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Letter

Φ-variable calculated for the mixture of thermal sources

M.K. Kirejczyk[a]

Institute of Experimental Physics, University of Warsaw, ul. Hoża 69, Warsaw, Poland

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Abstract. The $\Phi_{\rm pt}$-variable, a measure of statistical correlation of particle momenta, introduced by Mrówczyński and Gaździcki, has been calculated for the events simulated with a simple thermal toy model. It is shown that mixing two thermal sources of unequal temperature can result in negative values of $\Phi_{\rm pt}$.

PACS. 25.75.Gz Particle correlations and fluctuations – 24.60.-k Statistical theory and fluctuations

1 Introduction

The Φ-variable (or function) was proposed by Mrówczyński and Gaździcki [1]. It is considered to be a good measure of equilibration. The value of Φ equals zero if a given type of equilibration (chemical or thermal in terms of momentum distribution) is reached. If a nucleus-nucleus collision is just a superposition of individual nucleon-nucleon collisions (without any further interaction that would lead towards equilibration) its value should be the same as for the elementary nucleon-nucleon collisions. Intermediate values of Φ would give some insight into the degree of thermalisation in the system.

Φ is not necessarily positive definite, but the majority of published papers report positive values of Φ, as would be expected from the "equilibration" arguments. There are papers (for instance [2]), where negative values of Φ are reported. This effect may be caused by a number of reasons like two-particle correlations or conservation laws. In this paper it is argued that mixing particles coming from fully equilibrated sources of different temperatures may also produce such an effect. It has been shown for instance in [3] that in the energy regime of the heavy-ion synchrotron SIS-18 at GSI Darmstadt ($\sqrt{s} \sim 2.5\,\mathrm{A\,GeV}$) it is very difficult to separate particles originating from sources of different character (in that case central "fireball" and the spectators). It is then of interest to study the strength of the $\Phi_{\rm pt}$-observable for systems involving two or more equilibrated sources of different temperature.

To prove the point, a very simple Monte-Carlo–based toy model has been used. It generates particles with momenta drawn from Boltzmann-like distributions. It allows to construct "events" with an arbitrary number of particles coming from sources of arbitrary temperatures. In this study it is shown, that just by mixing particles from sources of different temperature negative values of $\Phi_{\rm pt}$ are obtainable.

In the following the Φ-variable is defined and discussed. The applied toy model is described in more detail. The results are presented and finally some conclusions are drawn.

2 Φ-variable

The Φ-variable is a measure of fluctuations of a certain observable x (*e.g.* transverse momentum p_t, charge, even baryonic number). Its idea is based on statistical considerations. The distribution of x is constructed and Φ is defined as the difference between widths of this distribution calculated event-wise and over the entire data set.

One can define z as the difference between x and its average, $z_{k,j} = x_{k,j} - \bar{x}$, where k marks an event number, j counts the particles in the event, and the bar denotes global inclusive averaging. By construction $\bar{z}$ equals 0. For each event one may then define $Z_k = \sum_{j=1}^{N_k} z_{k,j}$, by construction $\langle Z \rangle$ equals 0 ($\langle\ \rangle$ denote averaging over events). Finally, one defines Φ as

$$\Phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\bar{z^2}}. \qquad (1)$$

The averages of Z^2 and z^2 correspond to the variances of their respective distributions.

As a quick exercise $\Phi_{\rm pt}$ may be "calculated" for the nucleon-nucleon elastic scattering. In each collision one obtains two nucleons with p_t of the same size, therefore for each event $Z_k = 2 \cdot z_k$ and $N_k = 2$. By averaging and substituting those values in (1) one obtains $\Phi_{\rm pt}^{\rm (NN)_{el}} = (\sqrt{2}-1)\sqrt{\bar{z^2}}$. It is a positive, exact number depending on the energy.

[a] e-mail: mkir@fuw.edu.pl

3 Model description

In order to test the hypothesis that mixing of thermal sources can lead to negative values of Φ a simple toy model has been used. This model essentially allows to fold various distributions using a Monte Carlo technique.

The model generates "particles" with momentum according to simplest, non-relativistic, thermal-like distribution:

$$P(p) \sim p^2 \exp\left(-\frac{p^2}{2T}\right), \tag{2}$$

where P denotes the probability and T is a parameter (henceforth called "temperature"). Then two emission angles are drawn from the isotropic distribution, all 3 components of $\boldsymbol{p}$ are calculated, and p_t is deduced.

The events can contain particles from any number of sources, but in this study the number of sources was limited to two. The sources can be of any temperature. N_i, the number of particles coming from the i-th source, can either be the same in all events of a sample ("constant multiplicity"), or can be within each event selected according to the Poisson distribution ("Poisson multiplicity"). For this second case the N_i shown is the mean of Poisson distribution.

This model contains no explicit correlations, the conservation laws are also not imposed.

Once a sample of events is generated, $\Phi_{\rm pt}$ is calculated. Its uncertainty is estimated using the method of splitting the sample randomly into sub-samples of roughly equal sizes, calculating $\Phi_{\rm pt}$ for each sub-sample, and finally calculating the variance of their distribution.

Formula (2) implies a universal mass of produced particles. As this formula is non-relativistic, the mass must fulfill the condition $Mc^2 \gg T$.

The following conventions are used in the paper: subscripts denote source number, T_i denotes the temperature of the i-th source and N_i its multiplicity. $N = \sum_i N_i$ denotes the total (constant or average) multiplicity of the event.

The size of the event sample was chosen such that the estimated error is smaller than the size of the symbols or 1% relative.

4 Results

Figure 1 presents the results of an analysis of samples of events with the same total multiplicity of 100 split between two sources with constant multiplicities. The temperature T_1 was set to 100 MeV, T_2 was set to 8 values between 10 and 200 MeV, marked on the upper plot of fig. 1 with different symbols. The obtained values of $\Phi_{\rm pt}$ are presented as a function of N_2/N on the upper plot. For each T_2 the lowest value of $\Phi_{\rm pt}$ is extracted and shown on the lower plot as a function of T_2.

One can see from fig. 1 that $\Phi_{\rm pt}$ is negative definite, reaches a minimum for the admixture of around 55–60% of "softer" particles, and rises with rising difference between the source temperatures.

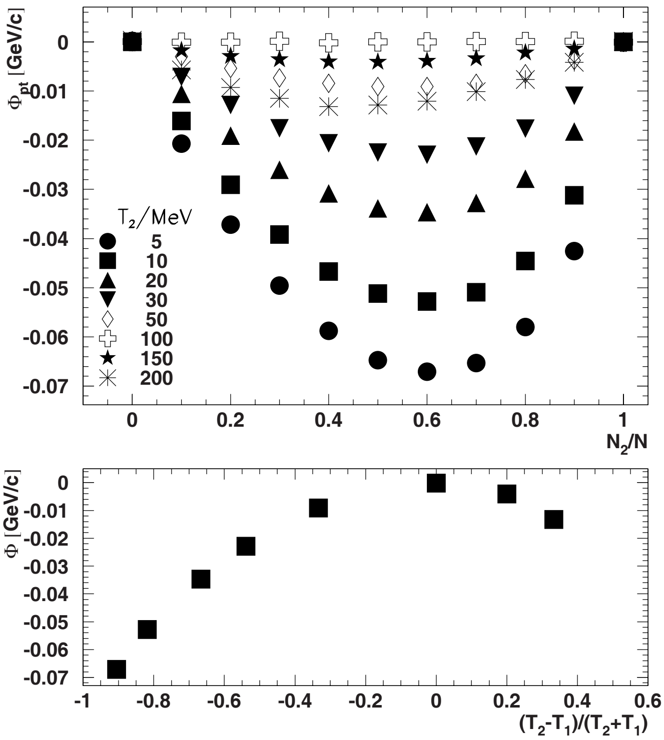

Fig. 1. The value of $\Phi_{\rm pt}$ for the mixed temperature source, with $T_1 = 100$ MeV and $N = 100$. T_2, N_2 and $N_1 = N - N_2$ is being varied. Upper plot: $\Phi_{\rm pt}$ as a function of N_2/N for different values of T_2. Lower plot: minimum value of $\Phi_{\rm pt}$ for a given T_2 as a function of the relative difference between T_1 and T_2.

Figure 2 shows the data obtained for two sources with temperatures set to 100 and 15 MeV, respectively. The upper plot shows the dependence of $\Phi_{\rm pt}$ on the total multiplicity, the results being presented in the same representation as in the upper plot of fig. 1. The symbols are spread in the horizontal direction to make the plot more readable, with different symbols denoting total multiplicities of 50, 100, 150 and 200. Within each sample N_1 and N_2 are constant.

The lower plot of fig. 2 shows how $\Phi_{\rm pt}$ depends on the way multiplicities are obtained. N_1 and N_2 are either constant or drawn from a Poisson distribution.

The results for constant multiplicities are insensitive to the total multiplicity. $\Phi_{\rm pt}$ depends on the differences in source temperatures and N_2/N. This is quite intuitive, as the widths of the p_t sub-distributions should not depend on the number of particles taken into account from the same underlying distribution.

On the other hand, the $\Phi_{\rm pt}$ value depends on the way source multiplicities are obtained, as shown in the lower plot of fig. 2. The circles follow the results from the upper plot, as the conditions are the same. However, once the event-wise distributions of the multiplicities N_i approach a Poissonian, the $\Phi_{\rm pt}$ values increase and actually reach zero for the case of both N_i being Poisson-like distributions.

To simulate the influence of limited detector efficiency the constant source multiplicity was folded with the bino-

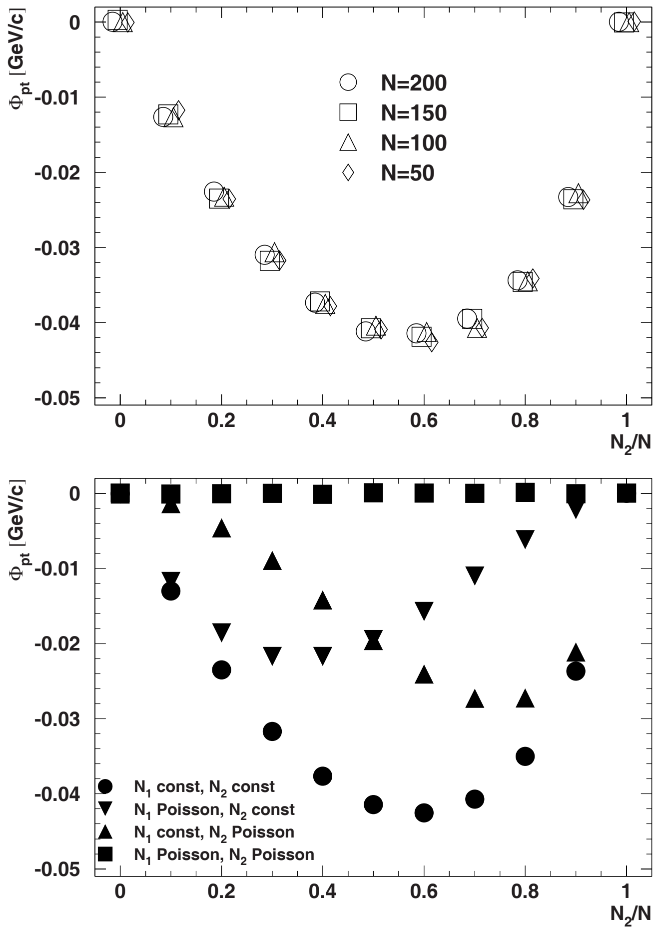

Fig. 2. The values of Φ_{pt} for the mixing of two sources with temperatures of 100 and 15 MeV as a function of the percentage of particles coming from the lower-temperature source. Upper plot —different total multiplicities; lower plot —same total multiplicity (100), N_1 and N_2 for the data point are either constant, or coming from a Poisson distribution.

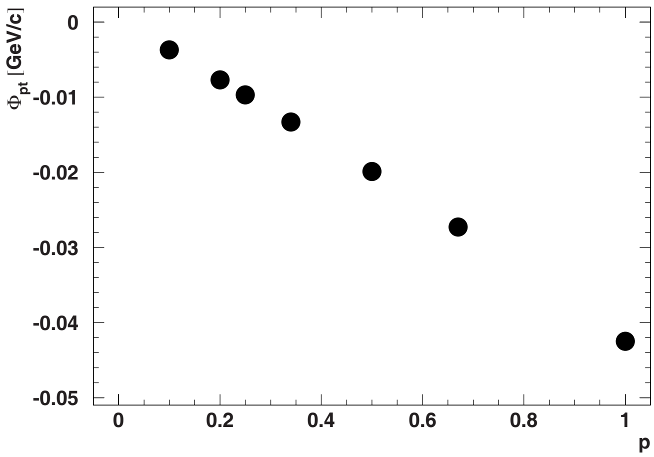

Fig. 3. The dependence of Φ_{pt} for the folding of the model events with binomial distribution, shown as a function of the particle acceptance probability p. $pN \approx 100$, $N_2/N = 0.6$, $T_1 = 100\,\mathrm{MeV}$ and $T_2 = 15\,\mathrm{MeV}$.

mial distribution by accepting each particle with a probability p. The results are presented in fig. 3. Events were produced for $T_1 = 100$, $T_2 = 15\,\mathrm{MeV}$, $N_2/N = 0.6$ and both multiplicities constant. The total multiplicity N and acceptance probability p were varied so that the product of those two numbers was about 100. The dependence of Φ_{pt} on p is close to linear. One may note, that the Poisson distribution is a limit of a binomial distribution for $p \to 0$ and $N \to \infty$ keeping pN constant, so one should expect the Poisson result for $p \to 0$, while the constant multiplicities result is reproduced for $p \to 1$.

5 Conclusions

It has been shown in principle that within the framework of a simple model of two equilibrated sources with different temperatures one can obtain negative values of the Φ_{pt}-variable, while $\Phi_{pt} = 0$ for a single source. Therefore it is important to take into account possible contributions of different sources while analyzing the experimental data. In the SIS-18 energy range those sources may consist of participating nucleons and spectators, in the SPS energy range perhaps of created mesons and originally existing baryons. In this light one may notice that the results quoted in [2] show negative values of Φ_{pt} for all and for positively charged particles, while for negatively charged ones they show $\Phi_{pt} \approx 0$ (and all negative particles are created).

The effect of negative Φ_{pt} values is present only for fixed source multiplicities, which is an idealized case. It disappears if multiplicities are smeared, and this would be the case for any real-life detector system. No detector can measure simultaneously all reaction products and limited acceptance causes smearing of multiplicities. So the discussed model cannot be applied directly to the data analysis. On the other hand if event selection is made only on the basis of multiplicity cut artificial results may be obtained, and this should be kept in mind.

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