



NMSSM with generalized deflected mirage mediation

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Abstract We propose to generate a realistic soft SUSY breaking spectrum for Next-to-Minimal Supersymmetric Standard Model (NMSSM) with a generalized deflected mirage mediation scenario, in which additional Yukawa and gauge mediation contributions are included to deflect the renormalization group equation trajectory. Based on the Wilsonian effective action obtained by integrating out the messengers, the NMSSM soft SUSY breaking spectrum can be given analytically at the messenger scale. We find that additional contributions to $m_{\tilde{g}}^2$ can possibly ameliorate the stringent constraints from the electroweak symmetry breaking and 125 GeV Higgs mass. Constraints from dark matter and fine-tuning are also discussed. The Barbieri–Giudice fine-tuning measure and electroweak fine-tuning measure in our scenario can be as low as $\mathcal{O}(1)$, which possibly indicates that our scenario is natural.

1 Introduction

Low energy supersymmetry (SUSY), which can solve elegantly the gauge hierarchy problem of the Standard Model (SM) and provide a viable dark matter (DM) candidate, has been regarded by many physicists as one of the most appealing candidates for the TeV-scale new physics. However, the reported data of the Large Hadron Collider (LHC) agree quite well with the SM predictions and no significant deviations have been observed in the electroweak precision measurements and the flavor physics. Besides, the lack of SUSY signals at the LHC [1–3] and the difficulty to accommodate the discovered 125 GeV Higgs [4,5] seem to indicate that the low energy SUSY spectrum should display an intricate structure. As the low energy soft SUSY breaking spectrum is determined by the SUSY breaking mechanism, it is interesting to survey the phenomenology related to the SUSY breaking and mediation mechanism from a top-down approach.

SUSY breaking mechanism from flux compactification of type IIB string theory can lead to interesting consequences. In the generalized Kahler-modulus dominated scenarios, the dilaton and complex structure moduli fields can be stabilized by the background NS and RR 3-form fluxes. Such super-heavy modes will be integrated out and eliminated from the low energy effective theory. The remaining Kahler moduli fields can be stabilized by non-perturbative effects, such as the gaugino condensation. To generate SUSY breaking in the observable sector and tune the cosmological constant to a tiny positive value, Kachru et al. propose to add an anti-D3 brane at the tip of the Klebanov–Strassler throat to explicitly break SUSY and lift the AdS universe to obtain a dS one [6]. Consequently, the F-component of the light Kahler moduli field could mediate the SUSY breaking effects to the visible sector and result in a mixed modulus-anomaly mediation SUSY breaking scenario [7,8]. It is interesting to note that the

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involved modulus mediated SUSY breaking contributions can be comparable to that of the anomaly mediation [9, 10]. With certain assumptions on the Yukawa couplings and the modular weights, the change of the SUSY breaking contributions (from the inputs) by the renormalization group equation (RGE) evolution and the anomaly mediation contributions could cancel each other at a ‘mirage’ scale, leading to an apparent pure modulus mediation scenario at such a mirage scale [11]. Such a mixed modulus-anomaly mediation SUSY breaking mechanism is dubbed as ‘mirage mediation’.

Anomaly mediation contribution is a crucial ingredient of such a mixed modulus-anomaly mediation mechanism. It is well known that purely anomaly mediation mechanism is bothered by the tachyonic slepton problem [12–18]. One of its non-trivial extensions with a messenger sector, namely the deflected anomaly mediation SUSY breaking (AMSB) scenario, can elegantly solve such a tachyonic slepton problem through the deflection of the RGE trajectory [19–25] by additional gauge [26–32] or Yukawa mediation contributions. Such a messenger sector can also be present in the mirage mediation scenarios and play an important role in generating a preferable low energy SUSY spectrum. In fact, the ‘mirage’ unification of gaugino masses at TeV scale can be possible in deflected mirage mediation scenarios, even with the simplest minimal KKLТ set-up [33–35]. In the mixed modulus-anomaly mediation mechanism, a straightforward extension of μ sector to predict a small $B\mu$ term needs some fine tuning [11]. Such a $\mu - B\mu$ problem can be solved naturally in the framework of Next-to-Minimal Supersymmetric Standard Model (NMSSM). The realization of NMSSM in TeV mirage mediation scenarios had already been discussed in the literature [36–39]. However, it was found that only a small portion of the parameter space can be consistent with the electroweak symmetry breaking (EWSB) conditions and at the same time accommodate the 125 GeV Higgs [36, 37]. So it is rather interesting to seek new ways to generate a realistic NMSSM spectrum.

Additional gauge or Yukawa mediation contributions in the mirage mediation [33–35] can deflect the RGE trajectory and change the low energy soft SUSY predictions. Analytical expressions for the soft SUSY breaking parameters at the messenger scale are given in the Wilsonian effective action approach [40] by one of the authors. Based on the general discussions in [40], we propose to generate the NMSSM spectrum by a generalized deflected mirage mediation mechanism [41] with additional gauge and Yukawa mediation contributions. We find that the inclusion of the messenger sector and non-trivial interactions can possibly alleviate the stringent constraints from the 125 GeV Higgs and the EWSB conditions.

This paper is organized as follows. We briefly review the mirage mediation mechanism in Sect. 2. We state our model and present the analytical expressions for the soft SUSY

parameters in Sect. 3. The numerical results are discussed in Sect. 4. Section 5 contains our conclusions.

2 Brief review of the mirage mediation mechanism

In Type IIB string theory compactified on a Calabi–Yau orientifold, the presence of background fluxes can fix the dilaton and the complex structure moduli, leaving only the Kahler moduli in the four-dimensional Wilsonian effective supergravity action (defined at the boundary scale Λ) after integrating out the superheavy complex structure moduli and dilaton. The low energy effective Lagrangian in terms of compensator field and a single Kahler modulus that parameterizes the overall size of the compact space [11] is given as

$$e^{-1}\mathcal{L} = \int d^4\theta \left[\phi^\dagger \phi \left(-3e^{-K/3} \right) - (\phi^\dagger \phi)^2 \bar{\theta}^2 \theta^2 \mathcal{P}_{lift} \right] + \int d^2\theta \phi^3 W + \int d^2\theta \frac{f_i}{4} W_i^a W_i^a, \quad (2.1)$$

with a holomorphic gauge kinetic term

$$f_i = \frac{1}{g_i^2} + i \frac{\theta}{8\pi}. \quad (2.2)$$

The Kahler potential involves the ‘no-scale’ kinetic form for the Kahler modulus while the superpotential involves the KKLТ setup

$$W = \left(\omega_0 - A e^{-aT} \right) + W_0, \quad (2.3)$$

where the first term is generated from the fluxes and the second term from non-perturbative effects, such as gaugino condensation from the non-abelian gauge sector or D3-brane instantons. W_0 denotes the superpotential terms involving the MSSM (NMSSM) sector as well as a possible messenger sector. The modulus T , which is not fixed by the background flux, can be stabilized by such a KKLТ-type superpotential with

$$a \Re\langle T \rangle \approx \ln \left(\frac{A}{\omega_0} \right) \approx \ln \left(\frac{M_{Pl}}{m_{3/2}} \right) \approx 4\pi^2, \quad (2.4)$$

up to $\mathcal{O}(\ln[M_{Pl}/m_{3/2}]^{-1})$. Note that in the KKLТ setup, the flux-induced SUSY breaking is dynamically canceled by the non-perturbative SUSY breaking that stabilizes the Kahler modulus T , leading to a SUSY-preserving AdS solution. In order to obtain a vacuum with a positive cosmological constant and break SUSY, KKLТ proposed to add an $\bar{D}3$ brane to provide an uplifting operator given by

$$\mathcal{P}_{lift} = D(T + T^\dagger)^{n_P}, \quad (2.5)$$

with a positive constant $D \sim \mathcal{O}(m_{3/2}^2 M_{Pl}^2)$. The uplifting operator, which represents the low energy consequence of the sequestered SUSY-breaking brane, is independent of visible matter fields and T (with $n_P = 0$) in the minimal KKLТ

set-up. Explicit SUSY breaking via anti-D3 branes can be replaced by typical D-term or F-term uplifting mechanisms [42–44].

With the uplifting low energy effective potential, we have the leading order F-terms of compensator field F_ϕ and Kahler modulus F_T [11]

$$F_\phi \approx m_{3/2} \approx \frac{\omega_0}{M_{Pl}^2 (T + T^*)^{3/2}}, \tag{2.6}$$

$$M_0 \equiv \frac{F_T}{T + T^*} \approx \frac{2}{a(T + T^*)} m_{3/2} \approx \frac{m_{3/2}}{\ln\left(\frac{M_{Pl}}{m_{3/2}}\right)}. \tag{2.7}$$

The non-zero F-term VEV of the heavy moduli H are given approximately by $F_H \sim m_{3/2}^2/m_U \ll m_{3/2}$. Therefore, it gives negligible contributions to SUSY breaking [45].

In the mirage mediation, we have $m_{3/2} \approx (4\pi^2)M_0$ numerically, which means that the loop induced anomaly mediation contributions are comparable to the modulus mediation contributions. The importance of the anomaly mediation contributions relative to the modulus mediation contributions can be parameterized by

$$\alpha' \equiv \frac{m_{3/2}}{M_0 \ln\left(M_{Pl}/m_{3/2}\right)}. \tag{2.8}$$

So $\alpha' \ll 1$ corresponds to the limit of pure modulus mediation, while $\alpha' \gg 1$ corresponds to the pure anomaly mediation. Although the minimal KKLT predicts $\alpha' \approx 1$, other values of $\alpha' \sim \mathcal{O}(1)$ can be obtained by assuming proper uplifting operator and proper forms for the potential of Kahler modulus [11]. So we leave the value α' as a free parameter in the following discussions.

3 NMSSM with deflected mirage mediation

The general form of the low energy effective action Eq. (2.1) can be amended to include new ingredients, for example, the NMSSM sector and a messenger sector. The Kahler potential will include not only the ‘no-scale’ kinetic form for the Kahler modulus but also additional kinetic terms for messenger fields

$$K = -3 \ln(T + T^\dagger) + Z_X(T^\dagger, T)X^\dagger X + Z_\Phi(T^\dagger, T)\Phi^\dagger \Phi + \sum_i Z_{P_i, \tilde{P}_i}(T^\dagger, T) \left[P_i^\dagger P_i + \tilde{P}_i^\dagger \tilde{P}_i \right], \tag{3.1}$$

with \tilde{P}_i, P_i the messenger superfields and Φ the NMSSM superfield. The Kahler metric for matter fields and messengers as well as holomorphic gauge kinetic functions are assumed to depend non-trivially on the Kahler modulus T

$$Z_X(T^\dagger, T) = \frac{1}{(T^\dagger + T)^{n_X}}, \quad Z_\Phi(T^\dagger, T) = \frac{1}{(T^\dagger + T)^{n_\Phi}},$$

$$f_i(T) = T^{l_i}, \quad Z_{P_i, \tilde{P}_i}(T^\dagger, T) = \frac{1}{(T^\dagger + T)^{n_P}}. \tag{3.2}$$

The values of n_X, n_Φ, n_P, l_i depend on the location of the fields on the D3/D7 branes. Besides, universal $l_i = 1$ are adopted in our scenario so that the gauge fields reside on the D7 brane and the gauge coupling unification can be preserved.

The W_0 term within the superpotential Eq. (2.3) is given as

$$W_0 = W_{NMSSM} + W_M, \tag{3.3}$$

with

$$W_{NMSSM} = \lambda S H_u H_d + \frac{1}{3} \kappa S^3 + W_{MSSM}, \tag{3.4}$$

the (Z_3 symmetric) NMSSM sector and

$$W_M = \sum_m \left[\lambda_X^T X \tilde{X}_m X_m + \lambda_X^D X \tilde{Y}_m Y_m \right] + \lambda_P^T S \tilde{X}_1 X_2 + \lambda_P^D S \tilde{Y}_1 Y_2 + W(X), \tag{3.5}$$

the messenger sector. The ‘ $2m$ ’ family of messengers can be fitted into $\mathbf{5}, \bar{\mathbf{5}}$ representation of $SU(5)$ gauge group

$$P_m(\mathbf{5}) = X_m(3, 1)_{-1/3} \oplus Y_m(1, 2)_{1/2},$$

$$\tilde{P}_m(\bar{\mathbf{5}}) = \tilde{X}_m(\bar{3}, 1)_{1/3} \oplus \tilde{Y}_m(1, \bar{2})_{-1/2}. \tag{3.6}$$

The VEV of the pseudo-modulus superfield X , which can be determined by $W(X)$ and other SUSY breaking effects, can set the messenger thresholds through the $X P_m \tilde{P}_m$ type couplings. The deflection parameter ‘ d ’, which characterizes the relative size of contributions between the gauge (Yukawa) mediation and anomaly mediation, can be readily obtained

$$dF_\phi \equiv \frac{F_X}{X} - F_\phi. \tag{3.7}$$

Similar to the deflected AMSB, a positive deflection parameter in the deflected mirage mediation, which can be generated by a carefully chosen superpotential etc. [21–25], may be preferable because less messenger species are needed so that the problem of strong gauge couplings below the GUT scale can be evaded. The purpose to introduce even number of messenger species is to forbid possible kinetic mixing between X and S , otherwise the tadpole term for S would destabilize the weak scale [46]. The discrete Z_3 breaking by EWSB may generate domain walls in the early universe which may lead to an unacceptably large anisotropy of CMB. To avoid such a problem, the Z_3 symmetry is assumed to be broken by some higher dimensional operators.

There are two approaches to obtain the low energy SUSY spectrum in the deflected mirage mediation:

- In the first approach, the deflected mirage mediation soft SUSY spectrum is given by the expressions in [11] at the boundary scale. Such a spectrum will receive additional contributions towards its RGE running to low energy scale, especially the threshold corrections related to the appearance of messengers [47].
- In the second approach which we will adopt, the soft SUSY spectrum at low energy scale is derived directly from the (low energy) effective action. The SUGRA description in Eq. (2.1) can be seen as the Wilsonian effective action after integrating out the superheavy complex structure moduli and dilaton field. After the pseudo-modulus X acquires a VEV and determines the messenger threshold, the messengers can be integrated out to obtain a low energy effective action below the messenger threshold scale. So we anticipate that the Kahler metric Z_ϕ and gauge kinetic f_i should depend non-trivially on the messenger threshold $M_{mess}^2/\phi^\dagger\phi$ and M_{mess}/ϕ , respectively. The resulting soft SUSY spectrum below the messenger threshold can be derived from the wavefunction renormalization approach [48–54]. The analytical expressions for the soft SUSY breaking parameters in the most general form of deflected mirage mediation scenarios are given in [40] by one of the authors.

3.1 The modular weight choices for NMSSM superfields

We need to specify the modular weights $n_i \equiv 1 - m_i$ for NMSSM superfields before we can carry out numerical analysis. In the NMSSM, successful EWSB and the solution to the μ -problem in general require a large VEV for the singlet S . So it is preferable to introduce a negative m_S^2 and/or large trilinear terms A_λ, A_κ for the singlet superpotential interactions. As a negative m_S^2 prefers vanishing modulus contributions, we set the modular weight $m_S = 0$. The choice of modulus weight for H_u, H_d can be understood from the EWSB conditions in NMSSM. From

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta^2}{\tan^2 \beta - 1} - \mu^2, \tag{3.8}$$

we can see that $m_{H_u}^2$ should be light to avoid a too large fine-tuning. On the other hand, $m_{H_u}^2 \ll m_{H_d}^2$ for $\tan \beta \gg 1$. So we can set $m(H_d) = 1$ or $1/2$ and $m_{H_u} = 0$.

The electroweak (EW) naturalness prefers relatively light stops. In the MSSM, light stops below 1 TeV are disfavored because it is difficult to accommodate the observed 125 GeV Higgs. However, large loop corrections involving stops are not necessarily required to interpret the 125 GeV Higgs in NMSSM. So light stops, which is preferable from the criterion of a low EW fine-tuning, are still allowed in NMSSM. Squarks of the first two generations should be heavy to avoid various SUSY CP and flavor constraints. We note that even

pure AMSB contributions can guarantee the heaviness of colored SUSY particles. Besides, it is preferable to ameliorate the $g_\mu - 2$ discrepancy in the framework of SUSY with light sleptons and electroweakinos. With the modular weight $l_i = 1$ for gauge fields, a positive deflection parameter ‘ d ’ can possibly guarantee the lightness of the electroweakinos.

The notorious tachyonic slepton problem in AMSB can be solved in our scenario. Positive slepton masses can be obtained by introducing a proper deflection parameter ‘ d ’ or by adding extra modulus mediation contributions. So we chose the following m_i in our scenario:

- Modular weights for sleptons are given by $m_{(L_L)^{1,2,3}} = m_{(E_L^c)^{1,2,3}} = 1/2$.
- Modular weights for other matter and messenger fields are given by

$$\begin{aligned} m_{H_u} &= m_S = m_{Q_L^3} = m_{t_L^c} = m_{b_L^c} = 0, \\ m_a &= \frac{1}{2}, \quad (a = Q_L^{1,2}, (U_L^c)^{1,2}, (D_L^c)^{1,2}), \\ m_{H_d} &= m_X = m_{\tilde{X}} = m_Y = m_{\tilde{Y}} = 1. \end{aligned} \tag{3.9}$$

- Double messenger species with $m = 1$ are adopted in our subsequent numerical study.

Note that the messenger modular weights also play a role and contribute to m_S^2 . The modular weights $n_i = 0$ correspond to matter fields on D7 branes while $n_i = 1$ on D3 branes. Modular weights $n_i = 1/2$ corresponds to fields on the intersections of the D3–D7 branes.

3.2 Analytical expressions for soft SUSY breaking parameters

Now we use the second Wilsonian approach in our analysis. From the analytical expressions for the generalized mirage mediation [40], the soft SUSY breaking parameters in NMSSM at the messenger scale M_{mess} after integrating out the messenger fields can be given explicitly. The gaugino masses are given as

$$M_i = l_i M_0 \frac{g_i^2(\mu)}{g_i^2(GUT)} + F_\phi \frac{\alpha_i}{4\pi} (b_i - d\Delta b_i), \tag{3.10}$$

with $l_i = 1$ and

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3 \right), \tag{3.11}$$

$$\Delta(b_1, b_2, b_3) = (2, 2, 2). \tag{3.12}$$

Within the expression, the relative size between the anomaly and modulus mediation contribution at the messenger scale

is determined by the free parameter

$$\alpha = \frac{F_\phi}{(16\pi^2)M_0}. \tag{3.13}$$

We define the modular weight $q_{y_{ijk}}$ as

$$\begin{aligned} q_{y_t} &= m_{Q_{L,3}} + m_{H_u} + m_{t_L^c}, \\ q_{y_b} &= m_{Q_{L,3}} + m_{H_d} + m_{b_L^c}, \\ q_{y_\tau} &= m_{L_{L,3}} + m_{H_d} + m_{\tau_L^c}, \\ q_\lambda &= m_S + m_{H_d} + m_{H_u}, \\ q_\kappa &= 3m_S. \end{aligned} \tag{3.14}$$

The trilinear soft terms at the messenger scale are given by

$$\begin{aligned} A_t &= q_{y_t} M_0 - \frac{M_0}{8\pi^2} \left[6y_t^2 \frac{q_{y_t}}{2} + y_b^2 \frac{q_{y_b}}{2} + \lambda^2 \frac{q_\lambda}{2} \right. \\ &\quad \left. - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 \right) \right] \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \\ &\quad + \frac{F_\phi}{16\pi^2} \left[6y_t^2 + y_b^2 + \lambda^2 - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 \right) \right], \end{aligned} \tag{3.15}$$

$$\begin{aligned} A_b &= q_{y_b} M_0 - \frac{M_0}{8\pi^2} \left[y_t^2 \frac{q_{y_t}}{2} + 6y_b^2 \frac{q_{y_b}}{2} + \lambda^2 \frac{q_\lambda}{2} \right. \\ &\quad \left. - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 \right) \right] \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \\ &\quad + \frac{F_\phi}{16\pi^2} \left[y_t^2 + 6y_b^2 + \lambda^2 - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 \right) \right], \end{aligned} \tag{3.16}$$

$$\begin{aligned} A_\tau &= q_{y_\tau} M_0 - \frac{M_0}{8\pi^2} \left[3y_b^2 \frac{q_{y_b}}{2} + \lambda^2 \frac{q_\lambda}{2} \right. \\ &\quad \left. - \left(3g_2^2 + \frac{9}{5}g_1^2 \right) \right] \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \\ &\quad + \frac{F_\phi}{16\pi^2} \left[3y_b^2 + \lambda^2 - \left(3g_2^2 + \frac{9}{5}g_1^2 \right) \right], \end{aligned} \tag{3.17}$$

$$\begin{aligned} A_\lambda &= q_\lambda M_0 - \frac{M_0}{8\pi^2} \left[3y_t^2 \frac{q_{y_t}}{2} + 3y_b^2 \frac{q_{y_b}}{2} + 4\lambda^2 \frac{q_\lambda}{2} \right. \\ &\quad \left. + 2\kappa^2 \frac{q_\kappa}{2} - \left(3g_2^2 + \frac{3}{5}g_1^2 \right) \right] \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \\ &\quad + \frac{F_\phi}{16\pi^2} \left[4\lambda^2 + 2\kappa^2 + 3y_t^2 + 3y_b^2 - \left(3g_2^2 + \frac{3}{5}g_1^2 \right) \right] \\ &\quad + \Delta A_\lambda, \end{aligned} \tag{3.18}$$

$$\begin{aligned} A_\kappa &= q_\kappa M_0 - \frac{M_0}{8\pi^2} \left[6\lambda^2 \frac{q_\lambda}{2} + 6\kappa^2 \frac{q_\kappa}{2} \right] \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \\ &\quad + \frac{F_\phi}{16\pi^2} \left[6\lambda^2 + 6\kappa^2 \right] + \Delta A_\kappa, \end{aligned} \tag{3.19}$$

with new contributions due to non-vanishing ΔG_S

$$\Delta A_\lambda = -d \frac{F_\phi}{16\pi^2} \left[3(\lambda_P^T)^2 + 2(\lambda_P^D)^2 \right], \tag{3.20}$$

$$\Delta A_\kappa = -3d \frac{F_\phi}{16\pi^2} \left[3(\lambda_P^T)^2 + 2(\lambda_P^D)^2 \right]. \tag{3.21}$$

The soft SUSY breaking parameters for the scalars can be parameterized by

$$m_{soft}^2 = \delta_m + \delta_d + \delta_I, \tag{3.22}$$

with each part given as follows

- The pure modulus contribution part

$$\begin{aligned} \delta_{\tilde{Q}_{L,3}}^m &= m_{Q_{L,3}} M_0^2 \\ &\quad - \frac{M_0^2}{8\pi^2} \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \left\{ (q_{y_t}^2 + q_{y_t})y_t^2 + (q_{y_b}^2 \right. \\ &\quad \left. + q_{y_b})y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{1}{15}g_1^2 \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left[-y_t^2 q_{y_t} K_{y_t} - y_b^2 q_{y_b} K_{y_b} \right] \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left[\frac{8}{3}b'_3 g_3^2 + \frac{3}{2}b'_2 g_2^2 + \frac{1}{30}b'_1 g_1^2 \right] \right. \\ &\quad \left. \times \ln \left(\frac{GUT}{M_{mess}} \right) \right\}, \end{aligned} \tag{3.23}$$

$$\begin{aligned} \delta_{\tilde{U}_{L,3}^c}^m &= m_{U_{L,3}^c} M_0^2 - \frac{M_0^2}{8\pi^2} \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \left\{ 2(q_{y_t}^2 + q_{y_t})y_t^2 \right. \\ &\quad \left. - \frac{16}{3}g_3^2 - \frac{16}{15}g_1^2 \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left[-2y_t^2 q_{y_t} K_{y_t} \right] \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left[\frac{8}{3}b'_3 g_3^2 + \frac{8}{15}b'_1 g_1^2 \right] \ln \left(\frac{GUT}{M_{mess}} \right) \right\}, \end{aligned} \tag{3.24}$$

$$\begin{aligned} \delta_{\tilde{D}_{L,3}^c}^m &= m_{D_{L,3}^c} M_0^2 - \frac{M_0^2}{8\pi^2} \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \left\{ 2(q_{y_b}^2 + q_{y_b})y_b^2 \right. \\ &\quad \left. - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2 \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left[-2y_b^2 q_{y_b} K_{y_b} \right] \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left[\frac{8}{3}b'_3 g_3^2 + \frac{2}{15}b'_1 g_1^2 \right] \ln \left(\frac{GUT}{M_{mess}} \right) \right\}, \end{aligned} \tag{3.25}$$

$$\begin{aligned} \delta_{\tilde{L}_{L,a}}^m &= m_{L_{L,a}} M_0^2 \\ &\quad - \frac{M_0^2}{8\pi^2} \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \left\{ -3g_2^2 - \frac{3}{5}g_1^2 \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left[\frac{3}{2}b'_2 g_2^2 + \frac{3}{10}b'_1 g_1^2 \right] \ln \left(\frac{GUT}{M_{mess}} \right) \right\}, \end{aligned} \tag{3.26}$$

$$\begin{aligned} \delta_{\tilde{E}_{L,a}^c}^m &= m_{E_{L,a}^c} M_0^2 \\ &\quad - \frac{M_0^2}{8\pi^2} \ln \left(\frac{M_{GUT}}{M_{mess}} \right) \left\{ -\frac{12}{5}g_1^2 \right. \\ &\quad \left. + \frac{1}{8\pi^2} \left(\frac{6}{5}b'_1 g_1^2 \right) \ln \left(\frac{GUT}{M_{mess}} \right) \right\}, \end{aligned} \tag{3.27}$$

$$\begin{aligned} \delta_{H_u}^m &= m_{H_u} M_0^2 \\ &\quad - \frac{M_0^2}{8\pi^2} \ln\left(\frac{M_{GUT}}{M_{mess}}\right) \left\{ 3(q_{y_t}^2 + q_{y_t})y_t^2 \right. \\ &\quad + (q_\lambda^2 + q_\lambda)\lambda^2 - 3g_2^2 - \frac{3}{5}g_1^2 \\ &\quad + \frac{1}{8\pi^2} \left[-3y_t^2 q_{y_t} K_{y_t} - \lambda^2 q_\lambda K_\lambda \right] \\ &\quad \left. + \frac{1}{8\pi^2} \left[\frac{3}{2}b'_2 g_2^2 + \frac{3}{10}b'_1 g_1^2 \right] \ln\left(\frac{GUT}{M_{mess}}\right) \right\}, \end{aligned} \tag{3.28}$$

$$\begin{aligned} \delta_{H_d}^m &= m_{H_d} M_0^2 - \frac{M_0^2}{8\pi^2} \ln\left(\frac{M_{GUT}}{M_{mess}}\right) \left\{ 3(q_{y_b}^2 + q_{y_b})y_b^2 \right. \\ &\quad + (q_\lambda^2 + q_\lambda)\lambda^2 - 3g_2^2 - \frac{3}{5}g_1^2 \\ &\quad + \frac{1}{8\pi^2} \left[-3y_b^2 q_{y_b} K_{y_b} - \lambda^2 q_\lambda K_\lambda \right] \\ &\quad \left. + \frac{1}{8\pi^2} \left[\frac{3}{2}b'_2 g_2^2 + \frac{3}{10}b'_1 g_1^2 \right] \ln\left(\frac{GUT}{M_{mess}}\right) \right\}, \end{aligned} \tag{3.29}$$

$$\begin{aligned} \delta_S^m &= m_S M_0^2 - \frac{M_0^2}{8\pi^2} \ln\left(\frac{M_{GUT}}{M_{mess}}\right) \left\{ 2(q_\lambda^2 + q_\lambda)\lambda^2 \right. \\ &\quad + 2(q_\kappa^2 + q_\kappa)\kappa^2 \\ &\quad \left. + \frac{1}{8\pi^2} \left[-2\lambda^2 q_\lambda K_\lambda - 2\kappa^2 q_\kappa K_\kappa \right] \right\}, \end{aligned} \tag{3.30}$$

with $b'_i = b_i + \Delta b_i$ being the beta function upon the messenger thresholds and

$$\begin{aligned} K_{y_t} &= \left[6y_t^2 q_{y_t} + y_b^2 q_{y_b} + \lambda^2 q_\lambda \right. \\ &\quad \left. - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 \right) \right] \ln\left(\frac{GUT}{M_{mess}}\right), \end{aligned} \tag{3.31}$$

$$\begin{aligned} K_{y_b} &= \left[y_t^2 q_{y_t} + 6y_b^2 q_{y_b} + \lambda^2 q_\lambda \right. \\ &\quad \left. - \left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 \right) \right] \ln\left(\frac{GUT}{M_{mess}}\right), \end{aligned} \tag{3.32}$$

$$\begin{aligned} K_\lambda &= \left[3y_t^2 q_{y_t} + 3y_b^2 q_{y_b} + 4\lambda^2 q_\lambda + 2\kappa^2 q_\kappa \right. \\ &\quad \left. - \left(3g_2^2 + \frac{3}{5}g_1^2 \right) \right] \ln\left(\frac{GUT}{M_{mess}}\right), \end{aligned} \tag{3.33}$$

$$K_\kappa = \left[6\lambda^2 q_\lambda + 6\kappa^2 q_\kappa \right] \ln\left(\frac{GUT}{M_{mess}}\right). \tag{3.34}$$

Expressions for the first two generations can be obtained by setting $y_t = y_b = 0$.

- The deflected anomaly mediation part

$$\delta_{\tilde{Q}_{L:1,2}}^d = \frac{F_\phi^2}{16\pi^2} \left[\frac{8}{3}G_3\alpha_3^2 + \frac{3}{2}G_2\alpha_2^2 + \frac{1}{30}G_1\alpha_1^2 \right], \tag{3.35}$$

$$\delta_{\tilde{U}_{L:1,2}}^d = \frac{F_\phi^2}{16\pi^2} \left[\frac{8}{3}G_3\alpha_3^2 + \frac{8}{15}G_1\alpha_1^2 \right], \tag{3.36}$$

$$\delta_{\tilde{D}_{L:1,2,3}}^d = \frac{F_\phi^2}{16\pi^2} \left[\frac{8}{3}G_3\alpha_3^2 + \frac{2}{15}G_1\alpha_1^2 \right], \tag{3.37}$$

$$\delta_{\tilde{L}_{L:1,2,3}}^d = \frac{F_\phi^2}{16\pi^2} \left[\frac{3}{2}G_2\alpha_2^2 + \frac{3}{10}G_1\alpha_1^2 \right], \tag{3.38}$$

$$\delta_{\tilde{E}_{L:1,2,3}}^d = \frac{F_\phi^2}{16\pi^2} \frac{6}{5}G_1\alpha_1^2, \tag{3.39}$$

$$\begin{aligned} \delta_{H_u}^d &= \frac{F_\phi^2}{16\pi^2} \left[\frac{3}{2}G_2\alpha_2^2 + \frac{3}{10}G_1\alpha_1^2 \right] \\ &\quad + \frac{F_\phi^2}{(16\pi^2)^2} \left[\lambda^2 G_\lambda + 3y_t^2 G_{y_t} \right] \\ &\quad - 2d \frac{F_\phi^2}{(16\pi^2)^2} \lambda^2 \left[3(\lambda_P^T)^2 + 2(\lambda_P^D)^2 \right], \end{aligned} \tag{3.40}$$

$$\begin{aligned} \delta_{H_d}^d &= \frac{F_\phi^2}{16\pi^2} \left[\frac{3}{2}G_2\alpha_2^2 + \frac{3}{10}G_1\alpha_1^2 \right] \\ &\quad + \frac{F_\phi^2}{(16\pi^2)^2} \left[\lambda^2 G_\lambda + 3y_b^2 G_{y_b} \right] \\ &\quad - 2d \frac{F_\phi^2}{(16\pi^2)^2} \lambda^2 \left[3(\lambda_P^T)^2 + 2(\lambda_P^D)^2 \right], \end{aligned} \tag{3.41}$$

with

$$\begin{aligned} G_{y_t} &= 6y_t^2 + y_b^2 + \lambda^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2, \\ G_{y_b} &= y_t^2 + 6y_b^2 + \lambda^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2, \\ G_\lambda &= 4\lambda^2 + 2\kappa^2 + 3y_t^2 + 3y_b^2 - 3g_2^2 - \frac{3}{5}g_1^2, \\ G_\kappa &= 6\lambda^2 + 6\kappa^2, \end{aligned} \tag{3.42}$$

and $N = 2$ in our scenario for

$$G_i = Nd^2 + 2Nd - b_i, \tag{3.43}$$

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3 \right). \tag{3.44}$$

For the third generation $\tilde{Q}_{L,3}, \tilde{U}_{L,3}^c$, we need to include the ‘ y_t, y_b ’ Yukawa contributions

$$\delta_{\tilde{Q}_{L,3}}^d = \delta_{\tilde{Q}_{L:1,2}}^d + F_\phi^2 \frac{1}{(16\pi^2)^2} \left[y_t^2 G_{y_t} + y_b^2 G_{y_b} \right], \tag{3.45}$$

$$\delta_{\tilde{U}_{L,3}}^d = \delta_{\tilde{U}_{L,1,2}}^d + F_\phi^2 \frac{1}{(16\pi^2)^2} 2y_t^2 G_{y_t}, \tag{3.46}$$

$$\delta_{\tilde{D}_{L,3}}^d = \delta_{\tilde{D}_{L,1,2}}^d + F_\phi^2 \frac{1}{(16\pi^2)^2} 2y_b^2 G_{y_b}. \tag{3.47}$$

The contributions to $\delta_{\tilde{S}}^I$ can be divided into three parts

$$\delta_{\tilde{S}}^d = \Delta_P^A + \Delta_P^G + \Delta_P^I. \tag{3.48}$$

We have the pure anomaly mediation part

$$\Delta_P^A = \frac{F_\phi^2}{(16\pi^2)^2} [2\lambda^2 G_\lambda + 2\kappa^2 G_\kappa]. \tag{3.49}$$

Besides, the $m_{\tilde{S}}^2$ term receives new contributions involving λ_P because G_S is not continuous across the messenger threshold

$$\Delta G_S = -\frac{1}{8\pi^2} [3(\lambda_P^T)^2 Z_{XX}^{-1} Z_{\tilde{X}\tilde{X}}^{-1} + 2(\lambda_P^D)^2 Z_{YY}^{-1} Z_{\tilde{Y}\tilde{Y}}^{-1}]. \tag{3.50}$$

So the Yukawa mediation contribution is

$$\Delta_P^G = -\frac{d^2 F_\phi^2}{4(8\pi^2)^2} [3(\lambda_P^T)^2 (G_{\lambda_P^T}^+) + 2(\lambda_P^D)^2 (G_{\lambda_P^D}^+)] + \frac{d^2 F_\phi^2}{16\pi^2} (\lambda^2 \Delta G_\lambda + \kappa^2 \Delta G_\kappa), \tag{3.51}$$

with

$$G_{\lambda_P^T}^+ = -\frac{1}{8\pi^2} [5(\lambda_P^T)^2 + 2(\lambda_P^D)^2 + 2\lambda^2 + 2\kappa^2 + 2(\lambda_X^T)^2 - (\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2)], \tag{3.52}$$

$$G_{\lambda_P^D}^+ = -\frac{1}{8\pi^2} [3(\lambda_P^T)^2 + 4(\lambda_P^D)^2 + 2\lambda^2 + 2\kappa^2 + 2(\lambda_X^D)^2 - (3g_2^2 + \frac{3}{5}g_1^2)], \tag{3.53}$$

$$\Delta G_\kappa = -\frac{1}{8\pi^2} 3 [3(\lambda_P^T)^2 + 2(\lambda_P^D)^2], \tag{3.54}$$

$$\Delta G_\lambda = -\frac{1}{8\pi^2} [3(\lambda_P^T)^2 + 2(\lambda_P^D)^2]. \tag{3.55}$$

The anomaly-gauge (Yukawa) mixing term is given by

$$\Delta_P^I = -\frac{2dF_\phi^2}{(16\pi^2)^2} \{2\lambda^2 [3(\lambda_P^T)^2 + 2(\lambda_P^D)^2] + 6\kappa^2 [3(\lambda_P^T)^2 + 2(\lambda_P^D)^2]\}. \tag{3.56}$$

- The interference terms involving the Kahler modulus ‘ T ’:

$$\delta_{\tilde{Q}_L}^I = \frac{M_0 F_\phi}{8\pi^2} \left[y_t^2 \left(q_{y_t} - \frac{1}{8\pi^2} K_{y_t} \right) + y_b^2 \left(q_{y_b} - \frac{1}{8\pi^2} K_{y_b} \right) - \left(\frac{8}{3} \frac{g_3^4}{g_3^2(GUT)} + \frac{3}{2} \frac{g_2^4}{g_2^2(GUT)} + \frac{1}{30} \frac{g_1^4}{g_1^2(GUT)} \right) \right], \tag{3.57}$$

$$\delta_{\tilde{U}_L}^I = \frac{M_0 F_\phi}{8\pi^2} \left[2y_t^2 \left(q_{y_t} - \frac{1}{8\pi^2} K_{y_t} \right) - \left(\frac{8}{3} \frac{g_3^4}{g_3^2(GUT)} + \frac{8}{15} \frac{g_1^4}{g_1^2(GUT)} \right) \right], \tag{3.58}$$

$$\delta_{\tilde{D}_L}^I = \frac{M_0 F_\phi}{8\pi^2} \left[2y_b^2 \left(q_{y_b} - \frac{1}{8\pi^2} K_{y_b} \right) - \left(\frac{8}{3} \frac{g_3^4}{g_3^2(GUT)} + \frac{2}{15} \frac{g_1^4}{g_1^2(GUT)} \right) \right], \tag{3.59}$$

$$\delta_{\tilde{L}_L}^I = -\frac{M_0 F_\phi}{8\pi^2} \left(\frac{3}{2} \frac{g_2^4}{g_2^2(GUT)} + \frac{3}{10} \frac{g_1^4}{g_1^2(GUT)} \right), \tag{3.60}$$

$$\delta_{\tilde{E}_L}^I = -\frac{M_0 F_\phi}{8\pi^2} \left(\frac{6}{5} \frac{g_1^4}{g_1^2(GUT)} \right), \tag{3.61}$$

$$\delta_{H_u}^I = \frac{M_0 F_\phi}{8\pi^2} \left[3y_t^2 \left(q_{y_t} - \frac{1}{8\pi^2} K_{y_t} \right) + \lambda^2 \left(q_\lambda - \frac{1}{8\pi^2} K_\lambda \right) - \left(\frac{3}{2} \frac{g_2^4}{g_2^2(GUT)} + \frac{3}{10} \frac{g_1^4}{g_1^2(GUT)} \right) \right], \tag{3.62}$$

$$\delta_{H_d}^I = \frac{M_0 F_\phi}{8\pi^2} \left[3y_b^2 \left(q_{y_b} - \frac{1}{8\pi^2} K_{y_b} \right) + \lambda^2 \left(q_\lambda - \frac{1}{8\pi^2} K_\lambda \right) - \left(\frac{3}{2} \frac{g_2^4}{g_2^2(GUT)} + \frac{3}{10} \frac{g_1^4}{g_1^2(GUT)} \right) \right], \tag{3.63}$$

$$\delta_{\tilde{S}}^I = \frac{M_0 F_\phi}{8\pi^2} \left[2\lambda^2 \left(q_\lambda - \frac{1}{8\pi^2} K_\lambda \right) + 2\kappa^2 \left(q_\kappa - \frac{1}{8\pi^2} K_\kappa \right) \right] + \Delta_P^{TX}(m_{\tilde{S}}^2). \tag{3.64}$$

Note that the expressions for sfermions are hold for the third generation, the first two generation can be obtained

by setting $y_t, y_b \rightarrow 0$. Within the expressions, modular weight $l_i = 1$ for gauge couplings are used.

The previous contributions are just the anomaly-modulus interference part. Possible modulus-gauge interference part will also appear in our scenario. The anomalous dimensions for all fields except S are continuous across the messenger threshold, so their modulus-gauge interference contributions vanish. As noted previously, the anomalous dimension for S is discontinuous across the messenger threshold, so we have the new T, X interference contributions to $m_{\tilde{S}}^2$

$$\Delta_{\tilde{S}}^{TX}(m_{\tilde{S}}^2) = -\frac{dM_0 F_\phi}{8\pi^2} \left[3(\lambda_P^T)^2 \left(q_{\lambda_P^T} - \frac{1}{8\pi^2} K_{\lambda_P^T} \right) + 2(\lambda_P^D)^2 \left(q_{\lambda_P^D} - \frac{1}{8\pi^2} K_{\lambda_P^D} \right) \right], \quad (3.65)$$

with

$$\begin{aligned} K_{\lambda_P^T} &= \left[5(\lambda_P^T)^2 q_{\lambda_P^T} + 2(\lambda_P^D)^2 q_{\lambda_P^D} + 2\lambda^2 q_\lambda + 2\kappa^2 q_\kappa + 2(\lambda_X^T)^2 q_{\lambda_X^T} - \left(\frac{16}{3} g_3^2 + \frac{4}{15} g_1^2 \right) \right] \ln \left(\frac{GUT}{M_{mess}} \right), \\ K_{\lambda_P^D} &= \left[3(\lambda_P^T)^2 q_{\lambda_P^T} + 4(\lambda_P^D)^2 q_{\lambda_P^D} + 2\lambda^2 q_\lambda + 2\kappa^2 q_\kappa + 2(\lambda_X^D)^2 q_{\lambda_X^D} - \left(3g_2^2 + \frac{3}{5} g_1^2 \right) \right] \ln \left(\frac{GUT}{M_{mess}} \right). \end{aligned} \quad (3.66)$$

Here

$$\begin{aligned} q_{\lambda_P^T} &= m_S + m_{\tilde{X}_1} + m_{X_2}, \\ q_{\lambda_P^D} &= m_S + m_{\tilde{Y}_1} + m_{Y_2}, \\ q_{\lambda_X^T} &= m_X + m_{\tilde{X}_m} + m_{X_m}, \\ q_{\lambda_X^D} &= m_X + m_{\tilde{Y}_m} + m_{Y_m}. \end{aligned} \quad (3.67)$$

4 Numerical results

After fixing the modular weights, the remaining free parameters in our scenario are

$$d, \alpha, M_{mess}, M_0, \lambda, \kappa, \lambda_P^D, \lambda_P^T, \lambda_X^D, \lambda_X^T \quad (4.1)$$

with $F_\phi/(16\pi^2) \approx \alpha M_0$ and the simplest choice $\lambda_P^D = \lambda_P^T = \lambda_X^D = \lambda_X^T = \lambda_0$ in our numerical study. Note that for later convenience, the definition of α is four times smaller than α' that appears in Eq. (2.8). The ratio α between $F_\phi/(16\pi^2)$ and M_0 holds in the messenger scale and in general is different from its value at the GUT scale. We choose a positive α in our numerical study. For a negative α , virtual mirage unification at a super-GUT energy scale will appear.

We need to check if successful EWSB conditions are indeed fulfilled. In fact, the soft SUSY mass $m_{H_u}^2, m_{H_d}^2, m_S^2$ can be reformulated into $\mu, \tan \beta, M_Z^2$ by the minimum conditions of the scalar potential. Usually, M_A can be used to replace A_κ

$$M_A^2 = \frac{2\mu_{eff}}{\sin 2\beta} B_{eff}, \quad \mu_{eff} \equiv \lambda \langle s \rangle, \quad B_{eff} = (A_\lambda + \kappa \langle s \rangle). \quad (4.2)$$

In order to transform $m_{H_u}^2, m_{H_d}^2, m_S^2$ into $\mu, \tan \beta, M_Z^2$, we use the following approximation

$$|\mu_{eff}|^2 = -\frac{M_Z^2}{2} - m_{H_u}^2 + \frac{1}{\tan^2 \beta} (m_{H_d}^2 - m_{H_u}^2) + \mathcal{O}(1/\tan^4 \beta), \quad (4.3)$$

$$\sin 2\beta = \frac{2B_{eff}\mu_{eff}}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu_{eff}|^2 + \lambda^2 v^2}, \quad (4.4)$$

to obtain μ and $\tan \beta$ iteratively.

We use NMSSMTools5.2.0 [55,56] to scan the whole parameter space. The parameters are chosen to satisfy:

$$\begin{aligned} 10^{15} \text{ GeV} &> M_{mess} > 10^5 \text{ GeV}, \\ 100 \text{ TeV} &> M_0 > 0.1 \text{ TeV}, \end{aligned} \quad (4.5)$$

$$\begin{aligned} 16 > \alpha > 0, \quad 5 > d > 0, \quad 0.7 > \lambda, \kappa > 0, \\ \sqrt{4\pi} > \lambda_0 > 0. \end{aligned} \quad (4.6)$$

In our scan, we impose the following constraints:

- (I) The conservative lower bounds on SUSY particles [57–60] from the LHC
 - Gluino mass: $m_{\tilde{g}} \gtrsim 1.8 \text{ TeV}$.
 - Light stop mass: $m_{\tilde{t}_1} \gtrsim 0.85 \text{ TeV}$.
 - Light sbottom mass $m_{\tilde{b}_1} \gtrsim 0.84 \text{ TeV}$.
 - Degenerated first two generation squarks $m_{\tilde{q}} \gtrsim 1.0 \sim 1.4 \text{ TeV}$.
- (II) The CP-even component S_2 in the Goldstone-‘eaten’ combination of H_u and H_d doublets corresponds to the SM Higgs. The S_2 dominated CP-even scalar should lie in the combined mass range for the Higgs boson: $122 \text{ GeV} < M_h < 128 \text{ GeV}$ from ATLAS and CMS data [4,5]. Note that the uncertainty is 3 GeV instead of the default 2 GeV because a large λ may induce an additional 1 GeV correction to m_h at two-loop level [61], which is not included in the NMSSMTools.
- (III) The relic density of the neutralino dark matter should satisfy the Planck data $\Omega_{DM} = 0.1199 \pm 0.0027$ [62] in combination with the WMAP data [63] (with a 10% theoretical uncertainty).
- (IV) The electroweak precision observables [64] and the lower bounds on neutralinos and charginos, including

the invisible decay bounds for Z-boson. The most stringent constraints of LEP require $m_{\tilde{\chi}^\pm} > 103.5\text{GeV}$ and the invisible decay width $\Gamma(Z \rightarrow \tilde{\chi}_0 \tilde{\chi}_0) < 1.71\text{ MeV}$, which is consistent with the 2σ precision EW measurement $\Gamma_{inv}^{non-SM} < 2.0\text{ MeV}$ [65].

(V) Flavor constraints [66] from B-meson rare decays

$$1.7 \times 10^{-9} < Br(B_s \rightarrow \mu^+ \mu^-) < 4.5 \times 10^{-9}, \tag{4.7}$$

$$0.85 \times 10^{-4} < Br(B^+ \rightarrow \tau^+ \nu) < 2.89 \times 10^{-4}, \tag{4.8}$$

$$2.99 \times 10^{-4} < Br(B_S \rightarrow X_s \gamma) < 3.87 \times 10^{-4}. \tag{4.9}$$

(VI) The tension between the theoretical prediction and the experimental value for the muon anomalous magnetic moment should be ameliorated by additional positive SUSY contributions. The E821 experimental result for the muon $g - 2$ at the Brookhaven AGS [67] was given by

$$a_\mu^{\text{expt}} = 116592089(63) \times 10^{-11}, \tag{4.10}$$

which is larger than the SM prediction [68,69]

$$a_\mu^{\text{SM}} = 116591834(49) \times 10^{-11}. \tag{4.11}$$

The deviation is about 3σ

$$\Delta a_\mu(\text{expt} - \text{SM}) = (255 \pm 80) \times 10^{-11}. \tag{4.12}$$

We adopt a conservative estimation $4.7 \times 10^{-10} \lesssim \Delta a_\mu \lesssim 52.7 \times 10^{-10}$ in our numerical results.

We should note that the numerical results depend crucially on whether the 125 GeV Higgs is the lightest CP-even scalar (Type **A**) or the second lightest CP-even scalar (Type **B**). We have the following discussions

- The low energy soft SUSY breaking spectrum of NMSSM, determined from a top-down approach by a UV-completed theory, is always bothered by the requirement to achieve successful EWSB. As noted previously, EWSB conditions in NMSSM in general require a large VEV for the singlet and consequently prefer a negative m_S^2 and/or large A_λ, A_κ for the singlet potential. However, ordinary mirage mediation scenarios always predict large positive values for m_S^2 and not very large A_λ, A_κ , suppressing the singlet VEV. In our scenario, because of possible negative contributions to m_S^2 from new Yukawa interactions, stringent constraints from successful EWSB can be ameliorated. The observed 125 GeV Higgs mass, whether it is the lightest or the second-lightest CP-even scalar in

NMSSM, can also be successfully accommodated in our scenario.

We can see from Fig. 1 that many samples of (λ, κ) can survive the EWSB conditions in NMSSM. In contrast to the numerical results of TeV mirage mediation in [36,37] within which the allowed (λ, κ) only take values near (0.7,0.1), some portion of (λ, κ) parameter space can survive all constraints in Type **A**.

We should note that such a difference has the following reasons

- The choices of the modular weights in [36,37], for example, the values of $m_{Q_L^3}$ and $m_{t_L^c}$ etc, are different to ours which are given in Eq. (3.9).
- New ingredients, such as the messenger sector which can cause additional deflection of the RGE trajectory, will introduce new free parameters. After all, the mirage mediation scenario can be seen as a special case of our scenario with gauge couplings (Yukawa couplings) being switched off.
- The mirage unification scale is set to lie at TeV scale in [36,37]. It is known that simple mirage unification for soft parameters would in general be spoiled in deflected mirage mediation scenario. However, ‘mirage’ unification for gaugino masses persists which can be proven in our Wilsonian approach (see Appendix A for details).

In our scenario, the mirage unification scale for gaugino masses is not constrained to lie at TeV scale. However, we can see from the mirage unification scales shown by different colors in Fig. 1 that even if such scales are required to lie at TeV scale, vast parameter space, which is larger than the numerical result of Refs. ([36,37]), can survive the constraints from (I–V).

Note that the vacuum stability bounds are also taken into account in our numerical studies, which impose stringent constraints on scenario in [36,37]. In Type B in which the 125 GeV Higgs is the second lightest CP-even scalar, the allowed (λ, κ) parameter space is also much bigger than that in [36,37]. The Higgs mass can be increased by 8 GeV through the mixing with the singlet component for large $\tan \beta$ and $\lambda \lesssim 0.04$.

It can be seen from Fig. 2 that the modulus mediation contribution M_0 is bounded to lie between 1 and 8.5 TeV for Type **A**. A small M_0 always prefers a low messenger scale M_{mess} . A large M_0 , which controls the whole soft SUSY breaking parameters to be heavy, can easily accommodate the SM-like Higgs mass because of large loop corrections from heavy stops in addition to the tree-level contributions involving λ . The value of M_0 is upper bounded to be less than about 5.5 TeV for Type **B**, which sets an upper bound for the soft SUSY breaking param-

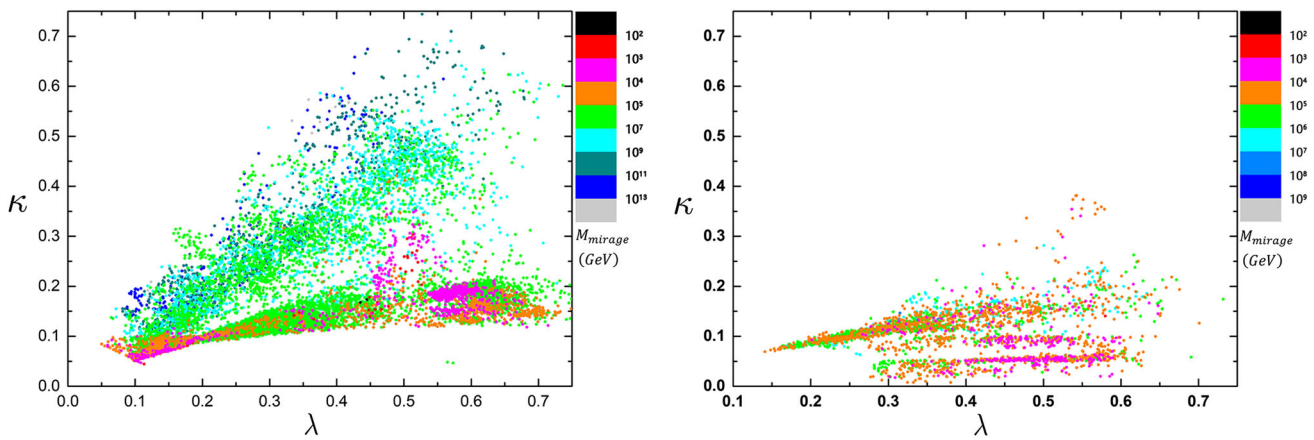


Fig. 1 The values of (λ, κ) that can satisfy the EWSB conditions and at the same time accommodate the 125 GeV Higgs boson as the lightest (left panel) or second lightest (right panel) CP-even scalar are shown.

The mirage unification points for gaugino masses are also shown in different colors. All points satisfy the constraints (I–V)

eters, especially for the gluino masses. The gluino mass, which is determined by the scale of M_0 , is bounded to below 16 TeV for Type **A** and below 8 TeV for Type **B**, respectively.

- The Barbieri–Giudice fine-tuning (BGFT) [70] measure is defined as

$$\Delta_{BG} \equiv \max_i \left| \frac{\partial \ln M_Z^2}{\partial \ln a_i} \right|, \tag{4.13}$$

where ‘ a_i ’ stands for the set of parameters defined at the input scale.

There are two mass scales for the soft SUSY parameters in our scenario, one is the scale that characterize the modulus contributions M_0 and the other is the scale that characterize the anomaly contribution F_ϕ . The latter one is rewritten into a dimensionless quantity α by Eq. (3.13). We can also calculate the electroweak fine-tuning (EWFT) measure Δ_{EW} of the survived points defined in [71, 72]

$$\Delta_{EW} \equiv \max_i (C_i) / \left(\frac{m_Z^2}{2} \right), \tag{4.14}$$

with the relevant terms (see [71, 72])

$$C_{H_u} = \left| -\frac{m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} \right|, \quad C_{H_d} = \left| \frac{m_{H_d}^2}{\tan^2 \beta - 1} \right|,$$

$$C_\mu = \left| -\mu_{eff}^2 \right|,$$

$$C_{\Sigma_d^u(\tilde{t}_{1,2})} = \frac{\tan^2 \beta}{\tan^2 \beta - 1} \left| \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \right.$$

$$\left. \times \left[y_t^2 - g_Z^2 \mp \frac{y_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_w)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right] \right|,$$

$$C_{\Sigma_d^d(\tilde{t}_{1,2})} = \frac{1}{\tan^2 \beta - 1} \left| \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \right.$$

$$\left. \times \left[g_Z^2 \mp \frac{y_t^2 \mu_{eff}^2 + 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_w)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right] \right|, \tag{4.15}$$

where $x_w = \sin^2 \theta_w$ and

$$\Delta_t = \frac{(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)}{2} + M_Z^2 \cos 2\beta \left(\frac{1}{4} - \frac{2}{3}x_w \right),$$

$$F(m^2) = m^2 \left(\log \frac{m^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} - 1 \right). \tag{4.16}$$

We can see from Fig. 2 that the BGFT measure in our scenario can range from $\mathcal{O}(1)$ to $\mathcal{O}(1000)$. In fact, the lowest BGFT can reach $\mathcal{O}(1)$ for Type **B**. Such a low fine-tuning possibly indicates that our scenario is natural. We also compare the BGFT measure of the survived points with their corresponding EWFT measure in the last two panels of Fig. 2. It can be seen that the calculated EWFT measure is positively correlated to the corresponding BGFT measure. In most of allowed parameter space, the EWFT and BGFT measures take values of the same order. Besides, the EWFT measure, which can be thought of as providing a lower bound on the electroweak fine-tuning, is always smaller than that of the BGFT measure [73].

In some of the allowed region, the EWFT measure as well as BGFT measure is rather low (being $\mathcal{O}(1)$). As emphasized in [73], EWFT measure is a necessary, albeit not sufficient, measure of electroweak fine-tuning. Low Δ_{EW} need not necessarily mean the model is not fine-tuned. Rather, it may indicate the possibility that some model might exist with low fine-tuning which might be hidden by the naive application of Δ_{BG} .

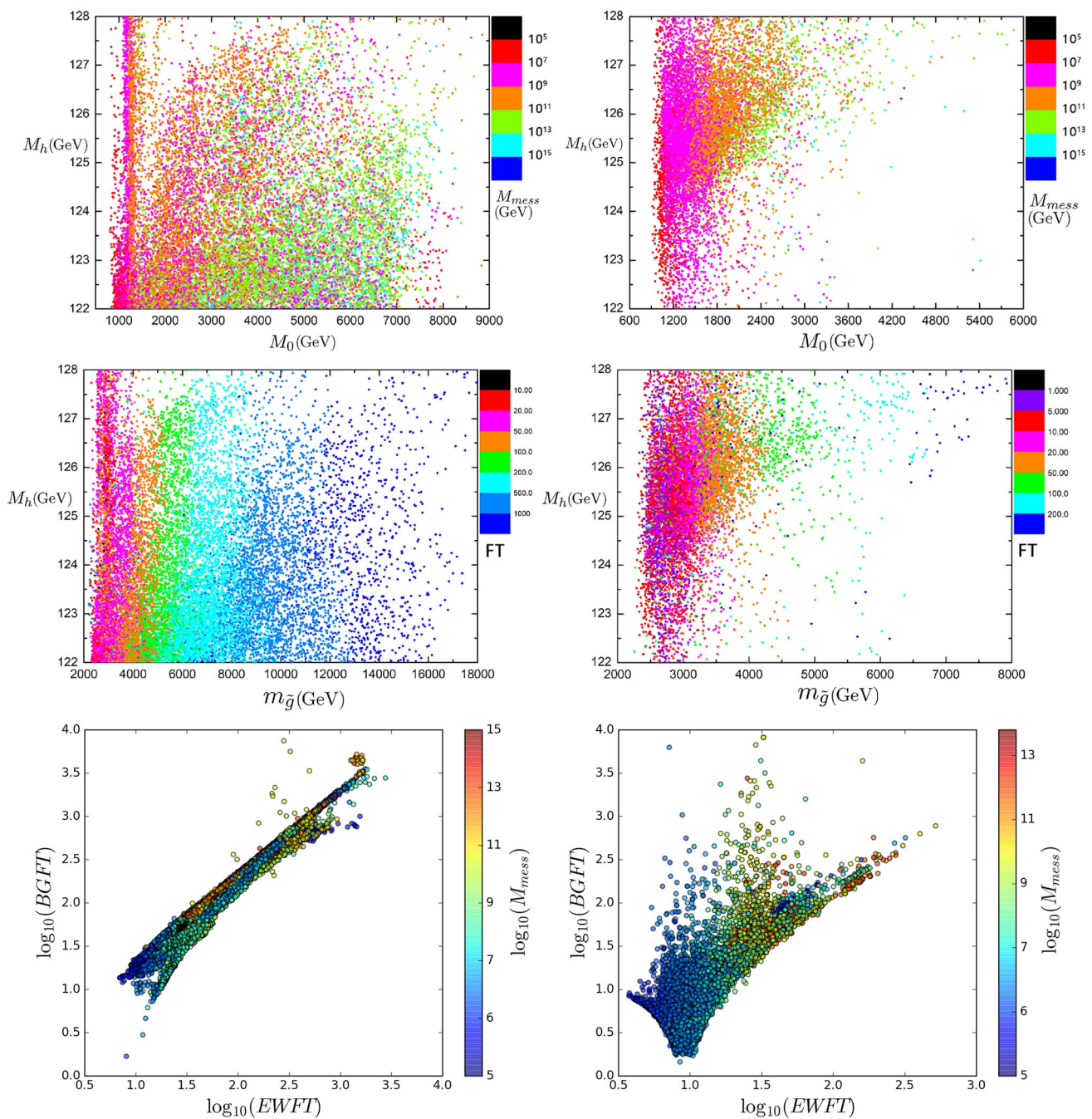


Fig. 2 Scatter plots of the SM-like Higgs mass m_h versus the modulus mediation parameter M_0 and gluino mass $m_{\tilde{g}}$ for Type A (left panels) or Type B (right panels). The fine-tuning measures given in the middle of the panels (m_h vs $m_{\tilde{g}}$) are the Barbieri–Giudice fine tuning (BGFT)

measures. The comparisons between the BGFT measure versus the electroweak fine tuning (EWFT) measure for Type A and Type B are shown in the last two panels, respectively. All samples satisfy the constraints (I–V)

It is known that low fine-tuning needs light stops as well as a small effective μ , which are naively determined by the dimensional parameter M_0 that controls the whole soft SUSY spectrum with moderate values of α . The lower the M_0 (consequently the lower gluino mass), the lower value of the BGFT(EWFT) measure.

- A positive deflection parameter ‘ d ’ is always favored to solve the tachyonic slepton problem in the deflected AMSB for fewer messenger species. In the deflected mirage mediation scenarios, if the modulus contribution is subdominant, a realistic model still prefers a positive deflection parameter ‘ d ’ with less messenger species. As

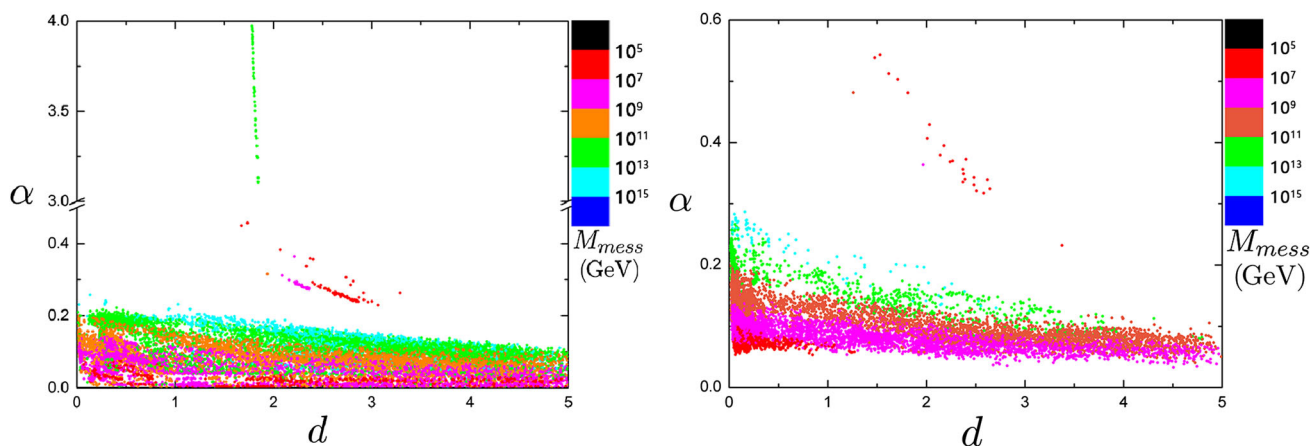


Fig. 3 The allowed regions for the deflection parameter ‘ d ’ versus α , which parametrize the relative size between the anomaly mediation and the modulus mediation. All samples satisfy the constraints (I–V)

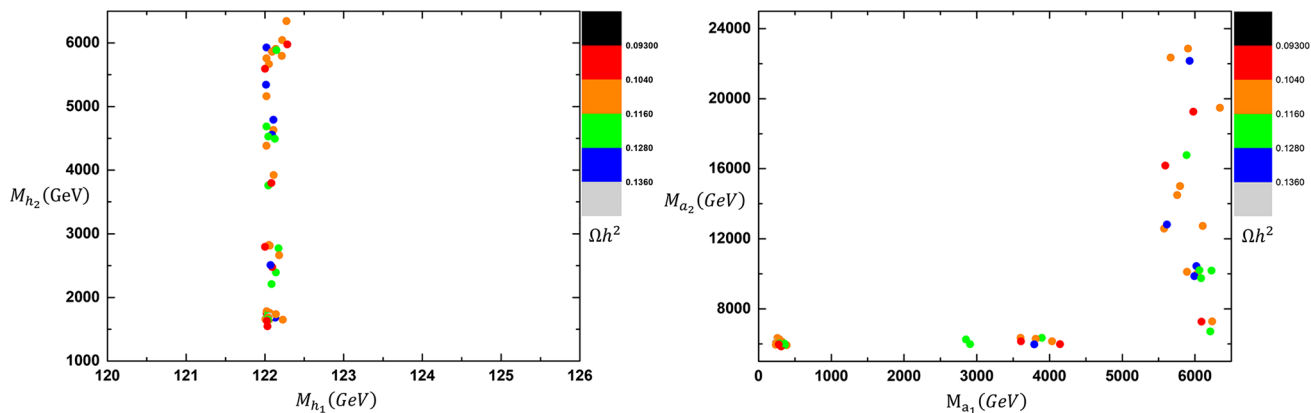


Fig. 4 The masses of the Higgs scalars with $d \approx 1.8$. All samples satisfy the constraints (I–V)

the parameter $\alpha \equiv \alpha'/4$ determines the relative size of the contributions between the anomaly mediation and the modulus mediation, a large value of α , which indicates small modulus mediation contributions, needs a large positive deflection parameter ‘ d ’ to avoid tachyonic sleptons. We check that large negative values of ‘ d ’ are mostly ruled out by the EWSB condition and tachyonic sfermions. It is obvious in the left panel of Fig. 3 that the deflection parameter is constrained to lie at about 1.8 to tune the tachyonic slepton masses to positive values by additional gauge and Yukawa mediation contributions in the region with a large α . It can be seen in Fig. 4 that the corresponding lightest CP-even Higgs mass should lie at a very narrow band centered at about 122.1 GeV with $d \approx 1.8$. Besides, the second-lightest CP-odd scalar a_2 is constrained to lie near 6000 GeV if the lightest CP-odd scalar a_1 is lighter than 5000 GeV while the lightest CP-odd scalar is constrained to lie near 6000 GeV if the second-lightest CP-odd scalar is heavier than 6000 GeV.

In our scenario, the quantity ‘ $4\alpha d$ ’ can approximately measure the relative size of deflection contributions (by gauge or Yukawa mediation) to the modulus mediation contributions. We can see from Fig. 3 that the deflection contributions can be dominant in a large portion of the surviving parameter space.

Besides, it can also be seen from Fig. 3 that in the modulus mediation dominated regions, that is small α with $d = 0$, realistic NMSSM spectra can be possible. This indicates that pure mirage mediation without deflection, which is a special case of our scenarios, can lead to realistic NMSSM spectrum even though it is stringently constrained by EWSB conditions and 125 GeV Higgs. This conclusion agrees with that of [36,37]. Additional deflection with $d \neq 0$ from messenger sector can enlarge the possible choice of α in mirage mediation, rendering the mirage-type scenarios more natural.

- From Eq. (3.10) in the appendix, we can see the gaugino ratio at the EW scale

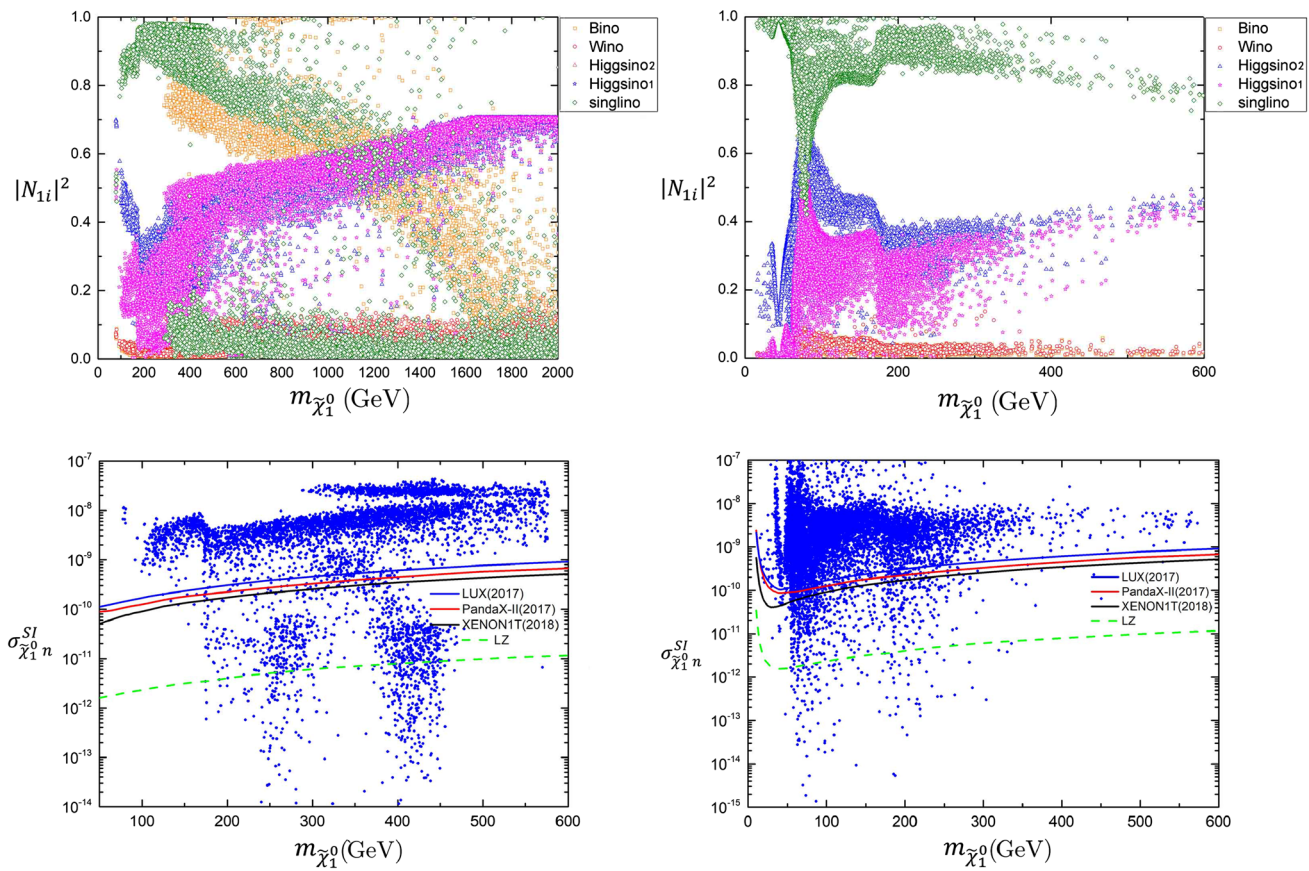


Fig. 5 The upper panels show the plots of DM mass vs the DM components for Type A (left panel) or Type B (right panel). Similarly, the lower panels show the plots of DM mass versus the spin-independent (SI) direct detection bounds. All samples satisfy the constraints from (I–V)

$$M_3 : M_2 : M_1 \approx 6 \cdot \left[\frac{1}{g_3^2} + \alpha(-3 - 2d) \right] : 2 \cdot \left[\frac{1}{g_2^2} + \alpha(1 - 2d) \right] : \left[\frac{1}{g_1^2} + \alpha(6.6 - 2d) \right] \quad (4.17)$$

where g_1, g_2, g_3 take values at the GUT scale. On the other hand, the singlino mass is determined by κ and $\langle s \rangle$, which rescales the effective μ_{eff} parameter by a factor $2\kappa/\lambda$. In general, a pure singlino-like LSP tends to have a too large relic density due to a comparatively small annihilation cross section because of its small couplings to SM particles. Non-negligible higgsino contents within singlino-dominated neutralino DM can open several annihilation channels and be helpful to reduce the DM relic density to right amount.

In Type A, the neutralino DM is either singlino-dominant or bino-dominant, each possibility contains non-negligible higgsino components. In the bino-dominant regions, the LSP annihilate dominantly into pairs of gauge bosons (W^+W^-, ZZ) and (doublet-like) Higgs bosons ($W^\pm H^\pm, ZH, HA$) via s-channel Z or Higgs exchange, as well as through t-channel neutralino and chargino

exchange processes. In the singlino-dominant regions, the singlino-like LSP (with the presence of non-negligible higgsino components) can annihilates via the t-channel χ_1^0 exchange into pairs of mostly singlet-like H_1 and A_1 by enhanced $\chi_1^0 \chi_1^0 H_1(A_1)$ couplings. Co-annihilation with heavier χ_2^0 (for $\Delta m \lesssim 10\text{GeV}$) will also efficiently reduce the singlino relic abundance to a proper Ω_{DM} . Besides, the annihilation channels $\chi_1^0 \chi_1^0 \rightarrow t\bar{t}, b\bar{b}$ can also be important. Similar DM annihilation channels exist for Type B in which the neutralino DM is mainly singlino-like with non-negligible higgsino components.

We know that mixed bino-Higgsino DM is severely constrained by direct detection constraints. For a singlino-dominant DM, the exchange of a light H_1 can possibly lead to a large direct detection cross section that will be accessible in the present generation of detectors. It can be seen from the lower panels of Fig. 5 that only a small portion of DM parameter space can survive the spin-independent (SI) DM direct detection constraints from the LUX [74], PANDAX [75] and Xenon1T [76]. In fact, direct detection constrained the DM mass to lie in the range [120, 470] GeV for Type A and [50, 400] GeV for Type B, respectively.

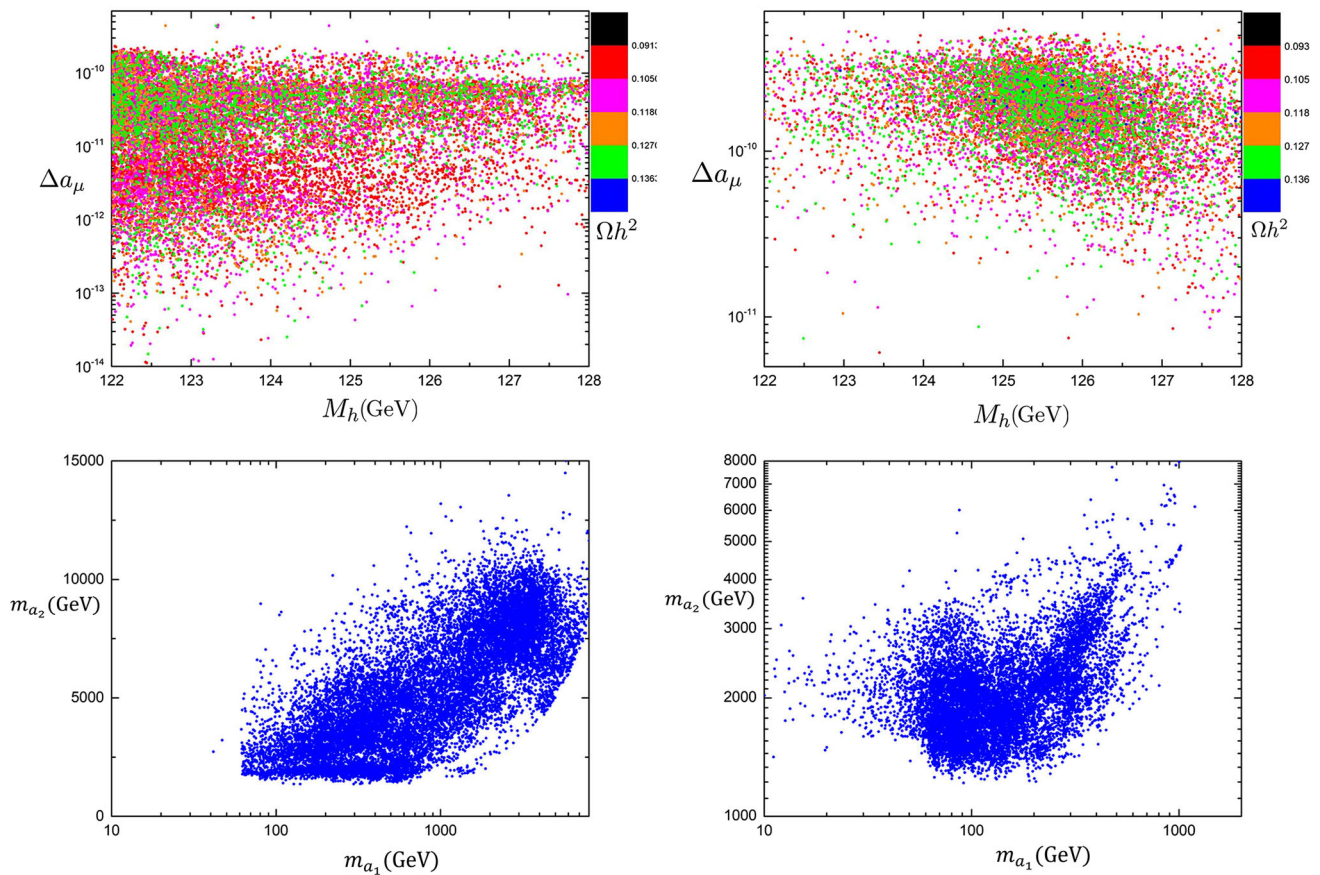


Fig. 6 The upper panels show the SUSY contributions to the muon anomalous magnetic moment Δa_μ for Type A (left) and Type B (right). The lower panels show the corresponding masses of the lightest CP-

odd scalar m_{a_1} , which can possibly give large two-loop contributions to Δa_μ . All samples satisfy the constraints (I–V)

- Figure 6 shows the SUSY contributions to the muon $g - 2$. It is known that the required SUSY contributions to Δa_μ can be achieved only if the relevant sparticles ($\tilde{\mu}, \tilde{\nu}_\mu, \tilde{B}, \tilde{W}, \tilde{H}$) are lighter than 600–700 GeV for $\tan\beta \sim 10$ in the MSSM [77]. The inclusion of the singlino in the NMSSM can not give sizable contributions to Δa_μ because of the suppressed couplings of singlino to MSSM sector. Although the two loop contributions involving the Higgs are negligible in the SM, new Higgs bosons in the NMSSM could have an important impact on Δa_μ if the lightest neutral CP-odd Higgs scalar is very light [78]. In fact, a positive two-loop contribution is numerically more important for a light CP-odd Higgs being a bit heavier than 3 GeV and the sum of both one-loop and two-loop contributions is maximal around $m_{a_1} \sim 6$ GeV. In our scenario, the lightest CP-odd Higgs a_1 is bounded to be heavier than 40 GeV and give negligible two-loop contributions to Δa_μ in Type A. The main contribution to Δa_μ is thus similar to that in the MSSM. In Type B, the lightest CP-odd Higgs a_1 can lie near 10

GeV and will give important two-loop contributions to increase Δa_μ to values favored by experiments.

- Before we finish our discussions on numerical results, we note that the gauge-modulus interference contribution for soft scalar masses will play an important role in phenomenological studies. In our scenario, such a contribution is non-vanishing only for S which is given in Eq. (3.65). This contribution will possibly change the EWSB condition, the dark matter relic density and other collider predictions. We show several benchmark points which are affected by the contributions of $\Delta_P^{TX}(m_{\tilde{g}_2})$. Table 1 shows several benchmark points, which should not survive the various constraints if the gauge-modulus interference contribution is absent. Table 2, in contrary, shows several benchmark points which should survive the various constraints if the gauge-modulus interference contribution is absent.

Table 1 Benchmark points, which should not survive the various constraints if the gauge-modulus interference contribution is absent. The quantities with mass dimension are in unit of GeV. Without the gauge-modulus interference contribution, the points will not survive the constraints because of the constraints shown in ‘Reasons’

	Sample I	Sample II	Sample III
d	0.326	0.257	0.036
α	0.102	0.075	0.115
M_{mess}	1.420×10^9	4.823×10^6	4.724×10^7
M_0	1278	1203	1222
λ	0.607	0.430	0.591
κ	0.205	0.147	0.193
λ_0	2.389	1.012	2.597
$m_{\tilde{Q}_{L:1,2}}$	2041	1452	1479
$m_{\tilde{Q}_{L:3}}$	1714	1081	1164
$m_{\tilde{U}_{L:1,2}}$	2096	1558	1584
$m_{\tilde{U}_{L:3}^c}$	1640	1096	1236
$m_{\tilde{D}_{L:1,2}^c}$	2113	1565	1598
$m_{\tilde{D}_{L:3}}$	1908	1322	1345
$m_{\tilde{L}_{L:1,2,3}}$	920.9	897.1	891.2
$m_{\tilde{E}_{L:1,2,3}}$	761.7	691.0	661.4
A_λ	1823	1523	1807
A_κ	-169.2	-163.2	-95.60
A_t	-2360	-2821	-2401
A_b	-3481	-3353	-3450
A_τ	1412	1453	1371
$M_{\tilde{g}}$	2971	2889	2775
μ_{eff}	646.3	200.9	635.2
m_{h_1}	125.8	125.7	126.9
$g_\mu - 2$	5.049×10^{-11}	2.170×10^{-10}	6.035×10^{-11}
$\Omega_\chi h^2$	0.114	0.118	0.120
$m_{\tilde{\chi}_1^0}$	442.0	122.6	418.5
σ_P^{SI}	$1.172 \times 10^{-11} pb$	$2.960 \times 10^{-12} pb$	$1.701 \times 10^{-12} pb$
Reasons	EWSB	EWSB	Collider; Higgs mass; Ωh^2

Table 2 Benchmark points, which should survive the various constraints if the gauge-modulus interference contribution is absent. The quantities with mass dimension are in unit of GeV. With the gauge-modulus interference contribution, the points will not survive the constraints because of the constraints shown in ‘Reasons’

	Sample I	Sample II	Sample III
d	1.114	0.307	1.102
α	0.056	0.102	2.804
M_{mess}	8.188×10^9	4.360×10^{12}	3.880×10^{12}
M_0	5049	2034	938.1
λ	0.005	0.097	0.105
κ	0.553	0.354	0.231
λ_0	1.597	2.279	0.381
$m_{\tilde{Q}_{L:1,2}}$	9087	3853	17,479
$m_{\tilde{Q}_{L:3}}$	7478	3155	15,378
$m_{\tilde{U}_{L:1,2}}$	9200	3836	17,234
$m_{\tilde{U}_{L:3}^c}$	6789	2756	12,746
$m_{\tilde{D}_{L:1,2}^c}$	9259	3867	17,223
$m_{\tilde{D}_{L:3}}$	8366	3482	17,057
$m_{\tilde{L}_{L:1,2,3}}$	3648	1465	1377
$m_{\tilde{E}_{L:1,2,3}}$	3249	1449	3455
A_λ	4657	1511	-2737
A_κ	-6511	-3614	-4573
A_t	-10,187	-3843	9530
A_b	-11,949	-4828	6936
A_τ	6368	2267	-4347
$M_{\tilde{g}}$	11,819	4523	17,463
μ_{eff}	439.2	288	4998
m_{h_1}	127.1	123.8	123.6
$g_\mu - 2$	1.580×10^{-11}	1.096×10^{-10}	-1.110×10^{-11}
$\Omega_\chi h^2$	0.022	0.010	0.004
$m_{\tilde{\chi}_1^0}$	458.3	294.5	421.0
σ_P^{SI}	$1.030 \times 10^{-10} pb$	$7.316 \times 10^{-10} pb$	$1.870 \times 10^{-12} pb$
Reasons	EWSB	EWSB	Collider; Higgs mass; Ωh^2

5 Conclusions

We propose to generate a realistic soft SUSY breaking spectrum for Next-to-Minimal Supersymmetric Standard Model with a generalized deflected mirage mediation scenario, in which additional Yukawa and gauge mediation contributions are included to deflect the RGE trajectory. Based on the Wilsonian effective action obtained by integrating out the messengers, the NMSSM soft SUSY breaking spectrum can be given analytically at the messenger scale. We find that additional contributions to $m_{\tilde{S}}^2$ can possibly ameliorate the

stringent constraints from the EWSB and 125 GeV Higgs mass. Constraints from dark matter and fine-tuning are also discussed. The Barbieri–Giudice fine-tuning measure and electroweak fine-tuning measure in our scenario can be as low as $\mathcal{O}(1)$, which possibly indicates that our scenario is natural.

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Appendix A: The mirage scale in deflected mirage mediation mechanism

The gaugino mass at scale μ below the messenger scale can be written as

$$M_i = M_0 \frac{g_i^2(\mu)}{g_i^2(GUT)} + F_\phi \frac{\alpha_i}{4\pi} (b_i - d\Delta b_i), \tag{A.1}$$

with

$$\begin{aligned} \frac{1}{g_i^2(\mu)} &= \frac{1}{g_i^2(GUT)} + \frac{b_i + \Delta b_i}{8\pi^2} \ln\left(\frac{M_G}{M}\right) \\ &\quad + \frac{b_i}{8\pi^2} \ln\left(\frac{M}{\mu}\right), \\ &= \frac{1}{g_i^2(M_Z)} - \frac{b_i}{8\pi^2} \ln\left(\frac{\mu}{M_Z}\right). \end{aligned} \tag{A.2}$$

Here M_G , M denote the gauge coupling unification scale and the messenger scale, respectively. We will show that apparent ‘mirage’ unification for gaugino masses will still be preserved after the introduction of messenger sector in deflected mirage mediation scenarios. Substituting $\Delta b_i \equiv N$ and the definition $F_\phi \equiv (16\pi^2)\alpha M_0$ into Eq. (A.1), the gaugino masses can rewrite as

$$\begin{aligned} M_i &= M_0 \frac{g_i^2(\mu)}{g_i^2(GUT)} + \alpha M_0 (b_i - dN) g_i^2(\mu), \\ &= \left[M_0 - \alpha M_0 dN g_i^2(GUT) \right] \\ &\quad \times \left[1 - \frac{b_i + N}{8\pi^2} g_i^2(\mu) \ln\left(\frac{M_G}{M}\right) \right. \\ &\quad \left. - \frac{b_i}{8\pi^2} g_i^2(\mu) \ln\left(\frac{M}{\mu}\right) \right] + \alpha M_0 b_i g_i^2(\mu), \\ &\approx \left(M_0 - \alpha M_0 dN g_i^2(GUT) \right) \end{aligned}$$

$$\begin{aligned} &\times \left[\left(1 - \frac{N}{8\pi^2} g_i^2(M_G) \ln\left(\frac{M_G}{M}\right) \right) \right. \\ &\quad \left. - \frac{b_i}{8\pi^2} g_i^2(\mu) \ln\left(\frac{M_G}{\mu}\right) \right] + \alpha M_0 b_i g_i^2(\mu), \\ &\equiv K_0 \left[c_0 - \frac{b_i}{8\pi^2} g_i^2(\mu) \ln\left(\frac{M_G}{\mu}\right) \right] + \alpha M_0 b_i g_i^2(\mu), \end{aligned} \tag{A.3}$$

with

$$\begin{aligned} K_0 &\equiv M_0 - \alpha M_0 dN g_i^2(GUT), \\ c_0 &= 1 - \frac{N}{8\pi^2} g_i^2(M_G) \ln\left(\frac{M_G}{M}\right). \end{aligned} \tag{A.4}$$

So we can see that mirage unification for gaugino masses will be satisfied at the scale μ determined by

$$\ln\left(\frac{M_G}{\mu}\right) = \frac{8\pi^2 \alpha M_0}{K_0}. \tag{A.5}$$

with the mirage unification values for gaugino masses as

$$M_i(\mu_{mirage}) = K_0 c_0. \tag{A.6}$$

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