

# Surrounded Vaidya black holes: apparent horizon properties

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**Abstract** We study the thermodynamical features and dynamical evolutions of various apparent horizons associated with the Vaidya evaporating black hole surrounded by the cosmological fields of dust, radiation, quintessence, cosmological constant-like and phantom. In this regard, we address in detail how do these surrounding fields contribute to the characteristic features of a surrounded dynamical black hole in comparison to a dynamical black hole in an empty background.

## 1 Introduction

The Vaidya solution [1–3] is one of the non-static solutions of Einstein field equations. This solution can be regarded as a dynamical generalization of the static Schwarzschild black hole solution. This solution is characterized by a dynamical mass function, i.e  $m = m(u)$  where  $u$  is the advanced time coordinate, and an outgoing null radiation flow. One can interpret this null radiation as high frequency electromagnetic or gravitational waves, massless scalar particles or neutrinos. Based on these properties, this solution has been used for studying the spherical symmetric gravitational collapse and for exploring the validity of the cosmic censorship conjecture [4–7], as a model for studying evaporating black holes as well as Hawking radiation [8–14], among the other applications. This solution has been generalized to the Vaidya-de Sitter solution [15], Bonnor–Vaidya solution [16], Bonnor–Vaidya–de Sitter solution [17–21], radiating dyon solution [22], and the surrounded Vaidya and Bonnor–Vaidya solutions [23, 24], see also [25, 26] for the other similar generalizations.

The connections between black holes and thermodynamics were initially discovered by Hawking [27–30] and Bekenstein [31]. Black holes behave as thermodynamic objects

emitting radiation, the well-known Hawking radiation, from their event horizons which is described in the context of quantum field theory in curved spacetime. From thermodynamical point of view, there are (i) a characteristic temperature for a black hole which is proportional to the surface gravity associated with its event horizon (EH), and (ii) an entropy equal to one quarter of the area of the EH. Interestingly, the apparent horizon (AH) and the event horizon EH coincide for the static or stationary black holes. Then, one cannot distinguish exactly where is the actual emission surface for the Hawking radiation; AH or EH? For the case of Vaidya black hole, it is found in [32] that the Hawking radiation originates always in regions having geometry near to the geometry in a close neighborhood of AH. Then, EH appears as radiating surface only if it passes through such regions, similar to what happens for the case of static Schwarzschild black hole. Considering the collapse of a spherical shell leading to a black hole with the Vaidya solution [33], it is suggested that in order to maintain a local thermodynamics, the entropy of the black hole should be identified as  $\mathcal{A}_{AH}/4$  where  $\mathcal{A}_{AH}$  is the area of the black hole's AH. Also, the fact that thermodynamics of a dynamical black hole can be built successfully on the AH is verified in [34, 35].

In this paper, we aim to study the properties of the various horizons, mainly the AH, associated with the Vaidya evaporating black hole solution surrounded by the cosmological fields of dust, radiation, quintessence, cosmological constant-like and phantom. In this regard, we explore the surrounded Vaidya solutions introduced in [23] using the approach used in [36, 37]. The organization of the paper is as follows. In Sect. 2, we review briefly the surrounded Vaidya black hole solutions introduced in [23]. In Sect. 3, we discuss on the proprieties of these solutions in a general form. In Sect. 4, we study in detail the properties of these solutions for the surrounding fields of dust, radiation, quintessence, cosmological constant-like and phantom. In Sect. 5, we give our concluding remarks with introducing our two new works.

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## 2 The surrounded evaporating Vaidya black holes

The Vaidya black hole surrounded by generic fields in the advanced time coordinate  $u$  is obtained in [23] as

$$dS^2 = - \left( 1 - \frac{2M(u)}{r} - \frac{N_s(u)}{r^{3\omega_s+1}} \right) du^2 + 2dudr + r^2 d\Omega_2^2, \tag{1}$$

where  $M(u)$ ,  $N_s(u)$  and  $\omega_s$  are the time dependant decreasing mass of the evaporating Vaidya black hole, the normalization parameter and the equation of state parameter of the surrounding field, respectively. The corresponding total energy-momentum tensor  $T^\mu_\nu$  is given by [23,24,38,39]

$$T^\mu_\nu = \tau^\mu_\nu + \mathcal{T}^\mu_\nu, \tag{2}$$

where  $\tau^\mu_\nu$  is the energy-momentum tensor associated to the Vaidya null radiation as

$$\tau^\mu_\nu = \sigma k^\mu k_\nu, \tag{3}$$

such that  $\sigma = \sigma(u, r)$  is the density of the “outgoing radiation flow” and  $k^\mu$  is a null vector field directed radially outward, and  $\mathcal{T}^\mu_\nu$  is the energy-momentum tensor of the surrounding dynamical field as

$$\begin{aligned} \mathcal{T}^0_0 &= \mathcal{T}^1_1 = -\rho_s(u, r), \\ \mathcal{T}^2_2 &= \mathcal{T}^3_3 = \frac{1}{2} (1 + 3\omega_s) \rho_s(u, r), \end{aligned} \tag{4}$$

where the energy density  $\rho_s(u, r)$  of the surrounding field is given by

$$\rho_s(u, r) = - \frac{3\omega_s N_s(u)}{r^{3(\omega_s+1)}}. \tag{5}$$

The weak energy condition on the energy density (5) of the surrounding field requires

$$\omega_s N_s(u) \leq 0, \tag{6}$$

implying that for the surrounding fields with  $\omega_s \geq 0$  and  $\omega_s \leq 0$ , we need  $N_s(u) \leq 0$  and  $N_s(u) \geq 0$ , respectively. Regarding (1) and (2), the original Vaidya evaporating black hole solution can be recovered by turning off the background field, i.e setting  $\rho_s(u, r) = 0$ .

Following [36,41,42], we can rewrite the metric of our surrounded Vaidya black hole spacetime (1), using the null-vector decomposition, in the following form

$$g_{\mu\nu} = -n_\mu l_\nu - l_\mu n_\nu + h_{\mu\nu}, \tag{7}$$

where  $n_\mu$  and  $l_\mu$  are ingoing and outgoing null vectors, respectively, given by

$$\begin{aligned} n_\mu &= -\delta^\mu_\mu, \\ l_\mu &= -\frac{1}{2} \left( 1 - \frac{2M(u)}{r} - \frac{N_s(u)}{r^{3\omega_s+1}} \right) \delta^\mu_\mu + \delta^r_\mu, \end{aligned} \tag{8}$$

and

$$h_{\mu\nu} = r^2 \delta^\theta_\mu \delta^\theta_\nu + r^2 \sin^2 \theta \delta^\phi_\mu \delta^\phi_\nu, \tag{9}$$

is the two dimensional metric for  $u = constant$  and  $r = constant$  surfaces such that  $n_\mu$ ,  $l_\mu$  and  $h_{\mu\nu}$  satisfy the following normalization conditions

$$\begin{aligned} n_\mu n^\mu &= l_\mu l^\mu = 0, \\ l_\mu n^\mu &= -1, \\ l^\mu h_{\mu\nu} &= n^\mu h_{\mu\nu} = 0. \end{aligned} \tag{10}$$

As we will see in the following section, the null vector decomposition of the spacetime metric is more appropriate for studying its various geometric properties.

## 3 The location and properties of the horizons

It is a well known fact that the black holes in general have not single horizons to characterize completely their local and global structures, see [36,40] as instances. They possess three important loci: (i) the future EH which is a null three-surface describing the locus of outgoing future-directed null geodesic rays that never reach arbitrarily large distances from the black hole, (ii) the AH which is the outermost marginally trapped three-surface for the outgoing photons. Classically, the AH can be either null or spacelike surfaces, and consequently it can move causally or non-causally. Considering the black hole quantum radiance, the AH can also be a timelike surface, and (iii) the timelike limit surface (TLS) which is a horizon-like locus. For the black holes possessing an exact timelike Killing vector field  $\partial/\partial t$  in their exterior regions, TLS is the surface given by  $g(\partial_t, \partial_t) = g_{tt} = 0$ . For black holes with a small dimensionless accretion or luminosity  $L = -\frac{dM}{du} = -\dot{M}(u)$ , one can define the TLS (or quasi-static limit) as the locus where  $g(\partial_u, \partial_u) = g_{uu} = 0$ ,  $\partial/\partial u$  being the timelike vector field for an observer sitting at rest at the far distance from the black hole. Generally, the TLS can be a null, timelike, or a spacelike three-surface. We note that when  $L$  refers to the quantum radiance, i.e given by the Hawking relation  $L = L_H \propto \hbar/M^2$ , it is usually very small. In this case, it represents a black hole radiating an small amount of quanta in any reasonable time interval. However, the fluctuations in the radiation could be very large, at least comparable to  $L$  itself. Thus, we consider  $L$  as an average value such that the terms with  $\mathcal{O}(L^2)$  are negligible. We also mention that for the

spherical symmetric solutions of  $R_{\mu\nu} = 0$ , there is a degeneracy for the horizons which means that all these three horizons coincide at  $r = 2M$ , known as the Schwarzschild radius. In general, such a coincidence of various horizons happens for the stationary situations. However, in the case of small accretion or luminosity, i.e for  $R_{\mu\nu} = \mathcal{O}(L)$ , this degeneracy is partially lifted even for the spherical symmetric spacetimes. For the case of  $R_{\mu\nu} = \mathcal{O}(L)$ , we have  $AH = TLS$  but  $EH \neq AH$ . For the case of  $R_{\mu\nu} = 0$  and without spherical symmetry but preserving stationarity, as for a Kerr hole, we have  $EH = AH$  but  $EH \neq TLS$ . So, we see that these three horizons do not coincide generally and they are very sensitive to small perturbations. Moreover, by including the generic background fields in our solution (1), we will consider  $\dot{N}_s(u)$  here also as an average value and the terms with  $\mathcal{O}(\dot{N}_s^2)$  are negligible.

In general, the structure and dynamics of various horizons for dynamical spacetimes can be understood using the behavior of null geodesic congruences, governed by the Raychaudhuri equation [42]

$$\frac{d\Theta}{du} = \mathcal{K}\Theta - \frac{1}{2}\Theta^2 - \sigma^2 + \omega^2 - R_{\mu\nu}l^\mu l^\nu, \tag{11}$$

where  $\mathcal{K}$ ,  $\Theta$ ,  $\sigma^2 = \sigma_{\mu\nu}\sigma^{\mu\nu}$ ,  $\omega^2 = \omega_{\mu\nu}\omega^{\mu\nu}$  and  $R_{\mu\nu}$  are the surface gravity, expansion scalar, shear scalar, vorticity scalar, and Ricci tensor, respectively. The expansion scalar for the null rays parameterized by the parameter  $u$  is given by [36,43]

$$\Theta = \nabla_\mu l^\mu - \mathcal{K}, \tag{12}$$

where  $\nabla_\mu$  is the covariant derivative operator corresponding to the metric (1). The surface gravity is given by the non-affinely parameterized geodesic equation with the null tangent vector  $l^\mu$  as [40,43–45]

$$l^\mu \nabla_\mu l^\nu = \mathcal{K}l^\nu. \tag{13}$$

Using the normalization conditions (10) on the null vectors  $n^\mu$  and  $l^\mu$ , one can obtain the surface gravity in (13) in the following form

$$\mathcal{K} = -n^\nu l^\mu \nabla_\mu l_\nu. \tag{14}$$

Spherically symmetric non-rotational spacetimes, such as our metric (1), are vorticity and shear free, i.e  $\sigma_{\mu\nu} = 0 = \omega_{\mu\nu}$ . Then, for these spacetimes, the location, structure and dynamics of the various horizons depend on the surface gravity  $\mathcal{K}$ , the expansion scalar  $\Theta$  and the Ricci tensor  $R_{\mu\nu}$ . One can follow York [36] and obtain the location of the AH and EH for a dynamical radiating black hole up to  $\mathcal{O}(L)$  as

- AH is defined at the surface such that  $\Theta \simeq 0$ ,

- EH is the null surface such that  $\frac{d\Theta}{du} \simeq 0$ .

Thus, one realizes that the EH can be obtained using the Raychaudhuri equation (11), and the AH can be determined by the Eq. (12).

Using the Eqs. (8) and (14), one can obtain the general dynamical surface gravity associated to our metric (1) as

$$\mathcal{K} = \frac{1}{2} \left( \frac{2M(u)}{r^2} + \frac{(3\omega_s + 1)N_s(u)}{r^{3\omega_s+2}} \right). \tag{15}$$

Then, by using the Eqs. (8), (12), and (15), the expansion scalar can be obtained as

$$\Theta = \frac{1}{r} \left( 1 - \frac{2M(u)}{r} - \frac{N_s(u)}{r^{3\omega_s+1}} \right). \tag{16}$$

Regarding (1) and (16), we see that the solution to  $g(\partial_u, \partial_u) = g_{uu} = 0$  representing the TLS is the same as the solution to  $\Theta = 0$  representing the AH location except the trivial solution  $r = \infty$  for the latter. Then, the AH and the TLS coincide for our spacetime (1).

The Hawking temperature on the horizon (apparent or event) of black hole can be determined using its corresponding surface gravity as

$$\mathcal{T}_H = \frac{\mathcal{K}}{2\pi}. \tag{17}$$

Then, regarding (15), we see that temperature is also a dynamical quantity with respect to the time coordinate  $u$ . Also, the semiclassical Bekenstein-Hawking entropy at the horizon is given by [31]

$$\mathcal{S}_H = \frac{\mathcal{A}_H}{4}, \tag{18}$$

where  $\mathcal{A}_H$  is the area of the horizon given by

$$\mathcal{A}_H = \int_0^{2\pi} \int_0^\pi \sqrt{g_{\theta\theta}g_{\phi\phi}} d\theta d\phi = 4\pi r_H^2. \tag{19}$$

In the following of this section, we show that the condition  $\frac{d\Theta}{du} \simeq 0$  determining the location of the EH up to  $\mathcal{O}(L)$  is equivalent to the requirement that the acceleration of the congruences of null geodesics vanishes at the EH, i.e

$$\ddot{r}_{EH}(u) = \frac{d^2r}{du^2} |_{r=r_{EH}} \simeq 0. \tag{20}$$

To prove this, using the metric (1), for a radial null geodesic parametrized by the parameter  $u$ , we have

$$\dot{r}(u) = \frac{dr}{du} = \frac{1}{2} \left[ 1 - \frac{2M(u)}{r} - \frac{N_s(u)}{r^{3\omega_s+1}} \right], \tag{21}$$

and consequently, we find

$$\ddot{r}(u) = \frac{1}{2} \left[ \frac{2L}{r} - \frac{\dot{N}_s(u)}{r^{3\omega_s+1}} + \dot{r}(u) \left( \frac{2M(u)}{r^2} + \frac{(3\omega_s + 1)N_s(u)}{r^{3\omega_s+2}} \right) \right]. \tag{22}$$

Then, using the Eqs. (15), (16) and (22), we can rewrite the Eq. (20) as

$$\mathcal{K}\Theta|_{r=r_{EH}} = -\frac{2L}{r_{EH}^2} + \frac{\dot{N}_s(u)}{r_{EH}^{3\omega_s+2}}. \tag{23}$$

On the other hand, using the metric (1) and the null vector  $l^\mu$  defined in (8), we find

$$R_{\mu\nu}l^\mu l^\nu|_{r=r_{EH}} = -\frac{2L}{r_{EH}^2} + \frac{\dot{N}_s(u)}{r_{EH}^{3\omega_s+2}}. \tag{24}$$

Thus, by using the Eqs. (23) and (24) and neglecting  $\Theta^2 = \mathcal{O}(L^2, \dot{N}_s^2, L\dot{N}_s)$  in the Raychaudhuri equation (11), we arrive at

$$\frac{d\Theta}{du}|_{r_{EH}} \simeq 0. \tag{25}$$

Hence, the condition (25) determining the location of the event horizon becomes equivalent to solving the Eq. (23) in  $\mathcal{O}(L, \dot{N}_s)$  resulting from the vanishing acceleration of the congruences of null geodesics at EH. Then, the discussion in [36] for the EH location is also valid and verified here for the surrounded evaporating black holes.

As mentioned previously, in contrast to the stationary spacetimes, the local definitions of the various horizons do not necessarily coincide with the location of the EH for dynamical black holes [40]. For such dynamical spacetimes, one is left with the question: “For which surface should one define the black hole area, surface gravity, temperature or entropy?” The canonical choice is to use the EH. However, as mentioned in the introduction, there are some evidences that it is the AH, and not the EH, that plays the key role in the Hawking radiation [32–35], see also [46–48]. This finding has become a key point in hopes to demonstrate the Hawking radiation in the laboratory using the models of analogue gravity [49]. Thus, regarding the importance of AH, we first introduce some specific subclasses of our general surrounded Vaidya black hole solution (1) and then, we investigate in detail the AH properties of these solutions including the AH location and dynamics as well as their associated Hawking temperature and Bekenstein-Hawking entropy. We consider the detailed study of EHS as our next work.

## 4 Apparent horizon properties for the evaporating Vaidya black hole in generic backgrounds

### 4.1 Evaporating Vaidya black hole in an empty background

In this section, we first explore the AH properties of the Vaidya black hole in empty background. This provides us with the possibility of comparing the properties of this black hole in the various backgrounds with its original form in an empty space.

The case of Vaidya black hole in an empty space corresponds to  $N_s(u) = 0$ . Then, the metric (1) takes the original Vaidya solution form as

$$dS^2 = -\left(1 - \frac{2M(u)}{r}\right) du^2 + 2dudr + r^2 d\Omega_2^2. \tag{26}$$

The surface gravity associated to the metric (26) is given by

$$\mathcal{K} = \frac{M(u)}{r^2}, \tag{27}$$

where the expansion scalar for the outgoing null geodesics reads as

$$\Theta = \frac{1}{r} \left(1 - \frac{2M(u)}{r}\right). \tag{28}$$

Neglecting the trivial solution of  $\Theta = 0$  at  $r = \infty$ , the location of the AH is given by

$$r_{AH}(u) = 2M(u). \tag{29}$$

Then, the dynamics of the single AH (29) is governed by the relation

$$\dot{r}_{AH}(u) = -2L. \tag{30}$$

This shows that the single AH is moving inward during the evaporation, i.e it is shrinking. For this case, the nature of the AH (29) can be determined by looking at the induced metric on the horizon as

$$dS_{AH}^2 = -4L du^2 + 4M^2(u) d\Omega_2^2, \tag{31}$$

showing that AH is a timelike surface for an evaporating BH. The Hawking temperature on the AH (29) can be determined using the surface gravity as

$$\mathcal{T}_{AH} = \frac{1}{2\pi} \frac{M(u)}{r_{AH}^2} = \frac{1}{8\pi} \frac{1}{M(u)}. \tag{32}$$

This relation implies that the temperature associated with the AH (29) rises up by decreasing the mass of black hole

during the evaporation. Finally, the semiclassical Bekenstein-Hawking entropy at AH (29) is given by

$$S_{AH} = \pi r_{AH}^2 = 4\pi M^2(u). \tag{33}$$

#### 4.2 Evaporating Vaidya black hole surrounded by the dust field

For the case of Vaidya black hole surrounded by the dust field with the equation of state parameter  $\omega_d = 0$  [38,50], the metric (1) takes the following form

$$dS^2 = - \left( 1 - \frac{2M(u)}{r} - \frac{N_d(u)}{r} \right) du^2 + 2dudr + r^2 d\Omega_2^2, \tag{34}$$

where  $N_d(u)$  is the normalization parameter for the dust field surrounding the back hole, with the dimension of  $[N_d] = l$  where  $l$  denotes the length. Regarding (34), the black hole in the dust background appears as a black hole with an effective dynamical mass  $2M_{eff}(u) = 2M(u) + N_d(u)$ . In the following, we see that how the presence of this effective mass modifies the thermodynamics, causal structure and Penrose diagrams of the original Vaidya black hole.

The surface gravity associated to the metric (34) is given by

$$\mathcal{K} = \frac{1}{2} \left( \frac{2M(u)}{r^2} + \frac{N_d(u)}{r^2} \right). \tag{35}$$

We see that the dust background appears with a positive surface gravity which increases the gravitational attraction of the black hole. The expansion scalar for the outgoing null geodesics reads as

$$\Theta = \frac{1}{r} \left( 1 - \frac{2M(u)}{r} - \frac{N_d(u)}{r} \right). \tag{36}$$

Then, we find the solution to  $\Theta = 0$  determining the location of the single physical AH as

$$r_{AH}(u) = 2M_{eff}(u), \tag{37}$$

where  $r_{AH}(u)$  corresponds to a modified larger AH compared with the case of black hole in an empty space given by (29). Then, the surrounding background dust field contributes positively to the black hole mass and size of AH. The dynamics of the AH (37) is governed by the relation

$$\dot{r}_{AH}(u) = 2\dot{M}_{eff}(u) = -2L + \dot{N}_d(u). \tag{38}$$

Then, the AH behaviours for the cases of evaporating black hole in a static and dynamical dust backgrounds are different. For this case, we have the following possibilities:

- For  $\dot{N}_d(u) = 0$ , representing the static surrounding dust field, the AH (37) is shrinking. Regarding (29), this case also shows that the dynamics of AH for an evaporating black hole surrounded by a static dust field is the same as the black hole in the empty space.
- For  $\dot{N}_d(u) = 2L$ , the AH (37) is frozen. This case shows that the black hole input by the surrounding field is equal to its output radiation.
- For  $-2L + \dot{N}_d(u) > 0$ , the AH (37) is expanding. This case requires  $\dot{N}_d(u) > 2L > 0$  which shows an evaporating black hole in a collapsing dust field.
- For  $-2L + \dot{N}_d(u) < 0$ , the AH (37) is shrinking. This case requires  $0 < \dot{N}_d(u) < 2L$  or  $\dot{N}_d(u) < 0$  showing an evaporating black hole in a slowly collapsing or a decaying dust field, respectively. The AH (37) with  $0 < \dot{N}_d(u) < 2L$  or  $\dot{N}_d(u) < 0$ , shrinks slower or faster than the AH of black hole in a static dust background.

The nature of AH (37) can be determined by looking at the induced metric on it as

$$dS_{AH}^2 = 2(-2L + \dot{N}_d(u)) du^2 + 4M_{eff}^2 d\Omega_2^2, \tag{39}$$

which implies that the AH (37) is a timelike surface for an evaporating black hole in the static dust field and for a dust field with  $-2L + \dot{N}_d(u) < 0$ , whereas it is a spacelike surface for a dust field with  $-2L + \dot{N}_d(u) > 0$ . For the specific case of  $\dot{N}_d(u) = 2L$ , AH (37) is a null surface.

The Hawking temperature of the AH (37) of black hole is given by

$$\mathcal{T}_{AH} = \frac{1}{2\pi} \frac{M_{eff}(u)}{r_{AH}^2} = \frac{1}{8\pi} \frac{1}{M_{eff}(u)}. \tag{40}$$

Then, we observe that the temperature associated with the AH (37) for the black hole in dust field is lower than the temperature of AH (29) of the black hole in an empty space. Finally, the semiclassical Bekenstein-Hawking entropy at the AH (37) is given by

$$S_{AH} = \pi r_{AH}^2 = 4\pi M_{eff}^2(u), \tag{41}$$

which implies that the entropy associated with the AH (37) in a dust field background is higher than the case of the black hole in an empty space.

#### 4.3 Evaporating Vaidya black hole surrounded by the radiation field

For the case of Vaidya black hole surrounded by a radiation field with the equation of state parameter  $\omega_r = \frac{1}{3}$  [38,50], the metric (1) takes the following form

$$dS^2 = - \left( 1 - \frac{2M(u)}{r} - \frac{N_r(u)}{r^2} \right) du^2 + 2dudr + r^2 d\Omega_2^2, \tag{42}$$

where  $N_r(u)$  denotes the normalization parameter for the radiation field surrounding the black hole, with the dimension of  $[N_r] = l^2$ . Regarding the positive energy condition on the surrounding radiation field, represented by the relation (6), it is required that  $N_r(u) < 0$ . Then, by defining the positive parameter  $\mathcal{N}_r(u) = -N_r(u)$ , we arrive at the following metric

$$dS^2 = - \left( 1 - \frac{2M(u)}{r} + \frac{\mathcal{N}_r(u)}{r^2} \right) du^2 + 2dudr + r^2 d\Omega_2^2. \tag{43}$$

This metric looks like a radiating charged Vaidya black hole with the dynamical charge  $Q^2(u) = \mathcal{N}_r(u)$ . Then, this result can be interpreted as the positive contribution of the characteristic feature of the surrounding radiation field as the effective charge-like term for the Vaidya black hole, see the Bonnor–Vaidya solution [16]. The appearance of such a charge-like term in the metric changes the causal structure and Penrose diagrams of this solution in comparison to the original Vaidya black hole. A similar effect in the causal structure of spacetime happens when one adds charge to the static Schwarzschild black hole leading to Reissner–Nordström black hole. Then, turning off the background radiation field which surrounds the dynamical Vaidya black hole is equal to turning off the charge in the static Reissner–Nordström case.

The surface gravity associated to the metric (43) is given by

$$\mathcal{K} = \frac{1}{2} \left( \frac{2M(u)}{r^2} - \frac{2\mathcal{N}_r(u)}{r^3} \right). \tag{44}$$

Then, we see that the radiation background appears with a negative surface gravity which decreases the gravitational attraction of the black hole, exactly the same as the charge effect in Reissner–Nordström black hole. The expansion scalar for the outgoing null geodesics can be obtained as

$$\Theta = \frac{1}{r} \left( 1 - \frac{2M(u)}{r} + \frac{\mathcal{N}_r(u)}{r^2} \right). \tag{45}$$

We can find the solutions to  $\Theta = 0$  determining the locations of the AH as

$$r_1(u) = M(u) - \sqrt{M^2(u) - \mathcal{N}_r(u)}, \tag{46}$$

and

$$r_2(u) = M(u) + \sqrt{M^2(u) - \mathcal{N}_r(u)}. \tag{47}$$

Then, depending on the sign of the discriminant  $\Delta(u) = M^2(u) - \mathcal{N}_r(u)$ , we have the following three situations.

- For  $\Delta(u) < 0$ , we have no real solutions. Then, there is no physical AH.
- For the extremal case of  $\Delta(u) = 0$ , equivalent to  $\mathcal{N}_r(u) = M^2(u)$ , the solutions (47) and (47) coincide with a single extremal AH, i.e  $r_1(u) = r_2(u) = M(u)$ .
- For  $\Delta(u) > 0$ , there are two real physical AHs as  $r_{AH^-}(u) = r_1(u)$  and  $r_{AH^+}(u) = r_2(u)$  for the black hole in the radiation background in contrast to the black hole in the empty space and in the dust field background having a single AH.

For the extremal case of  $\Delta(u) = 0$ , there is only one single extremal AH given by

$$r_{AH^{ext}}(u) = r_1(u) = r_2(u) = M(u), \tag{48}$$

which is half of the dynamical Schwarzschild radius. Then, the dynamics of the single extremal AH (48) is governed by the relation

$$\dot{r}_{AH^{ext}}(u) = -L. \tag{49}$$

This shows that the single extremal AH (48) for the black hole in the radiation background is moving inward during the evaporation, i.e it is shrinking. For this case, the nature of extremal AH (48) can be determined by looking at the induced metric on it as

$$dS_{AH^{ext}}^2 = -2L du^2 + M^2(u) d\Omega_2^2, \tag{50}$$

showing that the extremal AH (48) is a timelike surface for an evaporating BH in the radiation background. The Hawking temperature on the extremal AH (48) can be determined using the surface gravity as

$$\mathcal{T}_{AH^{ext}} = \frac{1}{4\pi} \left( \frac{2M(u)}{r_{AH^{ext}}^2} - \frac{2M^2(u)}{r_{AH^{ext}}^3} \right) = 0, \tag{51}$$

showing that the extremal AH (48) is a zero temperature surface. One may interpret this zero temperature as the suppressed or zero Hawking radiation for the extremal black hole in the radiation background. Finally, the semiclassical Bekenstein-Hawking entropy at AH (48) is given by

$$\mathcal{S}_{AH^{ext}} = \pi r_{AH^{ext}}^2 = \pi M^2(u). \tag{52}$$

Then, the entropy of the extremal AH (48) is a quarter of the entropy associated to the single AH of the black hole in an empty space.

For the case of  $\Delta(u) > 0$ , the location of the AHs (47) and (47) for the dilute radiation background, i.e.  $\mathcal{N}_r(u) \ll M(u)$ , are given by

$$r_{AH^-}(u) = \frac{\mathcal{N}_r(u)}{2M(u)} + \mathcal{O}(\mathcal{N}_r^2), \tag{53}$$

and

$$r_{AH^+}(u) = 2M(u) - \frac{\mathcal{N}_r(u)}{2M(u)} + \mathcal{O}(\mathcal{N}_r^2). \tag{54}$$

It is seen that the contribution of the surrounding background radiation field leads to (i) the existence of a small inner AH (53) vanishing with  $\mathcal{N}_r(u) \rightarrow 0$ , and (ii) an smaller outer AH (54) in comparison to the case of the AH (29) for the black hole in an empty background in which at the limit of  $\mathcal{N}_r(u) \rightarrow 0$  we recover the dynamical Schwarzschild radius  $r(u) = 2M(u)$ . The dynamics of the AHs (53) and (54) are given by

$$\dot{r}_{AH^-}(u) = \frac{L \mathcal{N}_r(u)}{2M^2(u)} + \frac{\dot{\mathcal{N}}_r(u)}{2M(u)} + \mathcal{O}(\mathcal{N}_r^2), \tag{55}$$

and

$$\dot{r}_{AH^+}(u) = -2L - \frac{L \mathcal{N}_r(u)}{2M^2(u)} - \frac{\dot{\mathcal{N}}_r(u)}{2M(u)} + \mathcal{O}(\mathcal{N}_r^2). \tag{56}$$

Then, the AH behaviours for the cases of evaporating black hole in a static and dynamical radiation backgrounds are different. For this case, we have the following possibilities:

- For  $\dot{\mathcal{N}}_r(u) = 0$ , representing the static surrounding radiation field, the inner AH (53) is moving outward (is expanding) while the outer AH (54) is moving inward (is shrinking) during the evaporation. By stopping the evaporation, if possible, both of the horizons will be frozen, i.e.  $\dot{r}_{AH^-}(u) = \dot{r}_{AH^+}(u) = 0$ . Another interesting result is that regarding (29) and (56), one realizes that the outer AH of the black hole in the static radiation field (54) is shrinking faster than the case of black hole in an empty background.
- For  $\dot{\mathcal{N}}_r(u) > 0$ , the inner and outer AHs (53) and (54) are moving outward and inward, respectively, but both moving faster than the AH of the black hole in the static radiation background. This case represents an evaporating black hole in a collapsing radiation field.
- For  $\dot{\mathcal{N}}_r(u) < 0$ , it is possible that the inner and outer AHs (53) and (54) move again outward and inward, respectively, but both will be slower than the AH of the black hole in the radiation backgrounds with  $\dot{\mathcal{N}}_r(u) = 0$  and  $\dot{\mathcal{N}}_r(u) > 0$ . In the particular case of  $L \mathcal{N}_r(u) = -\dot{\mathcal{N}}_r(u)M(u)$ , the inner AH (53) will be frozen while

the outer AH (54) will move inward exactly the same as AH (29) of the black hole in an empty background. Reversely, if the term  $\frac{\dot{\mathcal{N}}_r(u)}{2M(u)}$  overcomes the other terms in (56) and (56), the inner and outer AHs (53) and (54) will move inward and outward, respectively.

The nature of AHs (53) and (54) can be determined by looking at the induced metrics on them up to  $\mathcal{O}(\mathcal{N}_r)$  as

$$dS_{AH^-}^2 = 2 \left( \frac{L \mathcal{N}_r}{2M^2(u)} + \frac{\dot{\mathcal{N}}_r(u)}{2M(u)} \right) du^2 + r_{AH^-}^2 d\Omega_2^2, \tag{57}$$

and

$$dS_{AH^+}^2 = 2 \left( -2L - \frac{L \mathcal{N}_r}{2M^2(u)} - \frac{\dot{\mathcal{N}}_r(u)}{2M(u)} \right) du^2 + r_{AH^+}^2 d\Omega_2^2. \tag{58}$$

Then, for  $\dot{\mathcal{N}}_r(u) \geq 0$  the inner and outer AHs (53) and (54) are spacelike and timelike respectively while for  $\dot{\mathcal{N}}_r(u) < 0$ , we see that the natures of AHs (53) and (54) depend on the value of  $\frac{\dot{\mathcal{N}}_r(u)}{2M(u)}$  term relative to the other terms in the above parentheses. For the totally positive parentheses, AHs (53) and (54) are spacelike whereas they are timelike for the totally negative parentheses. For the specific case of  $L \mathcal{N}_r(u) = -\dot{\mathcal{N}}_r(u)M(u)$ , the inner AH (53) will be null, whereas the outer AH (54) will be timelike.

The Hawking temperature on the AHs (53) and (54) of the black hole can be determined as

$$\begin{aligned} \mathcal{T}_{AH^-} &= \frac{1}{2\pi} \left( \frac{M(u)}{(M(u) - \sqrt{M^2(u) - \mathcal{N}_r(u)})^2} \right. \\ &\quad \left. - \frac{\mathcal{N}_r(u)}{(M(u) - \sqrt{M^2(u) - \mathcal{N}_r(u)})^3} \right) \\ &= \frac{1}{2\pi} \left( -4 \frac{M^3(u)}{\mathcal{N}_r^2(u)} + 4 \frac{M(u)}{\mathcal{N}_r(u)} - \frac{1}{4M(u)} \right) + \mathcal{O}(\mathcal{N}_r^2(u)), \end{aligned} \tag{59}$$

and

$$\begin{aligned} \mathcal{T}_{AH^+} &= \frac{1}{2\pi} \left( \frac{M(u)}{(M(u) + \sqrt{M^2(u) - \mathcal{N}_r(u)})^2} \right. \\ &\quad \left. - \frac{\mathcal{N}_r(u)}{(M(u) + \sqrt{M^2(u) - \mathcal{N}_r(u)})^3} \right) \\ &= \frac{1}{2\pi} \left( \frac{1}{4M(u)} - \frac{1}{64} \frac{\mathcal{N}_r^2(u)}{M^5(u)} \right) + \mathcal{O}(\mathcal{N}_r^3(u)). \end{aligned} \tag{60}$$

Regarding (60) and (60), one realizes that the temperature of the inner AH (53) is always negative whereas the temperature of the outer AH (54) is positive. This situation for

the temperatures of the AHs is exactly similar to what happens for the Reissner–Nordström black hole where for the region  $r_{AH^-} < r < r_{AH^+}$ , the radial coordinate  $r$  is time-like and time coordinate  $t$  is spacelike. One may write the geodesic equation and see that the gravity force is repulsive at  $r = r_{AH^-}$  for the Reissner–Nordström black hole. That is, the inner horizon is repelling stuff while outer horizon is attracting them. In fact, the horizon temperature is related to surface gravity which is the gravitational acceleration of an object falling to the black hole (as measured by the asymptotic observer). Positivity of temperature then means that gravity force is attractive and vice versa. In our case, this point can be realized by the surface gravity given by (44). Here, the negative contribution of the background radiation field, playing the same role as the charge in the Reissner–Nordström black hole, leads to a negative surface gravity for small  $r$  values where the inner AH (53) lies there. This negative surface gravity yields the gravitational repulsion and the negative temperature (60) for the inner AH (53). One can also discuss about the gravitational repulsion of the inner AH (53), preventing anything to reach it, and its spacelike nature obtained in (57). Also regarding (32) and (60), we see that the temperature of the outer AH (54) of the black hole in the radiation background is lower than the temperature of the AH (29) in an empty background.

The semiclassical Bekenstein–Hawking entropy at the AHs (53) and (54) are given by

$$S_{AH^-} = \pi r_{AH^-}^2 = \pi \left( \frac{\mathcal{N}_r^2(u)}{4M^2(u)} \right) + \mathcal{O} \left( \mathcal{N}_r^3(u) \right), \quad (61)$$

and

$$S_{AH^+} = \pi r_{AH^+}^2 = \pi \left( 4M^2(u) - 2\mathcal{N}_r(u) \right) + \mathcal{O} \left( \mathcal{N}_r^2(u) \right). \quad (62)$$

Comparing (61) and (62), the inner AH (53) has a negligible entropy relative to the entropy associated with the outer AH (54) in a dilute radiation background. Also regarding (62), we see that the background radiation field contributes negatively in the entropy of the outer AH leading to a lower entropy compared to the entropy (33) of the single AH (29) in an empty background. In the limit of  $\mathcal{N}_r(u) \rightarrow 0$ , the entropy of the inner AH (53) vanishes as the horizon itself vanishes, while the entropy (62) of the outer AH (54) reduces to the entropy (33) of the evaporating black hole in an empty space.

#### 4.4 Evaporating Vaidya black hole surrounded by the quintessence field

For the case of Vaidya black hole surrounded by the quintessence field with the equation of state parameter  $\omega_q =$

$-\frac{2}{3}$  [38,50], the metric (1) takes the following form

$$dS^2 = - \left( 1 - \frac{2M(u)}{r} - N_q(u)r \right) du^2 + 2dudr + r^2 d\Omega_2^2, \quad (63)$$

where  $N_q(u)$  is the normalization parameter for the quintessence field surrounding the black hole, with the dimension of  $[N_q] = l^{-1}$ . Regarding the positive energy condition on the surrounding quintessence field, represented by the relation (6), it is required that  $N_q(u) > 0$ . Regarding the metric (63), one realizes the non-trivial contribution of the characteristic feature of the surrounding quintessence field to the metric of the Vaidya black hole. In this case, the background quintessence field changes the causal structure of this black hole solution in comparison to the original Vaidya black hole in an empty background. A rather similar effect happens when one immerses a Vaidya black hole in a de Sitter background [15] with the difference that here our spacetime tends asymptotically to the quintessence rather than the de Sitter asymptotics.

The surface gravity associated to the metric (63) is given by

$$\mathcal{K} = \frac{1}{2} \left( \frac{2M(u)}{r^2} - N_q(u) \right). \quad (64)$$

Then, we see that the quintessence background appears with a negative surface gravity contribution which decreases the gravitational attraction of the black hole. This characteristic feature of the quintessence field, namely the gravitational repulsion, is favored by its cosmological application as one of the candidates for the dark energy which is responsible for the current accelerating expansion of the universe [50].

The expansion scalar for the outgoing null geodesics reads as

$$\Theta = \frac{1}{r} \left( 1 - \frac{2M(u)}{r} - N_q(u)r \right). \quad (65)$$

Then, one can find the solutions to  $\Theta = 0$  determining the locations of the AH as

$$r_1(u) = \frac{1 - \sqrt{1 - 8M(u)N_q(u)}}{2N_q(u)}, \quad (66)$$

and

$$r_2(u) = \frac{1 + \sqrt{1 - 8M(u)N_q(u)}}{2N_q(u)}. \quad (67)$$

Then, depending on the sign of the discriminant  $\Delta(u) = 1 - 8M(u)N_q(u)$ , we have the following three situations.

- For  $\Delta(u) < 0$ , we have no real solutions. Then, there is no physical AH.
- For the extremal case of  $\Delta(u) = 0$ , equivalent to  $N_q(u) = \frac{1}{8M(u)}$ , the solutions (66) and (67) coincide to a single extremal AH, i.e  $r_1(u) = r_2(u) = 4M(u)$ .
- For  $\Delta(u) > 0$ , there are two real physical AHs as  $r_{AH^-}(u) = r_1(u)$  and  $r_{AH^+}(u) = r_2(u)$  for the black hole in the quintessence background in contrast to the black hole in the empty space and in the dust field background having a single AH.

For the extremal case of  $\Delta(u) = 0$ , there is only one single extremal AH given by

$$r_{AH^{ext}}(u) = r_1(u) = r_2(u) = 4M(u), \tag{68}$$

which is twice of the dynamical Schwarzschild radius. Then, the dynamics of the single extremal AH (68) is governed by the relation

$$\dot{r}_{AH^{ext}}(u) = -4L. \tag{69}$$

This shows that the single extremal AH (68) for the black hole in the quintessence background is moving inward during the evaporation, i.e it is shrinking. For this case, the nature of the extremal AH (68) can be determined by looking at the induced metric on it as

$$dS_{AH^{ext}}^2 = -8L du^2 + 16M^2(u)d\Omega_2^2, \tag{70}$$

showing that the extremal AH (68) is a timelike surface for an evaporating BH in the quintessence background. The Hawking temperature on the extremal AH (68) can be determined using the surface gravity as

$$\mathcal{T}_{AH^{ext}} = \frac{1}{4\pi} \left( \frac{2M(u)}{r_{AH^{ext}}^2} - \frac{1}{8M(u)} \right) = 0, \tag{71}$$

indicating that the extremal AH (68) is a zero temperature surface. One may interpret this zero temperature as the suppressed or zero Hawking radiation for the extremal black hole in the quintessence background. Finally, the semiclassical Bekenstein-Hawking entropy at AH (68) is given by

$$\mathcal{S}_{AH^{ext}} = \pi r_{AH^{ext}}^2 = 16\pi M^2(u). \tag{72}$$

Then, the entropy of the extremal AH (68) is four times the entropy (33) associated to the single AH (29) of the black hole in an empty space.

The location of two AHs (66) and (67) for the case  $\Delta(u) > 0$  with a small quintessence parameter, i.e  $N_q(u) \ll M(u)$ , are given by

$$r_{AH^-}(u) = 2M(u) + 4M^2(u)N_q(u) + \mathcal{O}\left(N_q^2(u)\right), \tag{73}$$

and

$$r_{AH^+}(u) = \frac{1}{N_q(u)} - 2M(u) - 4M^2(u)N_q(u) + \mathcal{O}\left(N_q^2(u)\right). \tag{74}$$

It is seen that the contribution of the surrounding background quintessence field leads to (i) the existence of an inner AH (73) larger than the dynamical Schwarzschild radius  $r(u) = 2M(u)$ , and (ii) a large outer (cosmological) AH (74) tending to infinity for  $N_q(u) \lll 1$ . Regarding (73) and (74), the quintessence field has a positive contribution to the inner AH while the black hole mass has a negative contribution to the outer AH. Then, as the black hole evaporation continues, the inner AH (73) shrinks to vanish while the outer AH (74) expands toward its asymptotic value of  $N_q^{-1}(u)$ .

For this case, the dynamics of the AHs (73) and (74) are given by

$$\dot{r}_{AH^-}(u) = -2L - 8LM(u)N_q(u) + 4M^2(u)\dot{N}_q(u) + \mathcal{O}\left(N_q^2(u)\right), \tag{75}$$

and

$$\dot{r}_{AH^+}(u) = 2L + 8LM(u)N_q(u) - \frac{\dot{N}_q(u)}{N_q^2(u)} - 4M^2(u)\dot{N}_q(u) + \mathcal{O}\left(N_q^2(u)\right), \tag{76}$$

respectively. Then one realizes the following points:

- For  $\dot{N}_q(u) = 0$  representing the static surrounding quintessence field, the inner and outer AHs (73) and (74) have equal velocities but with opposite signs. Then, the inner AH (73) shrinks while the outer (74) expands during the evaporation. Also, we observe that both of these AHs are moving faster than the case of single AH (29) for the black hole in an empty space.
- For  $\dot{N}_q(u) < 0$ , the inner AH (73) shrinks while the outer (74) expands, and these are faster than the cases in static quintessence background and in empty spaces.
- For  $\dot{N}_q(u) > 0$ , the behaviour of the AHs (73) and (74) depend on the  $M(u)$ ,  $L$ ,  $N_q(u)$  and  $\dot{N}_q(u)$  values. For the specific case of  $4M^2(u)\dot{N}_q(u) = 2L + 8LM(u)N_q(u)$ , the inner AH (73) is frozen while the outer AH (74) is moving inward.

The nature of the AHs (73) and (74) can be determined by looking at the induced metrics on them up to  $\mathcal{O}(N_q)$  as

$$dS_{AH^-}^2 = 2\left(-2L - 8LM(u)N_q(u) + 4M^2(u)\dot{N}_q(u)\right) du^2 + r_{AH^-}^2 d\Omega_2^2, \tag{77}$$

and

$$dS^2_{AH+} = 2 \left( 2L + 8LM(u)N_q - \frac{\dot{N}_q(u)}{N_q^2(u)} - 4M^2(u)\dot{N}_q(u) \right) du^2 + r^2_{AH+} d\Omega_2^2. \tag{78}$$

These show that for  $\dot{N}_q(u) \leq 0$ , the inner AH (73) is a time-like surface while the outer AH (73) is an spacelike surface. For  $\dot{N}_q(u) > 0$ , the nature of these AHs depend on the  $M(u)$ ,  $L$ ,  $N_q(u)$  and  $\dot{N}_q(u)$  values. For the specific case of  $4M^2(u)\dot{N}_q(u) = 2L + 8LM(u)N_q(u)$ , the inner AH (73) is null while the outer (74) is timelike.

The Hawking temperature on the AHs (73) and (74) of the black hole are

$$\begin{aligned} \mathcal{T}_{AH-} &= \frac{1}{4\pi} \left( \frac{8M(u)N_q^2(u)}{\left(1 - \sqrt{1 - 8M(u)N_q(u)}\right)^2} - N_q(u) \right) \\ &= \frac{1}{4\pi} \left( \frac{1}{2M(u)} - 3N_q(u) \right) + \mathcal{O} \left( N_q^2(u) \right), \end{aligned} \tag{79}$$

and

$$\begin{aligned} \mathcal{T}_{AH+} &= \frac{1}{4\pi} \left( \frac{8M(u)N_q^2(u)}{\left(1 + \sqrt{1 - 8M(u)N_q(u)}\right)^2} - N_q(u) \right) \\ &= \frac{1}{4\pi} \left( -N_q(u) + 2M(u)N_q^2(u) \right) + \mathcal{O} \left( N_q^3(u) \right). \end{aligned} \tag{80}$$

Then, regarding the condition  $8M(u)N_q(u) < 1$ , one realizes that the temperature of the inner AH (73) is always positive while the temperature of the outer AH (74) is negative. For the inner AH (73), the background quintessence field decreases the temperature of the AH relative to the case of black hole in empty space. Here, regarding (64), the negative temperature of the outer AH (74) is resulting from the fact that the quintessence background field effect dominates at large distances which leads to the negative surface gravity, namely the gravitational repulsion.

Finally, the semiclassical Bekenstein-Hawking entropy at the horizons (73) and (74) are given by

$$\begin{aligned} \mathcal{S}_{AH-} &= \pi r^2_{AH-} = \pi \left( \frac{1 - \sqrt{1 - 8N_q(u)M(u)}}{2N_q(u)} \right)^2 \\ &= \pi \left( 4M^2(u) + 16M^3(u)N_q(u) \right) + \mathcal{O} \left( N_q^2(u) \right), \end{aligned} \tag{81}$$

and

$$\begin{aligned} \mathcal{S}_{AH+} &= \pi r^2_{AH+} = \pi \left( \frac{1 + \sqrt{1 - 8N_q(u)M(u)}}{2N_q(u)} \right)^2 \\ &= \pi \left( \frac{1}{N_q^2(u)} - \frac{4M(u)}{N_q(u)} - 4M^2(u) - 16M^3(u)N_q(u) \right) \\ &\quad + \mathcal{O} \left( N_q^2(u) \right). \end{aligned} \tag{82}$$

Then, for the quintessence background with  $N_q(u) \ll 1$ , the inner AH (73) has a negligible entropy in comparison to the entropy (74) associated with the outer AH. Also regarding (81), we see that the background quintessence field contributes positively in the entropy of the inner AH (73), which leads to a higher entropy relative to the single AH (29) of the black hole in an empty background. In the limit of  $N_q(u) \rightarrow 0$ , the entropy (81) of the inner AH (73) tends to the entropy (33) of AH (29) of the black hole in an empty background while the entropy of the outer AH blows up to infinity, as the horizon itself goes to infinity. Also regarding (82), one realizes that since the entropy is a non-negative quantity, then it is required that the first term overcomes to all of the next negative terms. This requires the value of quintessence normalizing parameter  $N_q(u)$  to be very small. In fact, this entropy consideration on the cosmological AH (74) gives a direct restriction on the quintessence field normalization parameter  $N_q(u)$ .

#### 4.5 Evaporating Vaidya black hole surrounded by the cosmological constant-like field

For the case of Vaidya black hole surrounded by the cosmological constant-like field with the equation of state parameter  $\omega_c = -1$  [38,50], which we call it in the following as the cosmological field in short, the metric (1) takes the following form

$$dS^2 = - \left( 1 - \frac{2M(u)}{r} - N_c(u)r^2 \right) du^2 + 2dudr + r^2 d\Omega_2^2, \tag{83}$$

where  $N_c(u)$  is the normalization parameter for the cosmological field surrounding the black hole, with the dimension of  $[N_c(u)] = l^{-2}$ . Regarding the positive energy condition on the surrounding cosmological field, represented by the relation (6), it is required that  $N_c(u) > 0$ . Regarding the metric (83), one realizes the non-trivial contribution of the characteristic feature of the surrounding cosmological field to the metric of the Vaidya black hole. In this case, the background cosmological field changes the causal structure of the original Vaidya black hole in an empty background to the Vaidya-de Sitter space [15], similar to the case of Schwarzschild to Schwarzschild-de Sitter case. Notice to the difference between our more general solution here and that of obtained in [15] having a constant cosmological field.

The surface gravity associated with the metric (83) is given by

$$\mathcal{K} = \frac{M(u)}{r^2} - N_c(u)r. \tag{84}$$

Then, we see that the cosmological background appears with a negative surface gravity contribution which decreases the

gravitational attraction of the black hole. This characteristic feature of the cosmological field, namely the gravitational repulsion, is evidenced by its application as the most favored candidates for the dark energy as the field responsible for the current accelerating expansion of the universe [50].

The expansion scalar for the outgoing null geodesics reads as

$$\Theta = \frac{1}{r} \left( 1 - \frac{2M(u)}{r} - N_c(u)r^2 \right). \tag{85}$$

Then, one can find the solutions to  $\Theta = 0$  determining the locations of the AHs as

$$\begin{aligned} r_1(u) &= -\frac{1}{3^{\frac{1}{3}} \left( 9M(u)N_c^2(u) + \sqrt{-3N_c^3(u)\Delta(u)} \right)^{\frac{1}{3}} - \frac{\left( 9M(u)N_c^2(u) + \sqrt{-3N_c^3(u)\Delta(u)} \right)^{\frac{1}{3}}}{3^{\frac{2}{3}} N_c(u)}}, \\ r_2(u) &= \frac{1 - i\sqrt{3}}{2 \times 3^{\frac{1}{3}} \left( 9M(u)N_c^2(u) + \sqrt{-3N_c^3(u)\Delta(u)} \right)^{\frac{1}{3}} + \frac{\left( 1 + i\sqrt{3} \right) \left( 9M(u)N_c^2(u) + \sqrt{-3N_c^3(u)\Delta(u)} \right)^{\frac{1}{3}}}{2 \times 3^{\frac{2}{3}} N_c(u)}}, \\ r_3(u) &= \frac{1 + i\sqrt{3}}{2 \times 3^{\frac{1}{3}} \left( 9M(u)N_c^2(u) + \sqrt{-3N_c^3(u)\Delta(u)} \right)^{\frac{1}{3}} + \frac{\left( 1 - i\sqrt{3} \right) \left( 9M(u)N_c^2(u) + \sqrt{-3N_c^3(u)\Delta(u)} \right)^{\frac{1}{3}}}{2 \times 3^{\frac{2}{3}} N_c(u)}}, \end{aligned} \tag{86}$$

where  $\Delta(u) = 1 - 27M^2(u)N_c(u)$ . Then, depending on the sign of the discriminant  $\Delta(u)$  we have the following three situations:

- For  $\Delta(u) < 0$ , the first solution is negative while the second and third solutions are imaginary and complex conjugate. Then, there is no physical AH.
- For the extremal case  $\Delta(u) = 0$ , equivalent to  $N_c(u) = \frac{1}{27M^2(u)}$ , the first solution will be negative as  $r_1 = -\frac{2}{\sqrt{3}N_c(u)}$  and the second and third solutions coincide to a single extremal AH as  $r_2(u) = r_3(u) = \frac{1}{\sqrt{3}N_c(u)} = 3M(u)$ . Then, the first solution is not a physical horizon while the coinciding second and third solutions are representing a physical extremal AH.
- For  $\Delta(u) > 0$ , the first solution will be negative, and gives no physical AH while the second and third solutions give two physical inner and outer AHs.

For the extremal case of  $\Delta(u) = 0$ , there is only one single extremal AH given by

$$r_{AH^{ext}}(u) = r_{AH^-}(u) = r_{AH^+}(u) = 3M(u), \tag{87}$$

which is one and a half times of the dynamical Schwarzschild radius. Then, the dynamics of the single extremal AH (87) is governed by the relation

$$\dot{r}_{AH^{ext}}(u) = -3L. \tag{88}$$

This shows that the single extremal AH (87) for the black hole in the cosmological background is moving inward during the evaporation, i.e it is shrinking. For this case, the nature of the extremal AH (87) can be determined by looking at the induced metric on it as

$$dS_{AH^{ext}}^2 = -6L du^2 + 16M^2(u)d\Omega_2^2, \tag{89}$$

indicating that the extremal AH (87) is a timelike surface for an evaporating BH in the cosmological background. The Hawking temperature on the extremal AH (87) can be determined using the surface gravity as

$$\mathcal{T}_{AH^{ext}} = \frac{1}{2\pi} \left( \frac{M(u)}{r_{AH^{ext}}^2} - 3M(u)r_{AH^{ext}} \right) = 0, \tag{90}$$

indicating that the extremal AH (87) is a zero temperature surface. One may interpret this zero temperature as the suppressed or zero Hawking radiation for the extremal black hole in the cosmological background. Finally, the semiclassical Bekenstein-Hawking entropy at extremal AH (87) is given by

$$\mathcal{S}_{AH^{ext}} = \pi r_{AH^{ext}}^2 = 9\pi M^2(u). \tag{91}$$

Then, the entropy of the extremal AH (87) is  $\frac{9}{4}$  the entropy (33) associated to the single AH (29) of the black hole in an empty space.

For the case of  $\Delta(u) > 0$ , we have the following solutions

$$r_{AH^-}(u) = r_2(u) = 2M(u) + 8M^3(u)N_c(u) + \mathcal{O}\left(N_c^2(u)\right), \tag{92}$$

$$\begin{aligned} r_{AH^+}(u) = r_3(u) &= \frac{1}{\sqrt{N_c(u)}} - M(u) - \frac{3}{2}M^2(u)\sqrt{N_c(u)} \\ &\quad - 4M^3(u)N_c(u) + \mathcal{O}\left(N_c^{\frac{3}{2}}(u)\right), \end{aligned} \tag{93}$$

representing the inner and outer AH, respectively. It is seen that the contribution of the surrounding background cosmological field leads to (i) the existence of an inner AH (92) larger than the dynamical Schwarzschild radius  $r(u) = 2M(u)$ , and (ii) a large outer (cosmological) AH (93) tending to infinity for  $N_c(u) \lll 1$ . Regarding (92) and (93), similar

to the quintessence case, the cosmological field has a positive contribution to the inner AH while the black hole mass has a negative contribution to the outer AH. Then, as the black hole evaporation continues, the inner AH (92) shrinks to vanish while the outer AH (93) expands toward its asymptotic value of  $N_c^{-\frac{1}{2}}(u)$ .

For this case, the dynamics of the AHs (92) and (93) are given by

$$\dot{r}_{AH^-}(u) = -2L - 24LM^2(u)N_c(u) + 8M^3(u)\dot{N}_c(u) + \mathcal{O}(N_c^2), \tag{94}$$

and

$$\dot{r}_{AH^+}(u) = L + 3LM(u)\sqrt{N_c} + 12LM^2(u)N_c - \frac{\dot{N}_c(u)}{2N_c^{\frac{3}{2}}} - \frac{3}{4}M^2(u)\frac{\dot{N}_c(u)}{\sqrt{N_c(u)}} - 4M^3(u)\dot{N}_c(u) + \mathcal{O}(N_c^{\frac{3}{2}}), \tag{95}$$

respectively. Then, one realizes the following points:

- For  $\dot{N}_c(u) = 0$ , showing the static surrounding cosmological field, the inner AH (92) shrinks while the outer AH (93) expands. Also, we observe that the inner AH (92) is moving inward faster than the case of AH (29) for the black hole in an empty space.
- For  $\dot{N}_c(u) < 0$ , the inner AH (92) shrinks while the outer AH (93) expands, and these are faster than the cases in static cosmological field background and in empty space.

$$\mathcal{T}_{AH^+} = \frac{1}{2\pi} \left( \frac{M(u)}{\left( \frac{1}{\sqrt{N_c(u)}} - M(u) - \frac{3}{2}M^2(u)\sqrt{N_c(u)} - 4M^3(u)N_c(u) + \mathcal{O}\left(N_c^{\frac{3}{2}}(u)\right) \right)^2} - \frac{1}{2\pi} \left( N_c(u) \left( \frac{1}{\sqrt{N_c(u)}} - M(u) - \frac{3}{2}M^2(u)\sqrt{N_c(u)} + \mathcal{O}(N_c(u)) \right) \right) \right) = \frac{1}{2\pi} \left( -\sqrt{N_c(u)} + 2M(u)N_c(u) \right) + \mathcal{O}\left(N_c^{\frac{3}{2}}(u)\right), \tag{99}$$

- For  $\dot{N}_c(u) > 0$ , the behaviour of these AHs (92) and (93) depend on the  $M(u)$ ,  $L$ ,  $N_c(u)$  and  $\dot{N}_c(u)$  values. For the specific case of  $8M^3(u)\dot{N}_c(u) = 2L + 24LM^2(u)N_c(u)$ , the inner AH (92) is frozen while the outer AH (93) dynamics depends on the total value of the remaining terms in (95).

The nature of AHs (92) and (93) can be determined by looking at the induced metrics on them up to  $\mathcal{O}(N_c)$  as

$$dS_{AH^-}^2 = 2 \left( -2L - 24LM^2(u)N_c + 8M^3(u)\dot{N}_c(u) \right) du^2 + r_{AH^-}^2 d\Omega_2^2, \tag{96}$$

and

$$dS_{AH^+}^2 = 2 \left( L + 3LM(u)\sqrt{N_c} + 12LM^2(u)N_c - \frac{\dot{N}_c(u)}{2N_c^{\frac{3}{2}}} - \frac{3}{4}M^2(u)\frac{\dot{N}_c(u)}{\sqrt{N_c(u)}} - 4M^3(u)\dot{N}_c(u) \right) du^2 + r_{AH^+}^2 d\Omega_2^2. \tag{97}$$

These show that for  $\dot{N}_c(u) \leq 0$ , the inner AH (92) is a time-like surface while the outer AH (93) is spacelike surface. For  $\dot{N}_c(u) > 0$ , the nature of these AHs depends on the  $M(u)$ ,  $L$ ,  $N_c(u)$  and  $\dot{N}_c(u)$  values. For the specific case of  $8M^3(u)\dot{N}_c(u) = 2L + 24LM^2(u)N_c(u)$ , the inner AH (92) is null while the nature of the outer AH (93) depends on the total value of the remaining terms in the parenthesis in (97).

The Hawking temperature on the inner and outer AHs (92) and (93) of black hole can be determined using the surface gravity as

$$\mathcal{T}_{AH^-} = \frac{1}{2\pi} \left( \frac{M(u)}{\left( 2M(u) + 8M^3(u)N_c(u) + \mathcal{O}(N_c^2(u)) \right)^2} - N_c(u) \left( 2M(u) + 8M^2(u)N_c(u) + \mathcal{O}(N_c^2(u)) \right) \right) = \frac{1}{2\pi} \left( \frac{1}{4M(u)} - 4M(u)N_c(u) \right) + \mathcal{O}(N_c^2(u)), \tag{98}$$

and

respectively. One realizes that the temperature of the inner AH (92) is always positive while the temperature of the outer AH (93) is negative. For the inner AH (92), the background cosmological field decreases the temperature of the AH relative to the AH (29) of black hole in empty space. Here, regarding (84), the negative temperature of the outer AH (93) is resulting from the fact that the cosmological background field effect dominates at large distances which leads to the negative surface gravity, equivalently to the gravitational repulsion.

Finally, the semiclassical Bekenstein-Hawking entropy at the AHs (92) and (93) are given by

$$S_{AH-} = \pi r_{AH-}^2 = \pi \left( 4M^2(u) + 32M^4(u)N_c(u) \right) + \mathcal{O} \left( N_c^2(u) \right), \tag{100}$$

and

$$\begin{aligned} S_{AH+} &= \pi r_{AH+}^2 \\ &= \pi \left( \frac{1}{N_c(u)} - 2 \frac{M(u)}{\sqrt{N_c(u)}} - 2M^2(u) - 5M^3(u) \right. \\ &\quad \left. \times \sqrt{N_c(u)} - 16M^4(u)N_c(u) \right) + \mathcal{O} \left( N_c^{\frac{3}{2}}(u) \right). \end{aligned} \tag{101}$$

Then, for the cosmological field background with  $N_c(u) \lll 1$ , the inner AH (92) has a negligible entropy (100) in comparison to the entropy (101) associated with the outer AH (93). Also regarding (100), we see that the background cosmological field contributes positively to the entropy of the inner AH (92), which leads to a higher entropy relative to the single AH (29) of the black hole in an empty background. In the limit of  $N_c(u) \rightarrow 0$ , the entropy (100) of the inner AH (92) tends to the entropy (33) of AH (29) of the black hole in an empty background while the entropy (100) of the outer AH (93) blows up to infinity. Also regarding (101), one realizes that since the entropy is a non-negative quantity, then this relation requires that the first term overcomes to all of the next negative terms. This requires the value of cosmological field normalizing parameter  $N_c(u)$  to be very small. In fact, this entropy consideration on the cosmological AH (93) gives a restriction on the cosmological field normalization parameter  $N_c(u)$ .

#### 4.6 Evaporating Vaidya black hole surrounded by the phantom field

For the case of Vaidya black hole surrounded by the phantom field with the equation of state parameter  $\omega_p = -\frac{4}{3}$  [38, 50], the metric (1) takes the following form

$$dS^2 = - \left( 1 - \frac{2M(u)}{r} - N_p(u)r^3 \right) du^2 + 2dudr + r^2 d\Omega_2^2, \tag{102}$$

where  $N_p(u)$  is the normalization parameter for the phantom field surrounding the black hole, with the dimension of  $[N_p(u)] = l^{-3}$ . Regarding the positive energy condition on the surrounding phantom field, represented by the relation (6), it is required that  $N_p(u) > 0$ . Regarding the metric (102), one realizes the non-trivial contribution of the characteristic feature of the surrounding phantom field to the metric

of the Vaidya black hole. In this case, the background phantom field changes the causal structure of the original Vaidya black hole in an empty background to the case with a phantom asymptotic.

The surface gravity associated with the metric (102) is given by

$$\mathcal{K} = \frac{1}{2} \left( \frac{2M(u)}{r^2} - 3N_p(u)r^2 \right). \tag{103}$$

Then, we see that the phantom background appears with a negative surface gravity contribution which decreases the gravitational attraction of the black hole. This characteristic feature of the phantom field, namely the gravitational repulsion, is favored by its cosmological application as one of the candidates for the dark energy [50].

The expansion scalar for the outgoing null geodesics reads as

$$\Theta = \frac{1}{r} \left( 1 - \frac{2M(u)}{r} - N_p(u)r^3 \right). \tag{104}$$

Then, we find the solutions of  $\Theta = 0$  determining the locations of the AH as

$$\begin{aligned} r_1(u) &= -\frac{1}{2}\sqrt{\Gamma(u)} - \frac{1}{2}\sqrt{-\Gamma(u) - \frac{2}{N_p(u)\sqrt{\Gamma(u)}}}, \\ r_2(u) &= -\frac{1}{2}\sqrt{\Gamma(u)} + \frac{1}{2}\sqrt{-\Gamma(u) - \frac{2}{N_p(u)\sqrt{\Gamma(u)}}}, \\ r_3(u) &= \frac{1}{2}\sqrt{\Gamma(u)} - \frac{1}{2}\sqrt{-\Gamma(u) + \frac{2}{N_p(u)\sqrt{\Gamma(u)}}}, \\ r_4(u) &= \frac{1}{2}\sqrt{\Gamma(u)} + \frac{1}{2}\sqrt{-\Gamma(u) + \frac{2}{N_p(u)\sqrt{\Gamma(u)}}}, \end{aligned} \tag{105}$$

where

$$\begin{aligned} \Gamma(u) &= \frac{8 \left( \frac{2}{3} \right)^{\frac{1}{3}} M(u)}{\left( 9N_p(u) - \sqrt{81N_p^2(u)\Delta(u)} \right)^{\frac{1}{3}}} \\ &\quad + \frac{\left( 9N_p(u) - \sqrt{81N_p^2(u)\Delta(u)} \right)^{\frac{1}{3}}}{2^{\frac{1}{3}} 3^{\frac{2}{3}} N_p(u)}, \end{aligned} \tag{106}$$

and  $\Delta(u) = 1 - \frac{2048}{27}M^3(u)N_p(u)$ . Then, depending on the sign of the discriminant  $\Delta(u)$  we have the following three situations.

- For  $\Delta(u) < 0$ , the first and second solutions are negative while the third and fourth solutions are imaginary. Then, there is no physical AH.
- For the extremal case  $\Delta(u) = 0$ , equivalent to  $N_p(u) = \frac{27}{2048M^3(u)}$ , the first and second solutions coincide as

$r_1(u) = r_2(u) = \frac{8}{3} \left( \frac{27}{2048 N_p(u)} \right)^{\frac{1}{3}}$  while the third and fourth solutions read as  $r_3(u) = -r_4(u) = \frac{8}{3} i (\sqrt{2} + i) M(u)$ . Then, the first and second coinciding solutions are physical AH while the third and fourth coinciding solutions are not physical.

- For  $\Delta(u) > 0$ , the first and second solutions are imaginary and complex conjugate to each other and gives no physical AH, while the third and fourth solutions give two physical inner and outer AH.

For the extremal case of  $\Delta(u) = 0$ , there is only one single extremal AH given by

$$r_{AH^{ext}}(u) = r_1(u) = r_2(u) = \frac{8}{3} M(u), \tag{107}$$

which is 4/3 of the dynamical Schwarzschild radius. Then, the dynamics of the single extremal AH (107) is governed by the relation

$$\dot{r}_{AH^{ext}}(u) = -\frac{8}{3} L. \tag{108}$$

This shows that the single extremal AH (107) for the black hole in the quintessence background is moving inward during the evaporation, i.e it is shrinking. For this case, the nature of the extremal AH (107) can be determined by looking at the induced metric on it as

$$dS_{AH^{ext}}^2 = -\frac{16}{3} L du^2 + 16M^2(u) d\Omega_2^2, \tag{109}$$

indicating that the extremal AH (107) is a timelike surface for an evaporating BH in the phantom background. The Hawking temperature on the extremal AH (107) can be determined using the surface gravity as

$$\mathcal{T}_{AH^{ext}} = \frac{1}{4\pi} \left( \frac{2M(u)}{r_{AH^{ext}}^2} - 3 \left( \frac{27}{2048M^3(u)} \right) r_{AH^{ext}}^2 \right) = 0, \tag{110}$$

indicating that the extremal AH (107) is a zero temperature surface. One may interpret this zero temperature as the suppressed or zero Hawking radiation for the extremal black hole in the phantom background. The semiclassical Bekenstein-Hawking entropy at AH (107) is given by

$$\mathcal{S}_{AH^{ext}} = \pi r_{AH^{ext}}^2 = \frac{64\pi}{9} M^2(u). \tag{111}$$

Then, the entropy of the extremal AH (107) is  $\frac{16}{9}$  the entropy (33) associated to the single AH (29) of the black hole in an empty space.

For the case of  $\Delta(u) > 0$ , we have the following solutions

$$r_{AH^-}(u) = r_3(u) = 2M(u) + 16M^4(u)N_p(u) + \mathcal{O}(N_p^2), \tag{112}$$

$$r_{AH^+}(u) = r_4(u) = \frac{1}{N_p^{\frac{1}{3}}(u)} - \frac{2}{3}M(u) - \frac{8}{9}M^2(u)N_p^{\frac{1}{3}}(u) - \frac{160}{81}M^3(u)N_p^{\frac{2}{3}}(u) - \frac{16}{3}M^4(u)N_p(u) + \mathcal{O}\left(N_p^{\frac{4}{3}}(u)\right), \tag{113}$$

representing the inner and outer AHs, respectively. It is seen that the contribution of the background phantom field leads to (i) the existence of an inner AH (112) larger than the dynamical Schwarzschild radius  $r(u) = 2M(u)$ , and (ii) a large outer (cosmological) AH (113) tending to infinity for  $N_p \lll 1$ . Regarding (112) and (113), similar to the quintessence and cosmological field cases, the phantom field has a positive contribution to the inner AH (112) while the black hole mass has a negative contribution to the outer AH. Then, as the black hole evaporation continues, the inner AH (112) shrinks to vanish while the outer AH (113) expands toward its asymptotic value of  $N_p^{-\frac{1}{3}}(u)$ . For this case, the dynamics of the AHs (112) and (113) are given by

$$\dot{r}_{AH^-}(u) = -2L - 64LM^2(u)N_p(u) + 16M^4(u)\dot{N}_p(u) + \mathcal{O}\left(N_p^2(u)\right), \tag{114}$$

and

$$\begin{aligned} \dot{r}_{AH^+}(u) = & \frac{2}{3}L + \frac{16}{9}LM(u)N_p^{\frac{1}{3}}(u) + \frac{160}{27}LM^2(u)N_p^{\frac{2}{3}}(u) \\ & + \frac{64}{3}LM^3(u)N_p(u) \\ & - \frac{\dot{N}_p(u)}{3N_p^{\frac{4}{3}}(u)} - \frac{8}{27}\frac{\dot{N}_p(u)}{N_p^{\frac{2}{3}}(u)}M^2(u) - \frac{320}{243}\frac{\dot{N}_p(u)}{N_p^{\frac{1}{3}}(u)}M^3(u) \\ & - \frac{16}{3}M^4(u)\dot{N}_p(u) + \mathcal{O}\left(N_p^{\frac{4}{3}}(u)\right), \end{aligned} \tag{115}$$

respectively. Then one realizes the following points:

- For  $\dot{N}_p(u) = 0$  representing the static surrounding phantom field, the inner AH (112) shrinks while the outer AH (113) expands. Also, we observe that the inner AH (112) is moving inward faster than the case of AH (29) for the black hole in an empty space.
- For  $\dot{N}_p(u) < 0$ , the inner AH (112) shrinks while the outer AH (112) expands, and these are faster than the cases in static phantom field background and in empty space.
- For  $\dot{N}_p(u) > 0$ , the behaviour of the AHs (112) and (113) depend on the  $M(u)$ ,  $L$ ,  $N_p(u)$  and  $\dot{N}_p(u)$  values. For the specific case of  $16M^4(u)\dot{N}_p(u) = 2L +$

$64LM^2(u)N_p(u)$ , the inner AH (112) is frozen while the outer's behaviour depends on the total value of the remaining terms in (115).

The nature of the AHs (112) and (113) can be determined by looking at the induced metric on them up to  $\mathcal{O}(N_p(u))$  as

$$dS_{AH-}^2 = 2 \left( -2L - 8LM(u)N_q(u) + 4M^2(u)\dot{N}_q(u) \right) du^2 + r_{AH-}^2 d\Omega_2^2, \tag{116}$$

and

$$dS_{AH+}^2 = 2 \left( \frac{2}{3}L + \frac{16}{9}LM(u)N_p^{\frac{1}{3}}(u) + \frac{160}{27}LM^2(u)N_p^{\frac{2}{3}}(u) + \frac{64}{3}LM^3(u)N_p(u) - \frac{\dot{N}_p(u)}{3N_p^{\frac{4}{3}}(u)} - \frac{8}{27}\frac{\dot{N}_p(u)}{N_p^{\frac{2}{3}}(u)}M^2(u) - \frac{320}{243}\frac{\dot{N}_p(u)}{N_p^{\frac{1}{3}}(u)}M^3(u) - \frac{16}{3}M^4(u)\dot{N}_p(u) \right) du^2 + r_{AH+}^2 d\Omega_2^2. \tag{117}$$

These indicate that for  $\dot{N}_p(u) \leq 0$ , the inner AH (112) is a timelike surface while the outer AH (113) is spacelike surface. For  $\dot{N}_p(u) > 0$ , the nature of these AH depend on the  $M(u)$ ,  $L$ ,  $N_p(u)$  and  $\dot{N}_p(u)$  values. For the specific case of  $16M^4(u)\dot{N}_p(u) = 2L + 64LM^2(u)N_p(u)$ , the inner AH (112) is null while the nature of the outer AH (113) depends on the total value of the remaining terms in (117).

The Hawking temperature on the inner and outer AHs (112), (113) of black hole can be determined using the surface gravity as

$$\begin{aligned} \mathcal{T}_{AH-} &= \frac{1}{2\pi} \left( \frac{M(u)}{\left( 2M(u) + 16M^4(u)N_p(u) + \mathcal{O}(N_p^2(u)) \right)^2} - N_p(u) \left( 2M(u) + 16M^4(u)N_p(u) + \mathcal{O}(N_p^2(u)) \right)^2 \right) \\ &= \frac{1}{2\pi} \left( \frac{1}{4M(u)} - 10M^2(u)N_p(u) \right) + \mathcal{O}(N_p^2(u)), \end{aligned} \tag{118}$$

and

$$\begin{aligned} \mathcal{T}_{AH+} &= \frac{1}{2\pi} \frac{M(u)}{\left( \frac{1}{N_p^{\frac{1}{3}}(u)} - \frac{2}{3}M(u) - \frac{8}{9}M^2(u)N_p^{\frac{1}{3}}(u) - \frac{160}{81}M^3(u)N_p^{\frac{2}{3}}(u) - \frac{16}{3}M^4(u)N_p(u) + \mathcal{O}\left(N_p^{\frac{4}{3}}(u)\right) \right)^2} \\ &\quad - \frac{3}{2\pi} N_p(u) \left( \frac{1}{N_p^{\frac{1}{3}}(u)} - \frac{2}{3}M(u) - \frac{8}{9}M^2(u)N_p^{\frac{1}{3}}(u) - \frac{160}{81}M^3(u)N_p^{\frac{2}{3}}(u) - \frac{16}{3}M^4(u)N_p(u) + \mathcal{O}\left(N_p^{\frac{4}{3}}(u)\right) \right)^2 \\ &= \frac{1}{2\pi} \left( -\frac{3}{2}N_p^{\frac{1}{3}}(u) + 3M(u)N_p^{\frac{2}{3}}(u) + \frac{10}{3}M^2(u)N_p(u) \right) + \mathcal{O}\left(N_p^{\frac{4}{3}}(u)\right). \end{aligned} \tag{119}$$

respectively. One realizes that the temperature of the inner AH (112) is always positive while the temperature of the outer AH (113) is negative. For the inner AH (112), the background phantom field decreases the temperature of the AH relative to the case of black hole in empty space. Here, regarding (103), the negative temperature of the outer AH (113) is resulting from the fact that the phantom background field effect dominates at large distances which leads to the negative surface gravity, namely the gravitational repulsion.

Also, the semiclassical Bekenstein-Hawking entropy at the AHs (112) and (113) are given by

$$S_{AH-} = \pi r_{AH-}^2 = \pi \left( 4M^2(u) + 64M^5(u)N_p(u) \right) + \mathcal{O}\left(N_p^2(u)\right), \tag{120}$$

and

$$\begin{aligned} S_{AH+} &= \pi r_{AH+}^2 \\ &= \pi \left( \frac{1}{N_p^{\frac{2}{3}}(u)} - \frac{4}{3}\frac{M(u)}{N_p^{\frac{1}{3}}(u)} - \frac{4}{3}M^2(u) - \frac{224}{81}M^3(u)N_p^{\frac{1}{3}}(u) - \frac{1760}{243}M^4(u)N_p^{\frac{2}{3}}(u) - \frac{64}{3}M^5(u)N_p(u) \right) \\ &\quad + \mathcal{O}\left(N_p^{\frac{4}{3}}(u)\right). \end{aligned} \tag{121}$$

Then, for the phantom background with  $N_p(u) \ll 1$ , the inner AH (112) has a negligible entropy in comparison to the entropy associated with the outer AH (113). Also regarding (121), we see that the background phantom field contributes positively in the entropy of the inner AH (112), which leads to a higher entropy relative to the single AH (29) of the black hole in an empty background. In the limit of  $N_p(u) \rightarrow 0$ , the entropy (121) of the inner AH (112) tends to the entropy (33) of AH (29) of the black hole in an empty background while the entropy of the outer AH (113) blows up to infinity. Also regarding (121), one realizes that since the entropy is a non-negative quantity, then this relation requires that the first term overcomes to all of the next terms. This requires the value of phantom field normalizing parameter  $N_p(u)$  to be very small.

In fact, this entropy consideration on the cosmological AH (113) gives a restriction on the phantom field normalization parameter  $N_p(u)$ .

## 5 Conclusion

In this work, we have studied the thermodynamical features and dynamical evolutions of the various apparent horizons associated with the Vaidya evaporating black hole surrounded by the cosmological fields of dust, radiation, quintessence, cosmological constant-like and phantom. We have explored in detail how do these surrounding fields contribute to the characteristic features of a surrounded dynamical black hole. To this aim, using the null vector decomposition of the spacetime metric and following York [36], we have obtained the geometric properties of the solution in its general form. We have found that the solution to  $g(\partial_u, \partial_u) = g_{uu} = 0$  representing the TLS is the same as the solution to  $\Theta = 0$  representing the AH location, except the trivial solution  $r = \infty$ , for the latter. Then, the AH and the TLS coincide for our spacetime (1). Also, for the surrounded evaporating black hole solutions here, we have verified that the condition  $\frac{d\Theta}{du} \simeq 0$  determining the location of EH is equal to the vanishing acceleration of the congruences of null geodesics at EH. In the following, we summarize some of our results for the AH properties of the evaporating black hole in various backgrounds.

- For the black hole with surrounding dust field, the dust background appears with a positive surface gravity increasing the gravitational attraction of the black hole. For this case, the size of AH is larger than that of black hole in an empty space. For the black hole surrounded by a static dust field, i.e.  $\dot{N}_d(u) = 0$ , AH is timelike surface and its dynamics is the same as the dynamics of AH in black hole in the empty space in which it is shrinking during the evaporation. For a dynamical background with  $\dot{N}_d(u) = 2L$ , the AH is a frozen null surface. For the background with  $-2L + \dot{N}_d(u) > 0$ , AH is spacelike surface and expands during the evaporation while for  $-2L + \dot{N}_d(u) < 0$ , it is a timelike surface and shrinks during the evaporation. The temperature of AH for the black hole in dust field is lower than the temperature of AH of the black hole in an empty space. Finally, the entropy associated with the AH in a dust field background is higher than that of the black hole in an empty space.
- For the black hole with surrounding radiation field, the surrounding radiation field contributes as the effective charge-like term for the Vaidya black hole. For this case, the radiation background appears with a negative surface gravity decreasing the gravitational attraction of the black hole, exactly the same as the charge effect in

Reissner–Nordström black hole. For the extremal case of  $\Delta(u) = 0$ , there is only one single AH equal to half of the dynamical Schwarzschild radius which is a timelike and zero temperature surface, and shrinks during the evaporation. The entropy of the extremal AH is a quarter of the entropy associated to the single AH of the black hole in an empty space. For  $\Delta(u) < 0$ , we have no real solutions while for  $\Delta(u) > 0$ , there are two real physical AHs. For the latter case, the contribution of the background radiation field leads to (i) the existence of a small inner AH and (ii) an smaller outer AH in comparison to the case of the AH for the black hole in an empty background. For the static surrounding radiation field, i.e.  $\dot{N}_r(u) = 0$ , the inner AH is expanding while the outer AH is shrinking during the evaporation, faster than the case of black hole in an empty background. For  $\dot{N}_r(u) > 0$ , the inner and outer AH are expanding and shrinking, respectively, but both moving faster than the AH of the black hole in the static radiation background. For  $\dot{N}_r(u) < 0$ , it is possible that the inner and outer AH move again outward and inward, respectively, but both will be slower than the AH of the black hole in the radiation backgrounds with  $\dot{N}_r(u) = 0$  and  $\dot{N}_r(u) > 0$ . In the particular case of  $L\dot{N}_r(u) = -\dot{N}_r(u)M(u)$ , the inner AH will be frozen while the outer AH will move inward exactly the same as AH of the black hole in an empty background. For  $\dot{N}_r(u) \geq 0$  the inner and outer AH are spacelike and timelike respectively, while for  $\dot{N}_r(u) < 0$ , we see that the nature of AH depends on the value of  $\frac{\dot{N}_r(u)}{2M(u)}$ . For the specific case of  $L\dot{N}_r(u) = -\dot{N}_r(u)M(u)$ , the inner AH will be null while the outer AH will be timelike. The temperature of the inner AH is always negative while the temperature of the outer AH is positive, exactly the same as in the Reissner–Nordström black hole. The inner AH has a negligible entropy relative to the entropy associated with the outer AH in a dilute radiation background. The background radiation field contributes negatively in the entropy of the outer AH leading to a lower entropy relative to the entropy of the single AH in an empty background.

- For the black hole with surrounding quintessence field, the quintessence background contributes negatively to the surface gravity which decreases the gravitational attraction of the black hole. This characteristic feature of the quintessence field is favored by its cosmological application as one of the candidates for the dark energy. For the extremal case of  $\Delta(u) = 0$ , there is only one single extremal AH twice of the dynamical Schwarzschild radius which is a timelike and zero temperature surface and shrinks during the black hole evaporation. The entropy of the extremal AH is four times the entropy associated to the single AH of the black hole in an empty

space. For  $\Delta(u) < 0$ , we have no real solutions while for  $\Delta(u) > 0$ , there are two real physical AHs. For the latter case, the contribution of the background quintessence field leads to (i) the existence of an inner AH larger than the dynamical Schwarzschild radius  $r(u) = 2M(u)$ , and (ii) a large outer cosmological AH tending to infinity for  $N_q(u) \lll 1$ . For the static surrounding quintessence field, i.e.  $\dot{N}_q(u) = 0$ , the inner and outer AH have equal velocities but with opposite signs. Then, the inner AH is a timelike surface and shrinks while the outer is a spacelike surface and expands during the evaporation in which both of these AH are moving faster than the case of the single AH for the black hole in an empty space. For  $\dot{N}_q(u) < 0$ , the inner AH is a timelike surface and shrinks while the outer is a spacelike surface and expands in which these are faster than the cases in static quintessence background and in empty spaces. For  $\dot{N}_q(u) > 0$ , the behaviour and nature of the AH depend on the  $M(u)$ ,  $L$ ,  $N_q(u)$  and  $\dot{N}_q(u)$  values. For the specific case of  $4M^2(u)\dot{N}_q(u) = 2L + 8LM(u)N_q(u)$ , the inner AH is a frozen null surface while the outer AH is shrinking timelike surface. The temperature of the inner AH is always positive while the temperature of the outer AH is negative. For the inner AH, the background quintessence field decreases the temperature of the AH relative to the case of black hole in empty space. Finally, the inner AH has a negligible entropy in comparison to the entropy associated with the outer AH. Also, the background quintessence field contributes positively in the entropy of the inner AH, which leads to a higher entropy relative to the single AH of the black hole in an empty background.

- For the black hole with surrounding cosmological field, the cosmological field contributes negatively to the surface gravity decreasing the gravitational attraction of the black hole. This characteristic feature of the cosmological field is evidenced by its application as the most favored candidates for the dark energy. For the extremal case of  $\Delta(u) = 0$ , there is only one single extremal AH,  $3/2$  times the dynamical Schwarzschild radius, which is a timelike and zero temperature surface and shrinks during the black hole evaporation. The entropy of the extremal AH is  $\frac{9}{4}$  times the entropy associated to the single AH of the black hole in an empty space. For  $\Delta(u) < 0$ , we have no real solutions while for  $\Delta(u) > 0$ , there are two real physical AHs. For the latter case, the contribution of the background cosmological field leads to (i) the existence of an inner AH larger than the dynamical Schwarzschild radius  $r(u) = 2M(u)$ , and (ii) a large outer cosmological AH tending to infinity for  $N_c(u) \lll 1$ . For the static surrounding cosmological field, i.e.  $\dot{N}_c(u) = 0$ , the inner AH is a timelike surface and shrinks faster than the case of the single AH for the black hole in an empty space while

the outer is a spacelike surface and expands during the evaporation. For  $\dot{N}_c(u) < 0$ , the inner AH is a timelike surface and shrinks while the outer is a spacelike surface and expands in which these are faster than the cases in static cosmological background and in empty spaces. For  $\dot{N}_c(u) > 0$ , the behaviour and nature of the AH depend on the  $M(u)$ ,  $L$ ,  $N_c(u)$  and  $\dot{N}_c(u)$  values. For the specific case of  $8M^3(u)\dot{N}_c(u) = 2L + 24LM^2(u)N_c(u)$ , the inner AH is a frozen null surface while the outer AH is shrinking timelike surface. The temperature of the inner AH is always positive while the temperature of the outer AH is negative. For the inner AH, the background cosmological field decreases the temperature of the AH relative to the case of black hole in empty space. Finally, the inner AH has a negligible entropy in comparison to the entropy associated with the outer AH. Also, we find that the background cosmological field contributes positively in the entropy of the inner AH, which leads to a higher entropy relative to the single AH of the black hole in an empty background.

- For the black hole with surrounding phantom field, the phantom field contributes negatively to the surface gravity which decreases the gravitational attraction of the black hole. This characteristic feature of the phantom field is evidenced by its application as one of the candidates for the dark energy. For the extremal case of  $\Delta(u) = 0$ , there is only one single extremal AH,  $4/3$  times the dynamical Schwarzschild radius, which is a timelike and zero temperature surface and shrinks during the black hole evaporation. The entropy of the extremal AH is  $\frac{16}{9}$  times the entropy associated to the single AH of the black hole in an empty space. For  $\Delta(u) < 0$ , we have no real solutions while for  $\Delta(u) > 0$ , there are two real physical AHs. For the latter case, the contribution of the background phantom field leads to (i) the existence of an inner AH larger than the dynamical Schwarzschild radius  $r(u) = 2M(u)$ , and (ii) a large outer cosmological AH tending to infinity for  $N_p(u) \lll 1$ . For the static surrounding phantom field, i.e.  $\dot{N}_p(u) = 0$ , the inner AH is a timelike surface and shrinks faster than the case of the single AH for the black hole in an empty space while the outer is a spacelike surface and expands during the evaporation. For  $\dot{N}_p(u) < 0$ , the inner AH is a timelike surface and shrinks while the outer is a spacelike surface and expands in which these are faster than the cases in static cosmological background and in empty spaces. For  $\dot{N}_p(u) > 0$ , the behaviour and nature of the AHs depend on the  $M(u)$ ,  $L$ ,  $N_p(u)$  and  $\dot{N}_p(u)$  values. For the specific case of  $16M^4(u)\dot{N}_p(u) = 2L + 64LM^2(u)N_p(u)$ , the inner AH is a frozen null surface while the outer AH is shrinking timelike surface. The temperature of the inner AH is always positive while the temperature of the outer AH is negative. For the inner AH, the background

phantom field decreases the temperature of the AH relative to the case of black hole in empty space. Finally, the inner AH has a negligible entropy in comparison to the entropy associated with the outer AH. Also, we find that the background phantom field contributes positively in the entropy of the inner AH, which leads to a higher entropy relative to the single AH of the black hole in an empty background.

We aim to report elsewhere the detailed study of EH properties together with the back-reaction and metric fluctuation for the surrounded Vaidya black holes.

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