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PeV IceCube signals and H_0 tension in the framework of Non-Local Gravity

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Abstract We study possible effects of non-local gravity corrections on the recent discovery by the IceCube collaboration, reporting high-energy neutrino flux detected at energies of order PeV. Considering the 4-dimensional operator $\sim y_{\alpha\chi}\overline{L_\alpha}\, H\, \chi$, it is possible to explain both the IceCube neutrino rate and the abundance of Dark Matter, provided that non-local corrections are present in the cosmological background. Furthermore, the mechanism could constitute a natural way to address the H_0 tension issue.

1 Introduction

General Relativity (GR) provides the best theory to describe gravitational interaction confirmed by experiments and observations ranging from Solar System up to cosmology. Despite these successes, GR is affected by several shortcomings, arising both at ultraviolet (UV) and infrared (IR) scales. These suggest that GR is an effective theory not working at any energy scale and, therefore, it could be not the final theory of gravity. At fundamental level, space-time singularities emerge and the theory is inconsistent with a quantum field formulation. At astrophysical scales, discrepancies between theoretical and observed dynamics in galaxies and clusters of galaxies lead to the missing matter problem. In general, this issue is addressed by introducing Dark Matter (DM), which accounts for more or less 25% of the Universe content. At cosmological scales, current observations show that the present Universe is in an accelerating phase. This can be explained by means of the so-called Dark Energy (DE). However, at the moment, there is no final evidence for particles addressing DM and DE at fundamental scales so modifying the gravitational sector is becoming a realistic approach to the dark side problem (see, e.g. [1]).

A possibility to overcome GR shortcomings, both at UV and IR scales, is considering non-local theories of gravity which are characterized by actions of the form

$$\mathcal{S} = \int d^4x \sqrt{-g}\, R\left[1 + F(\Box^{-1}R)\right], \tag{1.1}$$

where R is the Ricci curvature scalar and $\Box$ the d’Alembert operator. From a fundamental physics point of view, non-local theories emerge in view to obtain renormalizable and unitary effective gravity models [2, 3]. Remarkably, the term $\Box^{-1}R$ can account for the late-time cosmic expansion without invoking any DE [4].

In [5, 6], it is shown that non-local terms are related to conserved quantities coming from the existence of Noether symmetries which select the form of the function $F(\Box^{-1}R)$. Specifically, if a Noether symmetry exists, the Lagrangian in (1.1) can result of the form

$$\mathcal{L} = R\left(c_1 + c_2\, e^{\frac{3q-1}{3(2q-1)}\Box^{-1}R}\right), \tag{1.2}$$

where $c_{1,2}$ are gravitational couplings, q is a free parameter related to the symmetry. Here the case $q = 1/2$ has to be excluded and this fact will be crucial for the discussion below. In a Friedman-Lemaître-Roberson-Walker cosmology, dynamics is exactly solved by

$$a(t) = a_0\, t^q \tag{1.3}$$

$$R(t) = 6q\left(\frac{1-2q}{t^2}\right) \tag{1.4}$$

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$$\Box^{-1}R = \Lambda - \frac{6q(2q-1)\log(t)}{3q-1} \tag{1.5}$$

We can recast the evolution of the scale factor of the Universe (1.3) as

$$a(t) = a_*\left(\frac{t}{t_*}\right)^q, \tag{1.6}$$

where the index $*$ points out the instant t_* (and the corresponding temperature T_*) at which the Universe starts to expand. The expansion rate $H = \dot{a}/a$, where dot indicates the derivative with respect to the cosmic time t, can be cast as [7]

$$H_{non-local}(T) = A(T)H_{GR}(T), \tag{1.7}$$

where H_{GR} is the expansion rate in GR ($H_{GR} = \sqrt{\frac{8\pi\rho}{3M_P^2}} = \sqrt{\frac{8\pi^3 g_*}{45}}\frac{T^2}{M_P}$), and $A(T)$ is an amplification factor. Using the power law solution and the conservation of entropy ($Ta = T_*a_* = T_0$, with T_0 the temperature of the present Universe at $a_0 = 1$), one can parameterize the amplification factor as

$$A(T) = \eta\left(\frac{T}{T_*}\right)^\nu, \qquad \nu = \frac{1}{q} - 2, \qquad \eta = 2q, \tag{1.8}$$

where $\{\eta, \nu\}$ characterize the given cosmological model[1]. To preserve the successful predictions of Big Bang Nucleosynthesis (BBN), one refers to the pre-BBN epoch which is not directly constrained by cosmological observations [7, 8]. Therefore, $A(T) \neq 1$ at early time ($T \gtrsim T_* \gg T_{BBN}$), and $A(T) \to 1$ at $T = T_*$ (before BBN begins).

We want to show that non-local gravity (1.1), in particular the model (1.2) selected by Noether symmetries, allows to get a consistent explanation of PeV-DM and IceCube neutrinos events reported by the IceCube Collaboration [13] with energies ~ 1 PeV [13].

Candidates for the generation of such high energy neutrino events are various astrophysical sources, although there is no clear correlations with the known astrophysical hot-spots like the supernova remnants or active galactic nuclei [14]. This suggests the possibility that neutrinos could arise from the decay of PeV-DM particles [16, 17]. To explain the PeV-DM relic density and the decay rate required for IceCube, one can take into account the minimal DM-neutrino (4-dimensional) interaction

$$\mathcal{L}^{d=4} = y_\alpha \, \bar{L}_\alpha \cdot H\chi, \qquad \alpha = e, \mu, \tau, \tag{1.9}$$

where α indicates the mass eigenstates of the three active neutrinos, χ the DM particle, H the Higgs doublet, L_α the left-handed lepton doublet, and $y_{\alpha\chi}$ the Yukawa couplings. Notice that the 4-dimensional operator (1.9) fails to explain both IceCube data and DM relic abundance, if the cosmological background evolves according to the standard Einstein-Friedman field equations [16, 17]. In the latter case, in fact, one finds that the relic abundance (induced by inverse decay) is [16]

$$\Omega_{DM}h^2 = 0.1188\left(\frac{106.75}{g_*}\right)^{3/2}\frac{\sum_\alpha |y_{\alpha\chi}|^2}{7.5\times 10^{-25}}. \tag{1.10}$$

From (1.10), it follows that in order to get the correct relic DM abundance ($\Omega_{DM}h^2 \sim 0.1188$), one needs

$$\sum_{\alpha=e,\mu,\tau} |y_{\alpha\chi}|^2 = 7.5\times 10^{-25}. \tag{1.11}$$

However, Eq. (1.10) is incompatible with the value of $\sum_{\alpha=e,\mu,\tau}|y_{\alpha\chi}|^2$ needed to explain the IceCube data. To fix this point, let us note that the DM lifetime $\tau_\chi = \Gamma_\chi^{-1}$ has to be larger that the age of the Universe, $\tau_\chi > t_U \simeq 4.35\times 10^{17}$sec. Moreover, IceCube spectrum sets constraints on lower bounds of DM lifetime $\tau_\chi^b \simeq 10^{28}$sec, i.e. $\tau_\chi \gtrsim \tau_\chi^b$, which is (approximatively) model-independent (see [16]). From (1.11), one obtains $\Gamma_\chi \simeq 4.5\times 10^4 \frac{m_\chi}{1\text{PeV}}\text{sec}^{-1}$, that is $\tau_\chi \simeq 2.2\times 10^{-5}\frac{1\text{PeV}}{m_\chi}\text{sec} \ll t_U$. Observations of IceCube collaboration require, however, that the DM decay lifetime has to be $\tau_\chi \sim 10^{28}$ sec, which implies

$$\sum_\alpha |y_{\alpha\chi}|^2 \simeq 10^{-58}, \tag{1.12}$$

which is ~ 33 order of magnitudes smaller than the value of $\sum_{\alpha=e,\mu,\tau}|y_{\alpha\chi}|^2 \sim 10^{-25}$ needed to explain the DM relic abundance, see (1.11). As a consequence, the IceCube high energy events and the DM relic abundance are not compatible with the DM production, if the latter is ascribed to the 4-dimensional operator $\overline{L_\alpha}H\chi$ and it is assumed that the evolution of the Universe is governed by GR. Such a discrepancy can be avoided if non-local gravity corrections are assumed in the cosmological evolution as we will show below.

[1] Investigations have been performed in different cosmological scenarios [7–9], where the parameter ν labels the cosmological model. It is $\nu = 2$ in Randall-Sundrum type II brane cosmology [10]; $\nu = 1$ in kination models [11]; $\nu = 0$ in cosmologies with an overall boost of the Hubble expansion rate [8]; $\nu = -0.8$ in scalar-tensor cosmology [8]; $\nu = 2/n - 2$ in $f(R)$ gravity with $f(R) = R + \alpha R^n$ [12].

2 PeV neutrinos and DM relic abundance in non-local gravity

Let us consider a *freeze-in* production where DM particles are never in thermal equilibrium because they interact very weakly and are gradually produced from the hot thermal bath. This occurs owing to a feeble coupling to the Standard Model particles (at $T \gg m_\chi$) allowing the DM particles to remain in the observed Universe due to small back-reaction rates and to slow decay processes. Therefore, a sizable DM abundance is allowed, at least, until the temperature falls down to $T \sim m_\chi$. Temperatures below m_χ are such that the phase-space of DM particles is kinematically difficult to access [15, 16].

The evolution of DM particles is governed by the Boltzmann equation. Let us denote with $Y_\chi = n_\chi/s$ the DM abundance, where n_χ is the number density of the DM particles and $s = \frac{2\pi^2}{45} g_*(T)T^3$ the entropy density (g_* denotes the degrees of freedom). By using the Boltzmann equation and assuming that the relativistic degrees of freedom are constant, i.e. $dg_*/dT = 0$, the DM relic abundance can be cast in the form

$$\Omega_{DM}h^2 = \frac{2m_\chi^2 s_0 h^2}{\rho_{cr}} \int_0^\infty \frac{dx}{x^2}\left(-\frac{dY_\chi}{dT}\Big|_{T=\frac{m_\chi}{x}}\right), \tag{2.1}$$

where $x = m_\chi/T$. Here $s_0 = \frac{2\pi^2}{45} g_* T_0^3 \simeq 2891.2/\text{cm}^3$ is the present value of the entropy density, and $\rho_{cr} = 1.054 \times 10^{-5} h^2 \text{GeV/cm}^3$ the critical density. In the case of 4-dimensional operator (1.9), the dominant contributions to DM production is given by the *inverse decay* processes $\nu_\alpha + H^0 \to \chi$ and $l_\alpha + H^+ \to \chi$ occurring when $m_\chi > m_H + m_{\nu,l}$. The interaction rate is [16]

$$\Gamma_\chi = \sum_\alpha \frac{|y_{\alpha\chi}|^2}{8\pi} m_\chi\,, \quad \alpha = e, \mu, \tau\,. \tag{2.2}$$

Therefore, according to (1.8), the inverse decay processes turn out to be

$$\frac{dY_\chi}{dT} = -\frac{m_\chi^2 \Gamma_\chi}{\pi^2 s H_{non-local}} K_1\left(\frac{m_\chi}{T}\right), \tag{2.3}$$

where $K_1(x)$ is a modified Bessell function of second kind. From (1.7) and (1.8), it is

$$sH_{non-local} = \frac{3.32\pi^2}{45} g_*^{3/2} \eta \left(\frac{m_\chi}{T_*}\right)^\nu \frac{1}{x^{5-\nu}}\,, \quad x \equiv \frac{T}{T_*}\,. \tag{2.4}$$

By inserting (2.3) into (2.1), one obtains the DM relic abundance

$$\Omega_{DM}h^2 = \frac{45h^2}{1.66\pi^2 g^{3/2}} \frac{s_0 M_{Pl}}{\rho_{cr}} \frac{\Gamma_\chi}{m_\chi} \frac{2^{2+\nu}}{\eta} \left(\frac{T_*}{m_\chi}\right)^\nu \Gamma\left(\frac{5+\nu}{2}\right)\Gamma\left(\frac{3+\nu}{2}\right) \tag{2.5}$$

$$\simeq 0.1188 \left(\frac{106,7}{g_*}\right)^{3/2} \frac{\sum_\alpha |y_{\alpha\chi}|^2}{10^{-58}}\,\Pi\,, \tag{2.6}$$

where $\Gamma(x)$ is the Gamma function, and

$$\Pi \equiv 10^{-33} \frac{2^\nu}{7.5\eta} \left(\frac{T_*}{m_\chi}\right)^\nu \frac{\Gamma\left(\frac{5+\nu}{2}\right)\Gamma\left(\frac{3+\nu}{2}\right)}{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{3}{2}\right)}\,. \tag{2.7}$$

The function Π depends on non-local gravity parameters ν and η, that is q. The DM relic abundance (requiring $\Omega_{DM}h^2 \sim 0.1188$ [18]) and the IceCube data (requiring $\sum_\alpha |y_{\alpha\chi}|^2 \sim 10^{-58}$ [13]) can be consistently explained provided $\Pi \simeq 1$. To search the values of q such that the condition $\Pi \sim 1$ is fulfilled, we parameterize the temperature T as

$$T = 10^\alpha \text{GeV}\,, \tag{2.8}$$

with $\alpha > 6$, that means we consider temperature greater than the DM mass, $T > m_\chi$ (and therefore greater than the BBN temperature $T_{BBN} \sim 1\text{MeV}$). In Fig. 1, we report Π vs q for fixed values of the parameter α. The latter ranges from $\alpha = 8$ to $\alpha = 15$ to account for scales from DM to GUT. As we can see, the parameter q varies in the range $0.07 \lesssim q \lesssim 0.18$. This is the result we want. It shows that, for $q < 1/2$, the expansion rate of the Universe is modified, as well as the Boltzmann equations and any contribution to the energy density due to matter and geometry sectors. The effect on the evolution makes the relic DM abundance and IceCube (1PeV) neutrino signals consistent with the observations. It is worth noticing that the presence of non-local term in the Lagrangian (1.2) exclude $q = 1/2$, corresponding to the radiation dominated epoch of the standard cosmological model. It is worth noticing that the presence of the operator $\Box^{-1}$ explains the current late-time accelerated cosmic expansion without invoking any Dark Energy. Specifically, it enables a delayed response to the radiation-matter transition which could explain the current cosmic acceleration [4]. Such a result opens possibilities to probe new physics beyond GR and it is a natural way to potentially address the H_0 tension issue [19].

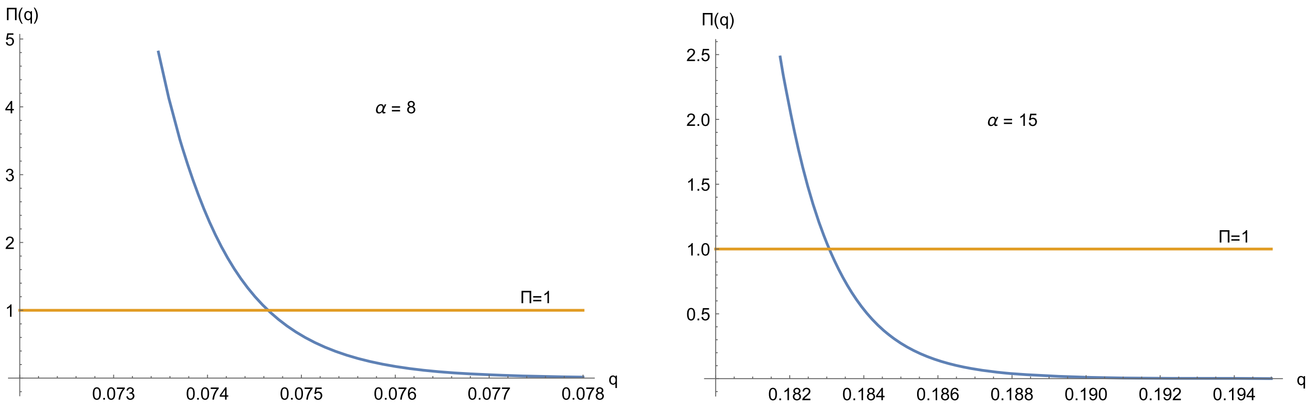

Fig. 1 Π vs q for $\alpha = 8, 15$

3 Conclusions

To reconcile the current bound on DM relic abundance with IceCube data, in terms of the 4-dimensional operator $\mathcal{L}_{d=4} = y_{\alpha\chi}\overline{L_\alpha}\, H\, \chi$, we considered cosmological solutions coming from non-local gravity. Specifically, we focused on cosmological power law solutions emerging from a non-local model selected by the presence of Noether symmetries. These exact solutions allow to account for IceCube observations (high energy neutrinos) and DM relic abundance, observed in a minimal particle physics model. The main idea relies on the fact that, in non-local cosmology, the expansion rate of the Universe can be cast in the form $H(T) = A(T)H_{GR}(T)$, encoding in $A(T)$ the parameters characterizing the cosmological model. As a consequence, also the thermal history of particles results modified. On the other hand, the PeV signals reported by the IceCube collaboration could be a straightforward signature for non-local effects emerging at cosmological level. Finally, the stretching induced by non-local term in the Hubble parameter, led by the function $A(T)$, could solve, in principle, the H_0 tension in the framework of fundamental physics. In a forthcoming paper, we will discuss in detail this topic from an observational point of view.

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