The effect of magnetic field on jet quenching parameter

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Abstract Using AdS/CFT correspondence, we investigate the effect of a constant magnetic field on the jet quenching parameter in strongly-coupled $\mathcal{N} = 4$ SYM plasma. We analyze the jet moving parallel and transverse to the magnetic field, respectively. For both cases, it is found that the jet quenching parameter is generally enhanced in the presence of a magnetic field, consistently with earlier findings.

1 Introduction

It is believed that the experiments at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) have produced a new state of matter so-called quark gluon plasma (QGP) [1–3]. One of the interesting properties of QGP is jet quenching: when a high energy parton propagates through the medium, it radiates gluons and consequently lose energy. Usually, this phenomenon can be characterized by the jet quenching parameter $\hat{q}$, which is defined as the average transverse momentum square transferred from the traversing parton, per unit mean free path. In the framework of weakly coupled theories, this parameter has been studied in many papers, see e.g. [4–8]. However, lots of evidences indicate that the QGP behaves as a strongly coupled fluid [3]. Therefore, it would be interesting to study jet quenching parameter with non-perturbative techniques. Such techniques are now available via the AdS/CFT correspondence.

AdS/CFT correspondence or more generally, the gauge/string duality [9–11], has yielded many important insights into the dynamics of strongly coupled gauge theories, see e.g. [12–25]. In this approach, Liu et al. have carried out the jet quenching parameter $\hat{q}_0$ for $\mathcal{N} = 4$ SYM plasma in their seminal work [26]. Therein, this parameter is obtained from the minimal surface of a world-sheet which ends on an orthogonal Wilson loop lying along two light-like lines and the results

show that the magnitude of $\hat{q}_0$ turns out to be closer to the value extracted from RHIC data [27, 28] than pQCD result for the typical value of the 't Hooft coupling, $\lambda \simeq 6\pi$, of QCD. Motivated by [26], there are many attempts to address the jet quenching parameter in this direction. For example, the effect of chemical potential on $\hat{q}$ is studied in [29–31]. The effect of electromagnetic field on $\hat{q}$ have been analyzed in [32, 33]. The anisotropy effects on $\hat{q}$ are investigated in [34]. Some corrections to $\hat{q}$ have been studied in [35–37]. Also, this parameter has been discussed in some AdS/QCD models [38, 39]. Other related results can be found, for example, in [40–45].

Moreover, it was argued that the early stage of noncentral ultrarelativistic heavy ion collisions may produce extremely large magnetic fields of the order of $eB \sim 15m_\pi^2$ at top LHC energies, and such strong magnetic field may be relevant at the time the QGP is formed during the spacetime evolution of the fastly expanding fireball produced in heavy ion collisions [46–50]. Given this, there are some works that regarding the effects of magnetic fields on some topological [51] and dynamical [52–55] properties of QGP. On the other hand, AdS/CFT can be as insightful in this issue and many quantities have already been studied. Such as entropy density [56], shear viscosity to entropy density ratio [57], conductivity [58], heavy quark potential [59] and energy loss [60–62]. Recently, the jet quenching parameter in a strongly coupled $\mathcal{N} = 4$ SYM plasma with a strong magnetic field was analyzed in [45], but the metric therein is valid only near the horizon, so their discussions are confined to infrared (IR) regime. In this work, we extend it to the case of all regimes by considering a general magnetic field. Specially, we would like to see how an arbitrary constant magnetic field affects $\hat{q}$. This is the motivation of the present work.

The organization of the paper is as follows. In the next section, we briefly review the asymptotic AdS_5 holographic Einstein–Maxwell model and introduce the background metric in the presence of a magnetic field given in [56]. In Sect. 3, we investigate the jet quenching parameter for the jet moving

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parallel and transverse to the magnetic field, in turn. The last part is devoted to discussion and conclusion.

2 Setup

The holographic model is Einstein gravity coupled with a Maxwell field, corresponding to strongly coupled $\mathcal{N} = 4$ SYM subjected to a constant and homogenous magnetic field. The bulk action is

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R + \frac{12}{L^2} - F^2 \right) + S_{body}, \quad (1)$$

where G_5 is the five-dimensional gravitational constant, L denotes the radius of the asymptotic AdS_5 spacetime. F stands for the Maxwell field strength 2-form. Moreover, the term S_{body} contains the Chern–Simons terms, Gibbons–Hawking terms and other contributions necessary for a well posed variational principle, but S_{body} does not affect the solutions considered here [56].

The equations of motion for (1) are given by the Einstein equations

$$R_{\mu\nu} + \frac{4}{L^2} g_{\mu\nu} + \frac{1}{3} F_{\rho\sigma} F^{\rho\sigma} g_{\mu\nu} - 2F_{\mu\rho} F_{\nu}^{\rho} = 0, \quad (2)$$

and the Maxwell's field equations

$$\nabla_{\mu} F^{\mu\nu} = 0. \quad (3)$$

Following [56], the ansatz for the finite temperature is

$$ds^2 = -H(r)dt^2 + e^{2P(r)}(dx_1^2 + dx_2^2) + e^{2V(r)}dx_3^2 + \frac{dr^2}{H(r)}, \quad (4)$$

with

$$F = B dx_1 \wedge dx_2, \quad (5)$$

where, for simplicity, we have set L as unity. For metric (4), the black hole horizon is located at $r = r_h$ with $H(r_h) = 0$ and the boundary is located at $r = \infty$. The constant B stands for the bulk magnetic field, pointing in the x_3 direction. In addition, the coefficients $H(r)$, $P(r)$, and $V(r)$ can be calculated from the equations of motion. For the sake of notation simplicity, henceforth we write $H(r)$, $P(r)$, $V(r)$ as H , P , V .

By virtue of (4), the set of linearly independent components of the Einstein equations read

$$H(P'' - V'') + (H' + H(2P' + V'))(P' - V') = -2B^2 e^{-4P}, \quad (6)$$

$$2P'' + V'' + 2(P')^2 + (V')^2 = 0, \quad (7)$$

$$\frac{1}{2}H'' + \frac{1}{2}H'(2P' + V') = 4 + \frac{2}{3}B^2 e^{-4P}, \quad (8)$$

$$2H'P' + H'V' + 2H(P')^2 + 4HP'V' = 12 - 2B^2 e^{-4P}, \quad (9)$$

where the derivations are with respect to r . For these coupled equations, it is difficult to obtain an analytic solution. But deep in the IR, an exact solution, which represents the product of a BTZ black hole times a two dimensional torus T^2 , can be found as

$$ds^2 = -\frac{r^2 f(r)}{\mathcal{R}^2} dt^2 + \mathcal{R}^2 \mathcal{B} (dx_1^2 + dx_2^2) + \frac{r^2}{\mathcal{R}^2} dx_3^2 + \frac{\mathcal{R}^2}{r^2 f(r)} dr^2, \quad (10)$$

with

$$f(r) = 1 - \frac{r_h^2}{r^2}, \quad (11)$$

where $\mathcal{B} = \sqrt{3}B$ is the physical magnetic field at the boundary. $\mathcal{R} = \frac{L}{\sqrt{3}} = \frac{1}{\sqrt{3}}$ refers to the BTZ black hole radius. In [45], the authors have applied (10) to discuss the effect of a strong magnetic field on the jet quenching parameter. But it should be noticed that this metric is only valid near the horizon, i.e., in the regime $r \ll \sqrt{\mathcal{B}}\mathcal{R}^2$ where the scale is much smaller than the magnetic field.

In this paper, we would like to use a solution that interpolates between (10) in the IR and AdS_5 in the UV. In the boundary theory, this refers to an RG flow between a $D = 3 + 1$ CFT at small r and a $D = 1 + 1$ CFT at large r . In the following section, we will study the behavior of jet quenching parameter for the background metric (4) by using numerical procedure.

3 Jet quenching parameter

In the field theory, the jet quenching parameter can be obtained from a Wilson line in the adjoint representation along a light cone direction [4]. While in the gravity dual description, it can be calculated from the minimal surface of a world-sheet which ends on an orthogonal Wilson loop lying along two light-like lines. Under the dipole approximation, which is valid for small transverse separation L_k and $L_k T \ll 1$, this parameter can be extracted from the following expression [26]

$$\langle W^A[C] \rangle \approx \exp \left[-\frac{1}{4\sqrt{2}} \hat{q} L_{-} L_k^2 \right], \quad (12)$$

where $\mathcal{C}$ denotes the null-like rectangular contour of size $L_k \times L_-$, with length L_- runs along the light-cone and the limit $L_- \rightarrow \infty$ is taken in the end. Using the relations $\langle W^A[C] \rangle \approx \langle W^F[C] \rangle^2$ and $\langle W^F[C] \rangle \approx \exp[-S_I]$, one has

$$\hat{q} = 8\sqrt{2} \frac{S_I}{L_- L_k^2}, \quad (13)$$

where $S_I = S - S_0$ with S the total energy of the heavy quark pair and S_0 the self-energy of the two single quarks.

Generally, to study the magnetic effect, one should consider different orientations of the jet velocity with respect to the direction of magnetic field, including parallel ($\theta = 0$), transverse ($\theta = \pi/2$) or arbitrary direction (θ). Here we analyze two extreme cases: parallel and transverse. For these cases, there are three different choices for the transverse momentum broadening [34]. The first one, $\hat{q}_{\parallel(\perp)}$, is for the jet moving along x_3 direction (magnetic field direction) while the momentum broadening happens along one transverse direction (x_1 or x_2 direction). The second one, $\hat{q}_{\perp(\parallel)}$, is for the jet moving along one transverse direction and the momentum broadening occurring along x_3 direction. The last one, $\hat{q}_{\perp(\perp)}$, is for the jet moving along one transverse direction while the momentum broadening is along the other transverse direction. Next we study $\hat{q}_{\parallel(\perp)}$, $\hat{q}_{\perp(\parallel)}$ and $\hat{q}_{\perp(\perp)}$ one by one.

3.1 Parallel to the magnetic field ($\theta = 0$)

In this subsection we analyze $\hat{q}_{\parallel(\perp)}$ by considering the jet moving along x_3 direction and the momentum broadening occurring along x_1 direction. We use the light-cone coordinates

$$dt = \frac{dx^+ + dx^-}{\sqrt{2}}, \quad dx_3 = \frac{dx^+ - dx^-}{\sqrt{2}}, \quad (14)$$

to rewrite metric (4) as

$$ds^2 = \frac{1}{2}(-H + e^{2V})((dx^+)^2 + (dx^-)^2) + e^{2P}(dx_1^2 + dx_2^2) - (H + e^{2V})dx^+dx^- + \frac{dr^2}{H}. \quad (15)$$

In this situation, the ansatz for the string configuration is

$$x^- = \tau, \quad x_1 = \sigma, \quad x^+ = \text{constant}, \\ x_2 = \text{constant}, \quad r = r(\sigma). \quad (16)$$

Under this assumption, metric (15) becomes

$$ds^2 = \frac{1}{2}(-H + e^{2V})d\tau^2 + \left(e^{2P} + \frac{\dot{r}^2}{H}\right)d\sigma^2, \quad (17)$$

with $\dot{r} = \frac{dr}{d\sigma}$.

The string is described by the Nambu–Goto action

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det g_{\alpha\beta}}, \quad (18)$$

with

$$g_{\alpha\beta} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}, \quad (19)$$

where X^μ and $G_{\mu\nu}$ represent the target space coordinates and the metric, respectively.

In terms of (17), the Nambu–Goto action reduces to

$$S = \frac{\sqrt{2}L_-}{2\pi\alpha'} \int_0^{\frac{L_k}{2}} d\sigma \sqrt{e^{2P}(e^{2V} - H) + \left(\frac{e^{2V}}{H} - 1\right)\dot{r}^2}, \quad (20)$$

where the boundary condition is $r(\pm \frac{L_k}{2}) = \infty$.

Since action (20) doesn't depend explicitly on σ , the corresponding Hamiltonian is a constant

$$\mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{r}} \dot{r} = \frac{e^{2P}(e^{2V} - H)}{\sqrt{e^{2P}(e^{2V} - H) + \left(\frac{e^{2V}}{H} - 1\right)\dot{r}^2}} = C, \quad (21)$$

results in

$$\dot{r}^2 = \frac{He^{2P}}{C^2} [e^{2P}(e^{2V} - H) - C^2]. \quad (22)$$

Note that Eq. (22) involves determining the zeros and the region of positivity of the right-hand side. It was argued [26] that the turning point satisfies $H = 0$ implying $\dot{r} = 0$ happens at the horizon $r = r_h$.

For the low energy limit ($C \rightarrow 0$), one integrates (22) to leading order of C^2 and gets

$$L_k = 2 \int_{r_h}^{\infty} dr \frac{d\sigma}{dr} = 2C \int_{r_h}^{\infty} dr \sqrt{\frac{1}{He^{4P}(e^{2V} - H)}}. \quad (23)$$

On the other hand, substituting (22) into (20), one has

$$S = \frac{\sqrt{2}L_-}{2\pi\alpha'} \int_{r_h}^{\infty} dr \frac{d\sigma}{dr} \sqrt{\frac{e^{4P}(e^{2V} - H)^2}{C^2}} \\ = \frac{\sqrt{2}L_-}{2\pi\alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{1}{\frac{H}{e^{2V} - H} \left(1 - \frac{C^2}{e^{2P}(e^{2V} - H)}\right)}}. \quad (24)$$

Similarly, one can expand (24) to leading order of C^2 as

$$S = \frac{\sqrt{2}L_-}{2\pi\alpha'} \int_{r_t}^{\infty} dr \left[1 + \frac{C^2}{2e^{2P}(e^{2V} - H)} \right] \sqrt{\frac{e^{2V}}{H} - 1}, \quad (25)$$

however, this action is divergent. To eliminate the divergence it needs to be subtracted by the self energy of the two single quarks, which is

$$\begin{aligned} S_0 &= \frac{2L_-}{2\pi\alpha'} \int_{r_h}^{\infty} dr \sqrt{g_{--}g_{rr}} \\ &= \frac{\sqrt{2}L_-}{2\pi\alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{e^{2V}}{H} - 1}. \end{aligned} \quad (26)$$

The normalized action is then given by

$$S_I = S - S_0 = \frac{\sqrt{2}L_-C^2}{4\pi\alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{1}{He^{4P}(e^{2V} - H)}}. \quad (27)$$

Therefore, by using (13) one ends up with

$$\hat{q}_{\parallel(\perp)} = \frac{I(q)^{-1}}{\pi\alpha'}, \quad (28)$$

where

$$I(q) = \int_{r_h}^{\infty} dr \sqrt{\frac{1}{He^{4P}(e^{2V} - H)}}. \quad (29)$$

We have checked that by taking $H = \frac{r^2}{L_-^2}(1 - \frac{r_h^4}{r^4})$, $e^{2P} = e^{2V} = \frac{r^2}{L_-^2}$ in (28), the jet quenching parameter for $N = 4$ SYM case [26] can be recovered, that is

$$\hat{q}_0 = \frac{\pi^{\frac{3}{2}}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3, \quad (30)$$

where we have used the relations $T = r_h/(\pi L_-^2)$ and $\sqrt{\lambda} = L_-^2/\alpha'$. Also, if one sets $H = \frac{r^2}{\mathcal{R}^2}(1 - \frac{r_h^2}{r^2})$, $e^{2P} = \mathcal{R}^2\mathcal{B}$, $e^{2V} = \frac{r^2}{\mathcal{R}^2}$ in (28), the jet quenching parameter of $N = 4$ SYM plasma in strong magnetic field [45] can be reproduced, that is

$$\hat{q}_{\parallel(\perp)} = \frac{4\sqrt{\lambda}BT}{3\log(\mathcal{B}/T^2)}, \quad (31)$$

where we have used $\frac{\mathcal{R}^2}{\alpha'} = \frac{\sqrt{3}}{3}$.

Now our goal is to study the jet quenching parameter from (28). As stated earlier, we can not solve H , P , V analytically and we have to resort to numerical methods. Before numerical calculations, we derive some useful equations at hand. By eliminating the B^2e^{-4P} term from (6)–(9), we have

$$\begin{aligned} 3H'' + 5(V' + 2P')H' \\ + 4(P'^2 + 2P'V')H - 48 = 0, \end{aligned} \quad (32)$$

$$3HP'' + 2HP'^2 - H'P' - 5HP'V' + 12 - 2H'V' = 0, \quad (33)$$

$$\begin{aligned} 3HV'' + 3HV'^2 + 4H'V' + 10HP'V' \\ + 2HP'^2 + 2H'P' - 24 = 0. \end{aligned} \quad (34)$$

For the sake of simplicity, we use rescale coordinates. First, we rescale $t \rightarrow \bar{t}$, $r \rightarrow \bar{r}$ and fix the horizon at $\bar{r}_h = 1$, so that

$$H(1) = 0, \quad H'(1) = 1, \quad (35)$$

note that by using the rescale coordinates the physical quantity in this model depend on the dimensionless ratio $T/\sqrt{B}$ [56].

Also, the Hawking temperature is fixed as

$$T = \frac{\sqrt{-g'_{\bar{t}\bar{t}}g'_{\bar{r}\bar{r}}}}{4\pi} \Big|_{\bar{r}=1} = \frac{1}{4\pi}. \quad (36)$$

Next we rescale x_1, x_2, x_3 coordinates such that

$$P(1) = V(1) = 0, \quad P'(1) = 4 - \frac{b^2}{3}, \quad V'(1) = 4 + \frac{b^2}{6}, \quad (37)$$

where b denotes the value of the magnetic field in the rescaled coordinates. Note that if $P' < 0$, the geometry will not be asymptotically AdS_5 , so the second equation in (37) implies $0 \leq b < 2\sqrt{3}$.

On the other hand, the geometry has the asymptotic behavior as $\bar{r} \rightarrow \infty$,

$$H(\bar{r}) \rightarrow \bar{r}^2, \quad e^{2P(\bar{r})} \rightarrow m(b)\bar{r}^2, \quad e^{2V(\bar{r})} \rightarrow n(b)\bar{r}^2, \quad (38)$$

where $m(b)$ and $n(b)$ are rescaling parameters which can be obtained numerically. In addition, the physical magnetic field B_0 is related to $m(b)$ as

$$B_0 = \sqrt{3}\frac{b}{m(b)}, \quad (39)$$

note that the interval of b can be analyzed from (39) as well. One can numerically check that $m(b)$ is a decreasing function of b and $m(b \rightarrow 2\sqrt{3}) \rightarrow 0$. Thus, one can cover in practice all values of B_0 for $0 \leq b < 2\sqrt{3}$. Here we present the numerical solutions of $m(b)$ and $n(b)$ versus b in the left panel of Fig. 1.

Finally, to have an asymptotic AdS_5 in the UV, one should rescale back to the original coordinate system by taking $(\bar{x}_1, \bar{x}_2, \bar{x}_3) \rightarrow (x_1/\sqrt{m(b)}, x_2/\sqrt{m(b)}, x_3/\sqrt{n(b)})$. The metric (4) then reads

Fig. 1 Left: The rescaling parameters $n(b)$ (dash curve) and $m(b)$ (solid curve) versus b . Right: Numerical solutions for the functions $\ln H(r)$ (solid curve), $V(r)$ (dash curve) and $P(r)$ (dot curve) versus r for $b = 2.7$

$$ds^2 = -H(\bar{r})dt^2 + \frac{e^{2P(\bar{r})}}{m(b)}(dx_1^2 + dx_2^2) + \frac{e^{2V(\bar{r})}}{n(b)}dx_3^2 + \frac{dr^2}{H(\bar{r})}, \quad (40)$$

after this, one can solve the coupled equations (32)–(34) with the boundary conditions (35) and (37).

For convenience, we drop from now on the bars in the rescaled coordinates. In the right panel of Fig. 1, we plot $\ln H(r)$, $P(r)$, $V(r)$ versus r for $b = 2.7$, we have found that it is consistent with the Fig. 3 in [59].

To analyze the effect of the magnetic field on the jet quenching parameter for the parallel case, we plot the curve of $\hat{q}_{\parallel(\perp)}/\hat{q}_0$ in terms of B_0/T^2 in Fig. 2. One can see that due to the presence of the magnetic field, the jet quenching parameter is larger than that of $N = 4$ SYM plasma. Moreover, the jet quenching parameter is almost linearly dependent on B_0/T^2 . In fact, one can find a nearly linear behavior from Eq. (31) although it is only valid near the horizon.

Moreover, we would like to compare the results with experimental data. Taking $\alpha' = 0.5$, which is reasonable for temperatures not far above the QCD phase transition [26] and $\lambda = 6\pi$ as well as $T = 300$ MeV, one gets $\hat{q}_{\parallel(\perp)} \simeq 4.5, 6.88, 33.09$ GeV²/fm for $B_0/T^2 = 0, 50, 500$, respectively. These results are in some agreement with the extracted values from RHIC data ($5 \rightarrow 25$ GeV²/fm) [63].

3.2 Transverse to the magnetic field ($\theta = \pi/2$)

In this subsection we consider the jet moving perpendicularly to the magnetic field. First we study $\hat{q}_{\perp(\parallel)}$ by considering the jet moving along x_1 direction and the momentum broadening occurring along x_3 direction. Using light-cone coordinates

Fig. 2 The plot of $\hat{q}_{\parallel(\perp)}/\hat{q}_0$ versus B_0/T^2

$$dt = \frac{dx^+ + dx^-}{\sqrt{2}}, \quad dx_1 = \frac{dx^+ - dx^-}{\sqrt{2}}, \quad (41)$$

one rewrites metric (4) as

$$ds^2 = \frac{1}{2}(-H + e^{2P})((dx^+)^2 + (dx^-)^2) + e^{2P}dx_2^2 + e^{2V}dx_3^2 - (H + e^{2P})dx^+dx^- + \frac{dr^2}{H}. \quad (42)$$

The ansatz for the string configuration is

$$x^- = \tau, \quad x_3 = \sigma, \quad x^+ = \text{constant}, \\ x_2 = \text{constant}, \quad r = r(\sigma), \quad (43)$$

then metric (42) reads

$$ds^2 = \frac{1}{2}(-H + e^{2P})d\tau^2 + \left(e^{2V} + \frac{\dot{r}^2}{H}\right)d\sigma^2, \quad (44)$$

with $\dot{r} = \frac{dr}{d\sigma}$.

The next analysis is similar to the previous subsection, we here show the final results. One obtains

$$\hat{q}_{\perp(\parallel)} = \frac{J(q)^{-1}}{\pi\alpha'}, \quad (45)$$

where

$$J(q) = \int_{r_h}^{\infty} dr \sqrt{\frac{1}{He^{4V}(e^{2P} - H)}}. \quad (46)$$

By setting $H = \frac{r^2}{\mathcal{R}^2}(1 - \frac{r_h^2}{r^2})$, $e^{2P} = \mathcal{R}^2\mathcal{B}$, $e^{2V} = \frac{r^2}{\mathcal{R}^2}$ in (45), one gets

$$\hat{q}_{\perp(\parallel)} = \frac{2\pi}{3}\sqrt{\lambda}\sqrt{\mathcal{B}T^2}, \quad (47)$$

which is exactly Eq. (3.59) in [45].

Also, applying the same procedure, one finds

$$\hat{q}_{\perp(\perp)} = \frac{K(q)^{-1}}{\pi\alpha'}, \quad (48)$$

where

$$K(q) = \int_{r_h}^{\infty} dr \sqrt{\frac{1}{He^{4P}(e^{2P} - H)}}, \quad (49)$$

and the leading-log result of $\hat{q}_{\perp(\perp)}$ is

$$\hat{q}_{\perp(\perp)} = \frac{\sqrt{\lambda}\mathcal{B}^{3/2}}{3\pi \log(\mathcal{B}/T^2)}. \quad (50)$$

To study the influence of the magnetic field on the jet quenching parameter for the transverse case, we also plot $\hat{q}_{\perp(\parallel)}/\hat{q}_0$ and $\hat{q}_{\perp(\perp)}/\hat{q}_0$ versus B_0/T^2 in Fig. 3. For the two cases, one can see that increasing the magnetic field leads to increasing the value of the jet quenching parameter.

We now summarize Sect. 3 as follows:

1. For the parallel case and transverse case, the jet quenching parameter is generally enhanced in the presence of a magnetic field.
2. By comparing Figs. 2 and 3, one gets

$$\hat{q}_{\perp(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{\parallel(\parallel)} > \hat{q}_0, \quad (51)$$

Fig. 3 The plots of $\hat{q}_{\perp(\parallel)}/\hat{q}_0$ and $\hat{q}_{\perp(\perp)}/\hat{q}_0$ versus B_0/T^2

which shows that the values of the jet quenching parameter depends strongly on the direction of the moving quark as well as the direction which the momentum broadening occurs. Specially, the jet quenching is stronger enhanced for a quark moving in the transverse plane while the momentum broadening occurs on the same plane. Also, one can find this behavior from Eqs. (31), (47) and (50).

4 Conclusion

Recently, the jet quenching parameter in a strongly coupled $\mathcal{N} = 4$ SYM plasma with a strong magnetic field has been studied in [45], but the metric therein is valid only near the horizon, so their discussions are restricted to IR regime. In this paper, we extended it to the case of all regimes by considering a general magnetic field. We considered the quark anti-quark pair moving parallel and transverse to the magnetic field, and analyzed the momentum broadening occurs on different directions. All cases show the same result: the jet quenching parameter is generally enhanced in the presence of magnetic field compared to the value of $\mathcal{N} = 4$ SYM plasma, which supports the findings of [45]. Moreover, it is shown that the jet quenching parameter is stronger enhanced for the jet moving in the transverse plane while the momentum broadening occurs on the same plane. Also, we compared the results with experimental data and find that with typical values of parameters the values of the jet quenching parameter are in some agreement with the extracted values from RHIC data.

Finally, it is relevant to mention that the drag force has been recently studied for the same background [62] as well and the results show that the magnetic field enhances the

drag force, implying the effects of the magnetic field on the jet quenching parameter and the drag force are consistent.

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