$\begin{aligned} \Delta\beta(r, t) &= \beta_M(r, t) - \beta(t) \\ &= \beta_0(t) e^{a(t)(\frac{r}{r_0} - 1)} + \delta\beta(t), \end{aligned} \quad (14)$$

where $\delta\beta(t) = \beta_0(t) - \beta(t)$. The Fourier Transform $\Delta\beta(k; t)$ now becomes

$$\begin{aligned} \Delta\beta(k; t) &= \Delta\beta_k \\ &= \frac{2\beta_0(t)}{(2\pi)^2 k} \int_0^R e^{a(t)(\frac{r}{r_0} - 1)} r \sin(kr) dr + \delta\beta(t) \delta(\mathbf{k}), \end{aligned} \quad (15)$$

where R is the system size (taken to be 15, 17.5 and 20 fm at proper times $\tau = 2.2, 5.1$ and 9.1 fm, respectively) and $\delta(\mathbf{k})$ is the Dirac delta function.

3 Results and discussion

As seen from eq. (15), the soft modes of β -fluctuation become dominant implying that towards the periphery (at

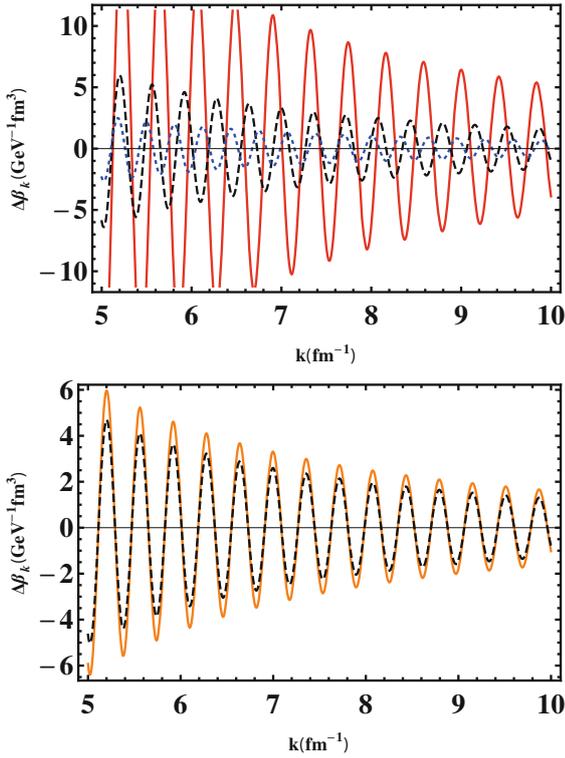


Fig. 3. (Color online) Variation of inverse temperature fluctuation (eq. (15)) in a viscous medium with k . (Upper panel) Red (solid): $\tau = 2.2$ fm/c, black (dashed): $\tau = 5.1$ fm/c, blue (dotted): $\tau = 9.1$ fm/c. $\eta/s = 0.08$ for all the figures. (Lower panel) Orange (solid): $\eta/s = 0.08$, black (dashed): $\eta/s = 0.3$ at $\tau = 5.1$ fm/c.

Table 1. Values of parameters extracted from the temperature profile shown in ref. [26] using eq. (12) and the \mathcal{R}_β values using eq. (16) at different τ with $\eta/s = 0.08$.

τ (fm/c)	β_0 (GeV $^{-1}$)	a	r_0 (fm)	\mathcal{R}_β
2.2	3.45	5.99	7.96	0.047
5.1	4.55	3.42	8.41	0.011
9.1	5.56	1.91	8.71	0.002

large system radius), the fluctuation is greater. The variation of the inverse temperature fluctuation in the momentum space is shown in the upper panel of fig. 3. The lower panel of fig. 3 shows the variation of fluctuation for different viscosities of the medium. As intuitively expected, higher viscosity favours lower fluctuations.

In the previous section, we have already defined the average $\beta(t)$ and fluctuation $\Delta\beta$ with the help of the set $\{\beta_M\}$. With the aid of the set $\{\beta_M\}$ generated from eq. (12) we can now define a relative β -fluctuation

$$\frac{\langle \beta_M^2 \rangle - \langle \beta_M \rangle^2}{\langle \beta_M \rangle^2} = \mathcal{R}_\beta. \quad (16)$$

Using the β_0 , a and r_0 values as tabulated in table 1, we compare the \mathcal{R}_β in the system produced in HICs at different stages of its evolution with the help of eq. (13).

Table 2. Relative fluctuations in β at $\tau = 5.1$ fm/c with change of viscosity.

η/s	\mathcal{R}_β
0.08	0.012
0.3	0.011

We observe that within any arbitrary choice of radius shell the relative fluctuations die down with time. For demonstration, we have chosen the shell ranging between the radii 14 fm to 15 fm in table 1, but our observation remains unaltered for any other shell. This behavior of temperature fluctuation has also been observed in refs. [19,20].

In table 2, we show the change in \mathcal{R}_β with viscosity (for a radius shell ranging between 14 fm and 15 fm). With increasing viscosity, the relative fluctuation decreases (see also refs. [19,20]), thereby leading to lower \mathcal{R}_β values.

As it turns out, the multiparticle production processes in high-energy electron-positron [14], hadronic and heavy-ion collisions [27–34] are quite accurately characterized by a Tsallis entropic parameter q [35], which is similar to \mathcal{R}_β , and lies typically in the range $1 < q < 1.2$ [36] in the context of high-energy collisions. Here, we would like to briefly mention some recent works done by the authors in [35,38] connecting the q -parameter and the temperature fluctuation or *non-extensivity* of thermal systems. The non-extensive nature is manifested once we find out that the simple addition of entropies (S) of two sub-parts (A and B) of a bigger system C does not give the entropy of the system C . Rather, $S(C) = S(A) + S(B) + (1-q)S(A)S(B)$, where q measures the degree of deviation from the additive domain. This leads to a proposal of modification of the usual Boltzmann-Gibbs formula to

$$G_q(x) = [1 + (q-1)\beta E]^{-\frac{1}{q-1}}. \quad (17)$$

As $q \rightarrow 1$, $G_q(x) \rightarrow e^{-\beta E}$, and we recover the usual Boltzmann-Gibbs formula. Therefore, q has also been dubbed as the non-extensivity parameter in the literature. In an elegant exposition of the same, Wilk [38] deduced that

$$G_q(x) = [1 + (q-1)\beta E]^{-\frac{1}{q-1}} = \int_0^\infty e^{-\beta' E} f(\beta') d\beta', \quad (18)$$

where the distribution function $f(\beta')$ is the usual chi-squared function given by

$$f(\beta') = \frac{\alpha\beta}{\Gamma(\alpha)} \left(\frac{\alpha\beta}{\beta'}\right)^{\alpha-1} \exp\left(-\frac{\alpha\beta}{\beta'}\right), \quad (19)$$

where $\alpha = \frac{1}{q-1}$. With respect to the above chi-squared distribution, we have the mean value $\langle \beta' \rangle = \beta$, and also the relative variance as

$$\frac{\langle \beta'^2 \rangle - \langle \beta' \rangle^2}{\langle \beta' \rangle^2} = q - 1. \quad (20)$$

Table 3. Comparison of \mathcal{R}_β at the boundary as obtained from eq. (12) with the $(q-1)$ value obtained from experiment [37].

\mathcal{R}_β	$(q-1)$
0.013	0.018 ± 0.005

We can therefore make a correspondence between the \mathcal{R}_β defined in eq. (16) and the $(q-1)$ defined in eq. (20). The non-zero values of q are associated not only with the relative β fluctuation in the system, but also with that during the hadronization process [38–40]. This also gains significance in the context of the QCD phase diagram and the search for critical point which, in fact, may be associated with large fluctuations in the q value [21]. Non-extensivity of any thermodynamic system is invariably linked to temperature fluctuation, and hence, the heat capacity/specific heat of the system. However, whether the converse statement holds is yet to be answered.

The q values for systems produced in high-energy collisions can be obtained [37] by fitting the experimentally observed particle spectrum which is obtained by averaging over similar events. Since the present calculation is based on a single event, (assuming the medium has frozen-out by $\tau = 9.1$ fm), we can study the temperature profile at $\tau = 9.1$ fm to compare the \mathcal{R}_β values with the experimentally observed $(q-1)$ values under similar conditions [37]. According to table 3, the \mathcal{R}_β value (~ 0.013) at the system boundary is comparable with the experimentally obtained value ($\sim 0.018 \pm 0.005$ for 0–10% central HICs at RHIC with $\sqrt{s_{NN}} = 200$ GeV [37]). This observation re-emphasizes the relationship between temperature fluctuation and the q parameter [38] and hints towards a possibility that the temperature fluctuation inherent in the quark-gluon system may give rise to the temperature fluctuation in the hadronic spectra obtained after freeze-out.

Recently, modification of eq. (20) has been suggested in ref. [41] where reservoir fluctuations are taken into account. The modified relation is given by

$$q-1 = \frac{\langle \beta'^2 \rangle - \langle \beta' \rangle^2}{\langle \beta' \rangle^2} - 1/C, \quad (21)$$

where C is the heat capacity of the reservoir. It will be interesting to extend the present analysis having this consideration. Additionally, some comments about connecting certain observables in HICs with the theory of cosmological perturbations are in order. Cosmological anisotropies reflect the energy content of the universe. The universe starts off as radiation dominated, changes over to being matter dominated, and is eventually conjectured to be purely governed by the cosmological constant (Λ) [23]. WMAP [42] and Planck [43] both provide a fairly precise representation of the energy distribution at our current epoch—via physical quantities like Ω_{matter} , Ω_Λ , $\Omega_{baryons}$, the acoustic scale, Hubble constant, neutrino fraction, reionization optical depth and other derived quantities. Since the anisotropies in temperature fluctuation are all time-dependent, in later epochs these fluctuations would die down, and theoretically one should expect a flat power

spectrum in the infinite future. However, authors in [44] have conjectured that the temperature fluctuation of our universe can be satisfactorily explained by the *modified Boltzmann-Gibbs* formula with $q = 1.045 \pm 0.005$. This is quite remarkable since the similar q -value that fits heavy-ion collision data also fits the data for cosmological fluctuations. This points to deep similarities between the physics of cosmic microwave background (CMB) radiation anisotropies and the flow anisotropies in relativistic heavy-ion collision experiments (RHICE). A relevant theoretical question would be: is the surface of last scattering for CMB radiation similar to the freeze-out surface in RHICE? This is a question we reserve for future work.

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References

1. S. Jeon, V. Koch, Phys. Rev. Lett. **85**, 2076 (2000).
2. M. Asakawa, U.W. Heinz, B. Müller, Phys. Rev. Lett. **85**, 2072 (2000).
3. M.A. Stephanov, K. Rajagopal, E.V. Shuryak, Phys. Rev. D **60**, 114028 (1999).
4. M.A. Stephanov, K. Rajagopal, E.V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).
5. NA49 Collaboration (S.V. Afanasiev *et al.*), Phys. Rev. Lett. **86**, 1965 (2001).
6. STAR Collaboration (B.I. Abelev *et al.*), Phys. Rev. C **75**, 064901 (2007).
7. PHENIX Collaboration (A. Adare *et al.*), Phys. Rev. D **83**, 052004 (2011).
8. PHENIX Collaboration (A. Adare *et al.*), Phys. Rev. C **83**, 064903 (2011).
9. ALICE Collaboration (K. Aamodt *et al.*), Phys. Lett. B **693**, 53 (2010).
10. CMS Collaboration (V. Khachatryan *et al.*), JHEP **02**, 041 (2010).
11. CMS Collaboration (V. Khachatryan *et al.*), Phys. Rev. Lett. **105**, 022002 (2010).
12. ATLAS Collaboration (G. Aad *et al.*), New J. Phys. **13**, 053033 (2011).
13. ALICE Collaboration (K. Aamodt *et al.*), Eur. Phys. J. C **73**, 2662 (2013).
14. I. Bediaga, E.M.F. Curado, J. Miranda, Physica A **286**, 156 (2000).
15. M.D. Azmi, J. Cleymans, Eur. Phys. J. C **75**, 430 (2015).
16. J. Cleymans, D. Worku, Eur. Phys. J. A **48**, 160 (2012).
17. B. Mohanty, Jan-e Alam, T.K. Nayak, Phys. Rev. C **67**, 024904 (2003).
18. M.A. Stephanov, Phys. Rev. D **81**, 054012 (2010).
19. S. Basu, R. Chatterjee, B.K. Nandi, T.K. Nayak, arXiv:1504.04502 [nucl-ex].

20. S. Basu, R. Chatterjee, B.K. Nandi, T.K. Nayak, arXiv:1405.3969 [nucl-ex].
21. D.K. Mishra, P. Garg, P.K. Netrakanti, A.K. Mohanty, J. Phys. G **42**, 105105 (2015).
22. H.B. Prosper, Am. J. Phys. **52**, 629 (1992).
23. S. Dodelson, *Modern Cosmology* (Academic Press, California, USA, 2003).
24. G. Sarwar, J.-e Alam, arXiv:1503.06019 [nucl-th].
25. R. Balescu, *Equilibrium and Non-equilibrium Statistical Mechanics* (John Wiley & Sons Inc., Hoboken, USA, 1975).
26. R. Baier, P. Romatschke, Eur. Phys. J. C **51**, 677 (2007).
27. W.M. Alberico, A. Lavagno, P. Quarati, Eur. Phys. J. C **12**, 499 (2000).
28. C.E. Aguiar, T. Kodama, Physica A **320**, 371 (2003).
29. T.S. Biró, A. Jakovac, Phys. Rev. Lett. **94**, 132302 (2005).
30. T.S. Biró, G. Purcsel, Phys. Rev. Lett. **95**, 162302 (2005).
31. M. Biyajima, M. Kaneyama, Eur. Phys. J. C **40**, 243 (2005).
32. O.V. Utyuzh, G. Wilk, Z. Włodarczyk, Acta Phys. Hung. A **25**, 65 (2006).
33. M. Biyajima, T. Mizoguchi, N. Nakajima, N. Suzuki, G. Wilk, Eur. Phys. J. C **48**, 597 (2007).
34. T. Osada, G. Wilk, Phys. Rev. C **77**, 044903 (2008).
35. C. Tsallis, J. Stat. Phys. **52**, 479 (1988).
36. C. Beck, Physica A **286**, 164 (2000).
37. Z. Tang, Y. Xu, L. Ruan, G.v. Buren, F. Wang, Z. Xu, Phys. Rev. C **79**, 051901 (2009).
38. G. Wilk, Z. Włodarczyk, Phys. Rev. Lett. **84**, 2770 (2000).
39. C. Beck, Phys. Rev. Lett. **87**, 180601 (2001).
40. C. Beck, E.G.D. Cohen, Physica A **322**, 267 (2003).
41. T.S. Biró, P. Ván, G.G. Barnaföldi, K. Ürmössy, Entropy **16**, 6497 (2014).
42. E. Komatsu *et al.*, Astrophys. J. **192**, 2 (2011).
43. Planck Collaboration, arXiv:1502.01589 [astro.ph-CO].
44. A. Bernui, C. Tsallis, T. Villela, Phys. Lett. A **356**, 426 (2006).