Temporal and Spatio-Temporal Dynamic Instabilities: Novel Computational and Experimental Approaches

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This special issue contains a concise account of significant research results presented at the international workshop on Advanced Computational and Experimental Techniques in Nonlinear Dynamics, which was held in Cusco, Peru in August 2015. The meeting gathered leading experts, as well as new researchers, who have contributed to different aspects of Nonlinear Dynamics. Particularly significant was the presence of many active scientists from Latin America. The topics covered in this special issue range from advanced numerical techniques to novel physical experiments, and reflect the present state of the art in several areas of Nonlinear Dynamics. It contains seven review articles, followed by twenty-one regular papers that are organized in five categories, namely (1) Nonlinear Evolution Equations and Applications, (2) Numerical Continuation in Self-sustained Oscillators, (3) Synchronization, Control and Data Analysis, (4) Hamiltonian Systems, and (5) Scaling Properties in Maps.

The first review article by Proctor, Brunton, and Kutz, considers recent methodological advances toward the control of complex systems. The ability to collect high-fidelity data and to perform targeted experiments with these systems offers significant opportunities for data-driven, equation-free modeling. The review focuses on recent adaptations of these data-driven methods for handling systems with inputs, where two perspectives on analyzing complex systems are offered: model-based dimensionality reduction and equation-free methods.

The review by Leblond, Grelu, Mihalache, and Triki, studies several propagation models in the optical domain that do not rely on the slowly-varying-envelope approximation (SVEA): modified Korteweg-de Vries, sine-Gordon, and mKdV-sG models.

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These models can be considered as fundamental systems addressing the formation and propagation of few-cycle pulsed field structures and solitary waves arising in the course of intense ultrashort optical pulse evolution. The non-SVEA models numerically predict supercontinuum generation over several octaves, and the authors have identified several mechanisms in the process. The review shows that a definite advantage of non-SVEA models is that they do not require a large number of terms and coefficients. They also offer the possibility to explore few-cycle to sub-cycle ultrashort pulse dynamics. The tuning of non-SVEA models and parameters to realistic material properties and actual experiments requires further study.

The third review, by Mitschke, Hause, and Mahnke, discusses the feasibility of using multi-soliton compounds in dispersion-managed optical fiber lines, in order to increase the number of transmitted coding symbols. The authors report experimental and theoretical evidence that multi-soliton localized solutions can at least double the transmission rate obtained by one-soliton one-zero coding.

The next article in this issue is a tutorial on continuation and bifurcation methods for the numerical analysis of dissipative PDEs, by Sánchez Umbría and Net. Their tutorial is self-contained, including a brief introduction to the numerical linear algebra tools needed for linear systems and eigenvalue problems, for continuation, and inexact Newton methods. The main focus of the tutorial is Newton-Krylov methods for the computation of steady states, periodic orbits, bifurcation loci, and invariant tori, for systems of nonlinear parabolic PDEs.

The review article by Ramírez and Kurths focuses on how the phenomenon of response to synchronization arises in sets of pulse-coupled dissimilar oscillators. In this work the authors extended the response to synchronization observed in fireflies to systems of dissimilar pulse-coupled oscillators. In particular, two interesting phenomena have been considered: control of firings and response to synchronization. The article also underlines the most important effects found in the systems under study, which include bursting and non-bursting oscillators, master-slave and mutually coupled configurations.

The review article on multidimensional solitons by Malomed gives a brief account of the broad area of 2D and 3D solitons, both fundamental (zero-vorticity) and vortical ones, which have their most important physical realizations in Bose-Einstein condensates (BEC) and nonlinear optics. It includes two particular topics, one well-established, *viz.*, the stabilization of the 3D and 2D modes with vorticities $S = 0$ and 1 in a trapping harmonic-oscillator potential (generally anisotropic). It also includes a recent topic, namely the creation of stable 2D and 3D solitons, which mix terms with $S = 0$ and 1 (semi-vortices) or $S = 0$ and ± 1 (mixed modes) in a two-component system that realizes the spin-orbit coupling in BEC for stabilizing these states against collapse and splitting.

The review paper by Satija considers the connection between the Hofstadter butterfly, which encodes the quantum Hall effect, and the set of integer Apollonian gaskets, named in honor of Apollonius of Perga (before 300 BC). The underlying idea is to associate an Apollonian gasket with each self-similar copy of the Hofstadter butterfly, and a construction via Ford circles and their duals for butterflies that are symmetric with respect to zero energy. It is argued that a systematic study that relates the butterfly to the set of Integral Apollonian gaskets is an open problem, and that many subtle questions arise within the mathematical framework of conformal and Möbius transformations.

“Nonlinear Evolution Equations and Applications” is the first group of regular articles, and concerns various aspects of the dynamics of coherent structures in nonlinear partial differential equations. Applications include chemical reactions in fluids, and localized structures in dissipative nonlinear wave systems. In the article entitled “Forced Snaking, Localized structures in the real Ginzburg-Landau equation with

spatially periodic parametric forcing” Ponedel and Knobloch study localized solutions in a spatially forced fifth order Landau-Ginzburg equation used to model pattern formation. They report snaking effects due to inhomogeneous forcing: front-like structures with oscillations that invade the domain following a snaking bifurcation diagram, and also pulse-like structures that fill the domain using a novel form of snake-like bifurcation diagram with a foliated structure that apparently adds an unbounded number of pulses.

The articles by Vilela and Vasquez, and by Guzman and Vasquez, use numerical techniques to study chemical reaction fronts coupled to different fluid motions in a two dimensional tube. In their article “The effects of fluid motion on oscillatory and chaotic fronts”, Vilela and Vasquez show that Poiseuille and Hele-Shaw-cell flows change the regions of periodic and chaotic reaction front velocity regimes seen in unforced fronts. The authors also report similar results when using a simpler forced Kuramoto-Sivashinsky model for the interface profile. The article “Surface tension driven on a thin reaction front”, by Guzman and Vasquez, studies a similar setup, but under the assumption that the flow in the regions separated by the front is Stokes, and is forced by changes in the surface tension along the front (the Marangoni effect). The resulting convective flows in the two regions is seen to increase the speed of the flows, for both signs of the Marangoni parameter. The article by Alarcón and coauthors is entitled “Growth rate and shape as possible control mechanisms for the selection of mode development in optimal biological branching processes”. It considers a mathematical model of lung formation and discusses the role that shape and growth might have as control mechanisms in the process of lung development in experiments, in particular during the branching process of lung formation in mice.

Of the regular articles in this special issue, six are classified under “Numerical Continuation in Self-sustained Oscillators”. The first of these is the article entitled “Effects of time-delay in a model of intra- and inter-personal motor coordination” by Slowinski, Tsaneva-Atanasova, and Krauskopf, who present a bifurcation study of a two-oscillator model of human motor coordination. Specifically, the model incorporates a delay in the coupling, which is shown to have a significant effect on the observed dynamics, and in particular on bistability properties. A numerical investigation of transient spike-adding behavior in a phenomenological model of plateau bursting oscillations is presented in the article “Transient spike adding in the presence of equilibria”, by Farjami, Kirk, and Osinga. Their work focuses on spike adding in the presence of additional equilibria, and on the effect of varying the duration and magnitude of the perturbation that causes the transient response. Building on their previous work, Doedel and Pando present a paper entitled “Phase Locked Periodic Solutions and Synchronous Chaos in a Model of Two Coupled Molecular Lasers”, in which they extend the analysis of a CO₂ laser with saturable absorber to the case of two mutually coupled lasers. In their article “Multistability in nonlinearly coupled ring of Duffing systems”, Jaros, Kapitaniak, and Perlikowski study a system of three oscillators that are unidirectionally linked in a ring. In particular, the authors discuss how the bifurcation structure depends on the three coefficients of a cubic polynomial coupling function. Dvorak, Astakhov, Perlikowski, and Kapitaniak present a study of “Nonlinear resonance and synchronization in the ring of unidirectionally coupled Toda oscillators”, in which they show how the nonlinear resonance affects the main synchronization region, inside which they observe coexistence of periodic and quasi-periodic behavior. The article “Global isochrons of a planar system near a phaseless set with saddle equilibria”, by Hannam, Krauskopf, and Osinga, uses accurate continuation techniques to compute isochrons for a planar system of ordinary differential equations. Isochrons are curves of initial conditions that converge to a stable periodic orbit, while having the same asymptotic phase. Their computations show that isochrons can have complicated and very interesting geometric properties.

“Synchronization, Control and Data Analysis” constitutes the third group of regular articles. The effects of different coupling mechanisms and external stimuli on nonlinear oscillators are considered, in particular synchronization phenomena, where long time series are analyzed. The first of six articles is entitled “A discrete-time chaos synchronization system for electronic locking devices” by Minero-Ramales and co-authors, who present a novel electronic locking key based on discrete-time chaos synchronization. Two chaotic systems are synchronized using a technique from nonlinear control theory, in order to perform the encryption/decryption of the signal to be transmitted. The article “Synchronization in arrays of coupled self-induced friction oscillators”, by Marszał, Saha, Jankowski, and Stefanski, investigates synchronization effects in systems of self-induced dry friction oscillators. Systems of two, three and four oscillators coupled by linear springs are considered to determine synchronization thresholds, using the master stability function approach. In their article “Phase synchronization based on a Dual-Tree Complex Wavelet Transform”, Ferreira, Oliveira Domingues, and Macau study a system of two coupled chaotic Lorenz systems. They consider the applicability of the Discrete Complex Wavelet Approach (DCWA) to explore the phenomenon of phase synchronization. The article “Analysis of noise-induced temporal correlations in neuronal spike sequences”, by Reinoso, Torrent, and Masoller, studies temporal correlations in sequences of noise-induced neuronal spikes arising from two neuron models with correlated noise. Their work uses a popular symbolic method, known as ordinal analysis, where the time series is transformed into a sequence of ordinal patterns. The fifth article in this group, by Bustos-Espinoza and Ramírez-Ávila, is entitled “Synchronization Conditions of Coupled Maps Using Periodicities”. The authors use new indicators, the synchrony factor and its periodicity, to identify not only the regimes of complete synchronization but also those of in-phase synchronization, in a model of diffusively coupled logistic maps. The final article in this group, by Jaimes-Reátegui and co-authors, is entitled “Synchronization of unidirectionally delay-coupled chaotic oscillators with memory”. The authors study synchronization phenomena in two Rössler-like oscillators, coupled with time delay in a master-slave configuration, and with delayed feedback in the slave oscillator. The regimes of small and strong coupling are considered.

The articles of the fourth group represent some of the many active areas in the study of the dynamics of Hamiltonian systems, with related physical applications. The article by Martínez-Galicia and Panayotaros, entitled “Localization and coherent states in a quantum DNLS trimer”, studies the trimer discrete nonlinear Schrödinger equation system that arises in nonlinear optics and in Bose-Einstein condensates. They compare some localized solutions of the classical and the quantized system, reporting recurrence phenomena for the distance between trajectories obtained using exact and approximate “semiclassical” evolution rules. Olvera and de la Rosa study the phenomenon of magnetic levitation of a rotating dipole in the Levitron. In particular they consider the effect of weak dissipation in their article entitled “Mechanical stabilization of the Levitron’s realistic model”. They propose a parametric forcing that prolongs the flight time. They also use novel numerical diagnostic tools to detect an underlying Arnold tongue structure in the dynamics, which partially explains their numerical observations. The article by Calleja, Doedel, and García-Azpeitia, entitled “Symmetry-breaking for a restricted n-body problem in the Maxwell-ring configuration”, studies bifurcations from relative equilibria of a generalized restricted N-body problem that arises from Maxwells classic work on the rings of Jupiter. New results on global bifurcations of periodic orbits in the 3-D problem are compared to numerical continuation results. The latter reveal secondary bifurcations that are characterized by different symmetry breaking properties. The global bifurcation theory mentioned in this work can be also used to locate 3-D choreographies.

The two articles classified under “Scaling Properties in Maps” show how scaling laws reflect the underlying dynamics. The article by de Oliveira, da Costa and Leonel, entitled “Survival probability for chaotic particles in a set of area preserving maps”, studies the rate of diffusion of the action in a class of symplectic maps on a 2-D cylinder that generalizes the standard (Taylor-Chirikov) map. They report that the function quantifying the rate of escape from the vicinity of a given value of the action is universal over a range of parameters and models. The second article of this group is entitled “Sums of variables at the onset of chaos, replenished”, by Diaz-Ruelas and Robledo. The authors consider distributions formed by sums of ensembles of positions of trajectories, with uniformly-distributed initial conditions through the total phase space of the logistic map, as the nonlinear parameter changes. Very large and finite size sums of ensembles are considered, where a plethora of complex behavior is shown.

In summary, this special issue highlights the spectrum of current research in Non-linear Dynamics, including both theory and experiments, and offers state of the art computational techniques and experimental methods in nonlinear physics. We hope that the articles will be valuable to physicists, engineers, and applied mathematicians who are interested in nonlinear phenomena. We thank all authors in this special issue for their outstanding contributions. We also wish to express our gratitude to the referees of the articles, for their often thorough and thoughtful reviews. We thank the managing editor Sabine Lehr, Nicolas Puyaubreau and the entire EPJ-ST editorial staff for their kind assistance in preparing this issue. Special thanks are due to EPJST Editor Jürgen Kurths, for his support and for bringing the opportunity of a special issue to our attention.