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Editorial

Application of variable-order fractional calculus in neural networks: where do we stand?

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Abstract After decades of evolution and advancement, artificial intelligence has profoundly affected all fields of science and engineering, and has started to revolutionize all aspects of today's life. In this regard, neural networks, which are a stepping stone in the search for artificial intelligence, play an important role. Motivated by this, the current special issue aims to explore recent trends and developments in the modeling, analysis, synchronization, and practical application of chaotic variable-order fractional neural networks.

1 Introduction

The concept of artificial intelligence is based on the assumption that machines can imitate the human thought process. Even before the industrial era, speculation of artificial intelligence could be seen in different civilizations. In 1956 at Dartmouth College, the term "artificial intelligence" was coined for the first time by American computer scientist John McCarthy, and it was formally accepted as an academic discipline [1]. From the first day, this field of study faced several fluctuations; sometimes, it attracted considerable attention, and sometimes it experienced winters due to technological shortcomings. Nevertheless, throughout the decades, the field of artificial intelligence did not completely disappear. After each winter, artificial intelligence came into the picture, attracted significant attention, and became an important research field. There is a simple explanation for today's rise of artificial intelligence: now, thanks to the ever-improving ability of fast computation, artificial intelligence-based methods work perfectly in many applications [2, 3].

Neural networks, which are a stepping stone in the search for artificial intelligence, considerably influence the pace of advances in artificial intelligence [4]. It has been demonstrated that in some conditions, neural networks can show chaotic behaviors and complicated dynamics. Due to the broadband application of chaotic neural networks as well as their random-like behavior, the development of novel methods and new models, and extending existing techniques for analysis of these systems, is of crucial importance, and more studies must be carried out in this field of study.

Finding reliable tools for modeling real-world phenomena can be considered as one of the most critical challenges for scientists [5]. Fractional calculus provides a suitable facility to reach this goal. In this respect, mathematical modeling, dynamical investigation, and applications of fractional-order chaotic systems have recently attracted considerable attention [6–9]. In comparison with integer-order derivatives, the constant-order fractional ones have advantages in describing the long memory of systems. However, recently, some studies have proven that several engineering and scientific phenomena cannot be precisely illustrated by means of constant fractional derivatives. For instance, in some physical models, systems exhibit the change of memory property over time. Therefore, by using a bounded function that depends on time, we could expand the conventional fractional derivative to be the so-called variable-order fractional derivative.

Considering the importance of neural networks as an inseparable part of artificial intelligence, as well as the promising results of variable-order fractional calculus in different fields of study, the current special issue has been devoted to the application of variable-order fractional calculus in neural networks. In what follows, the contributions of the papers presented in the current collection are briefly explained.

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2 Modeling and dynamical investigation

One can use a conventional model with integer-order-based equation to be performed on all types of networks; while it is simple to model, it is obviously not accurate for all real-world networks [10]. Since the fractional calculus is a generalized form of the integer one and can capture many time-dependent phenomena, it is a more practical and precise approach to the use of the fractional models, especially with time-varying fractional order. Considering this point, in the current collection, some studies have investigated modeling and dynamical investigation of neural networks using variable-order fractional calculus [11–16].

In [17], a variable-order fractional Hopfield-like neural network has been introduced. A general basin of attraction for the system has been presented, demonstrating that its dynamical behaviors are extremely sensitive to initial conditions, resulting in different periodic orbits and chaotic attractors' coexistence. In [18], a variable-order fractional Hopfield neural network under electromagnetic radiation has been proposed for the first time in the literature. In [19] a novel chaotic-fractional discrete neural network model has been offered.

A variable-order fractional model of HIV/AIDS has been proposed and studied in [20]. Afterward, using a new algorithm that uses particle swarm optimization and a long short-term memory neural network, the time-varying derivative of the model has been estimated. In [21], non-integer Hopfield neural network dynamics introducing the hyperbolic tangent transfer function generalized by the Mittag–Leffler function and the M-truncated derivative with constant and variable order has been analyzed. The novel neural network's behaviors have been studied through phase portraits and the 0-1 test. In [22], a method for studying the boundedness of the solutions of Hopfield neural network systems with fractional-like derivatives has been proposed. In [23], a new variable-order fractional network of the hepatitis B virus (HBV) model has been proposed. Considering the fact that most communities interact with each other, and the rate of disease spread is affected by this factor, the proposed network can provide more accurate insight for the modeling of the disease.

3 Engineering application

Nowadays, with the increasing usage of the internet, information security has become critical. Data-hiding strategies are one approach used to ensure secure information transfer. Many different methods for data hiding are reported in the literature. In this issue, by means of fractional-order neural networks, some interesting approaches have been proposed [24, 25]. In [18], the realization of a variable-order fractional chaotic system over a PC's sound card has been done for the first time in the literature. This application has been realized in an effective, simple, and low-cost way compared with the digital circuit implementation applications presented in the literature. It has been shown that the proposed methodology can be physically implemented with a digital circuit. In [26], a substitution-box (S-Box)-based video stenography algorithm has been offered. In the design of the developed system, a chaotic variable-order fractional Hopfield neural network system has been implemented, and the analysis of the system has been carried out.

4 Control

Since variable-order fractional systems show more complex characteristics and more degrees of freedom due to the time-varying fractional derivatives, their control is more challenging in comparison with their integer-order counterparts. In the current collection, robustness and stability, which are important measures in assessing the controller's performance, have been taken into account. In [27], taking advantage of a type 2 fuzzy and chatter-free approach, a new controller has been proposed to control and synchronize variable-order fractional networks. The behavior of a variable-order fractional system has been investigated, and the complex dynamics of the system have been controlled. The proposed control method used in this research has proved that using a modified fuzzy type 2 observer and a sliding mode technique can control a nonlinear fractional-order network, even in the presence of unknown external disturbances and uncertainties in the system's dynamic. The simulations demonstrate its strong robustness. Furthermore, exerting the proposed control method, chattering in the system has been effectively eradicated. In [17], an adaptive control scheme has been proposed for the variable-order fractional system. The main advantages of the proposed controller are its guaranteed stability, robustness against uncertainties, and simplicity. Simulations in this study show the excellent performance of the proposed controller for the variable-order fractional Hopfield-like neural network. In [21], two scenarios to anti-synchronize the master and slave system have been proposed. In [12], backstepping techniques with constant and variable-order derivatives have been used to develop a synchronization scheme for two identical one-scroll systems. In [28], a new variable-order fractional model has been proposed to investigate the effect of antiviral treatment over viral mutation to control disease transmission. Through the genetic algorithm, optimal treatment is presented, and its numerical simulations are illustrated. In [29], a fractional-order controller has been designed to reach synchronization using an optimal cuckoo search algorithm.

5 Trends and opportunities

To recapitulate briefly, in the current issue, the following aspects of variable-order fractional neural networks have been studied:

- Modeling and analysis
- Applications in various areas of engineering and physics
- Experimental application and validation
- Synchronization and anti-synchronization techniques using novel methods
- Special features.

In most of the studies presented in the current collection, shortcomings in the field, room for more study, possible future trends, and approaches for improvements in existing methods have been delineated and discussed. In addition, in review papers [19, 30], the architecture of the systems, the control strategies, and the fractional derivatives used for artificial neural networks in the literature have been summarized, and several future research topics arising on artificial neural networks involved with fractional calculus have been recommended to the research community.

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