

On a possible interpretation of the $X(3872)$ as a 1^1D_2 state in a constituent-quark model based on a generalized Fermi-Breit equation

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Abstract. A recent experimental analysis suggested to represent the $X(3872)$ -resonance as a $c\bar{c} 1^1D_2$ state but this attribution is being hotly debated. We calculate the mass values for that state by means of a previously studied constituent-quark model. The different contributions of the model Hamiltonian to the total mass are also explicitly shown.

1 Introduction

In a previous work [1] that will be denoted as (CSI) in the following, we studied the charmonium ($c\bar{c}$) spectrum by means of a semirelativistic constituent-quark model in which the interaction operator was derived by using a covariant procedure that generalizes the standard Fermi-Breit reduction. Such procedure gives for the relativistic corrections (of order c^{-2}) two kinds of contributions:

- i) the standard terms, *i.e.* the Darwin, spin-orbit, spin-spin and tensor operators;
- ii) some specific momentum-dependent (or nonlocal), Hermitian, terms.

The form of *all* these relativistic correction operators is determined by the c^0 potentials, according to the tensor rank of the underlying interacting field. In particular, for the numerical calculations of the model a purely vector and a mixture of vector and scalar interactions were considered.

The main objective in CSI was to determine, with the Hamiltonian of the model, the general feature of the charmonium spectrum. For this reason we used for the fitting procedure the 12 $c\bar{c}$ resonances with well-established quantum numbers [2], denoted as (WE) resonances.

As for the resonance $X(3872)$, whose quantum numbers were not clearly determined, a theoretically original *molecular* interpretation [3] was shortly discussed; we also considered the hypothesis of a standard $c\bar{c}$ state of

the type $\chi_{c1} : 2^3P_1$, that gives the quantum numbers $J^{PC} = 1^{++}$. These quantum numbers were considered, at that time, the most probable experimental assignment for the $X(3872)$. A thorough discussion about this point is given in a recent review on the heavy quarkonium states [4].

The results of our model for the mass of the $\chi_{c1} : 2^3P_1$ state (that, given the general uncertainties, was not included the fit) were given in table 1 of CSI; here we recall their values: column VR - 3907 MeV for a relativistic vector interaction and the fit performed over the 12 WE resonances; column VSR - 3912 MeV for a relativistic vector and scalar interaction and the fit performed over the 12 WE resonances; column BTR - 3915 MeV for a relativistic vector and scalar interaction and the fit performed only on the 8 WE resonances below the open charm threshold. As will be explained in sect. 2, the results of the columns VSN and VCR are not relevant for the present discussion.

Very recently the properties of the $X(3872)$ have been deeply re-examined both theoretically and experimentally, motivating the development of the present work.

The experimental relevant novelty is represented by the evidence, reported by the BABAR Collaboration, for the decay mode $X(3872) \rightarrow J/\psi \omega$ [5]. Their analysis favors the assignation $J^{PC} = 2^{-+}$ (in contrast to the previous one $J^{PC} = 1^{++}$) and the authors explicitly suggest the interpretation as a charmonium $\eta_{c2}(1D)$ state.

This result is analyzed in a theoretical study [6] where all the properties of $X(3872)$ are critically examined in the framework of various phenomenological models. The

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authors show that the BABAR quantum number assignment disfavor the molecular interpretation. As for the conventional $c\bar{c}$ interpretation, the authors highlight several difficulties; among them:

- i) the production cross-section is predicted much smaller than the experimental value;
- ii) the decay rates, both hadronic and radiative, are not easily reproduced;
- iii) different quark models do not reproduce the resonance mass.

A criticism about the η_{c2} assignment is also given in a recent theoretical work [7].

In this context it is of some interest to calculate now the mass of the $\eta_{c2} : 1^1D_2$ state within the quark model introduced in CSI. That state was not considered relevant before the recent re-examination of the $X(3872)$ properties.

We shall not discuss the production and decay processes that require theoretical tools not contained in our quark model.

The next section is devoted to show and comment the results and to draw some tentative conclusions.

2 The results of the model

We have performed exactly the same calculations of CSI (with the same parameters), for the mass of the $\eta_{c2} : 1^1D_2$ state, that was omitted in CSI. The results are shown in table 1. In more detail, we give the mass of this state for the VR, VSR and BTR versions of the model, as defined in CSI and recalled here in sect. 1. Two versions of the model of CSI have been ignored here: the VSN version, with a nonrelativistic kinetic energy, because in CSI it did not give new physical information with respect to the relativistic one; the VCR version, with a Cornell potential, because it could not reproduce accurately the WE resonances.

We also give in the same table 1 the contributions of the different terms of the Hamiltonian to the total mass; more precisely, these terms are: the relativistic kinetic energy, T^{rel} , standardly introduced in eq. (13) of CSI; the interaction terms introduced in eqs. (2) and (10) of CSI, for the vector and scalar interaction, respectively. In more detail, these interaction terms give rise to the following contributions listed in the table 1: the potential V^0 , the Darwin term V^D , the spin-orbit term V^{so} , the spin-spin term V^{ss} , the tensor term V^{te} and, finally, the momentum-dependent terms, collectively denoted as V^{md} ; V^0 represents the interaction operator of order c^0 while all the other interaction terms are of order c^{-2} .

The sum of all these contributions gives the total mass M of the resonance, that is

$$M = T^{rel} + V^0 + V^D + V^{so} + V^{ss} + V^{te} + V^{md}. \quad (1)$$

Finally, in order to make a comparison with another D state, we also give in the table 1 the corresponding results for the $\psi : 1^3D_1$ state, whose total mass was given in CSI.

Table 1. Results of the CSI quark model for the states 1^1D_2 and 1^3D_1 . All the energies are in MeV.

$\eta_{c2} : 1^1D_2$		$M_{exp} = 3872.3 \pm 0.8$		
	VR	VSR	BTR	
M	3860	3838	3843	
T^{rel}	3082	2738	2721	
V^0	583	818	849	
V^D	2	3	3	
V^{so}	0	0	0	
V^{ss}	-5	-6	-5	
V^{te}	0	0	0	
V^{md}	198	285	275	
$\psi : 1^3D_1$		$M = 3772.92 \pm 0.35$		
	VR	VSR	BTR	
M	3806	3789	3796	
T^{rel}	3132	2759	2752	
V^0	509	767	777	
V^D	3	3	3	
V^{so}	-64	-58	-60	
V^{ss}	2	2	2	
V^{te}	-5	-4	-4	
V^{md}	229	320	326	

As for our new results, we note that the model with only a vector interaction VR gives a mass value very close to that of the $X(3872)$, that is only 12 MeV lower. The VSR model gives a value that is 34 MeV lower than the experimental one. Finally, the result of the BTR model is 29 MeV lower. We have also tried to vary the parameters of the VSR model including the $X(3872)$ in the fit, using a weight equal to 1/2 of that used for the other resonances. In this way only a small improvement is obtained, that is $M(1^1D_2) = 3844$ MeV, without changing significantly the masses of the other resonances.

We point out that the momentum-dependent terms of our model V^{md} give a relevant contribution to the masses of the two states of table 1. Furthermore, the spin-orbit term V^{so} , that is exactly vanishing for the $\eta_{c2} : 1^1D_2$, gives a sizable contribution to the $\psi : 1^3D_1$ state, while the Darwin V^D and spin-spin V^{ss} contributions are small for both states; the tensor contribution V^{te} is exactly vanishing for the $\eta_{c2} : 1^1D_2$ and very small for the $\psi : 1^3D_1$ state.

We can now draw the following conclusions. Our quark model does not exclude that the $\eta_{c2} : 1^1D_2$ can represent the $X(3872)$ -resonance. Our predictions are lower than the experimental value, but closer to it than some predictions quoted in ref. [6]. We point out that, however, the semirelativistic constituent-quark potential models usually parametrize in an effective way some selected field-theoretical effects. This objective is accomplished by using several parameters to reproduce accurately the WE resonances. In consequence, the predictive capability of these

models should not be overestimated, in particular for the high-energy resonances.

Further theoretical investigations are needed to establish a closer link between the quark models and the underlying field theory. Only in this way reliable results about the resonance properties can be obtained.

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