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Analysis of black hole thermodynamics with a new higher order generalized uncertainty principle

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Abstract This research explores a unified expression for the black hole thermodynamics in the presence of a special generalized uncertainty principle (GUP). We have considered a heuristic analysis versus behavior of a particle which is absorbed by the black hole. Then we investigate the thermodynamic properties of the topological charged black hole under the consideration of GUP and we obtain the GUP-corrected temperature, entropy and heat capacity and we compare the behaviors of the usual temperature and the corrected temperature for the charged black hole in the spherical horizon case.

1 Introduction

The generalized uncertainty principle (GUP) raised of quantization of gravity is an adequate tool for describing black holes and their thermodynamic properties. By considering quantum gravity effects, thermodynamic quantities of black hole may be changed for example we can observe the temperature and entropy have corrected via GUP. The uncertainty principle is corresponding to the Heisenberg algebra, and so any modification of the uncertainty principle such as GUP will deform the Heisenberg algebra [1–8,8–34]. In Refs. [31,32], the deformed Schrodinger–Newton equation under GUP has been analyzed .

In Refs. [33,34], the production of mini black holes under GUP and the deformation of the Heisenberg algebra, consistent with both GUP and doubly special relativity (DSR) have been inspected respectively. Furthermore, based on GUP, the deformation of the second and third quantized theories by deforming the canonical commutation relations have been investigated [35] and this has been done for the Wheeler–DeWitt (WDW) equation [36,37]. Also this deformed WDW equation has been considered for analyzing quantum black holes [38,39]. In Ref. [40], one type of GUP that it is compatible with specific quantum gravity scenarios with a fundamental minimal length and Lorentz violation is obtained. In Ref. [41], GUP into field theories is incorporated with Lifshitz scaling. Also a supersymmetric field theory deformed by generalized uncertainty principle and Lifshitz scaling have been studied [42]. In Refs. [43,44], the effect of a new version of GUP has been investigated on the inflationary cosmology and the dynamics of the Universe respectively. Based on the universality of the entropy-area relation of a black hole, it is argued that the GUP-corrected entropyarea relation is universal for all black objects [45]. In Ref. [46], the role of GUP in the context of black hole complementarity is studied.

On the other hand, the modification to the thermodynamics of a black hole in higher dimensions because of GUP is calculated [47,48]. In Ref. [49], the modified Hawking temperature of the black hole by using the modified Klein-Gordon equation based on GUP is obtained. In Ref. [50], the authors studied effects of the simple form of GUP on the thermodynamics and critical behavior of black hole in Ads space and in Ref. [51], a heuristic method is introduced for investigating the usual form of GUP effects on the thermodynamics of a static black hole. This simple form of GUP has satisfied minimal length while it does not show the maximal momentum. The maximal momentum idea was first applied by considering modified commutation relation [9–11].

As one of the motivations for applying the higher order form of GUP with the new properties and different algebraic structure, we can mention to solving some important problems in limit of the usual GUP, such as the divergence of the energy spectrum raised in the position operator. Owing to our considered structure, the main purpose is to investigate the effects of the special form GUP on thermodynamics of the topological charged black hole.

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The outline of the present work is as follows. In Sect. 2 a new higher order GUP has been introduced. In Sect. 3 we have investigated the mass-temperature relation for black hole and in Sect. 4 we have calculated the GUP-corrected thermodynamic quantities of the charged black hole and simply discuss their properties. In Sect. 5, we give some numerical analyses and discuss the results. Finally, a conclusion is presented in Sect. 6.

2 Introduce a new higher order GUP

A new survey on GUP started with the advent of the high order GUP of the form [13]

$$[x, p] = \frac{i\hbar}{1-\beta^2 p^2}, \tag{1}$$

this commutation relation satisfies the algera corresponding to the usual form GUP [1], and this contains a singularity at $p^2 = 1/\beta^2$. Dependence above equation to the term p inserts the maximal momentum as well as the minimal length.

In this paper, we have considered the simpler form of higher order generalized (gravitational) Uncertainty Principle (GUP). [52]

$$[x, p] = \frac{i\hbar}{1-\beta|p|}. \tag{2}$$

It is so satisfied the above mentioned conditions. The following uncertainty relation that arises from GUP is given by

$$\Delta x \geqslant \frac{\hbar}{2}\left[-\beta + \frac{1}{(\Delta p)(1-\beta(\Delta p))}\right]. \tag{3}$$

By considering m_p as Planck mass, we have $\beta = \frac{\beta_0}{m_p c}$ where β_0 is constant and

$$\frac{\beta}{c} \sim 10^{-19}\ \mathrm{Gev}^{-1}. \tag{4}$$

Our analysis is based on very useful heuristic method for a particle which is captured by the black hole. In this method, a black hole appears as a sink for absorbing particles. The crucial technic is that the black hole's size and temperature are identified via Δx and Δp parameters and one of the main results is that the black hole mass is not allowed to be less than a scale order plank mass.

3 Black hole thermodynamics

From the first law of black hole mechanics and classical general relativity, we can define

$$dM = \frac{\kappa}{8\pi} dA + \Phi dQ, \tag{5}$$

where M, Q, A and κ are the black hole's mass, electric charge, area and the surface gravity, respectively and the electric potential measured at infinity with reference to the horizon is $\Phi = \frac{Q}{r_0}$.

However, from the thermodynamic point of view and with respect to the Hawking, the above equation can be rewrite as follows

$$dM = TdS + \Phi dQ, \tag{6}$$

where T is the temperature of a black hole and it is proportional to its surface gravity. S parameter has considered as black hole entropy. In more general situations, the entropy of a black hole is assumed to be a function of its area [52]. Following from (5) and the definition of thermodynamics, the temperature of a black hole can be generally expressed as

$$T = \left(\frac{\partial M}{\partial S}\right)_Q = \frac{dA}{dS} \times \left(\frac{\partial M}{\partial A}\right)_Q = \frac{\kappa}{8\pi} \times \frac{dA}{dS}. \tag{7}$$

When the particle disappears or absorbs by black hole, on one hand, its information is lost to an observer outside the horizon; on the other hand, the smallest increase in the area of a black hole is given by

$$A \sim Xm, \tag{8}$$

where X and m are the particle's size and mass, respectively. According to information defined by the theory, we can consider $(\Delta S)_{\min} = \mathrm{Ln}2$ [51]. However in quantum mechanics, the momentum uncertainty is not allowed to be greater than the mass ($\Delta p \leqslant m$) and we have investigated a particle by a wave packet that the width of wave packet is decribed as the standard deviation of X distribution i.e. the position uncertainty, which can be denoted as the characteristic size of the particle ($X \sim \Delta x$). Thus the representation (8) rewrite as follows

$$A \sim Xm \geqslant \Delta x \Delta p. \tag{9}$$

4 A class of static and spherically black holes

We consider a static and spherical black hole as follows [51]

$$dS^2 = -F(r)dt^2 + F^{-1}(r)dr^2 + r^2 d\Omega^2, \tag{10}$$

where

$$F(r) = K - \frac{8\pi GM}{\Sigma_K r} + \frac{16\pi^2 G^2 Q^2}{\Sigma_K^2 r^2} + \frac{r^2}{\ell^2}, \tag{11}$$

where r is cosmological radius, $d\Omega^2$ is line element of a two-dimensional Einestein space and M, Σ_K and Q have satisfied mass, volume and electric charge and ℓ is cosmological radius and for simplicity, we have considered $\frac{4\pi G}{\Sigma_K} = 1$. Without loss of generality, we can consider $K = 1$ for spherical horizon, $K = 0$ for planar/toroidal horizon and $K = -1$ for hyperbolic horizon.

According to the metric function in (10) and $F(r_0) = 0$ with defining cosmological constant $\Lambda = \frac{3}{\ell^2}$, the black hole mass is

$$M = \frac{r_0}{2} + \frac{Q^2}{2r_0} - \frac{\Lambda}{6} r_0^3, \tag{12}$$

where r_0 denotes the position of the event horizon of the black hole. If we have considered the thermodynamic pressure as $P = -\Lambda/8\pi$ [50]. Then the surface gravity of the black hole is

$$\kappa = \frac{F'(r_0)}{2} = \frac{-Q^2 + r_0^2 + 8\pi P r_0^4}{2r_0^3}, \tag{13}$$

The considered line element in (10) illustrates a class of static and spherically symmetric black holes, such as Schwarzschild, Reissner–Nordstrom and their partners in (anti-)de Sitter spacetime. When a particle is absorbed by black hole, Δx should not be greater than a specific scale which minimizes ΔA. This characteristic size should be related to the black hole, if $\Delta A_{\min}$ is expected to describe an intrinsic property of the horizon. For a static and spherically symmetric black hole, it is distinguished with the twice radius of horizon, i.e.

$$2r_0 \geqslant \Delta x \geqslant \frac{\hbar}{2}\left[-\beta + \frac{1}{(\Delta p)(1-\beta(\Delta p))}\right]. \tag{14}$$

From Eq. (3), we have

$$\Delta x \Delta p \geqslant \frac{\hbar}{2}\left[-\beta(\Delta p) + \frac{1}{1-\beta(\Delta p)}\right]. \tag{15}$$

Therefore the momentum uncertainty is obtained as follows

$$\begin{aligned} &\frac{\beta\hbar + 4r_0 - \sqrt{-3\beta^2\hbar^2 - 8\beta\hbar r_0 + 16r_0^2}}{2\beta(\beta\hbar + 4r_0)} \\ &\leqslant \Delta p \\ &\leqslant \frac{\beta\hbar + 4r_0 + \sqrt{-3\beta^2\hbar^2 - 8\beta\hbar r_0 + 16r_0^2}}{2\beta(\beta\hbar + 4r_0)}. \end{aligned} \tag{16}$$

Substituting Eqs. (14) and (16) into (9), the increase in area satisfies

$$A \geqslant \eta\hbar', \tag{17}$$

where η is a calibration factor and we have

$$\hbar' = \frac{4r_0\hbar}{\beta\hbar + 4r_0 + \sqrt{-3\beta^2\hbar^2 - 8\beta\hbar r_0 + 16r_0^2}}. \tag{18}$$

In the semi-classical case, the temperature and entropy for the black hole are

$$S_0 = \frac{A}{2\hbar}, \quad T_0 = \frac{\kappa\hbar}{4\pi}. \tag{19}$$

According to Heisenberg uncertainty principle, one can derive

$$\frac{dA}{dS} = \frac{(\Delta A)_{\min}}{(\Delta S)_{\min}} = 4\hbar', \tag{20}$$

where $\hbar'$ is the effective Planck constant and is defined as Eq. (18). Thus in our opinion, GUP changes the semiclassical framework to a certain context, and the semi classical black hole temperature (19) should suffer a the GUP-corrected black hole temperature

$$\begin{aligned} T' &= \frac{\kappa\hbar'}{2\pi} \\ &= \frac{\hbar(-Q^2 + r_0^2 + 8\pi P r_0^4)}{\pi r_0^2\left[\beta\hbar + 4r_0 + \sqrt{-3\beta^2\hbar^2 - 8\beta\hbar r_0 + 16r_0^2}\right]}. \end{aligned} \tag{21}$$

Therefore, by defining $\eta = 4\mathrm{Ln}2$ [51], the GUP-corrected entropy of the black hole can be derived.

$$S = \int \frac{dS}{dA} dA \simeq \int \frac{(\Delta S)_{\min}}{(\Delta A)_{\min}} dA \simeq \int \frac{dA}{4\hbar'}. \tag{22}$$

By considering Eqs. (18) and (20), we obtain

$$\begin{aligned} S = \frac{\pi}{2\hbar}\Bigg[&\beta\hbar r_0 + 2r_0^2 + \frac{(\beta\hbar + 4r_0)}{8} \\ &\sqrt{-3\beta^2\hbar^2 - 8\beta\hbar r_0 + 16r_0^2} - \frac{\beta^2\hbar^2}{2} Ln \\ &\left(-\beta\hbar + 4r_0 + \sqrt{-3\beta^2\hbar^2 - 8\beta\hbar r_0 + 16r_0^2}\right)\Bigg]. \end{aligned} \tag{23}$$

Here the effect of GUP leads to a logarithmic term, which also exists in many other quantum corrected entropy. As a thermodynamic system, the thermodynamic quantities of the black hole should satisfy the thermodynamic identity. Now the heat capacity of the GUP black hole can be defined as

$$\begin{aligned} C &= T\frac{\partial S}{\partial T} \\ &= \frac{\kappa\hbar'}{2\pi} \cdot \frac{\partial S}{\partial A} \cdot \frac{\partial A}{\partial T} \\ &= \frac{\kappa\hbar'}{2\pi} \cdot \frac{(\Delta S)_{\min}}{(\Delta A)_{\min}} \cdot \frac{\partial A}{\partial T}, \\ &= \frac{\kappa}{4}\left(\frac{1}{\kappa}\frac{\partial A}{\partial \hbar'} + \frac{1}{\hbar'}\frac{\partial A}{\partial \kappa}\right). \end{aligned} \tag{24}$$

By introducing $R = \sqrt{-3\beta^2\hbar^2 - 8\beta\hbar r_0 + 16r_0^2}$ in the context of the GUP, direct calculation for the heat capacity gives

$$\begin{aligned} C &= \frac{1}{4}\left(\frac{\partial \hbar'}{\partial A} + \frac{\hbar'}{\kappa}\frac{\partial \kappa}{\partial A}\right)^{-1} \\ &= -\frac{\pi}{4\hbar}[Rr_0(\beta\hbar + 4r_0 + R)^2 \\ &\quad \times (8\pi P r_0^4 + r_0^2 - Q^2)]/[3\hbar^2\beta^2(8\pi P r_0^4 + Q^2) \\ &\quad - 2r_0(4r_0 + R)(8\pi P r_0^4 - r_0^2 + 3Q^2) \end{aligned}$$

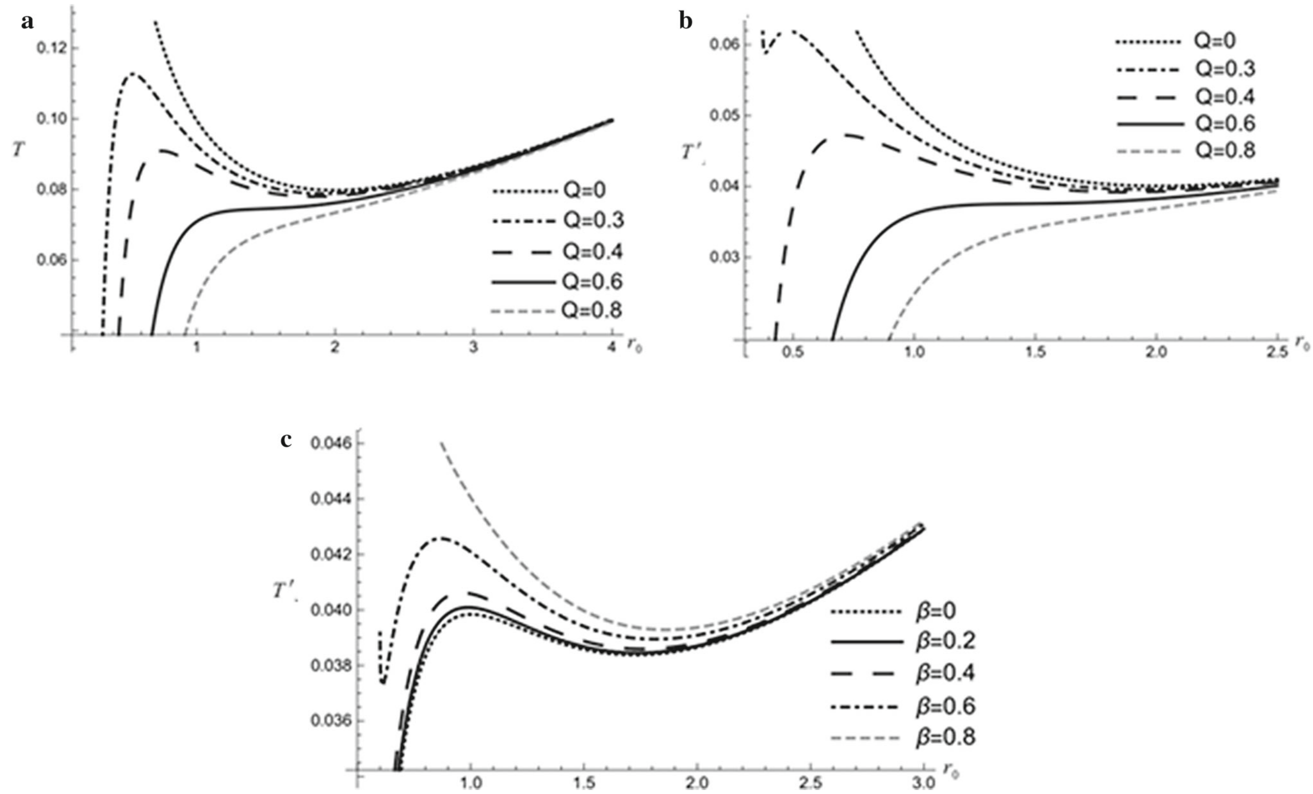

Fig. 1 **a** The standard temperature of the black hole vs. r_0. **b** The GUP-corrected temperature vs. r_0 with $\beta = 0.5$. **c** The GUP-corrected temperature vs. r_0 with $Q = 0.5$. In all cases we take $\hbar = 1$, $P = 0.01$

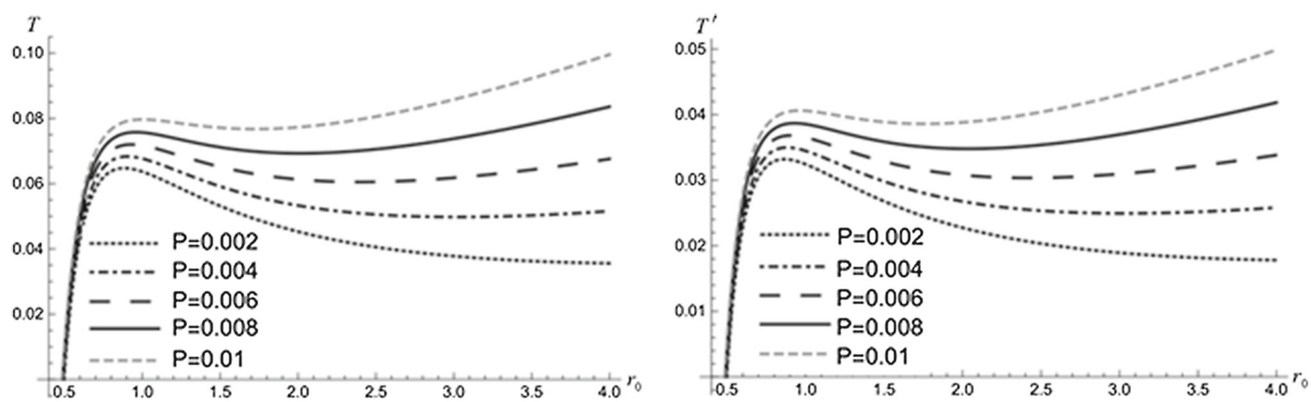

Fig. 2 The left panel: the standard temperature of the black hole vs. r_0. The right panel: the GUP-corrected temperature vs. r_0 with $\beta = 0.5$. In both cases we take $\hbar = 1$, $Q = 0.5$

$$+ 10\hbar\beta Q^2 r_0 - \hbar\beta Q^2 R - 2\hbar\beta r_0^3 \times (1 + 4\pi P r_0(-6r_0 + R))]. \tag{25}$$

5 Summery and numerical results

We have investigated in Fig. 1 the temperature of a black hole for two limits of the usual temperature of the black hole and the GUP-corrected charged black hole temperature. Figure 1 is included to give a better insight of the GUP-corrected black hole temperature vs. the electric charge Q and it reports the role of the electric charge in the temperature spectrum. We can see from Fig. 1a–c that although black hole temperature obtained in view of GUP increases with increasing β, it decrease with increasing the electric charge Q.

Figure 1 have interesting behavior via r_0, as far as there is a sharp increasing of the temperature of a black hole or a pick in the chart for a certain value of r_0, then the temperature decreases with increasing r_0.

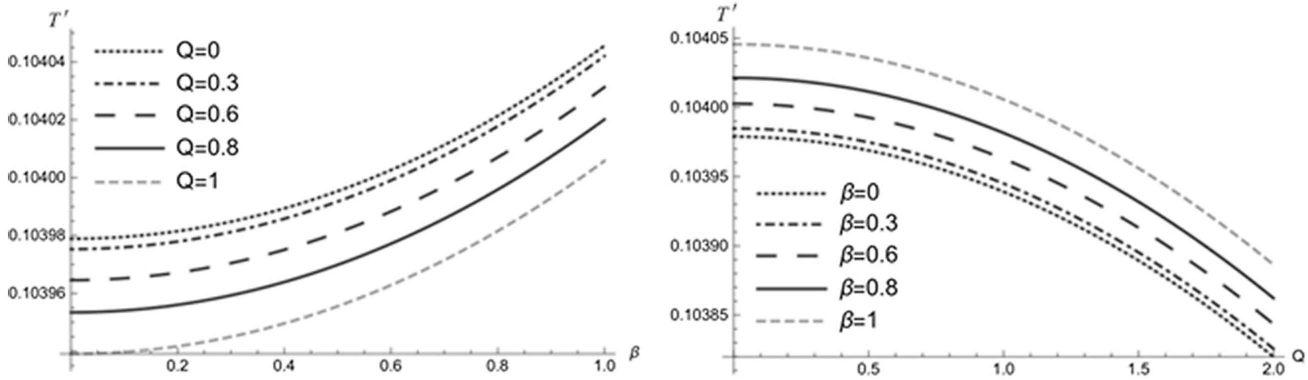

Fig. 3 The left panel: the GUP-corrected temperature vs. β. The right panel: the GUP-corrected temperature vs. Q. In both cases we take $\hbar = 1,\ P = 0.01,\ r_0 = 10$

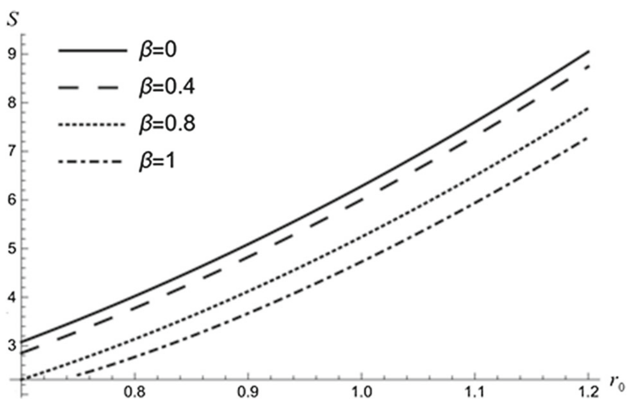

Fig. 4 Entropy vs. r_0 for $\hbar = 1$

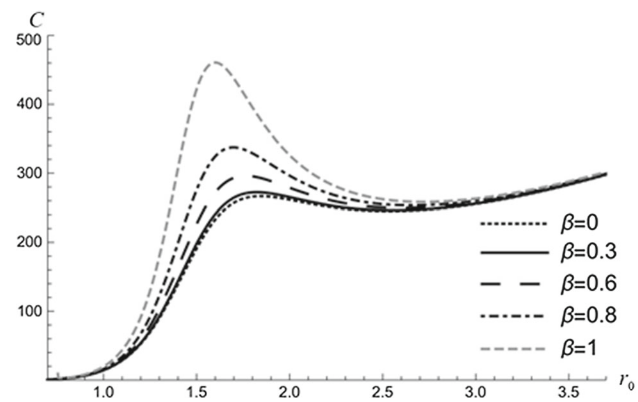

Fig. 5 The heat capacity vs. r_0 for $\hbar = 1,\ Q = 0.7,\ P = 0.01$

By keeping all the other parameters fixed, we have changed the the thermodynamic pressure parameter in order to investigate how the black hole temperature are sensitive to parameter P with increasing r_0. The results are presented in Fig. 2. From Fig. 2, it is readily observed that the the temperature spectrum increases with increasing P for both of the usual form of GUP i.e. $\beta = 0$ and the GUP-corrected case.

By taking a set of parameter $\hbar = 1, r_0 = 10$ and $P = 0.01$, we have investigated the effects of the β-parameter and the electric charge Q on the black hole temperature under the conditions of the GUP-corrected. Figure 3 shows that although the magnitude of the GUP-corrected black hole temperature increases as β increases, it decreases as Q increases.

Also, we depict in Fig. 4 the entropy vs. r_0 and we can see by increasing r_0 (the position of the event horizon of the black hole) the entropy has increased. Finally, we have portrayed the heat capacity for various values of β vs. r_0 in Fig. 5. Figure 5 have interesting and weird behavior vs. r_0, as far as there is a sharp increasing of the heat capacity of a black hole or a pick in the chart for a certain value of r_0, then the heat capacity decreases with increasing r_0. As seen there, GUP influences on the temperature, the entropy and the heat capacity.

6 Conclusions

We have considered a new higher order generalized (gravitational) uncertainty principle (GUP) in the form $[x, p] = i\hbar/(1 - \beta\,|p|)$ with $|p| = \sqrt{|p^2|}$. After considering GUP for the charged black hole with general topology ($k = 1$), we have also investigated the temperature of a black hole under the conditions of the semiclassical case and the GUP-corrected case respectively for both $\beta = 0$ and $\beta \neq 0$.

We have fund that the temperature of the black hole, not like its semiclassical counterpart, that it has more fruitful structures. The GUP-corrected entropy is always positive and is independent of the electric charge Q.

Our numerical data describe the temperature of a black hole in detail under the position of the event horizon of the black hole r_0. As seen in numerical results, GUP influences on the temperature, the entropy and the heat capacity.

The results of our work have been compared with the previous work of other authors in the literature and they are consistent and are reduced to the work reported, when $\beta = 0$.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: In this paper by plotting behavior of the GUP corrected temperature, entropy and heat capacity vs. black hole's characteristics, we investigate the thermodynamic properties of the topological charged black hole under the consideration of GUP. However, this manuscript has no associated data.]

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