Evolution of thick domain walls in inflationary and $p = w\rho$ universe

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Received: 17 August 2018 / Accepted: 12 October 2018 / Published online: 25 October 2018
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Abstract We study the evolution of thick domain walls in the different models of cosmological inflation, in the matter-dominated and radiation-dominated universe, or more generally in the universe with the equation of state $p = w\rho$. We have found that the domain wall evolution crucially depends on the time-dependent parameter $C(t) = 1/(H(t)\delta_0)^2$, where $H(t)$ is the Hubble parameter and δ_0 is the thickness of the wall in flat space-time. For $C(t) > 2$ the physical thickness of the wall, $a(t)\delta(t)$, tends with time to δ_0 , which is microscopically small. Otherwise, when $C(t) \leq 2$, the wall steadily expands and can grow up to a cosmologically large size.

1 Introduction

Creation of the baryon asymmetry of the universe and possible existence of the cosmological antimatter crucially depends upon the version of C and CP violation realized in the early universe. In this connection spontaneous CP violation suggested in Ref. [1] is of particular interest. This beautiful model, however, suffers from the domain wall problem [2, 3] – the energy density of the walls between domains with different sign of CP violation is so large that they would either overclose the universe or destroy the observed (near-) isotropy of the microwave background radiation. To avoid this problem the mechanism of the wall destruction was proposed, see e.g. [4] and references therein. If this mechanism is operative, it should get rid of remnants of the walls not earlier than the baryogenesis was accomplished and the universe acquired the necessary baryon asymmetry. Double-well shape of the potential would already vanish to this period and

the CP -violating scalar field φ , which formed the domain wall, would evolve down to zero. Baryogenesis in this scenario took place when φ rolled down to zero, but still before it reached the mechanical equilibrium point at $\varphi = 0$. Different related mechanisms of C and CP violation in cosmology are reviewed in Ref. [5].

Another problem of the baryogenesis based on spontaneous CP violation is the thickness of the wall. In flat space-time the thickness of the wall is microscopically small and if the walls with such or similar thicknesses were created in the cosmological situation, the matter-antimatter domains would be in close contact with each other. It would lead to very large annihilation rate and to unacceptably high background of the annihilation products in the universe. However, the cosmological expansion may lead to much larger separation of the domains eliminating or smoothing down this problem.

The evolution of the domain wall thickness in de Sitter universe, which is an approximation to the inflationary universe, was studied in the papers [6, 7]. It was shown there that for sufficiently small value of the parameter

$$C(t) = 1/(H(t)\delta_0)^2, \quad (1.1)$$

where $H(t)$ is the Hubble parameter and δ_0 is the thickness of the wall in flat space-time, the wall thickness would exponentially rise and the matter-antimatter domains may be safely separated.

In this work we studied the evolution of thick domain walls in the different models of inflation, as well as for arbitrary cosmological expansion regimes with the matter satisfying the equation of state $p = w\rho$ with constant parameter w . We have shown that there exists some range of the inflation parameters leading to a large domain separation prior to successful baryogenesis.

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2 Evolution of thick domain walls in de Sitter universe

In this section we briefly remind how the thick domain walls evolve in de Sitter universe [6, 7].

We consider a model of real scalar field φ with a simple double-well potential. The Lagrangian of such model is the following,

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\lambda}{2} (\varphi^2 - \eta^2)^2. \quad (2.1)$$

The equation of motion of field φ can be easily obtained from (2.1) and looks as

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi) = -2\lambda\varphi (\varphi^2 - \eta^2). \quad (2.2)$$

In flat space-time with the metric, $ds^2 = dt^2 - (dx^2 + dy^2 + dz^2)$, and in static one-dimensional case, $\varphi = \varphi(z)$, the Eq. (2.2) has the form,

$$\frac{d^2\varphi}{dz^2} = 2\lambda\varphi (\varphi^2 - \eta^2). \quad (2.3)$$

This equation has a kink-type solution, describing a static infinite flat domain wall in xy -plane. Without loss of generality we can assume that the wall is situated at $z = 0$,

$$\varphi(z) = \eta \tanh \frac{z}{\delta_0}, \quad (2.4)$$

where $\delta_0 = 1/(\sqrt{\lambda}\eta)$ is the thickness of the wall in flat space-time (see Fig. 1). Since $\sqrt{\lambda}\eta$ is essentially the mass of the Higgs-like boson, δ_0 is microscopically small. Otherwise, if δ_0 would be cosmologically large, this boson would have a tiny mass and thus it would generate long range forces which are strongly restricted by experiment.

The evolution of thick domain walls in spatially flat section of the de Sitter universe with the metric $ds^2 = dt^2 - e^{2Ht} (dx^2 + dy^2 + dz^2)$, with a constant Hubble parameter, $H > 0$, was considered previously in Refs. [6, 7]. A remarkable feature of this problem is that all parameters can be combined into a single positive constant, $C = 1/(H\delta_0)^2 = \lambda\eta^2/H^2 > 0$. Therefore, it is quite natural that the evolution of domain walls is determined only by the value of C . In the case of very thin domain walls, whose thickness is much smaller than the de Sitter horizon, $\delta \ll H^{-1}$, i.e. $C \gg 1$, the solution $\varphi(z)$ is well approximated by the flat-spacetime solution (2.4). However, as the flat-spacetime thickness parameter δ_0 increases, a deviation of the solution from the flat-spacetime solution increases as well. For sufficiently large value of parameter C , $C > 2$, the initial kink configuration in a de Sitter background tends to the stationary solution which depends only on physical distance $l = a(t)z$, i.e.

$\varphi = \eta f(Hl) = \eta f(e^{Ht}Hz)$. The thickness of the stationary wall rises with decreasing value of C .

Above the critical value, $\delta_0 \geq H^{-1}/\sqrt{2}$, i.e. $C \leq 2$, there are no stationary solutions at all. But if one allows for an arbitrary dependence of the solution on z and t , the solution exists for any C , and the case of $C \leq 2$ leads to the expanding kink with rising thickness. In other words, the thickness of the wall infinitely grows with time. For $C \lesssim 0.1$ the rise is close to the exponential one, therefore transition regions between domains might be cosmologically large.

3 Evolution of thick domain walls in inflationary universe

3.1 Φ^2 -inflation

In this section we consider a simple model of inflation with quadratic inflaton potential (see Fig. 2)

$$U(\Phi) = \frac{m^2\Phi^2}{2}, \quad (3.1)$$

(the inflaton field Φ should not be confused with the field φ which forms the domain wall). The model of inflation with a quadratic potential is strongly constrained in view of the recent observational data (for a review see e.g. [8]), but for our conclusion only the very fact of exponential expansion is important, regardless of the specific mechanism of inflation.

We assume that potential energy U of the inflaton gives the main contribution to the cosmological energy density ρ . Therefore, the Hubble parameter and the scale factor are completely determined by the inflaton field.

We consider the evolution of domain wall during the slow-roll regime of inflation, when the inflaton field Φ slowly rolls down from some initial value Φ_i to the minimum $\Phi = 0$ of the potential $U(\Phi)$. Slow-roll regime ends when at least one of the slow-roll parameters $\epsilon(\Phi)$ or $\eta(\Phi)$ becomes of the order of 1. In the model under consideration the slow-roll parameters coincide,

$$\epsilon(\Phi) = \frac{m_{Pl}^2}{16\pi} \left(\frac{U'(\Phi)}{U(\Phi)} \right)^2 = \frac{1}{4\pi} \frac{m_{Pl}^2}{\Phi^2}, \quad (3.2)$$

$$\eta(\Phi) = \frac{m_{Pl}^2}{8\pi} \frac{U''(\Phi)}{U(\Phi)} = \frac{1}{4\pi} \frac{m_{Pl}^2}{\Phi^2} = \epsilon(\Phi), \quad (3.3)$$

where $m_{Pl} \approx 1.2 \times 10^{19}$ GeV is the Planck mass. Therefore, the slow-roll regime of inflation ends when $\Phi \simeq m_{Pl}/\sqrt{4\pi} \approx 0.3 m_{Pl}$. It should be noted that in the models of inflation that we consider in this paper the end of slow-roll regime means the end of inflation itself.

Fig. 1 Domain wall in flat space-time

Fig. 2 Inflaton potential $U(\Phi)$ in the Φ^2 -model. Black dots correspond to $\Phi_i = 2.0 m_{Pl}$ and $\Phi_e = 0.3 m_{Pl}$

In contrast to de Sitter universe (see Sect. 2) the Hubble parameter now depends on time,

$$\begin{aligned} H(t) &= \sqrt{\frac{8\pi\rho(t)}{3m_{Pl}^2}} \approx \sqrt{\frac{8\pi}{3m_{Pl}^2} \frac{m^2\Phi^2(t)}{2}} \\ &= \sqrt{\frac{4\pi}{3}} \frac{m}{m_{Pl}} \Phi(t) \neq const, \end{aligned} \quad (3.4)$$

so at the end of the slow-roll regime, $H \simeq m/\sqrt{3}$.

The equation of motion of the inflaton in the slow-roll regime is the following,

$$\dot{\Phi}(t) \approx -\frac{m^2\Phi(t)}{3H(t)} \approx -\frac{m_{Pl}m}{\sqrt{12\pi}}. \quad (3.5)$$

This equation is easily integrated,

$$\Phi(t) = \Phi_i - \frac{m_{Pl}m}{\sqrt{12\pi}}t, \quad (3.6)$$

where Φ_i is the initial value of the inflaton field.

The Hubble parameter and the scale factor can also be easily found,

$$H(t) = \sqrt{\frac{4\pi}{3}} \frac{m}{m_{Pl}} \Phi_i - \frac{1}{3}m^2t, \quad (3.7)$$

$$a(t) = a_0 \cdot \exp\left(\sqrt{\frac{4\pi}{3}} \frac{m}{m_{Pl}} \Phi_i t - \frac{1}{6}m^2t^2\right). \quad (3.8)$$

These formulas are valid only till the end of the slow-roll regime,

$$t \lesssim t_e = \left(\sqrt{12\pi} \frac{\Phi_i}{m_{Pl}} - \sqrt{3}\right)m^{-1}. \quad (3.9)$$

The equation of motion (2.2) in the case when the field φ is a function of two independent variables, z and t , is written as

$$\frac{\partial^2 f}{\partial t^2} + 3H(t) \frac{\partial f}{\partial t} - \frac{1}{a^2(t)} \frac{\partial^2 f}{\partial z^2} = \frac{2}{\delta_0^2} f (1 - f^2), \quad (3.10)$$

where $f(z, t) = \varphi(z, t)/\eta$.

Since $H(t)$ has the form (3.7), it is convenient to use $1/m$ units in equation of motion (3.10):

$$H(t) = \frac{m}{3}(mt_e - mt) + \frac{m}{\sqrt{3}}, \quad (3.11)$$

$$a(t) = a_e \cdot \exp\left(-\frac{(mt_e - mt)^2}{6} - \frac{(mt_e - mt)}{\sqrt{3}}\right) \text{ with} \\ a_e = a_0 \cdot e^{m^2 t_e^2 / 6 + m t_e / \sqrt{3}}, \quad (3.12)$$

$$\frac{\partial^2 f}{\partial (t \cdot m)^2} + \left((mt_e - mt) + \sqrt{3}\right) \frac{\partial f}{\partial (t \cdot m)} \\ - \frac{1}{a^2(t)} \frac{\partial^2 f}{\partial (z \cdot m)^2} = \frac{2}{(m \cdot \delta_0)^2} f(1 - f^2). \quad (3.13)$$

The boundary conditions for the kink-type solution should be

$$f(0, t) = 0, f(\pm\infty, t) = \pm 1, \quad (3.14)$$

and we choose the initial configuration as the domain wall with physical thickness δ_0 and zero time derivative:

$$f(z, t_i) = \tanh \frac{z \cdot a(t_i)}{\delta_0}, \left. \frac{\partial f(z, t)}{\partial t} \right|_{t=t_i} = 0. \quad (3.15)$$

In numerical calculations we use the following values: $\Phi_i = 2m_{Pl}$, $t_i = 0$, and $a_0 = 1$.

Time dependence of physical thickness of the wall, $a(t)\delta(t)$, for different values of the initial wall thickness, δ_0 , is shown in Fig. 3. Here $\delta(t)$ is the coordinate thickness of the wall. It is defined as the value of the coordinate z at the position where $f(z, t) = \tanh 1 \approx 0.76$.

It turns out that the domain wall evolution is basically determined by the parameter

$$C(t) = \frac{1}{(H(t)\delta_0)^2}. \quad (3.16)$$

Since $H(t)$ is decreasing, $C(t)$ increases with time. As is known from Sect. 2, when $C < 2$ the thickness of the wall increases rapidly while for $C > 2$ the wall thickness tends to its stationary value which for $C \gg 2$ coincides with δ_0 .

Time t_C at which $C(t_C) = 2$ in the model under consideration is determined from the relation

$$mt_C = \sqrt{12\pi} \frac{\Phi_i}{m_{Pl}} - \frac{3}{\sqrt{2}m\delta_0}. \quad (3.17)$$

Parameter $C(t)$ can be equal 2 only if $t_C \geq t_i = 0$, i.e. if $m\delta_0 \geq \sqrt{3}m_{Pl}/(\sqrt{8\pi}\Phi_i) \approx 0.173$ for our choice of Φ_i . From (3.9) and (3.17) it follows that $t_C < t_e$ if $m\delta_0 < \sqrt{3/2} \approx 1.225$.

When δ_0 is so small that $C(t) > 2$ during all the time of inflation, i.e. $m\delta_0 < \sqrt{3}m_{Pl}/(\sqrt{8\pi}\Phi_i)$, then the domain

wall thickness, $a(t)\delta(t)$, after some damped oscillations tends to constant value, δ_0 , which is microscopically small.

For larger δ_0 , such that $\sqrt{3}m_{Pl}/(\sqrt{8\pi}\Phi_i) \leq m\delta_0 \leq \sqrt{3/2}$, initially one has $C(t) \leq 2$, but at the end of the slow-roll regime $C(t) \geq 2$. Therefore, the domain wall thickness grows initially, reaches the maximum and then diminishes.

Finally, if δ_0 is quite large, $m\delta_0 > \sqrt{3/2}$, then $C(t) < 2$ during all the time of inflation. In such case domain wall thickness could grow to cosmologically large size by the end of inflation. However, for domain walls, which were thick initially, $m\delta_0 \gg 1$, coordinate thickness almost does not change, $\delta(t) \approx \delta_0$, and domain wall expansion is entirely due to growth of scale factor, $a(t)$ (see Fig. 3h).

3.2 “Hilltop”-inflation

Since the Φ^2 -inflation is in tension with recent observational data let us consider below the other inflation models which are in agreement with observations. One of such models is the so-called “hilltop” model [9]. The inflaton potential for quite small values of Φ looks as follows (see Fig. 4)

$$U(\Phi) = \Lambda^4 \left(1 - \frac{\Phi^4}{\mu^4}\right). \quad (3.18)$$

We take $\mu = 2.5m_{Pl}$ to be consistent with the Planck data [10].

The slow-roll parameters are

$$\epsilon(\Phi) = \frac{m_{Pl}^2}{16\pi} \left(\frac{U'(\Phi)}{U(\Phi)}\right)^2 = \frac{m_{Pl}^2}{\pi} \frac{\Phi^6}{(\mu^4 - \Phi^4)^2}, \quad (3.19)$$

$$\eta(\Phi) = \frac{m_{Pl}^2}{8\pi} \frac{U''(\Phi)}{U(\Phi)} = -\frac{3m_{Pl}^2}{2\pi} \frac{\Phi^2}{\mu^4 - \Phi^4}. \quad (3.20)$$

It is seen that $\epsilon(\Phi), |\eta(\Phi)| \ll 1$ for $\Phi \ll \mu \sim m_{Pl}$. The slow-regime of inflation ends when $\epsilon(\Phi) \sim 1$ or $|\eta(\Phi)| \sim 1$, so in this model we take $\Phi_e = 2.3m_{Pl}$.

The Hubble parameter is

$$H(t) \simeq \sqrt{\frac{8\pi}{3m_{Pl}^2} U(\Phi(t))}, \quad (3.21)$$

and correspondingly the scale factor is

$$a(t) = a_0 \cdot \exp\left(\int_0^t H(t') dt'\right), \quad (3.22)$$

in the calculations we use $a_0 = 1$, therefore $a(0) = 1$.

We take $\Lambda = 10^{-3}m_{Pl}$ in agreement with measured value of the amplitude of the scalar density perturbations, $\Delta_R \approx 5 \cdot 10^{-5}$,

Fig. 3 Time dependence of physical thickness of the wall, $a(t) \cdot \delta(t) \cdot m$, for different values of the initial wall thickness, δ_0 , in the Φ^2 -model. Dashed horizontal line corresponds to δ_0 . The time at which the inflation ends is shown by vertical dashed line with the corresponding label.

In Fig. 3d–f there is a vertical dashed line which corresponds to $t = t_C$. In Fig. 3g, h there is a green (dash-dotted) line which corresponds to $a(t) \cdot \delta_0 \cdot m$. In Fig. 3h this line coincides with $a(t) \cdot \delta(t) \cdot m$, which means that $\delta(t) \approx \delta_0$ till the end of inflation with very good accuracy

$$\Delta_R = \frac{3H^3}{2\pi|U'(\Phi)|} = \sqrt{\frac{8\pi}{3}} \frac{\Lambda^2 \mu^4}{\Phi^3 m_{Pl}^3} \left(1 - \frac{\Phi^4}{\mu^4}\right)^{\frac{3}{2}}, \quad (3.23)$$

here the value of Φ should be taken at the moment of 50–60 e-foldings before the end of inflation.

The number of e-foldings till the end of inflation is $N_e(\Phi) \simeq \pi \mu^4 / (m_{Pl}^2 \Phi^2)$ for $\Phi \ll \mu \sim m_{Pl}$. Initial value $\Phi_0 = 0$ leads to divergence, so in such models one takes $\Phi_0 = H(0) \ll m_{Pl}$. In such a case $N_e(\Phi_0) \simeq 10^{13}$, so the condition $N > 60$ of successful inflation is well fulfilled.

Fig. 4 Inflaton potential $U(\Phi)$ in the “hilltop” model. Black dots correspond to $\Phi_i = 1.4 m_{Pl}$ and $\Phi_e = 2.3 m_{Pl}$

We choose the time when the domain wall was formed not at the very beginning of inflation but rather near the end of inflation, $\Phi_i = \Phi(0) = 1.4 m_{Pl}$ (see Fig. 4).

Solution $\Phi(t)$ of the equation of motion, $3H(t)\dot{\Phi}(t) \approx -U'(\Phi)$, during the slow-roll regime of inflation can be expressed as implicit function,

$$\begin{aligned} & \arctan \sqrt{\frac{\mu^4}{\Phi^4(t)} - 1} - \sqrt{\frac{\mu^4}{\Phi^4(t)} - 1} \\ &= \sqrt{\frac{8}{3\pi}} \frac{\Lambda^2}{\mu^2} m_{Pl} t \\ &+ \arctan \sqrt{\frac{\mu^4}{\Phi_i^4} - 1} - \sqrt{\frac{\mu^4}{\Phi_i^4} - 1}. \end{aligned} \quad (3.24)$$

From this equation we can find the duration of the inflation for our choice of parameters: $t_e \approx 11.6 \cdot m_{Pl}/\Lambda^2$.

Since $U(\Phi) \propto \Lambda^4$, the Hubble parameter $H \propto \Lambda^2/m_{Pl}$. Therefore, m_{Pl}/Λ^2 is the natural choice of units for t and $\delta(t)$ in this model, like m^{-1} units in Φ^2 -model, see (3.13).

Now we can find numerically $\Phi(t)$, $H(t)$, and $a(t)$ at given t , and therefore solve the equation describing the evolution of the field φ (3.10). The boundary conditions as usually are (3.14) and we choose the initial configuration as the domain wall with physical thickness δ_0 and zero time derivative (3.15).

As it was in the Φ^2 -model, we expect that the evolution of the wall is defined by $C(t)$. The values of δ_0 for which $C(t)$ can be equal to 2 during the wall evolution can be found from the inequality $0 \leq t_C \leq t_e$, and lie between the following values:

$$\begin{aligned} t_C = 0 \text{ for } \delta_0 &= \frac{1}{H(0)\sqrt{2}} \\ &= \sqrt{\frac{3}{16\pi \left(1 - \frac{\Phi_i^4}{\mu^4}\right)}} \cdot \frac{m_{Pl}}{\Lambda^2} \approx 0.2573 \cdot \frac{m_{Pl}}{\Lambda^2}, \end{aligned} \quad (3.25)$$

$$\begin{aligned} t_C = t_e \text{ for } \delta_0 &= \frac{1}{H(t_e)\sqrt{2}} \\ &= \sqrt{\frac{3}{16\pi \left(1 - \frac{\Phi_e^4}{\mu^4}\right)}} \cdot \frac{m_{Pl}}{\Lambda^2} \approx 0.4587 \cdot \frac{m_{Pl}}{\Lambda^2}. \end{aligned} \quad (3.26)$$

The results of numerical calculation for the time dependence of the physical thickness of the wall, $a(t)\delta(t)$, for different values of parameter δ_0 are presented in Fig. 5.

The results are similar to what we obtained in Φ^2 -model. Very thin domain walls do not expand significantly, oscillating near δ_0 , see Fig. 5a–c. For a wall of medium size, such that t_C is after the beginning of inflation but before the end, there are periods when the wall thickness increases and decreases very fast, see Fig. 5d, e. Finally, a very thick wall, such that t_C is after the end of inflation, grows with the universe, see Fig. 5h.

3.3 R^2 -inflation

One more inflation model consistent with observational data is R^2 -inflation proposed and developed by Starobinsky in the late 1970s [11, 12] (for review see e.g. [13]). The modified theory of gravity with the action

$$S = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{R^2}{6M^2} \right) \quad (3.27)$$

leads to the inflationary expansion of the universe. Here the parameter M has the dimension of mass, and the scalar curvature is defined as $R = g^{\mu\nu} g^{\alpha\beta} R_{\mu\alpha\nu\beta}$.

Such R^2 -gravity is similar to the general relativity with a scalar field Φ [14] with the potential (see Fig. 6)

$$U(\Phi) = \frac{3M^2 m_{Pl}^2}{32\pi} \left(1 - e^{-4\sqrt{\pi/3} \Phi/m_{Pl}} \right)^2. \quad (3.28)$$

Fig. 5 Time dependence of physical thickness of the wall, $a(t) \cdot \delta(t)$, for different values of the initial wall thickness, δ_0 , in the “hilltop” model. Here t , $a(t)\delta(t)$ and δ_0 are shown in $m_{Pl}/\Lambda^2 = (10^{-6}m_{Pl})^{-1}$ units. Dashed horizontal line corresponds to δ_0 . The time at which the

inflation ends is shown by vertical dashed line with the corresponding label. In Fig. 5d–f there is a vertical dashed line which corresponds to $t = t_C$. In Fig. 5g and h there is a green (dash-dotted) line which corresponds to $a(t) \cdot \delta(t) \cdot m$

The slow-roll parameters are

$$\epsilon(\Phi) = \frac{m_{Pl}^2}{16\pi} \left(\frac{U'(\Phi)}{U(\Phi)} \right)^2 = \frac{4}{3} \frac{1}{\left(e^{4\sqrt{\pi/3}\Phi/m_{Pl}} - 1 \right)^2}, \quad (3.29)$$

$$\eta(\Phi) = \frac{m_{Pl}^2}{8\pi} \frac{U''(\Phi)}{U(\Phi)} = -\frac{4}{3} \frac{e^{4\sqrt{\pi/3}\Phi/m_{Pl}} - 2}{\left(e^{4\sqrt{\pi/3}\Phi/m_{Pl}} - 1 \right)^2}. \quad (3.30)$$

One can see that $\epsilon(\Phi), |\eta(\Phi)| > 1$ for $\Phi < 0$, therefore the slow-roll inflation takes place only if $\Phi > 0$. The inflation

Fig. 6 Inflaton potential $U(\Phi)$ in the R^2 -inflation. Black dots correspond to $\Phi_i = 0.9 m_{Pl}$ and $\Phi_e = 0.2 m_{Pl}$

ends when $\epsilon(\Phi) \sim 1$ or $|\eta(\Phi)| \sim 1$, so in our model we choose $\Phi_e = 0.2 m_{Pl}$.

The Hubble parameter is

$$H(t) \simeq \sqrt{\frac{8\pi}{3m_{Pl}^2} U(\Phi(t))} = \frac{M}{2} \left(1 - e^{-4\sqrt{\pi/3} \Phi(t)/m_{Pl}}\right). \quad (3.31)$$

The amplitude of the scalar density perturbations is

$$\Delta_R = \frac{3H^3}{2\pi|U'(\Phi)|} = \sqrt{\frac{3}{\pi}} \frac{M}{m_{Pl}} \sinh^2 \left(2\sqrt{\frac{\pi}{3}} \frac{\Phi}{m_{Pl}} \right), \quad (3.32)$$

here the value of Φ should be taken at the moment of 50–60 e-foldings before the end of inflation. Using the measured magnitude of $\Delta_R \simeq 5 \cdot 10^{-5}$ one can calculate the value of mass parameter, $M \simeq 2.6 \cdot 10^{-6} m_{Pl}$.

We choose the time when the domain wall is formed not at the very beginning of inflation, $\Phi_i = \Phi(0) = 0.9 m_{Pl}$ (see Fig. 6).

Solution $\Phi(t)$ of the equation of motion, $3H(t)\dot{\Phi}(t) \approx -U'(\Phi)$, during the slow-roll regime of inflation can be found analytically,

$$\Phi(t) = \sqrt{\frac{3}{\pi}} \frac{m_{Pl}}{4} \log \left(e^{4\sqrt{\pi/3} \Phi_i/m_{Pl}} - \frac{2Mt}{3} \right). \quad (3.33)$$

From this equation we can find the duration of the inflation: $t_e \approx 56.3 \cdot M^{-1}$.

Substituting (3.33) into (3.31) one obtains

$$H(t) = \frac{M}{2} \left(1 - \frac{1}{e^{4\sqrt{\pi/3} \Phi_i/m_{Pl}} - \frac{2Mt}{3}} \right), \quad (3.34)$$

and correspondingly the scale factor is

$$\begin{aligned} a(t) &= a_0 \cdot \exp \left(\int_0^t H(t') dt' \right) \\ &= a_0 \cdot e^{Mt/2} \cdot \left(1 - \frac{2Mt}{3} e^{-4\sqrt{\pi/3} \Phi_i/m_{Pl}} \right)^{3/4}, \end{aligned} \quad (3.35)$$

in calculations we use $a_0 = 1$, therefore $a(0) = 1$.

Since the Hubble parameter has the form (3.34), it is convenient to use M^{-1} units for t and $\delta(t)$ in this model, like we used m^{-1} units in Φ^2 -model, see (3.13).

Now we can solve numerically the equation describing the domain wall (3.10). The boundary conditions as usually are (3.14) and we choose the initial configuration as the domain wall with physical thickness δ_0 and zero time derivative (3.15).

With the help of (3.31) we obtain the values of δ_0 for which we have $t_C = 0$ and $t_C = t_e$:

$$\begin{aligned} t_C = 0 \text{ for } \delta_0 &= \frac{1}{H(0)\sqrt{2}} \\ &= \frac{\sqrt{2}}{1 - e^{-4\sqrt{\pi/3} \Phi_i/m_{Pl}}} \cdot M^{-1} \approx 1.4507 \cdot M^{-1}, \end{aligned} \quad (3.36)$$

$$\begin{aligned} t_C = t_e \text{ for } \delta_0 &= \frac{1}{H(t_e)\sqrt{2}} \\ &= \frac{\sqrt{2}}{1 - e^{-4\sqrt{\pi/3} \Phi_e/m_{Pl}}} \cdot M^{-1} \approx 2.5300 \cdot M^{-1}. \end{aligned} \quad (3.37)$$

The results of numerical calculation for the time dependence of physical thickness of the wall, $a(t)\delta(t)$, for different values of parameter δ_0 are presented in Fig. 7.

The results are similar to what we have seen in Φ^2 and hilltop models, i.e. the plots can be separated into three categories: very thin walls, see Fig. 7a–d; medium size walls, see Fig. 7e, f; very thick walls, see Fig. 7g, h. In the latter case the thickness of the wall is growing almost with the size of the universe. Summing up, we conclude that the particular model of inflation is not that important, and in the case of fast expansion of the universe the walls that are thick enough will be growing with the scale factor.

Fig. 7 Time dependence of physical thickness of the wall, $a(t) \cdot \delta(t)$, for different values of the initial wall thickness, δ_0 , in the R^2 -inflation. Here t , $a(t)\delta(t)$ and δ_0 are shown in M^{-1} units. Dashed horizontal line corresponds to δ_0 . The time at which the inflation ends is shown by ver-

tical dashed line with the corresponding label. In Fig. 7e and 7f there is a vertical dashed line which corresponds to $t = t_C$. In Fig. 7g, h there is a green (dash-dotted) line which corresponds to $a(t) \cdot \delta_0 \cdot m$

4 Evolution of thick domain walls in $p = w\rho$ universe

Now let us study how the domain wall evolves in an postinflationary universe with the equation of state of matter $p = w\rho$, where constant $w > -1$. In such universe the scale factor increases as some power of time,

$$a(t) = \text{const} \cdot t^\alpha, \text{ where } \alpha = \frac{2}{3(1+w)} > 0, \quad (4.1)$$

and the Hubble parameter decreases as inverse time,

$$H(t) = \frac{\dot{a}}{a} = \frac{\alpha}{t}. \quad (4.2)$$

The values $w = 0$ ($\alpha = 2/3$) and $w = 1/3$ ($\alpha = 1/2$) correspond to the matter-dominated and radiation-dominated universe, respectively.

After the substitution $\tau = t/\delta_0$, $\zeta = z/\delta_0$ into the equation of motion (3.10) we get

$$\frac{\partial^2 \tilde{f}}{\partial \tau^2} + \frac{3}{\sqrt{C(\tau)}} \frac{\partial \tilde{f}}{\partial \tau} - \frac{1}{\tilde{a}^2(\tau)} \frac{\partial^2 \tilde{f}}{\partial \zeta^2} = 2\tilde{f}(1 - \tilde{f}^2), \quad (4.3)$$

where $\tilde{f}(\zeta, \tau) = f(\zeta \cdot \delta_0, \tau \cdot \delta_0)$, $\tilde{a}(\tau) = a(\tau \cdot \delta_0)$, and

$$C(\tau) = (H(\tau \cdot \delta_0) \cdot \delta_0)^{-2} = H^{-2}(\tau). \quad (4.4)$$

Here we used the explicit expression (4.2) for $H(t)$, so the relation $C(\tau) = H^{-2}(\tau)$ is true only for $p = w\rho$ universe. For general dependence $H(t)$ the equality $H(t)\delta_0 = H(t/\delta_0)$ does not hold, so the behaviour of thick and thin domain walls can be completely different like it was in the case of de Sitter universe, $H = \text{const}$ (see Sect. 2). However, due to this feature of the $p = w\rho$ universe there is no explicit dependence on δ_0 in (4.3). This means that the equation of motion is the same for different δ_0 if t and z are measured in units of δ_0 . Till the end of this section we will use (t, z) notations considering $\delta_0 = 1$.

For the scale factor we choose the following form: $a(t) = a_0 \cdot (t/t_i)^\alpha$ with the constant $a_0 = 1$, so $a(t_i) = 1$ which means that the initial coordinate and physical distances are the same.

The results of numerical calculation for the time dependence of physical thickness of the wall, $a(t)\delta(t)$, in radiation-dominated and matter-dominated universe are presented in Fig. 8 along with the plots for other values of w . In Fig. 8a, b the initial time is chosen to be $t_i/\delta_0 = 0.5$ and $t_i/\delta_0 = 1.0$, respectively. We can see that these two plots look very much alike but the curves for $t_i/\delta_0 = 0.5$ look similar to the curves for $t_i/\delta_0 = 1.0$ with another value of w (i.e. curve with $w = -2/3$ and $t_i/\delta_0 = 0.5$ reaches approximately the same maximum value as the curve with $w = -7/9$ and $t_i/\delta_0 = 1.0$). Let us explain this.

The evolution of the domain wall is basically determined by the parameter $C(t)$, which increases in the $p = w\rho$ universe as

$$C(t) = \frac{1}{(H(t)\delta_0)^2} = \frac{t^2}{(\alpha\delta_0)^2} \propto t^2. \quad (4.5)$$

Since $C = 2$ is the critical value, let us introduce the time t_C at which $C(t_C) = 2$. In $p = w\rho$ universe,

$$\frac{t_C}{\delta_0} = \sqrt{2\alpha}. \quad (4.6)$$

With the help of (4.1) we obtain that $t_C > t_i$ for

$$w < \frac{2\sqrt{2}}{3} \frac{\delta_0}{t_i} - 1. \quad (4.7)$$

Let us consider $t_i/\delta_0 = 1.0$ (see Fig. 8b). Using (4.7) we get that $t_C > t_i$ for $w < 2\sqrt{2}/3 - 1 \approx -0.057$. In this case $C(t)$ is larger than 2 during all the time of the wall evolution for $w = 1/3$ and $w = 0$. If we consider the value of parameter w which is different from $1/3$ or 0 , e.g. $w = -1/3$ ($\alpha = 1$, $\dot{a}(t) = \text{const}$), $w = -2/3$ ($\alpha = 2$, $\ddot{a}(t) = \text{const}$), $w = -7/9$ ($\alpha = 3$, $\ddot{\ddot{a}}(t) = \text{const}$), then at the initial moment parameter C can be smaller than 2. In Fig. 8b one can see that in this case, while $C < 2$, the physical thickness of the wall, $a(t)\delta(t)$, increases rapidly. Moreover, when the parameter w approaches (-1) , the thickness of the wall is growing much faster. However, the parameter $C(t)$ increases with time and eventually becomes larger than 2. After that the wall thickness starts to decrease.

In case of $t_i/\delta_0 = 0.5$ (see Fig. 8a), for the same w the wall thickness is growing for a longer period of time, $t_C - t_i$, since t_C does not depend on t_i . We also can say that with different t_i the period of growth is the same for different values of w . This explains why curves corresponding to different values of w do look alike in Fig. 8a, b.

As for behaviour at $t \rightarrow \infty$, one sees in Fig. 8 that the physical thickness oscillates with slowly decreasing amplitude around the value δ_0 . Therefore, when $t \rightarrow \infty$ the field configuration tends to

$$f(z, t) = \tanh \frac{z \cdot a(t)}{\delta_0}. \quad (4.8)$$

This asymptotic behaviour can be understood from (3.10). If the first two terms in the l.h.s. of (3.10) can be neglected, then this equation becomes kink-type one, and has the solution (4.8).

In Fig. 8 we present the domain walls with the initial physical thickness of the ones in stationary universe ($H = 0$). By doing that, we separate the influence of cosmological expansion from the natural shrinking/expanding of the wall when it approaches stationary solution. However, we can consider the evolution of the wall thickness from different initial configurations. In Fig. 9 we can see the evolution of the wall thickness from the configurations stretched along z by factor k :

$$f(z, t_i) = \tanh \frac{z \cdot a(t_i)}{k \cdot \delta_0}. \quad (4.9)$$

For $w = 1/3$ (so $t_C < t_i = 1 \cdot \delta_0$) all initial configurations quickly approach their common stationary solution. On the other hand, for $w = -2/3$ (so $t_C > t_i = 1 \cdot \delta_0$) there is

Fig. 8 Time dependence of the physical thickness of the wall, $a(t)\delta(t)$, for different values of parameter w . Dashed horizontal line corresponds to δ_0 . Vertical dashed lines correspond to the moment t_C at which $C(t_C) = 2$ (the correspondence between color and w is the same)

a period of growth for all initial configurations though it is worth noting that this growth depends on k .

To conclude this section, we note that in $p = w\rho$ universe it is difficult to obtain domain walls with cosmologically large thickness (w should be really close to -1 for that). Such domain walls can exist at the beginning of $p = w\rho$ stage. However, in such universe the parameter $C(t)$ increases, so at some moment the wall thickness starts to shrink and eventually goes to the constant value, δ_0 , which is microscopically small.

5 Conclusions

We have found that at inflationary epoch the thickness of the domain walls exponentially rises when the parameter $C(t)$

is smaller than the critical value $C = 2$. Therefore, the wall thickness might rise up to cosmologically large scales if $C(t)$ remained smaller than 2 till the end of inflation. However, when inflation is over and the expansion turns into the power law regime with H decreasing with time as $1/t$, the parameter $C(t) \sim 1/H^2(t) \sim t^2$ at some moment becomes larger than 2, and the wall started to shrink.

Our original scenario [4] spans throughout the inflation and ends in the beginning of the reheating stage where the baryogenesis is supposed to take place. That is why we are interested in the wall evolution during all the inflation and at the beginning of the $p = w\rho$ stage. The main result of our calculations is that there exists sufficiently wide range of the parameters for which the domain wall remains astronomically thick during the baryogenesis epoch. This ensures large

Fig. 9 Time dependence of the physical thickness of the wall, $a(t)\delta(t)$, for different values of stretch parameter k . Dashed horizontal line corresponds to δ_0 . For both plots $t_i/\delta_0 = 1.0$. Vertical dashed lines correspond to the moment t_C at which $C(t_C) = 2$

separation between domains of matter and antimatter and prevents from catastrophic annihilation. This result allows to determine the range of the parameters of realistic baryogenesis models with safe separation between matter and antimatter.

An efficient baryogenesis could start only after inflation, and moreover in our model [4] it should take place before the field φ rolled down to zero (here φ is the field which made the wall). This can easily be achieved with a proper choice of the model parameters, as is shown in [4]. The particular mechanism of inflation is not essential for our results. The only thing which we need is the exponential cosmological expansion.

We have considered here the evolution of a single domain wall, rather than that of a whole network of walls. Indeed,

knowledge of the wall speeds and interaction or annihilation rates would be important only if the domain size is smaller than the cosmological horizon. Normally the size of the domain is much larger than the thickness of the wall. So the size of the domain can be easily much larger than the horizon at the baryogenesis epoch. Soon after it the domain walls disappeared completely and their evolution became not essential. Here is a substantial difference between the usually considered case of the non-destructible domain walls and our model [4] with disappearing domain walls.

The fact that the wall thickness was much larger than the horizon also means that it is much larger than the diffusion length of the nucleons or quarks and the effects of their annihilation are not essential prior the domain wall disappearance.

As is argued in Ref. [15], the size of the domain walls cannot be much smaller than ~ 10 Mpc, since otherwise the annihilation would be too strong. On the other hand, it cannot be noticeably larger than the same 10 Mpc to avoid too large angular fluctuations of CMB above the diffusion (Silk) damping scale. Based on this result the authors of [15] have concluded that the nearest antimatter domain should be at the distance of a few Gigaparsec away. This result is true in baryosymmetric universe. However, if the cosmological fraction of antibaryons is much smaller or larger than that of baryons the limit would be proportionally weakened.

Note in this connection that the spontaneous CP violation does not necessarily lead to equal cosmological densities of matter and antimatter but to an excess or deficit of one with respect to the other, see e.g. the lectures [5].

Another possible concern about the result of Ref. [15] is that it has been derived in the approximation of uniform CMB temperature. Different rates of the cosmological expansion near the wall and far from it could lead to different temperatures at the scales below the diffusion damping. This effect might accelerate or slow down the proton or antiprotons diffusion into “hostile” antimatter or matter environment and respectively either amplify or inhibit the annihilation. This problem is under investigation now.

As rather far fetched but exciting idea let us mention the possibility that the maybe-observed deficit of baryons in the 300 Mpc local universe [16] can be explained in the model of mildly inhomogeneous baryogenesis with thick domain walls.

Acknowledgements We acknowledge support of the RSF Grant No. 16-12-10037. SG is also supported under the grants RFBR No. 16-32-60115, 16-32-00241, 16-02-00342. In addition, SG is grateful to Dynasty Foundation for support.

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