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Letter

Inadequacy of the statistical model: Some evidence for compound nuclei in the $A \approx 150$ and $E_x \approx 100$–200 MeV region

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Abstract. Light charged-particle multiplicities in the evaporation residue (ER) and fission (FF) channels from the reaction 200 MeV $^{32}S + ^{100}Mo$ as well as ER and FF channel cross-sections have been measured and compared to the predictions of the statistical model (SM) to estimate the fission time scale. The statistical model fails in reproducing the whole set of data and no convincing estimate is possible. In particular, while pre-scission multiplicities can be reproduced, the model strongly overestimates proton and alpha particle multiplicities in the ER channel, irrespective of the SM input parameters and prescriptions used for the level density and the transmission coefficients. Same calculations performed on data from literature in the $A \approx 150$ and excitation energy $E_x \approx 100$–200 MeV region, for the ER channel, provide similar conclusions. These findings repropose the problem of the reliability of the SM in describing the compound-nucleus decay and have a relevant impact on the extraction of the fission delay time through the use of the SM.

1 Introduction

The Statistical Model (SM) of nuclear decay has been extensively used to study fission dynamics [1] and, in particular, to estimate fission time scales on the basis of the particle emission from hot fissioning nuclei. A common procedure is to measure the light particle and/or gamma-ray pre-scission multiplicities and to try to reproduce the values of these observables with the SM. It turns out that the above multiplicities are in most of the cases under predicted. The widely used solution to resolve this problem is to introduce a mechanism in the SM that reduces the fission strength with respect to the particle evaporation strength. In the simplest fashion, the SM is modified to include another parameter, the fission delay τ_d: during the evaporative decay the fission width is kept to zero for a time τ_d, above which the fission width is set to the full value, given by Bohr-Wheeler model. Consequently, light particle evaporation is favored at the beginning of the decay, which results in a larger pre-scission multiplicity. In this scenario light particles are considered as a clock for the fission process. Estimates of τ_d are obtained by the fit of the experimental multiplicities with those predicted by the SM which includes this new parameter. The majority of these studies [1] have been performed with heavy composite nuclei ($A > 200$) and by fitting only neutron multiplicities. The reported values of the fission delay time are, however, quite different and range from zero [2] to 500×10^{-21} s [3], depending on the excitation energy and the experimental probe. Moreover, such estimates are made more uncertain by the fact that different sets of input parameters can provide equally good fits to the data [2,4].

It was pointed out in ref. [5] that in order to obtain a more reliable estimate of the fission delay it is necessary to search for additional observables with the aim to provide more constraints for the relevant input parameters

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Table 1. Proton and alpha particle multiplicities in the ER and pre-scission channels together with the FF and ER cross-sections for the 200 MeV $^{32}S + ^{100}Mo$ reaction. The calculation is performed with the parameters which best reproduce the FF channel data. See the text for details.

	ER channel		FF channel		σ_{FF} (mb)	σ_{ER} (mb)
	M_p	M_α	M_p	M_α		
Exp.	0.90 ± 0.14	0.56 ± 0.09	0.055 ± 0.007	0.038 ± 0.005	130 ± 13	828 ± 50
Present calc.	1.44	1.64	0.058	0.034	143	813

used in the model. It was further proposed [5] that the systems of intermediate fissility ($A \approx 100$–180) could provide additional observables for a more reliable picture of the occurrence of transient effects in the fission process. Compared to the heavier systems, those of intermediate fissility have larger pre-scission charged-particle multiplicities as well as comparable fission and evaporation residue (ER) cross-sections. Since the possibility of a hindered fission results in an increment of the ER cross-section, the light particle multiplicities in the ER channel, along with the ER integral cross-section, are proposed as potentially informative observables on the fission process. However, this hypothesis is founded on the reliability of the statistical model to reproduce the light particle multiplicities in the ER channel given all the necessary experimental constraints, and this has yet to be demonstrated.

In this letter we show the inadequacy of the SM for some systems when the evaporation residue channel is included as a further constraint in the procedure used to estimate the fission delay time. In particular, this analysis is applied to our recent experimental data and to other data found in the literature. Our experimental data concern the reaction 200 MeV $^{32}S + ^{100}Mo$ leading to the composite system ^{132}Ce at $E_x = 122$ MeV with fusion angular momentum $L_{fus} = 72\,\hbar$, derived from the measured fusion cross-section in the sharp cut-off approximation.

The experiment was performed at the Tandem-ALPI complex of the Laboratori Nazionali di Legnaro (LNL). We used the 8πLP apparatus [6] to detect light charged particles in coincidence with evaporation residues and fission fragments. The fission fragments were detected in the telescopes of the two most forward rings of the BALL section. Evaporation residues have been detected by means of four Parallel Plate Avalanche Counter modules. In a separate experiment the ER cross-section was measured by means of the electrostatic deflector of LNL [7] and the fission cross-section was measured with the double-arm time-of-flight spectrometer CORSET [8,9] at LNL as well.

For this system we have measured most of the relevant quantities in the ER and FF channels: proton and alpha particle energy spectra and multiplicities, ER and FF cross-sections. The extraction procedure of the proton and alpha particle multiplicities from the energy spectra measured in coincidence with ER and FF is the same as in refs. [10–12]. Here we concentrated on the analysis of the charged-particle multiplicities and cross-sections for both channels within the statistical model by changing multiple physical ingredients of the model. In a forthcoming paper a detailed description of all experimental data and calculations will be given.

The set of data is shown in table 1 together with the results of the SM calculations performed with the code PACE2_N97 [13], an extensively revised version of the PACE2 code, with the inclusion of new options for the level density and the transmission coefficients as well as the possibility to account for a fission delay according to the prescriptions widely reported in the literature [1]. If we limit our analysis to the FF channel, trying only to reproduce the multiplicities in the FF channel as usually done [1], the data shown in table 1 can be reasonably well reproduced assuming $a_\nu = A/9$, $a_f/a_\nu = 1.04$, the liquid drop model (LDM) yrast line and optical model (OM) transmission coefficients [14–16], without any delay. The parameter a_ν is the Fermi gas level density parameter for particle evaporation and a_f is the level density parameter for fission. From this result one could conclude that no transient effects take place in this decay, in contrast with the systematics [17]. A different combination of input parameters does not exclude, however, the presence of a relatively small fission delay. On the other hand, with the same parameters, the model strongly overestimates the ER particle multiplicities even though it reproduces the ER cross-section. This is an evident contradiction: if the model is not able to reproduce the light charged-particle (LCP) multiplicities in the ER channel, once the ER cross-section is well accounted for, the same model cannot be considered a reliable tool to estimate the fission time scale through the pre-scission light particle multiplicities.

In order to explore the possibility of reproducing the data of both channels with a unique set of input parameters we performed an extensive analysis with different prescriptions of the level density parameter and transmission coefficients, appropriate for the mass and excitation energy of the system under study. Calculations were carried out adopting three different and well-known prescriptions for the yrast line: 1) Gilbert-Cameron [18], 2) LDM and 3) sharp rigid sphere (RS) with radius parameter $r_0 = 1.2$ fm. Different prescriptions were also used for the level density parameter a_ν: 1) a constant value ranging from $A/6$ to $A/12$, which provides a reasonable reproduction of the proton and alpha particle spectra; 2) the inclusion of shell effects [19] with a damping term [20,21] as a function of the excitation energy and 3) a temperature-dependent prescription [22]. Transmission coefficients derived from 1) optical model and 2) fusion systematics (FS) [23] were used. To modulate the particle-fission competition, different values of the fission delay and a_f/a_ν were adopted as well. Calculations were constrained by the sum of the measured evaporation residue and fission cross-sections $\sigma_{fus} = \sigma_{ER} + \sigma_{FF} = 958$ mb.

Table 2. Comparison of the experimental data and calculated results for the ER emission from the compound nuclei in the $A \approx 150$ and $E_x \approx 100$–200 MeV region.

Reaction	Ref.	E_{lab} (MeV)	E_x (MeV)	L_{fus}	Exp.		Our calc.	
					M_p	M_α	M_p	M_α
$^{32}S + ^{109}Ag \to ^{141}Eu$	[5]	180	90	75	1.30 ± 0.30	0.60 ± 0.10	1.90	0.80
$^{121}Sb + ^{27}Al \to ^{148}Gd$	[24]	905	135	84	1.16 ± 0.26	0.73 ± 0.17	1.70	0.95
$^{40}Ar + ^{nat}Ag \to ^{147,9}Tb$	[10]	247	128	94	1.02 ± 0.20	0.50 ± 0.10	2.05	0.98
$^{40}Ar + ^{nat}Ag \to ^{147,9}Tb$	[10]	337	194	103	2.10 ± 0.60	1.40 ± 0.40	3.10	1.46
$^{60}Ni + ^{100}Mo \to ^{160}Yb$	[25]	550	251	78	1.10 ± 0.15	0.58 ± 0.15	3.34	3.18
$^{32}S + ^{100}Mo \to ^{132}Ce$	Present work	200	122	72	0.90 ± 0.14	0.56 ± 0.09	1.38	0.83

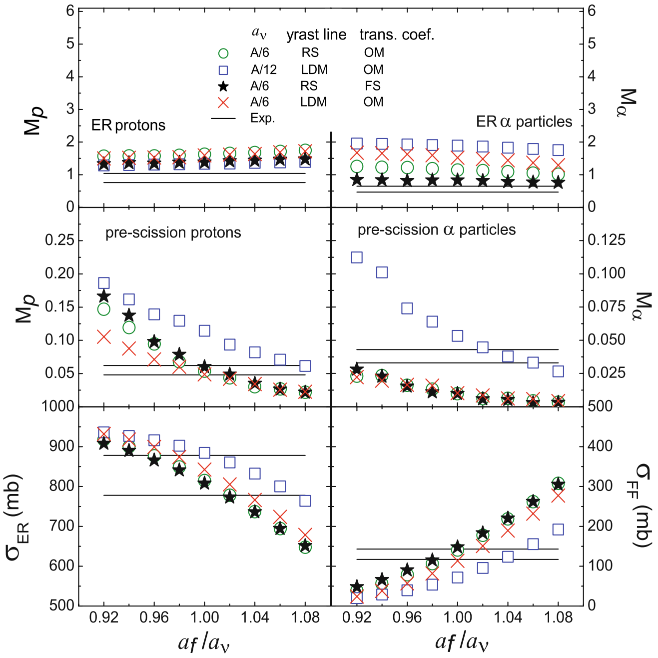

Fig. 1. Measured evaporative (ER) and pre-scission (PRE) charged-particle multiplicities together with the FF and ER cross-sections (full lines indicating lower and upper limits of the uncertainty), compared to the predictions of the statistical model changing: the level density parameter (a_ν), the yrast line, and the transmition coefficients. For details see text.

In fig. 1 we show the multiplicities for protons and alpha particles in the ER and FF channels, as well as the measured channel cross-sections, compared to the calculated values, as a function of the ratio a_f/a_ν. We report in the figure the results corresponding to four prescriptions. These latter have been chosen among the many combinations for which calculations were performed as they allow the exploration of the full range of variability of the calculated values of the observables under examination. The shell and temperature effects on the a_ν parameter and the Gilbert-Cameron prescription for level density produced only minor changes in the calculated observables, and therefore, these results are not presented here. No fission delay has been included in the calculations. From fig. 1 we infer that the model is not able to reproduce the observables altogether, the larger deviations being in the ER channel.

Here we can briefly highlight some of the deviations and expected trends in the ER channel. As a general behavior, for a fixed yrast line, higher values of a_ν reduce alpha particle multiplicities while those for protons are enhanced (square and cross). Compared to OM transmission coefficients, those derived from FS provide lower values for both proton and alpha particle multiplicities (circle and star). The dependence of the calculated multiplicities on a_f/a_ν appears to be relatively weak. Finally, we observe, as expected, a strong sensitivity of the alpha particle multiplicity on the yrast line: by using the RS yrast line (circle), we obtain a strong reduction of this quantity, with respect to that obtained with LDM yrast line (cross). It must be pointed out that the last prescription sets an upper limit for the yrast line, owing to the value used for the radius parameter r_0.

The main result of this analysis is that the model strongly overestimates proton and alpha particle multiplicities in the ER channel for this system, irrespective of the input parameters and the prescriptions used for the level density and transmission coefficients. The same result is confirmed by the calculations performed with the well-known codes Lilita_N97 [26] and Gemini [27]. Furthermore, the inclusion of a time delay to further suppress the fission does not change the overall pattern of the calculated data with respect to the experimental data. At the same time, the influence of nuclear deformation would further enhance the statistical model particle multiplicity predictions, resulting in a larger overestimation. On the other hand, the comparison of the measured proton and alpha particle energy spectra with the statistical model predictions, which will be discussed in detail in a forthcoming article, shows no evidence of nuclear deformation.

In order to gain insight on the extent of the above discrepancy in the ER channel, we compared the experimental data taken from the literature with the predictions of our code PACE2_N97. In table 2 we summarize the results for the few cases found in the literature. Once again the SM calculations overestimate protons and alpha particle multiplicities in the ER channel. Excluding the alpha particle multiplicity for the $^{147,9}Tb$ at $E_x = 194$ MeV, which is essentially reproduced, we observe overestimations up to a factor of 5.5.

The causes for such an unexpected behavior of the SM can be searched along two lines: either the competition between the different decay channels is not properly accounted for, or we are missing some decay channel, or both. Indications toward the first hypothesis would come from the neutron multiplicity in the ER channel that, unfortunately, is rarely measured. In the case of $^{60}\mathrm{Ni} + {}^{100}\mathrm{Mo}$, studied in ref. [25], where neutron multiplicity is also measured, our SM calculations, in agreement with those of ref. [28], enhance proton and alpha multiplicities and suppress neutron emission. This might be the case for the other reactions studied here. A rough indication of how much the SM branching ratios should be changed in favor of the neutron emission might be taken from the experimental multiplicities. However, since the branching ratios are strongly dependent on the decay step, empirical constant factors to reduce the strengths do not represent a reasonable approach to this problem, and would open the question on how to use these new parameters in the FF channel. The measurement of the neutron multiplicities in the ER channel is at this point a mandatory task [5]. There is also the possibility of other decay channels not presently considered in the SM code PACE2_N97, like the Intermediate Mass Fragments (IMF). Given the low probability of such emission we do not expect the IMF channel to be important for the reactions taken under consideration here except for the Ni + Mo case, where the excitation energy is high enough to allow for IMF emission. We remark that this is also the only case where the SM gives the largest deviation for both protons and alpha particles. In any case we did not observe any IMF in our data from the system $^{32}\mathrm{S} + {}^{100}\mathrm{Mo}$ at $E_{\mathrm{lab}} = 200\,\mathrm{MeV}$ in coincidence with ER.

Summarizing, the statistical model fails in reproducing simultaneously the proton and alpha particle multiplicities in the ER and FF channels for the system $^{32}\mathrm{S} + {}^{100}\mathrm{Mo}$ at $E_{\mathrm{lab}} = 200\,\mathrm{MeV}$ as well as in other experimental data, no matter what prescription is used for the level density and transmission coefficients among the models chosen. In particular, proton and alpha particle multiplicities in the ER channel are strongly overestimated irrespective of the input parameters. Prescission proton and alpha particle multiplicity and fission cross-section can be reproduced indicating no delay in the fission process, although this result is made questionable by the findings in the ER channel. The behavior observed in the ER channel, present for different systems from the literature in the $A \approx 150$ and $E_x \approx 100$–$200\,\mathrm{MeV}$ region, leaves an additional open question on the proper usage of the SM to predict fission delays.

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References

1. P. Paul, M. Thoennesen, Annu. Rev. Nucl. Part. Sci. **44**, 65 (1994) and references therein.
2. J.P. Lestone, Phys. Rev. C **59**, 1540 (1999).
3. D.J. Hofman, B.B. Back, P. Paul, Nucl. Phys. A **599**, 23c (1996).
4. A. Chatterjee, A. Navin, S. Kailas, P. Singh, D.C. Biswas, A. Karnik, S.S. Kapoor, Phys. Rev. C **52**, 3167 (1995).
5. E. Vardaci *et al.*, in *Proceedings of the International Symposium on Dynamics Aspects of Nuclear Fission, Casta-Papiernicka, Slovak Republic, Oct 1998*, edited by Yu.Ts. Oganessian, J. Kliman, S. Gmuca (World Scientific, Singapore, 2000) p. 261.
6. E. Fioretto *et al.*, IEEE Trans. Nucl. Sci. **44**, 1017 (1997).
7. S. Beghini *et al.*, Nucl. Instrum. Methods A **239**, 585 (1985).
8. E.M. Kozulin *et al.*, Prib. Tekh. Eksp. **1**, 51 (2008).
9. E.M. Kozulin *et al.*, Instrum. Exp. Tech. **51**, 44 (2008).
10. R. Lacey *et al.*, Phys. Rev. C **37**, 2540 (1988).
11. R. Lacey *et al.*, Phys. Rev. C **37**, 2561 (1988).
12. G. La Rana *et al.*, Eur. Phys. J. A **16**, 199 (2003).
13. PACE2_N97 is an extensively modified version of the original PACE2 program made by A. Gavron, Phys. Rev. C **21**, 230 (1980).
14. J.R. Huizenga, G. Igo, Nucl. Phys. **29**, 462 (1961).
15. F.G. Perey, Phys. Rev. **131**, 745 (1963).
16. D. Willmore, P.E. Hudson, Nucl. Phys. **55**, 673 (1964).
17. M. Thoennessen, G.F. Bertsch, Phys. Rev. Lett. **71**, 4303 (1993).
18. A. Gilbert, A.G.W. Cameron, Can. J. Phys. **43**, 1446 (1965).
19. S.K. Kataria, V.S. Ramamurthy, S.S. Kapoor, Phys. Rev. C **18**, 549 (1978).
20. A.V. Ignatyuk, G.N. Smirenkin, A.S. Tishin, Yad. Fiz. **21**, 485 (1975).
21. A.V. Ignatyuk, G.N. Smirenkin, A.S. Tishin, Sov. J. Nucl. Phys. **21**, 255 (1975).
22. J.P. Lestone, Phys. Rev. C **52**, 1118 (1995).
23. L.C. Vaz *et al.*, Z. Phys. A **318**, 231 (1984).
24. W.E. Parker *et al.*, Nucl. Phys. A **568**, 633 (1994).
25. M. Gonin *et al.*, Phys. Lett. **217**, 406 (1989).
26. Lilita_N97 is an extensively modified version of the original Lilita program made by J. Gomez del Campo, R.G. Stockstad, Oak Ridge National Laboratory Report No. TM7295, 1981 (unpublished).
27. R.J. Charity, Computer code GEMINI (unpublished).
28. J.M. Alexander *et al.*, in *Proceedings of the Symposium on Nuclear Dynamical and Nuclear Disassembly, Dallas, Texas, USA, 1989*, edited by J.B. Natowitz (World Scientific, Singapore, 1989) p. 211.