Planck and electroweak scales emerging from conformal gravity

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Abstract We show that both the Planck and electroweak mass scales can be generated from conformal gravity via the Coleman–Weinberg mechanism of dimensional transmutation. At the first step, the Planck scale is generated via the Coleman–Weinberg mechanism in the sector of conformal gravity, which means that radiative corrections associated with gravitons induce spontaneous symmetry breakdown of a local conformal symmetry. At the second step, the vacuum expectation value of a scalar field is transmitted to the sector of the standard model via a potential involving the conformally invariant part and the contribution from the Coleman–Weinberg mechanism, thereby generating the electroweak scale. The huge hierarchy between the two scales can be explained in terms of a very tiny coupling constant between the scalar and the Higgs field in a consistent way.

1 Introduction

The observation [1, 2] of a relatively light Higgs particle with properties consistent with the standard model (SM) has concluded the quest to complete the particle spectrum of the SM, but the SM itself still leaves many of unsolved questions. For instance, the crucial question remains why the vacuum expectation value of the Higgs field and its mass are much smaller than any high scale of new physics or the Planck scale. In particular, the non-observation of supersymmetric particles around the TeV scale has cast some doubts on the idea of naturalness that the Higgs boson is accompanied with its supersymmetric partners which protect the lightness of Higgs boson from the huge radiative corrections.

With the discovery of the Higgs boson and the absence of any sign of new particles at the LHC, it seems that we are now in an era of the long overdue paradigm-shift from the view that the SM is an incomplete theory and should be replaced with a completely new theory like supersymmetry or GUT

to the view that the overall structure of the SM is true and we should look for its minimal extension which largely preserves the structure of the SM. To put differently, over the past years after the discovery of the Higgs boson at the LHC, we have learnt that the SM is equipped with a most economical and beautiful structure and we are watching mounting evidence that its minor extension is enough to survive to the highest energy such as the Planck energy scale where a theory of quantum gravity (QG) possibly unifies gravity with the other interactions in nature.

In recent years, the idea that instead of supersymmetry a global scale symmetry might play an important role in the naturalness problem has been put forward by Bardeen [3] and afterwards various interesting models, which we call the scale invariant SM, have been constructed.¹ In the scenario of “great desert” mentioned above, the SM or physics beyond the SM (BSM) at the electroweak scale is directly unified with the gravitational theory at the Planck scale. Then it is natural to conjecture that the gravitational theory might also have a global scale symmetry as in the scale invariant SM.

However, no-hair theorem [11] of quantum black holes suggests that global additive conservation laws such as baryon and lepton number conservation cannot hold in any consistent quantum gravity theory. Indeed, in string theory, we never get any additive conservation laws and at least in known string vacua, the additive global symmetries turn out to be either gauge symmetries or explicitly violated. By contrast, gauge symmetries such as $U(1)$ electric charge conservation law cause no trouble for black hole physics. Thus, in the gravitational theory, the global scale symmetry should be promoted to a local scale symmetry, so a scale invariant gravity would be replaced with conformal gravity. In this article, we would like to pursue such a possibility that a model

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¹ There is an extensive literature on this subject, a small selection being [4–10].

involving the SM and gravity is constrained by the local conformal symmetry.²

The rest of this paper is organised as follows: In the next section, we review conformal gravity. In Sect. 3, we consider the Coleman–Weinberg mechanism [15] in conformal gravity, and see that the Einstein–Hilbert action of general relativity is indeed induced via spontaneous symmetry breakdown of the local conformal symmetry by radiative corrections associated with massive gravitons. In Sect. 4, the electroweak scale is generated by a potential affected by Coleman–Weinberg mechanism. Finally, we conclude in Sect. 5.

2 Review of conformal gravity

In this section, let us review some salient features of conformal gravity (or Weyl gravity). The basic building block of conformal gravity is the conformal tensor (or Weyl tensor) $C_{\mu\nu\rho\sigma}$ which is defined as

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - (g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}) + \frac{1}{3}g_{\mu[\rho}g_{\sigma]\nu}R, \quad (1)$$

where $A_{[\mu}B_{\nu]} = \frac{1}{2}(A_{\mu}B_{\nu} - A_{\nu}B_{\mu})$, $\mu, \nu, \dots = 0, 1, 2, 3$, $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R are the Riemann tensor, the Ricci tensor and the scalar curvature, respectively.³ The conformal tensor shares the same symmetric properties of indices as those of the Riemann tensor $R_{\mu\nu\rho\sigma}$ and is in addition trace-free on all its indices. It also behaves in a very simple way under conformal transformation⁴; $C^{\mu}{}_{\nu\rho\sigma} \rightarrow C^{\mu}{}_{\nu\rho\sigma}$ under $g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu}$ (notice the position of indices on $C^{\mu}{}_{\nu\rho\sigma}$).

With these properties, it is easy to see that the following action of conformal gravity is invariant under the conformal transformation

$$\begin{aligned} S_W &= -\frac{1}{2\xi^2} \int d^4x \sqrt{-g} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \\ &= -\frac{1}{2\xi^2} \int d^4x \sqrt{-g} (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 2R_{\mu\nu} R^{\mu\nu} + \frac{1}{3}R^2) \\ &= -\frac{1}{\xi^2} \int d^4x \sqrt{-g} (R_{\mu\nu} R^{\mu\nu} - \frac{1}{3}R^2), \end{aligned} \quad (2)$$

where the coupling constant ξ is a dimensionless constant reflecting a conformal symmetry, and in the last equality we have used the fact that $\sqrt{-g}(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2)$ is a total derivative. The classical field equation for conformal gravity in vacuum can be derived from the action (2):

$$\begin{aligned} g^{\mu\nu} \nabla^2 R - 6 \nabla^2 R^{\mu\nu} + 2 \nabla^\mu \nabla^\nu R - g^{\mu\nu} R^2 \\ + 3g^{\mu\nu} R_{\rho\sigma}^2 + 4R R^{\mu\nu} - 12R^{\mu\rho} R^\nu{}_\rho = 0, \end{aligned} \quad (3)$$

where $\nabla^2 \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$. We can verify that any Einstein metric, which satisfies $R_{\mu\nu} = \Lambda g_{\mu\nu}$, is a solution to this equation. Thus all the classical solutions of Einstein gravity in vacuum are also solutions of conformal gravity.

In the weak field approximation we can expand

$$g_{\mu\nu} = \eta_{\mu\nu} + \xi h_{\mu\nu}, \quad (4)$$

where $\eta_{\mu\nu}$ is a flat Minkowski metric and $|h_{\mu\nu}| \ll 1$. Then, up to quadratic terms in $h_{\mu\nu}$ the action (2) can be cast to

$$S_W = -\frac{1}{4} \int d^4x h^{\mu\nu} P_{\mu\nu,\rho\sigma}^{(2)} \square^2 h^{\rho\sigma}, \quad (5)$$

where $P_{\mu\nu,\rho\sigma}^{(2)}$ is the projection operator for spin-2 modes⁵ and $\square \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu$. The linearized equations of motion therefore become

$$P_{\mu\nu,\rho\sigma}^{(2)} \square^2 h^{\rho\sigma} = 0. \quad (6)$$

It is known that the linearized equations of motion for conformal gravity possess six physical degrees of freedom which consist of massless spin-2 and spin-1 normal modes and a massless spin-2 ghost mode [18, 19].

The advantages of conformal gravity compared to Einstein gravity are that conformal gravity is renormalizable [20] and asymptotically free [21, 22] so that it is a ultra-violet (UV) complete quantum field theory. Moreover, the square of conformal tensor, which appears in the action of conformal gravity, is certainly positive definite in the Euclidean signature. For each topological class, the action is therefore bounded from below and the saddle-point approximation can be applied in order to understand non-perturbative phenomena such as instantons [23].

However, the price we have to pay is the presence of a ghost violating the unitarity. In this article, we will not address the issue of the ghost in detail but briefly comment on it in section 4.⁶ Another problem of conformal gravity is related to conformal anomaly [25, 26]. It has been thought for a long time that conformal gravity is not a consistent quantum field theory owing to the existence of conformal anomaly. However, it is nowadays an established fact that there exists a quantization procedure of preserving the local conformal symmetry at the quantum level though the theory in essence becomes non-renormalizable [27–36].⁷ In this sense, we do not have to pay special attention to the issue of conformal anomaly any longer. As a last problem, on the

² We have already considered such models with scale symmetries at the classical level [12–14].

³ We will follow the conventions and notation by Misner et al. [11].

⁴ In this article, conformal transformation (or Weyl transformation) is defined as the *local* scale transformation.

⁵ We have used the definitions of projection operators in [16, 17].

⁶ For recent progress on this topics, see a review article [24]. This article also includes a full set of the one-loop renormalization group equations for the conformal SM.

⁷ The renormalizability is not essential for the validity on this point.

phenomenological grounds, the macroscopic behavior of the gravitational field is governed by the Einstein–Hilbert action, and not the one of conformal gravity. Thus, if quantum conformal gravity has any physical relevance, its effective action must include the induced Einstein–Hilbert action. It is natural to conjecture that the Einstein–Hilbert term arises when radiative corrections break the local conformal symmetry. In the next section, we will explicitly show that this is indeed the case.

To close this section, let us comment on the conformally invariant scalar–tensor gravity whose action reads [37]

$$S_{ST} = \int d^4x \sqrt{-g} \left(\frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right), \quad (7)$$

where ϕ is a real scalar field.⁸ This action is invariant under conformal transformation

$$g_{\mu\nu} \rightarrow \Omega^2(x) g_{\mu\nu}, \quad \phi \rightarrow \Omega^{-1}(x) \phi. \quad (8)$$

Note that the scalar field ϕ is not a normal field but a ghost. However, this ghost can be eliminated by taking the gauge condition for conformal transformation⁹

$$\phi(x) = \sqrt{6} M_{pl}, \quad (9)$$

with M_{pl} being the reduced Planck mass, and this gauge condition reduces the action (7) to the conventional Einstein–Hilbert action

$$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g} R. \quad (10)$$

Accordingly, when a real scalar field ϕ coexists with gravity, the most general action which is conformally invariant, is of form

$$S_C = \int d^4x \sqrt{-g} \left(-\frac{1}{2\xi^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right). \quad (11)$$

3 Coleman–Weinberg mechanism in conformal gravity

The standard model (SM) of particle physics without the Higgs mass is known to have an extra symmetry, which is a *global* scale symmetry. This global scale symmetry can be enlarged to a *local* scale symmetry, which we call *conformal symmetry* in this article, by introducing the conformally invariant coupling with gravity. In this section, we therefore wish to work with this conformally invariant SM

with the more general, conformally invariant gravity action (11), and see how the Coleman–Weinberg mechanism generates the Einstein–Hilbert action through spontaneous symmetry breaking of conformal symmetry by radiative corrections associated with gravitational fluctuations.

The Lagrangian density we study is

$$\begin{aligned} \frac{1}{\sqrt{-g}} \mathcal{L}_C = & -\frac{1}{2\xi^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \\ & + \frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \\ & - \frac{1}{6} (H^\dagger H) R - g^{\mu\nu} (D_\mu H)^\dagger (D_\nu H) \\ & + V(\phi, H) + L_m, \end{aligned} \quad (12)$$

where H is the Higgs doublet, D_μ is a covariant derivative including the SM gauge fields, and L_m denotes the remaining Lagrangian density of the SM without the Higgs mass term. Moreover, the new potential $V(\phi, H)$ beyond the SM, which is conformally invariant, is added and has the form

$$V(\phi, H) = \frac{\lambda_\phi}{4!} \phi^4 + \lambda_{H\phi} (H^\dagger H) \phi^2 + \frac{\lambda_H}{2} (H^\dagger H)^2, \quad (13)$$

where all the coupling constants λ_ϕ , $\lambda_{H\phi}$ and λ_H are dimensionless and we will assume that $\lambda_{H\phi} < 0$ and $|\lambda_{H\phi}| \ll 1$. The physical meaning of these two assumptions will be mentioned in the next section. Since we assume $|\lambda_{H\phi}| \ll 1$, we can envision the process of symmetry breaking as two independent steps. At the first step of symmetry breaking, the Coleman–Weinberg mechanism would generate a vacuum expectation value (VEV) for the scalar field ϕ around the Planck scale. Then, at the second step, it is expected that this huge VEV is transmitted to the SM sector through the full form of the potential $V(\phi, H)$, thereby generating the Higgs mass term. In this section, we will focus on the first step of symmetry breaking.

To this aim, let us first expand the scalar field and the metric around a classical field ϕ_c and a flat Minkowski metric $\eta_{\mu\nu}$ like

$$\phi = \phi_c + \varphi, \quad g_{\mu\nu} = \eta_{\mu\nu} + \xi h_{\mu\nu}, \quad (14)$$

where we take ϕ_c to be a constant since we are interested in the effective potential depending on the constant ϕ_c . Then, as seen in Eq. (5), the Lagrangian density corresponding to the action S_W takes the form

$$\mathcal{L}_W = -\frac{1}{4} h^{\mu\nu} P_{\mu\nu, \rho\sigma}^{(2)} \square^2 h^{\rho\sigma}, \quad (15)$$

and in a similar manner, up to quadratic terms in $h_{\mu\nu}$, the Lagrangian density corresponding to the action S_{ST} in Eq. (7) reads

$$\mathcal{L}_{ST} = -\frac{1}{6} \xi \phi_c \varphi \left(\eta_{\mu\nu} - \frac{1}{\square} \partial_\mu \partial_\nu \right) \square h^{\mu\nu}$$

⁸ The generalization to a complex scalar field is straightforward as seen later.

⁹ Alternatively, this gauge fixing procedure can be interpreted as a field redefinition from the conformally invariant metric $g'_{\mu\nu} = \frac{1}{6M_{pl}^2} \phi^2 g_{\mu\nu}$ to the metric $g_{\mu\nu}$.

$$\begin{aligned}
& + \frac{1}{48} \xi^2 \phi_c^2 h^{\mu\nu} \left(P_{\mu\nu, \rho\sigma}^{(2)} - 2P_{\mu\nu, \rho\sigma}^{(0,s)} \right) \square h^{\rho\sigma} \\
& - \frac{1}{2} \varphi \square \varphi.
\end{aligned} \quad (16)$$

Hence, adding the two Lagrangian densities, we have

$$\begin{aligned}
\mathcal{L}_W + \mathcal{L}_{ST} = & \frac{1}{4} h^{\mu\nu} \left[\left(-\square + \frac{1}{12} \xi^2 \phi_c^2 \right) P_{\mu\nu, \rho\sigma}^{(2)} \right. \\
& - \frac{1}{6} \xi^2 \phi_c^2 P_{\mu\nu, \rho\sigma}^{(0,s)} \left. \right] \square h^{\rho\sigma} \\
& - \frac{1}{6} \xi \phi_c \varphi \left(\eta_{\mu\nu} - \frac{1}{\square} \partial_\mu \partial_\nu \right) \square h^{\mu\nu} \\
& - \frac{1}{2} \varphi \square \varphi.
\end{aligned} \quad (17)$$

Here let us set up the gauge-fixing conditions. For diffeomorphisms, we adopt the gauge condition

$$\chi_\mu \equiv \partial^\nu (h_{\mu\nu} - \frac{1}{4} \eta_{\mu\nu} h) = 0, \quad (18)$$

which is invariant under conformal transformation. The Lagrangian density for this gauge condition and its corresponding FP ghost term is given by

$$\mathcal{L}_{GF+FP} = -\frac{1}{2\alpha} \chi_\mu \chi^\mu + \bar{c}_\mu (\square \eta^{\mu\nu} + \frac{1}{2} \partial^\mu \partial^\nu) c_\nu, \quad (19)$$

where α is a gauge parameter. Let us recall that in case of the higher derivative gravity, one usually works with a more general gauge-fixing term $\mathcal{L}_{GF} = \chi_\mu Y^{\mu\nu} \chi_\nu$ where $Y^{\mu\nu}$ is the weight function involving derivatives and gauge parameters [22]. However, for the present purpose it turns out to be sufficient to choose $Y^{\mu\nu} = -\frac{1}{2\alpha} \eta^{\mu\nu}$. Next, we fix the gauge freedom corresponding to conformal transformation by the gauge condition

$$h \equiv \eta^{\mu\nu} h_{\mu\nu} = 0. \quad (20)$$

Since this gauge fixing condition and the conformal transformation contain no derivatives, one can neglect the corresponding FP ghost term in the one-loop approximation.

Consequently, we can obtain a quantum Lagrangian density up to quadratic terms in $h_{\mu\nu}$:

$$\begin{aligned}
& \mathcal{L}_W + \mathcal{L}_{ST} + \mathcal{L}_{GF+FP} \\
& = \frac{1}{4} h^{\mu\nu} \left(-\square + \frac{1}{12} \xi^2 \phi_c^2 \right) P_{\mu\nu, \rho\sigma}^{(2)} \square h^{\rho\sigma} - \frac{1}{2} \varphi' \square \varphi' \\
& - \frac{1}{2\alpha} (\partial^\nu h_{\mu\nu})^2 - \bar{c}_\mu (\square \eta^{\mu\nu} + \frac{1}{2} \partial^\mu \partial^\nu) c_\nu,
\end{aligned} \quad (21)$$

where we have simplified this expression by using the gauge condition (20) and introducing φ' defined as

$$\varphi' \equiv \varphi - \frac{1}{6} \xi \phi_c \square^{-1} \partial_\mu \partial_\nu h^{\mu\nu}, \quad (22)$$

which corresponds to the massless Nambu–Goldstone boson. Based on this quantum Lagrangian density (21), we can evaluate the one-loop effective action by integrating out quan-

tum fluctuations associated with $h_{\mu\nu}$. Then, up to a classical potential, the effective action $\Gamma[\phi_c]$ reads

$$\Gamma[\phi_c] = i \frac{5}{2} \log \det \left(-\square + \frac{1}{12} \xi^2 \phi_c^2 \right). \quad (23)$$

In this expression, the factor 5 comes from the fact that a massive spin-2 state possesses five physical degrees of freedom and we have ignored the part of the effective action which is independent of ϕ_c .

To calculate $\Gamma[\phi_c]$, we can proceed by following the same line of the arguments as in [38]. First of all, let us note that $\Gamma[\phi_c]$ can be rewritten as follows:

$$\begin{aligned}
\Gamma[\phi_c] & = i \frac{5}{2} \text{Tr} \log \left(-\square + \frac{1}{12} \xi^2 \phi_c^2 \right) \\
& = i \frac{5}{2} \sum_k \log \left(k^2 + \frac{1}{12} \xi^2 \phi_c^2 \right) \\
& = (VT) i \frac{5}{2} \int \frac{d^4 k}{(2\pi)^4} \log \left(k^2 + \frac{1}{12} \xi^2 \phi_c^2 \right) \\
& = (VT) \frac{5}{2} \frac{\Gamma(-\frac{d}{2})}{(4\pi)^{\frac{d}{2}}} \left(\frac{1}{12} \xi^2 \phi_c^2 \right)^{\frac{d}{2}},
\end{aligned} \quad (24)$$

where (VT) denotes the space-time volume and in the last equality we have used the Wick rotation and the dimensional regularization.

Next, recall that the conventional method to calculate the effective potential is to introduce the counter-terms to subtract UV-divergences and then impose the renormalization conditions to fix the finite part. However, if we want to visualize the modification of the lowest-order results which is generated by radiative corrections, we can apply some renormalization scheme which can be implemented more easily [38]. For instance, the minimal subtraction scheme is simply to remove the $\frac{1}{\epsilon}$ poles (where $\epsilon \equiv 4 - d$) in divergent expressions. In this article, we will adopt the intermediate method; we first subtract the $\frac{1}{\epsilon}$ poles and then fix the finite part by imposing the renormalization conditions. By subtracting the $\frac{1}{\epsilon}$ poles, the effective potential $V_{eff}(\phi_c)$ in the one-loop approximation becomes

$$\begin{aligned}
V_{eff}(\phi_c) & \equiv -\frac{1}{VT} \Gamma[\phi_c] \\
& = \frac{5}{9216\pi^2} \xi^4 \phi_c^4 \left(\log \frac{\phi_c^2}{\mu^2} + c \right),
\end{aligned} \quad (25)$$

where μ is the renormalization mass scale and c is a constant to be determined by the renormalization conditions:

$$V_{eff}|_{\phi_c=0} = \frac{d^2 V_{eff}}{d\phi_c^2} \Big|_{\phi_c=0} = \frac{d^4 V_{eff}}{d\phi_c^4} \Big|_{\phi_c=\mu} = 0. \quad (26)$$

As a result, we have the effective potential

$$V_{eff}(\phi_c) = \frac{5}{9216\pi^2} \xi^4 \phi_c^4 \left(\log \frac{\phi_c^2}{\mu^2} - \frac{25}{6} \right). \quad (27)$$

Finally, by adding the classical potential we can arrive at the effective potential in the one-loop approximation

$$V_{eff}(\phi_c) = \frac{\lambda_\phi}{4!} \phi_c^4 + \frac{5}{9216\pi^2} \xi^4 \phi_c^4 \left(\log \frac{\phi_c^2}{\mu^2} - \frac{25}{6} \right). \quad (28)$$

It is easy to see that this effective potential has a minimum at $\phi_c = \langle \phi \rangle$ away from the origin where the effective potential, $V_{eff}(\langle \phi \rangle)$, is negative. Since the renormalization mass μ is arbitrary, we will choose it to be the actual location of the minimum, $\mu = \langle \phi \rangle$ [15]:

$$V_{eff}(\phi_c) = \frac{\lambda_\phi}{4!} \phi_c^4 + \frac{5}{9216\pi^2} \xi^4 \phi_c^4 \left(\log \frac{\phi_c^2}{\langle \phi \rangle^2} - \frac{25}{6} \right). \quad (29)$$

Since $\phi_c = \langle \phi \rangle$ is defined to be the minimum of V_{eff} , we deduce

$$\begin{aligned} 0 &= \left. \frac{dV_{eff}}{d\phi_c} \right|_{\phi_c=\langle \phi \rangle} \\ &= \left(\frac{\lambda_\phi}{6} - \frac{55}{6912\pi^2} \xi^4 \right) \langle \phi \rangle^3, \end{aligned} \quad (30)$$

or equivalently,

$$\lambda_\phi = \frac{55}{1152\pi^2} \xi^4. \quad (31)$$

This relation is very similar to $\lambda = \frac{33}{8\pi^2} e^4$ in Eq. (4.8) in the case of the scalar QED in Ref. [15]. Thus, as in the paper, the perturbation theory holds for very small ξ .

The substitution of Eq. (31) into V_{eff} in (29) leads to

$$V_{eff}(\phi_c) = \frac{5}{9216\pi^2} \xi^4 \phi_c^4 \left(\log \frac{\phi_c^2}{\langle \phi \rangle^2} - \frac{1}{2} \right). \quad (32)$$

Thus, the effective potential is now parametrized in terms of ξ and $\langle \phi \rangle$ instead of ξ and λ_ϕ ; it is nothing but dimensional transmutation, i.e., a dimensionless coupling constant λ_ϕ is traded for a dimensional quantity $\langle \phi \rangle$ via spontaneous symmetry breakdown of the *local* conformal symmetry.

With $\phi = \langle \phi \rangle + \varphi$, the Lagrangian density of the conformally invariant scalar–tensor gravity plus the classical potential term produces the Einstein–Hilbert term and a massive scalar ghost like

$$\begin{aligned} \frac{1}{\sqrt{-g}} \mathcal{L}' &\equiv \frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda_\phi}{4!} \phi^4 \\ &= \frac{1}{12} \langle \phi \rangle^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\lambda_\phi}{4} \langle \phi \rangle^2 \varphi^2 + \dots \\ &= \frac{M_{pl}^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m_\varphi^2 \varphi^2 + \dots, \end{aligned} \quad (33)$$

where the ellipses stand for the higher-order interaction terms, and the reduced Planck mass and the mass of a scalar ghost are respectively defined as

$$M_{pl}^2 = \frac{1}{6} \langle \phi \rangle^2, \quad m_\varphi^2 = \frac{\lambda_\phi}{2} \langle \phi \rangle^2. \quad (34)$$

Equation (34) clearly shows that the minimum $\phi = \langle \phi \rangle$ of the effective potential is located near the Planck scale, and if we suppose $\lambda_\phi \sim \mathcal{O}(0.1)$ the mass of the scalar ghost is of order of the Planck mass scale. In other words, in our model spontaneous symmetry breaking of the conformal symmetry happens around the Planck scale via the Coleman–Weinberg mechanism in a natural manner.

As a final remark, in the above we have mentioned that conformal symmetry was spontaneously broken via the Coleman–Weinberg mechanism, but precisely speaking this is not spontaneous but explicit symmetry breaking of the conformal symmetry. This can be certified in the fact that the scalar φ becomes massive because of radiative corrections as in Eq. (33) although it is massless at the tree level as in Eq. (21). (In this respect, the difference between φ' and φ makes no sense.) Since there is only one mass scale, which is the Planck mass, at this first stage of symmetry breakdown, it is natural to the scalar field to have the Planck scale. In order to have the true spontaneous symmetry breaking of the conformal symmetry, we should adopt a manifestly conformal invariant regularization where the renormalization scale μ is promoted to a dynamical field $\mu = z\phi$ with z being a constant [27–36].

4 Emergence of the electroweak scale

Including the Higgs sector, after spontaneous symmetry breakdown of conformal symmetry, we have now an effective potential

$$\begin{aligned} V_{eff}(\phi, H) &= \frac{5}{9216\pi^2} \xi^4 \phi^4 \left(\log \frac{\phi^2}{\langle \phi \rangle^2} - \frac{1}{2} \right) \\ &\quad + \lambda_{H\phi} (H^\dagger H) \phi^2 + \frac{\lambda_H}{2} (H^\dagger H)^2. \end{aligned} \quad (35)$$

Inserting the minimum $\phi = \langle \phi \rangle$ to Eq. (35) and completing the square, the effective potential reduces to

$$\begin{aligned} V_{eff}(\langle \phi \rangle, H) &= \frac{\lambda_H}{2} \left(H^\dagger H + \frac{\lambda_{H\phi}}{\lambda_H} \langle \phi \rangle^2 \right)^2 \\ &\quad - \frac{1}{2} \left(\frac{\lambda_{H\phi}^2}{\lambda_H} + \frac{5}{9216\pi^2} \xi^4 \right) \langle \phi \rangle^4. \end{aligned} \quad (36)$$

Owing to $\lambda_H > 0$, this potential has a minimum at $H^\dagger H = -\frac{\lambda_{H\phi}}{\lambda_H} \langle \phi \rangle^2$. Taking the unitary gauge $H^T = \frac{1}{\sqrt{2}}(0, v + h)$, this fact implies that the square of the VEV v and the Higgs mass m_h is given by

$$v^2 = \frac{2|\lambda_{H\phi}|}{\lambda_H} \langle \phi \rangle^2, \quad m_h^2 = \lambda_H v^2. \quad (37)$$

Using Eqs. (34) and (37), the magnitude of the coupling constant $\lambda_{H\phi}$ reads

$$|\lambda_{H\phi}| = \frac{1}{12} \left(\frac{m_h}{M_{pl}} \right)^2 \sim \mathcal{O}(10^{-33}). \quad (38)$$

This relation certainly supports our previous assumption $|\lambda_{H\phi}| \ll 1$.

At this stage, it is worthwhile to recall and reflect what we have done so far. One appealing point in our formalism is that starting with a conformally invariant gravity, we have generated the Einstein–Hilbert action via the Coleman–Weinberg mechanism at the one-loop level. In the starting Lagrangian density (12), the scalar field ϕ is a ghost with opposite sign kinetic term. The difficulty associated with (12) is then that, with both normal and ghost fields, the energy of the theory is unbounded from below, and therefore the vacuum $\langle 0|\phi|0\rangle = 0$ is unstable. Vacuum decay will inevitably occur at the quantum level. However, radiative corrections coming from massive ghost-like gravitons with spin 2 tame this instability and as a result the Einstein–Hilbert term describing massless normal gravitons with spin 2 is generated.

However, in the process of spontaneous symmetry breakdown of conformal symmetry, we still get a massive scalar ghost φ . Since this ghost field is in essence a conformal mode of the graviton, by following the pragmatic attitude in [39,40], let us perform the Wick rotation over the conformal mode, $\varphi \rightarrow i\varphi$, or equivalently, $\phi \rightarrow i\phi$, thereby the ghost becoming a tachyon. (Here note that the minimum $\langle \phi \rangle$ is fixed in such a way that the square of the reduced Planck mass is positive as seen in Eq. (34).) At the same time, the classical potential is changed as

$$\begin{aligned} V(\phi, H) &= \frac{\lambda_\phi}{4!} \phi^4 + \lambda_{H\phi} (H^\dagger H) \phi^2 + \frac{\lambda_H}{2} (H^\dagger H)^2 \\ &\rightarrow \frac{\lambda_\phi}{4!} \phi^4 - \lambda_{H\phi} (H^\dagger H) \phi^2 + \frac{\lambda_H}{2} (H^\dagger H)^2, \end{aligned} \quad (39)$$

where only the sign in front of the second term is modified. Then, the positivity of this potential requires us to select $\lambda_{H\phi} < 0$. Thus, in this interpretation on the basis of the Wick rotation, we can explain in a natural way why we had to choose $\lambda_{H\phi} < 0$ at the beginning. Of course, it is not clear at present that we could perform the Wick rotation of the conformal factor of the graviton. Nevertheless, at least the present consideration clarifies the reason of why $\lambda_{H\phi} < 0$ is. Moreover, it is suggested that the scalar field ϕ , which appears in the classical potential, is not a normal field but might be a ghost or tachyon.

5 Discussion

In this article, we have discussed a locally scale invariant standard model which includes conformal gravity and the conformally invariant scalar–tensor gravity in addition to the

SM but the Higgs mass term. One of the interesting aspects in the model at hand is that the whole system is constrained by gauge symmetries and in fact it is invariant under not a global but a local conformal symmetry. The enlargement of symmetry from the global scale symmetry to the local conformal one is expected from the consideration of the no-hair theorem of quantum black holes.

The main purpose behind the present study is to understand the hierarchy problems, in particular, the gauge hierarchy problem and the cosmological constant problem based on conformal symmetry instead of supersymmetry. However, to do so, the conformal symmetry must be maintained at the quantum level in all orders of perturbation theory. Since our world is not conformally invariant at least in the low energy regime, the conformal symmetry must be broken spontaneously, thereby leading to a massless dilaton. Note that in this case the advantage of the conformal invariance is preserved at the quantum level and the quantum conformal symmetry could play an important role in solving the hierarchy problems. As a future work, we wish to construct such a model where the conformal symmetry is spontaneously broken by using the manifestly conformal invariant regularization.

Another appealing point of our formalism is that starting with conformal gravity Einstein's general relativity is induced through radiative corrections associated with massive ghost-like gravitons. Even if we have performed this derivation in a flat Minkowski background, it would be possible to perform a similar calculation in a general curved background.

Of course, there is a big problem to be solved in future; the issue of ghosts.¹⁰ If the Wick rotation for the conformal factor were allowed, the scalar ghost would be transformed to a benign tachyon, which triggers spontaneous symmetry breakdown at the electroweak scale. But there still remain ghosts, i.e., massive ghost-like gravitons with spin 2. In the one-loop level, the ghosts supply us the non-trivial VEV for the scalar field ϕ through quantum effects. It is of interest to investigate whether this situation holds in the higher-order levels as well.

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¹⁰ For a different view, see Refs. [41,42].

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