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Letter

Glueball dynamics in the hot plasma

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Abstract. We discuss the glueball contribution to the equation of state (EoS) of hot gluon matter below and above T_c. We show that the strong variation of the masses of the scalar and pseudoscalar glueballs near T_c is determining the thermodynamics of the $SU(3)$ gauge theory. We provide arguments to justify that these glueballs become massless at $T_G \approx 1.1T_c$, a phenomenon which is crucial to understand the mysterious behavior of the trace anomaly found in lattice calculations.

The description of the thermodynamics of the pure $SU(3)$ gauge theory is one of the benchmarks of the our understanding of the properties of the Quark-Gluon Plasma (QGP). Recently very precise lattice results for the EoS of this theory at finite T, below and above deconfinement temperature T_c, were presented [1]. They challenge our understanding of QCD dynamics at finite temperatures. One of the more puzzling behaviors of these results is the temperature dependence of the trace anomaly, $I/T^4 = (\epsilon - 3p)/T^4$. More precisely, just above T_c the trace anomaly grows rapidly up to $T_G \approx 1.1T_c$ and then it decreases as $I/T^4 \sim 1/T^2$ up to $T \approx 5T_c$. Such behaviour was found for the first time by Pisarski analyzing the pioneer lattice calculation of ref. [2], and named it the manifestation of "fuzzy bags" [3] (see also the discussion of this phenomenon within an AdS/QCD approach in ref. [4]). A non-zero value of trace anomaly shows the deviation of the EoS from an ideal gluon gas, and therefore, gives important information regarding the interaction between the gluons. It is known that below T_c the EoS for pure $SU(3)$ is described rather well by a gas of the massive glueballs [5,1]. In this case the masses and spins of the glueballs carry information on the interaction of the gluons inside the glueballs (see review [6]). What happens with the glueballs and how their properties change above T_c is still an open question and only few studies have been done in this direction [7–9].

In this Letter we provide arguments to support the importance of the scalar and pseudoscalar glueballs in the EoS of the pure $SU(3)$ gauge theory above T_c. In particular, it will be shown that the most likely scenario is that these two glueballs become massless at $T_G \approx 1.1T_c$ and this phenomenon is crucial to understand the behavior of the trace anomaly above T_c.

Our starting point is the relation between the lowest scalar glueball mass, m_G, and the gluon condensate, $G^2 = \langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ at $T = 0$, which appeares naturally in the dilaton approach [10,11]

$$m_G^2 f_G^2 = \frac{11N_c}{6}\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle, \tag{1}$$

where f_G is the glueball coupling constant to gluons. Lattice calculations show that the gluon condensate decreases roughly by factor two at $T = T_c$ [12–15] due to the strong suppression of its electric component, while slightly above T_c the condensate vanishes very rapidly due to the cancellation between its magnetic and electric components [14]. The temperature behavior of the condensate at $T_G \geq T \geq T_c$ can be described by the equation [15]

$$G^2(T) = G^2\left[1 - \left(\frac{T}{T_G}\right)^n\right] \tag{2}$$

with $n = 4$. For $T > T_G$ the gluon condensate vanishes[1]. If the relation eq. (1) is valid also above T_c, then the mass of the scalar glueball should go to zero at $T = T_G$. The possibility of collapse of the 0^{++} glueball to a massless state above T_c due to strong attraction between two gluons in this channel, induced by both the perturbative gluon exchange and the non-perturbative interaction with instanton-antiinstanton molecules, was pointed out in refs. [16,9]. Based on these considerations, we assume that the mass of lowest scalar glueball goes to zero at

[1] We should mention that the particular value of n is not very important in our qualitative discussion on the EoS below.

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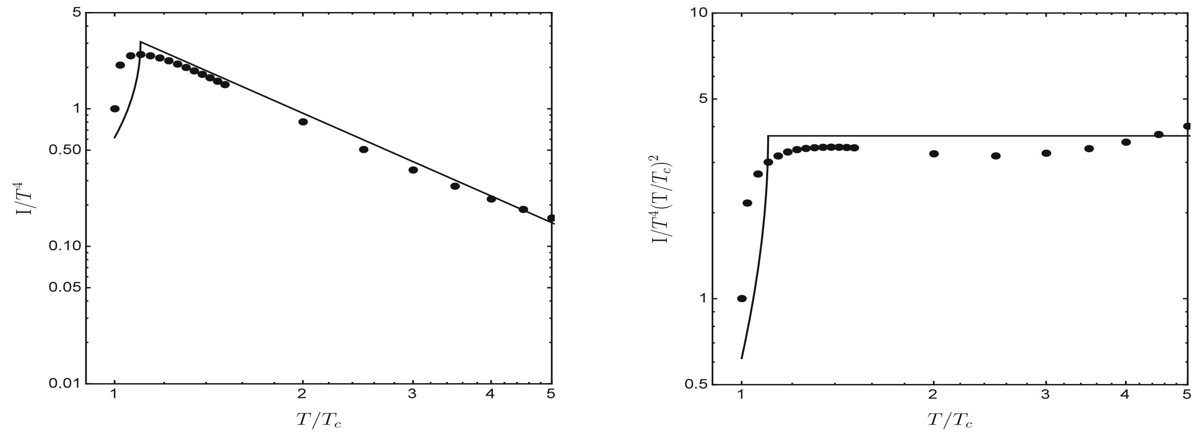

Fig. 1. The trace anomaly $I(T)/T^4$ (left panel), and its scaling value $(I(T)/T^4)(T/T_c)^2$ (right panel) as a function of T/T_c. The lattice data presented by the bold spots are from ref. [1].

$T = T_G$. We would like to emphasise that Debye screening cannot lead to the breaking of the lowest mass scalar glueball state in the plasma because even at $T = 0$ the size of this glueball is very small $R_G \leq 0.2\,\mathrm{fm}$ [17–19] (see discussion in [9]).

The important question next is what happens with the other glueballs above T_c? In ref. [9] it was argued that the lowest pseudoscalar glueball 0^{-+} should be degenerate with lowest scalar glueball at $T > T_c$ because the difference between the two masses might be related to the admixture of the so-called random instanton contribution to the vacuum state which is strongly suppressed for $T > T_c$ [20–23]. Therefore, the lowest preudoscalar glueball becomes also massless at $T = T_G$. It is therefore evident that just after T_c all others heavy glueballs decay very fast to these light glueballs. We can thus conclude that the thermodynamics of the theory for $T_G \geq T \geq T_c$ is determined only by the light glueballs. In this phase the gluons are still confined inside the two light glueballs and the pressure is given by

$$P(T)_{T_G>T>T_c} = -T\frac{N_G}{2\pi^2}\int_0^\infty k^2\mathrm{d}k \times \mathrm{Log}\left[1-\exp\left(-\frac{\sqrt{k^2+m^2(T)}}{T}\right)\right], \tag{3}$$

where $N_G = 2$ for two glueball states. The trace anomaly

$$\Delta(T) = \frac{I(T)}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{P}{T^4}\right) \tag{4}$$

for the case of a temperature dependent boson mass in a hot plasma is given by [24]

$$\Delta(T)=\frac{N_G}{2\pi^2}\int_0^\infty \mathrm{d}x\frac{x^2}{\sqrt{x^2+m(T)^2/T^2}(e^{\sqrt{x^2+m(T)^2/T^2}}-1)} \times\frac{m(T)^2}{T^2}\left[1-\frac{T}{m(T)}\frac{\mathrm{d}m(T)}{\mathrm{d}T}\right], \tag{5}$$

where the mass function, according to eqs. (1) and (2), for $T_G \geq T \geq T_c$ is given by

$$m(T) = m_0\sqrt{1-\left(\frac{T}{T_G}\right)^4}. \tag{6}$$

Here $\Delta(T)$ is a monotonically increasing function of T due to the decreasing behavior of the glueball mass. At $T = T_G$ the mass of glueball is zero and a new phase begins. This behavior is in the agreement with lattice data [1] (see fig. 1). Above T_G the massless glueballs can dissociate to massless gluons due to the process $G+G \rightarrow gluon+gluon$, therefore, at $T > T_G$ one should have a mixed glueball-gluon phase.

At very large temperature we have a free gluon gas and the contribution of glueballs to the pressure should be suppressed with respect to the free gluon case by a factor $(T_G/T)^\lambda$ with $\lambda > 0$. Based on the scaling in the trace anomaly $I(T)/T^4(T/T_c)^2$ observed in lattice calculation [1] we use $\lambda = 2$. With this assumption the pressure above T_G is the sum of glueball and gluon pressures

$$P(T)_{T>T_G} = \frac{N_G\pi^2}{90}T^4\left(\frac{T_G}{T}\right)^2+\frac{N_g\pi^2}{90}T^4\left[1-\left(\frac{T_G}{T}\right)^2\right], \tag{7}$$

where $N_g = 2(N_c^2-1) = 16$ is the number of the gluon degrees of freedom. By using eq. (7), we obtain for the scale anomaly at $T > T_G$

$$\Delta(T) = \frac{(N_g-N_G)\pi^2}{45}\left(\frac{T_G}{T}\right)^2. \tag{8}$$

Therefore, in this region the anomaly decreases fast with increasing temperature. This is also in agreement with lattice data [1] (see fig. 1). In our model, this phenomenon results from the decrease of the fraction of glueballs in the hot gluon plasma with increasing temperature. We emphasize that T_G is an important milestone since there the masses of both scalar and pseudoscalar mesons vanish. Furthermore, the trace anomaly increases with growing temperature before T_G and decreases after

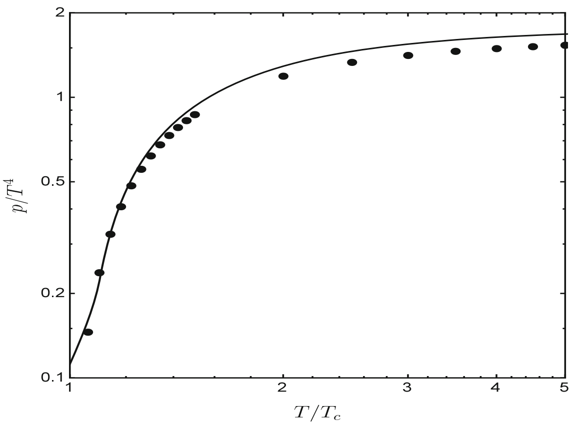

Fig. 2. The pressure p/T^4 as a function of T/T_c compared with the lattice data of ref. [1].

T_G, thus the trace anomaly has a maximum at T_G, (see fig. 1 (left panel)). The value of T_G can be determined from lattice results, $T_G \approx 1.1T_c$. Moreover, at $T = T_G$ the value of trace anomaly in the light glueball phase is given by

$$\Delta^G(T \to T_G) = \frac{N_G\zeta(2)}{\pi^2}\frac{m_0^2}{T_G^2} \tag{9}$$

and should be equal to its value in the mixed glueball-gluon phase

$$\Delta^{Gg}(T \to T_G) = \frac{(N_g - N_G)\pi^2}{45}, \tag{10}$$

where $\zeta(2) = \pi^2/6$ is the Riemann zeta function. This condition determines the parameter m_0

$$m_0 = \sqrt{\frac{2(N_g - N_G)}{15N_G}}\pi T_G. \tag{11}$$

For $T_c \approx 270\,\text{MeV}$ for pure $SU(3)$ gauge theory we obtain $m_0 = 901\,\text{MeV}$ and $m(T_c) = 507\,\text{MeV}$. Thus indeed in the temperature range $T_G > T > T_c$ we have very light scalar and pseudoscalar glueballs, $m(T_c) > m(T) > 0$.

In figs. 1 and 2 the result of the trace anomaly and pressure calculations are presented. One can see that they are in qualitative agreement with the lattice data at $5T_c \geq T \geq T_c$ [1]. We would like to emphasize that our simple model does not include perturbative QCD corrections to the EoS. Therefore, it might be possible to improve the agreement of the model with the data if we take into account such corrections. It should be also mentioned that our model for $T > T_G$, eq. (7), predicts for the trace anomaly a small violation of the $2(N_c^2 - 1)$ scaling observed in lattice calculations for the large N_c [25][2]. It would be interesting to check this prediction with a more precise lattice calculation.

In summary, we can conclude that there are three phases in the pure $SU(3)$ gauge theory at finite T. At $T < T_c$ there is the gas of the massive glueballs. Just above T_c at $T_G \geq T \geq T_c$ the phase is dominated by the light scalar and pseudoscalar mesons. After $T_G \approx 1.1T_c$ we have a mixed phase of massless gluons and scalar-pseudoscalar massless glueballs. It is evident that the above description should also influence the EoS of the full QCD including light quarks. In the latter case, the mixing of the glueballs with the quark-antiquark states should be taken into account as well. The study of mechanisms is the subject of our future investigation.

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