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Letter

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The small axial charge of the N(1535)-resonance

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Abstract. There is a natural cancellation between the contributions of the qqq and $qqqq\bar{q}$ components to the axial charge of the $N(1535)$-resonance. While the probability of the former is larger than that of the latter, its coefficient in the axial charge expression is exceptionally small. The magnitude of two of the corresponding coefficients of the $qqqq\bar{q}$ components is in contrast large and has the opposite sign. This result provides a phenomenological illustration of the recent unquenched lattice calculation result that the axial charge of the $N(1535)$-resonance is very small, if not vanishing (T.T. Takahashi, T. Kunihiro, arXiv: 0801.4707 [hep-lat]). The result sets an upper limit on the magnitude of the probability of $qqqq\bar{q}$ components as well.

PACS. 12.39.Jh Nonrelativistic quark model – 14.20.Gk Baryon resonances with $S = 0$

1 Introduction

A number of phenomenological failures of the constituent quark model for the baryons may be repaired by extending the model space beyond that of the basic three-quark configurations qqq [1–3]. The question of key interest is then that of the relative magnitude of the sea-quark configurations, and in particular of the most obvious $qqqq\bar{q}$ configurations. For most electromagnetic- and strong-decay observables, this is difficult to estimate, because of the very strong contribution from the transition matrix elements between the qqq and $qqqq\bar{q}$ components [4]. The axial current operator of the baryon resonances is an exception, as for this the transition matrix elements are suppressed —*i.e.* they involve the small components of the spinors— with respect to the diagonal matrix elements, so that the axial charges, to a good approximation, may be expressed as a sum of the diagonal matrix elements of all possible configurations, which takes the form of numerical coefficients A_n times the corresponding probabilities P_n:

$$g_A^* \simeq \sum_n A_n P_n \,. \tag{1}$$

The (diagonal) axial charges of baryon resonances are however not accessible experimentally. It is in this regard that the recent result, obtained numerically by an unquenched QCD lattice calculation, that the axial charge of the $N(1535)$ actually may vanish in the two-flavor case, is so interesting [5]. As that result appears to be insensitive to the quark mass (the magnitude of the value extrapolated to 0 is less than 0.2), it may be taken as a substitute for an experimental value. While the statistical error margins of the calculated values of the axial charge of the $N(1535)$ are not yet sufficiently narrow to exclude the small value $-1/9$ given by the conventional constituent quark model with only qqq configurations [6], it is interesting to explore the phenomenological consequences of a vanishing axial charge.

The smallness of the axial charge of the $N(1535)$ also appears to be unique, as the lattice calculation value for the axial charge of the following $1/2^-$-resonance, the $N(1650)$, is ~ 0.5 [5], which is close to the qqq quark model value $5/9$ [6]. These values suggest that the configuration mixing between these resonances is small [7].

If the axial charge of the $N(1535)$ vanishes, it implies that the sea-quark configurations shall have to cancel the (small) contribution of the qqq configuration. This makes it possible to put constraints on the sea-quark configurations in the $N(1535)$. To illustrate this possibility we consider below the contributions from all the $qqqq\bar{q}$ components, which may exist in the $N(1535)$.

2 $qqqq\bar{q}$ components in the N(1535)

The $qqqq\bar{q}$ components that are compatible with the quantum numbers of the $N(1535)$-resonance have been enumer-

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Table 1. The $qqqq\bar{q}$ configurations in the $N(1535)$ and the corresponding axial charge coefficient A_n (1).

Configuration	Flavor-spin	C_{FS}	Color-spin	C_{CS}	A_n
1	$[31]_{FS}[211]_F[22]_S$	-16	$[31]_{CS}[211]_C[22]_S$	-16	0
2	$[31]_{FS}[211]_F[31]_S$	$-40/3$	$[31]_{CS}[211]_C[31]_S$	$-40/3$	$+5/6$
3	$[31]_{FS}[22]_F[31]_S$	$-28/3$	$[22]_{CS}[211]_C[31]_S$	$-16/3$	$-1/9$
4	$[31]_{FS}[31]_F[22]_S$	-8	$[211]_{CS}[211]_C[22]_S$	0	$-4/15$
5	$[31]_{FS}[31]_F[31]_S$	$-16/3$	$[211]_{CS}[211]_C[31]_S$	$+8/3$	$+17/18$

ated in ref. [8]. As all 5 constituents in a $qqqq\bar{q}$ configuration in the negative parity $N(1535)$ may be in the ground state, the orbital state of the 4 quarks may be assumed to be completely symmetric. Then either the spin-flavor state has to have the mixed flavor-spin symmetry $[31]_{FS}$ or alternatively the color-spin state has to have one of the mixed flavor symmetries $[31]_{CS}$, $[22]_{CS}$ or $[211]_{[CS]}$. There are 5 different $qqqq\bar{q}$ configurations in the $N(1535)$ that have an appropriate symmetry structure and spin and isospin 1/2. These are listed in table 1.

The numbering of these configurations are in order of increasing energy, if the hyperfine interaction between the quarks is assumed to depend either on flavor and spin or on color and spin. In the table the matrix elements of the schematic hyperfine splitting operator

$$C_{kS} = -\sum_{i,j} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \tag{2}$$

are listed for both the cases where the operators $\boldsymbol{\lambda}$ represent either the generators of the color $SU(3)$ ($k = C$) or the flavor $SU(3)$ group ($k = F$), respectively. (Here the spatial structure of the interaction has been neglected, as all the constituents are in the same orbital ground state.) Note that because of their mixed flavor symmetry $[211]_F$ both the configurations (1) and (2) in table 1 have to contain a strange quark-antiquark pair. This is as expected on the basis of the observed large $N\eta$ decay branch of the $N(1535)$.

The general expression in the flavor-spin coupling scheme for these 5-quark wave functions is

$$\begin{aligned}\psi_{t,s}^{(i)} = &\sum_{a,b,c}\sum_{Y,y,T_z,t_z}\sum_{S_z,s_z} C^{[1^4]}_{[31]_a[211]_a} C^{[31]_a}_{[F^{(i)}]_b[S^{(i)}]_c}\\ &\times [F^{(i)}]_{b,Y,T_z}[S^{(i)}]_{c,S_z}[211;C]_a(Y,T,T_z,y,\bar{t},t_z|1,1/2,t)\\ &\times (S,S_z,1/2,s_z|1/2,s)\bar{\chi}_{y,t_z}\bar{\xi}_{s_z}\varphi_{[5]}\,.\end{aligned} \tag{3}$$

Here i is the number of the $qqqq\bar{q}$ configuration in table 1, $\bar{\chi}_{y,t_z}$ and $\bar{\xi}_{s_z}$ represent the isospinor and the spinor of the antiquark, respectively, and $\varphi_{[5]}$ represents the completely symmetrical orbital wave function. The first summation involves The symbols $C^{[\cdot]}_{[..][...]}$, which are S_4 Clebsch-Gordan coefficients for the indicated color ([211]), flavor-spin ([31]) and flavor ([F]) and spin ([S]) wave functions of the $qqqq$ system. The second summation runs over the flavor indices in the $SU(3)$ Clebsch-Gordan coefficient (with 9 symbols) and the third over the spin indices in the standard $SU(2)$ Clebsch-Gordan coefficient. In the case of the spin configuration [22] the total spin of the $qqqq$ system vanishes, so that $S = S_z = 0$. These wave functions are given in explicit form in ref. [9].

3 The axial charge of the N(1535)

Note first that the energetically most favorable configuration (1) in table 1 has zero total spin, and that, as a consequence, the mixed flavor symmetry requires that the corresponding antiquark be strange, it cannot contribute any matrix element to the axial charge operator $\sum_i \sigma_z(i)\tau_z(i)$ (calculated here as the matrix element of the third component of the axial vector current). In the table the matrix elements of the axial charge of these configurations, combined with the wave function of the antiquark are also listed.

With the results in table 1, the explicit expression for the axial charge of the $N(1535)$ takes the form

$$g_A(N(1535)) = -\frac{1}{9}P_3 + \frac{5}{6}P_5^{(2)} - \frac{1}{9}P_5^{(3)} - \frac{4}{15}P_5^{(4)} + \frac{17}{18}P_5^{(5)}\,. \tag{4}$$

Here P_3 is the probability for the conventional qqq configuration, while $P_5^{(i)}$ represents the probabilities of the $qqqq\bar{q}$ configurations in table 1.

The fact that the two $qqqq\bar{q}$ contributions in (4), which are positive, have large coefficients ~ 1, while the coefficient of the qqq contribution is small and negative ($-1/9$) immediately suggests the possibility for a considerable cancellation between the qqq valence and the $qqqq\bar{q}$ sea-quark contributions, as the probability of the latter is likely to be considerably smaller than that of the former. If only the first two terms in the expression (4) are taken into account $g_A(N(1535))$ would vanish if $P_5^{(2)} = 2/15P_{qqq}$, which may be a fairly reasonable assumption. The last two remaining $qqqq\bar{q}$ configurations are expected to have very small probability, as they are energetically unfavorable (table 1).

In ref. [9] it was in fact found that the quark model prediction for the helicity amplitude $A_{1/2}$ for $N(1535) \to N\gamma$ could be brought qualitatively into line with the empirical values if $P_{qqq} \simeq 0.55$ and $P_5^{(1)} \simeq 0.45$. Since the $qqqq\bar{q}$ configurations (1) and (2) in table 1 are similar in that both involve an $s\bar{s}$ pair, but the latter is energetically disfavored by the matrix elements of the hyperfine interaction (2), the helicity amplitude should be similar if the probability $P_5^{(2)}$ for the configuration (2) in table 1, which has a large axial charge coefficient (4), falls in the range (0.25–0.3)$P_5^{(1)}$. With these numbers $g_A(N(1535))$ comes out to lie in the range 0.03–0.06. If, on the other hand, one considers both the configurations (2) and (3) in table 1 as equally probable: $P_5^{(2)} = P_5^{(3)}$ and the probabilities to fall within the range $(0.12–0.15)P_5^{(1)}$, the numerical value for $g_A(N(1535))$ falls in the range -0.02 to -0.05. This shows that the likely range of values for $g_A(N(1535))$ in the extended quark model, which includes explicit $qqqq\bar{q}$ components $-0.05\ldots+0.06$, brackets 0. This range would bracket 0 also in the case where the relative qqq probability were increased to $P_3 = 0.7$ and $P_5^{(2)} = 0.3$. It does, in any case, not appear possible to reach the value 0 for $g_A(N(1535))$, with an overall $qqqq\bar{q}$ probability that is larger than 0.45.

4 Conclusion

Above the axial charge was calculated as the matrix element of the operator $\sum_i \sigma_z(i)\tau_z(i)$. If this operator is replaced by the corresponding Dirac operator $-i\sum_i \gamma_z(i)\gamma_5(i)\tau_z(i)$ the calculated result would be somewhat smaller. In the case of the qqq configuration the reduction of the calculated axial current matrix element has been found to be at most $\sim 34\%$ in the extant forms of relativistic kinematics and [10]. Taking this into account suggests a corresponding narrowing of the estimated range of the calculated axial charge of the $N(1535)$ and the upper limit of the probability of the $qqqq\bar{q}$ component above.

The conclusion is therefore that the very small or possibly vanishing axial charge of the $N(1535)$ already at the present level of accuracy constrains the magnitude of the probability of the sea-quark components in the $N(1535)$ to be less than 45%. The axial charge of the $N(1535)$-resonance is apparently a very special case as other resonances, as, *e.g.*, the $N(1440)$ and the $N(1650)$ do not have similarly small axial charge values [6,5].

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